Effective Field Theories in a nutshell

Benjamin Fuks

LPTHE / Sorbonne Université

Iwate Collider School @ Appi

27 February 2025

Outline

1. UV complete theories and Effective Field Theories

2. Matching EFTs and UV theories, operator mixing and running

3. The SMEFT as a generic BSM framework

4. Positivity bounds

[Some inspiration from previous MG5/FR schools]

Looking for new resonances at the LHC

LHC ≡ discovery machine

- Many existing ATLAS and CMS searches for new physics
- Interpretation within popular frameworks (MSSM, *etc.*) and simplified models

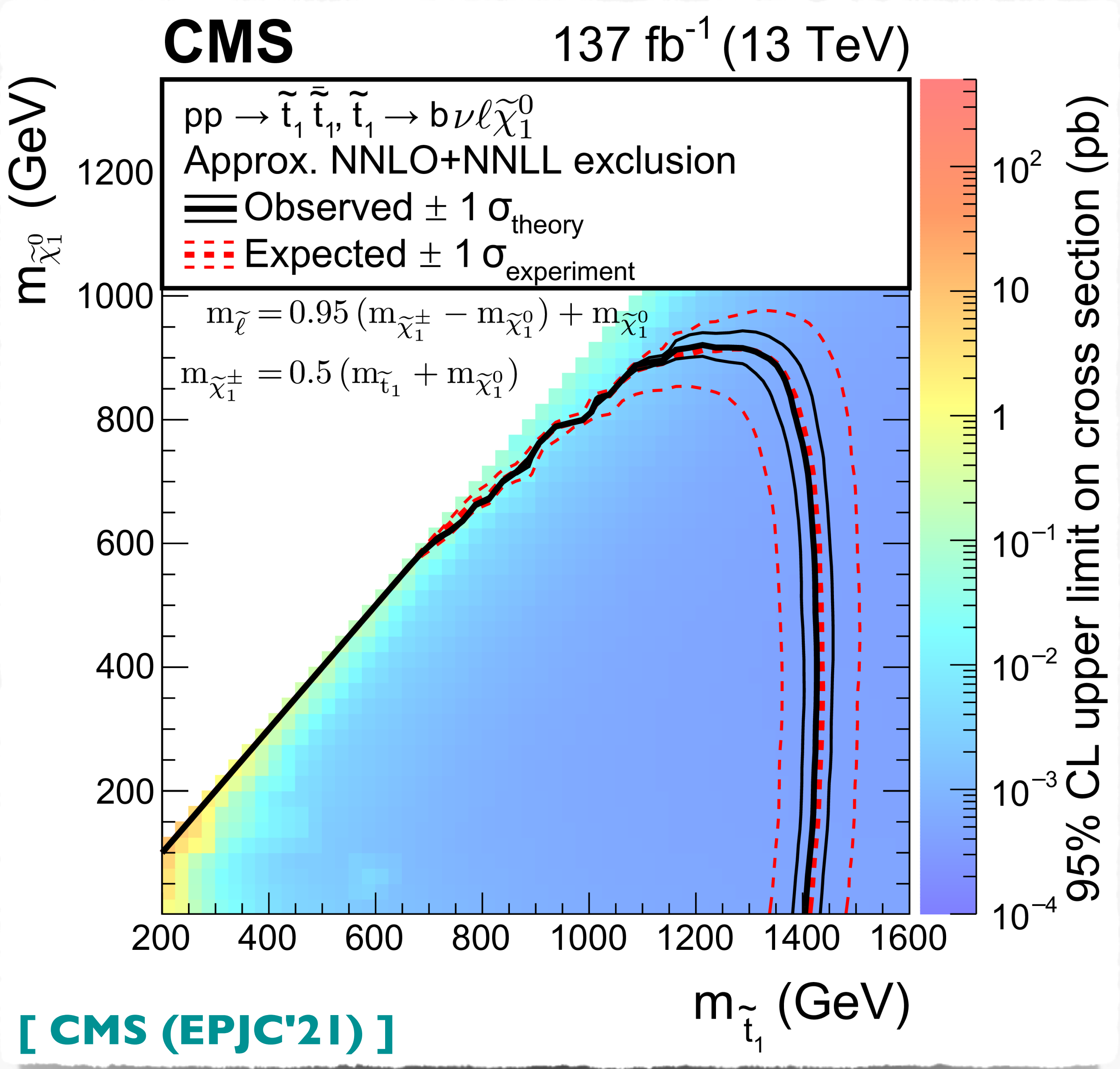
➙ (more or less) model dependent

Looking for new resonances at the LHC

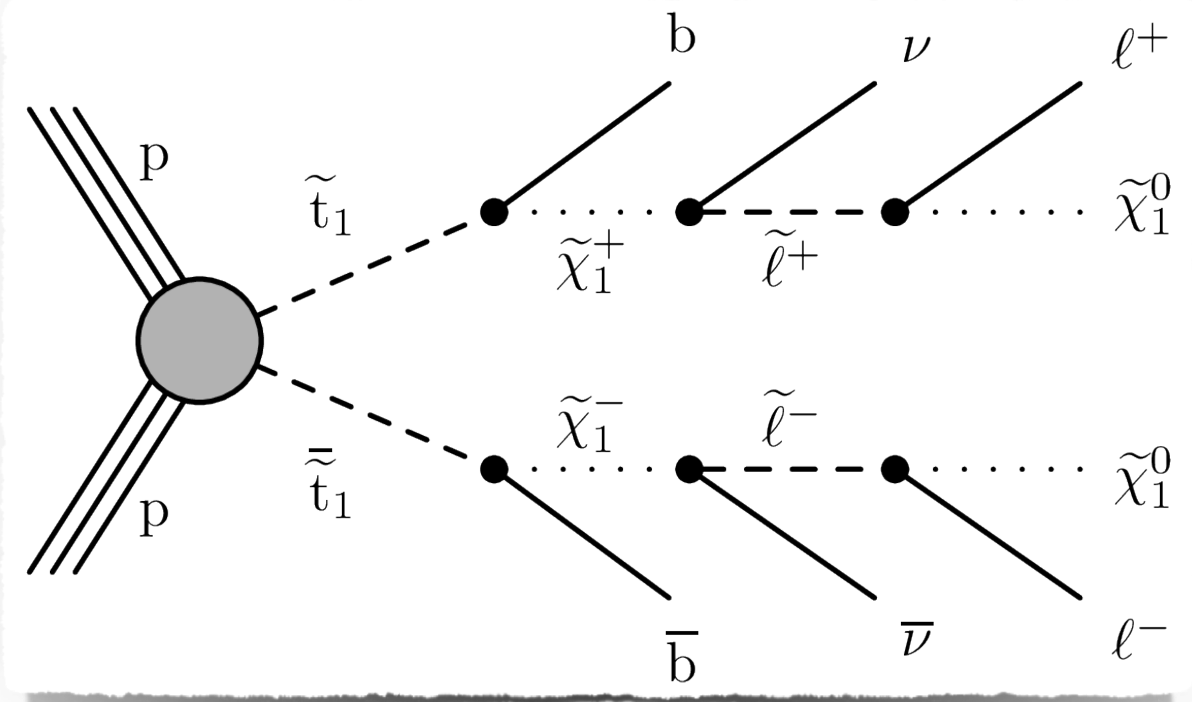
LHC ≡ discovery machine

- Many existing ATLAS and CMS searches for new physics
- Interpretation within popular frameworks (MSSM, *etc.*) and simplified models
  ➙ (more or less) model dependent

Example: CMS search for stops in the di-lepton channel



[CMS (EPJC'21)]

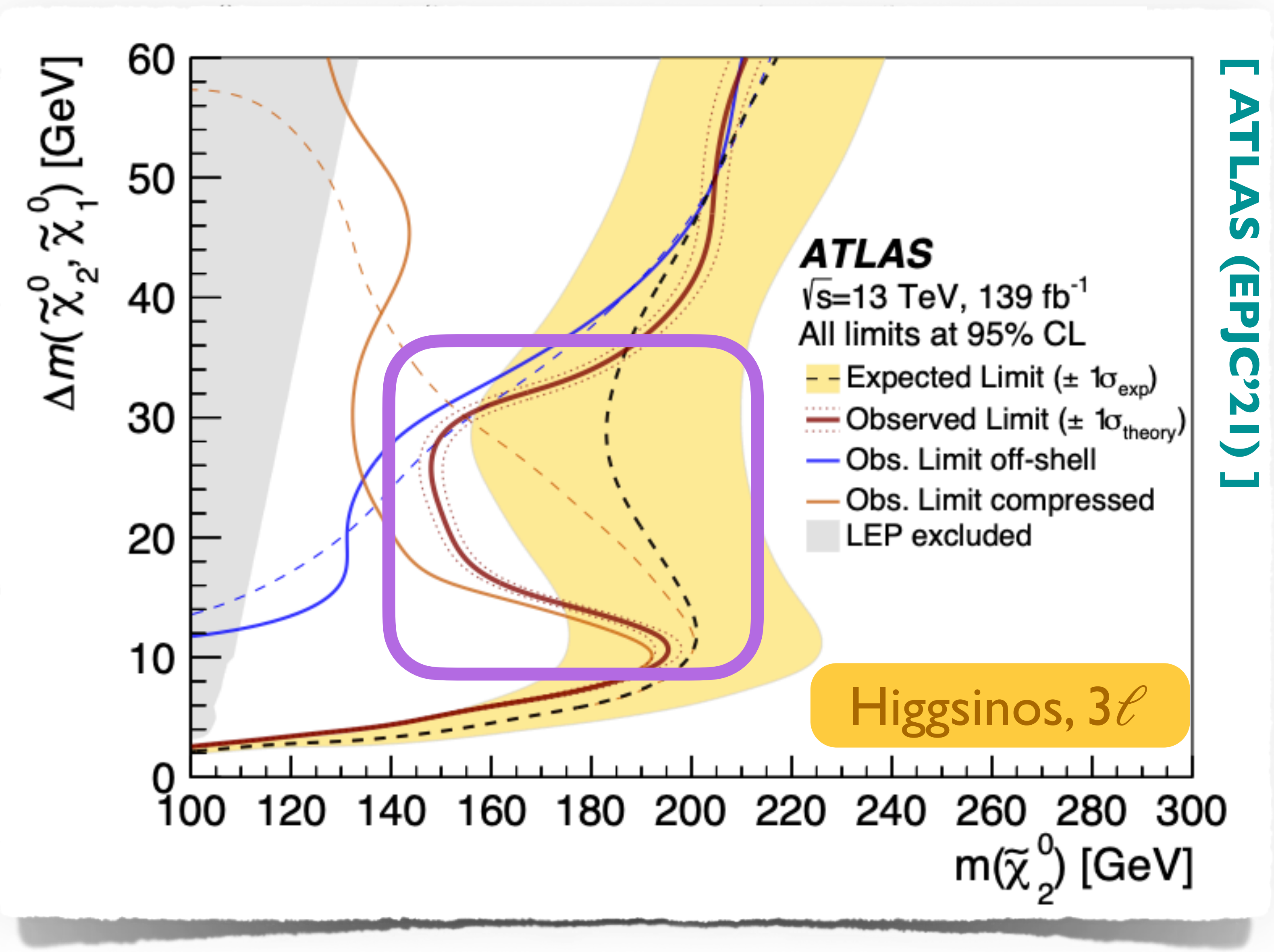


- No sign of new physics
- Bounds set on new state masses
  ➙ Stop mass $\gtrsim$ 1.4 TeV
  ➙ Neutralino mass $\gtrsim$ 600 − 800 GeV
- Exceptions possible (compressed spectra)

Effective Field Theories ↔ probe for BSM

SM-like measurements (except for a few anomalies)

- No leading candidate theory
- New physics is heavy
- EFTs as an alternative option for new physics

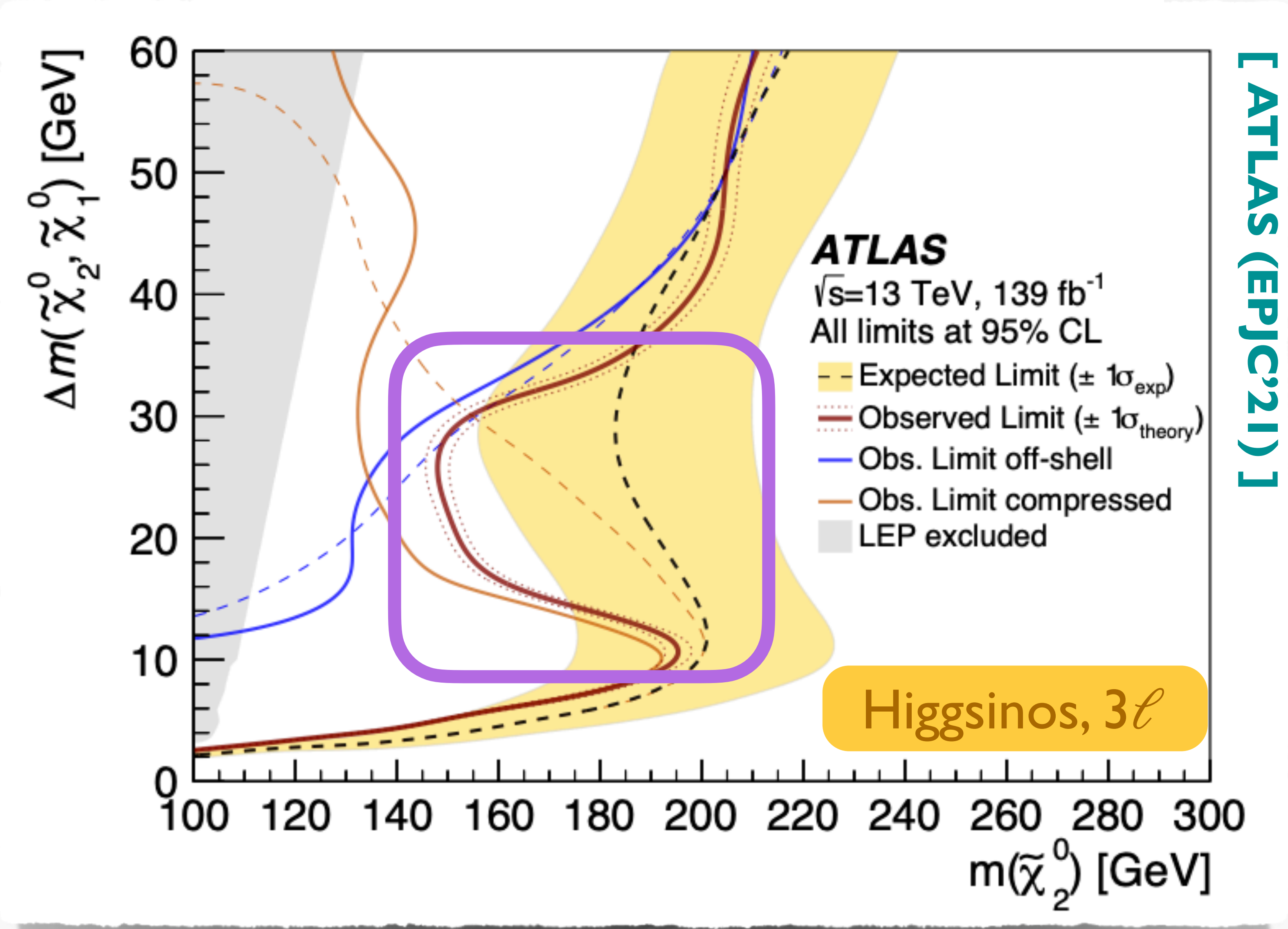


[ATLAS (EPJC'21)]

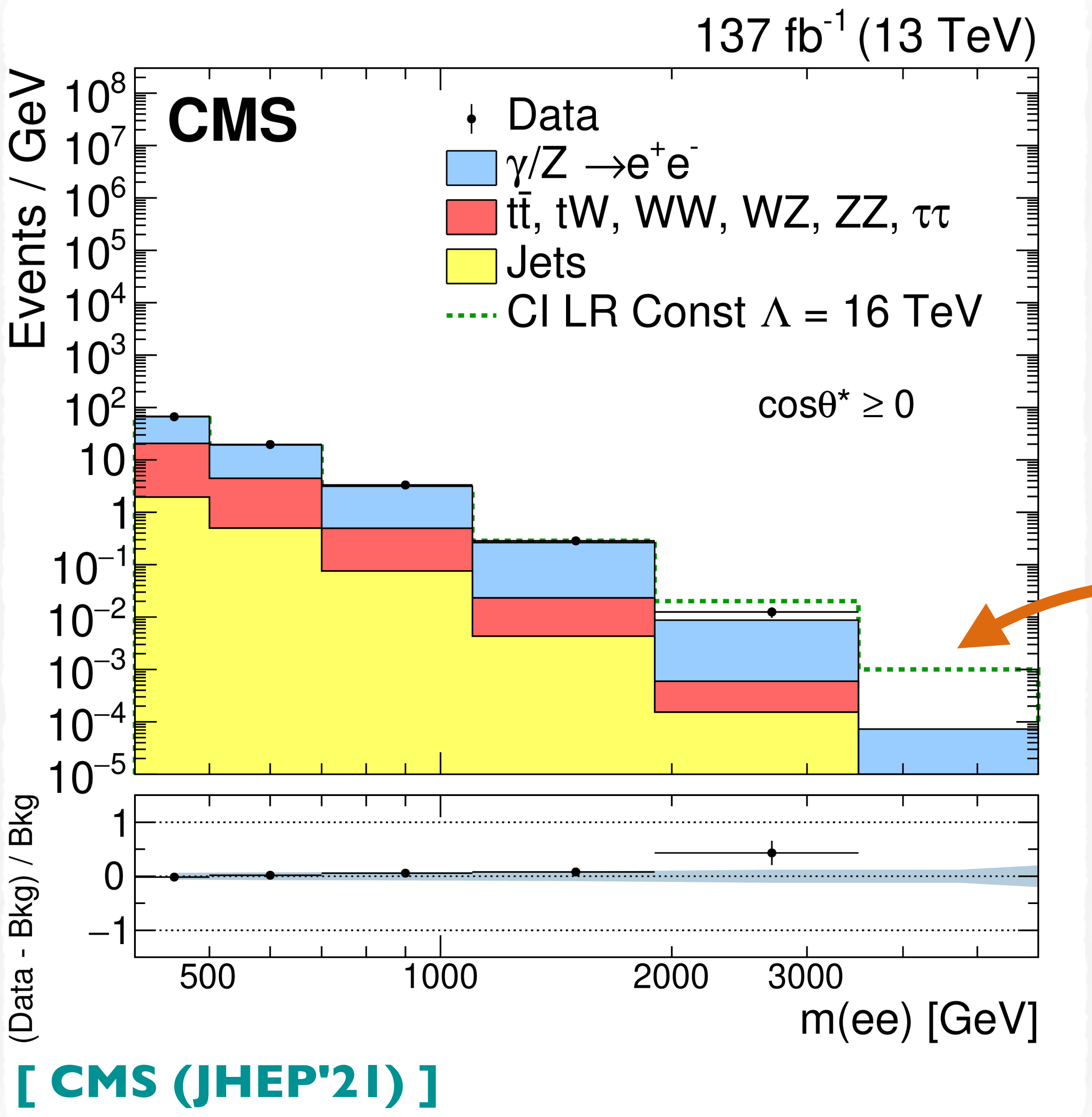
Effective Field Theories ↔ probe for BSM

SM-like measurements (except for a few anomalies)

- No leading candidate theory
- New physics is heavy
- EFTs as an alternative option for new physics



Example: CMS (Drell-Yan) di-lepton analysis



- BSM ≡ small deviations from the SM
  - ➞ Suppressed new interactions
  - ➞ Modifications of existing interactions
- One possibility:

$$\mathcal{L} = \frac{g_{\rm NP}}{\Lambda^2}\ \bar q_L \gamma^\mu q_L\ \bar\ell_R\gamma_\mu\ell_R$$

- Λ is the new physics scale

Scale control as a key concept

Any problem associated with a typical scale ⬌ other scales irrelevant

- Neutrino masses irrelevant for atomic physics
- General relativity irrelevant for the motion of a car
- $m_Z \equiv$ scale for (on-shell) Z-boson production
- $m_{\ell\ell} \equiv$ scale for high-energy Drell-Yan production

Particle masses, momentum transfers, $\sqrt{s}, \ldots$
$\equiv$ relevant scales for collider processes

Scale control as a key concept

Any problem associated with a typical scale ⬌ other scales irrelevant

- Neutrino masses irrelevant for atomic physics
- General relativity irrelevant for the motion of a car
- m_Z ≡ scale for (on-shell) Z-boson production
- $m_{\ell\ell}$ ≡ scale for high-energy Drell-Yan production

Particle masses, momentum transfers, $\sqrt{s}, \ldots$
≡ relevant scales for collider processes

Scale separation ⬌ EFT solution to problems [Appelquist & Carrazzone (PRD'75)]

- High-mass scale M and low-mass scale m
- Heavy physics (M) ≡ small impact at low energy ($E \sim m$) or momenta ($p \sim m$)
  - ➞ Perturbative treatment of its effects
  - ➞ Expansion in $(E/M)^n$
- Heavy physics decouples at low momenta
  - ➞ shifts of $\mathcal{O}\left(p^2/M^2\right)$

Decoupling new physics

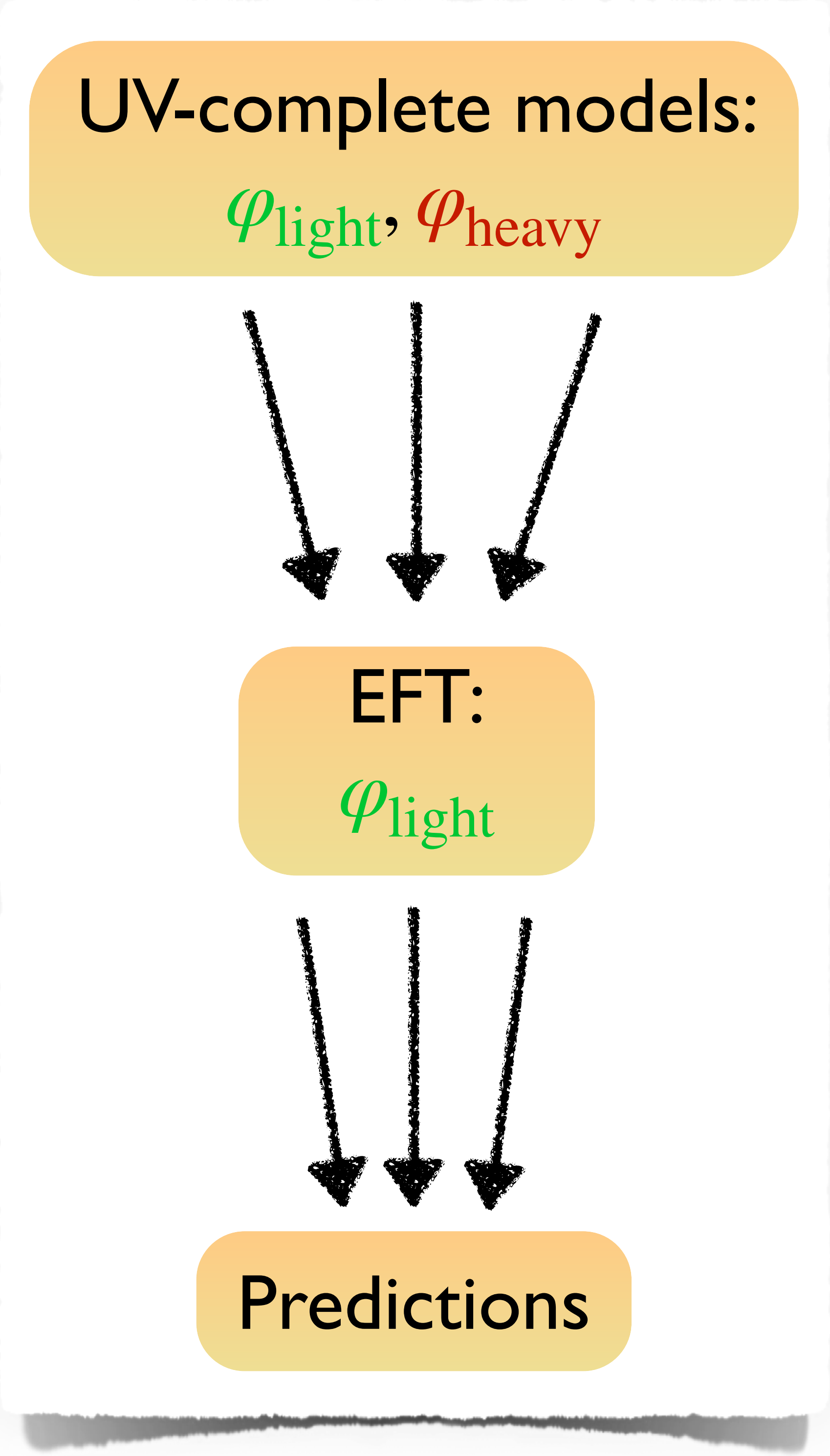
EFTs are successful predictive frameworks

- Heavy new physics is decoupled
  - ➙ probing new physics (M) with known light states (m)
  - ➙ connection with the null results of the LHC
- Valid at some energy scale
- Predictions at arbitrary levels of precision (⚠️ loops ⊕ EFTs not trivial)

Decoupling new physics

EFTs are successful predictive frameworks

- Heavy new physics is decoupled
  - ➙ probing new physics (M) with known light states (m)
  - ➙ connection with the null results of the LHC
- Valid at some energy scale
- Predictions at arbitrary levels of precision (⚠ loops ⊕ EFTs not trivial)



Why EFTs?

- Simplification of the calculations (known 'full' theory)
  - ➙ heavy fields integrated out

Top-down

$$\mathcal{L}_{\rm full}\big(\underset{m}{\varphi_{\rm light}}, \underset{M}{\varphi_{\rm heavy}}\big) \Rightarrow \mathcal{L}_{\rm full}\big(\varphi_{\rm light}\big) + \mathcal{L}_{\rm eff}\big(\varphi_{\rm light}\big)$$

- Universal parametrisation of heavy new physics (unknown 'full' theory)
  - ➙ most general expression for $\mathscr{L}_{\rm eff}\,(\varphi_{\rm light})$

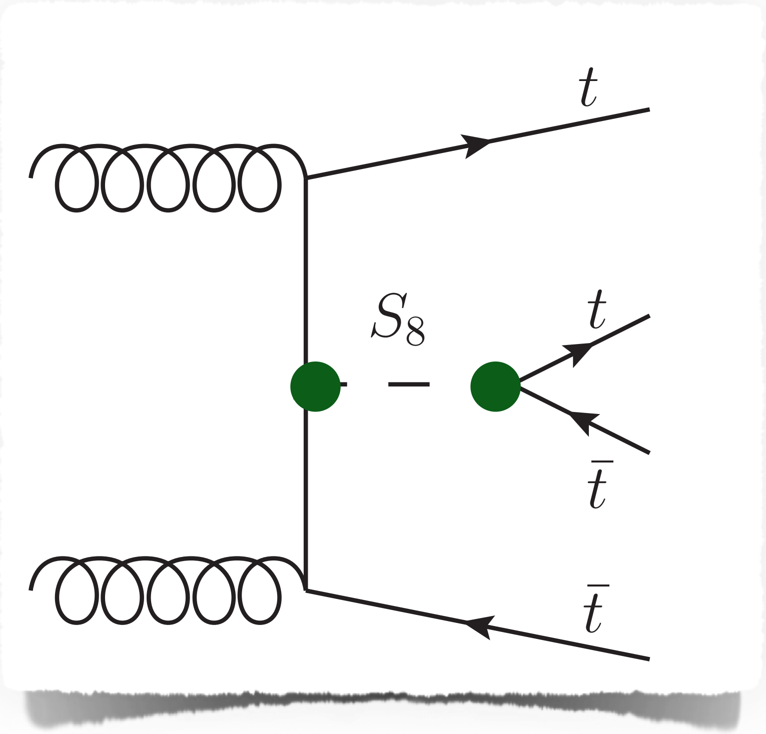
Bottom-up

- Fine-prints
  - ➙ Range of validity (*cf.* scales)
  - ➙ Typical E/M values: small (power counting)

Example 1 – a heavy top-philic scalar gluon

Four-top production through a heavy scalar gluon

Energy



Full theory: $\mathscr{L}_{\rm full}\big(\varphi_{\rm SM}, S_8\big)$

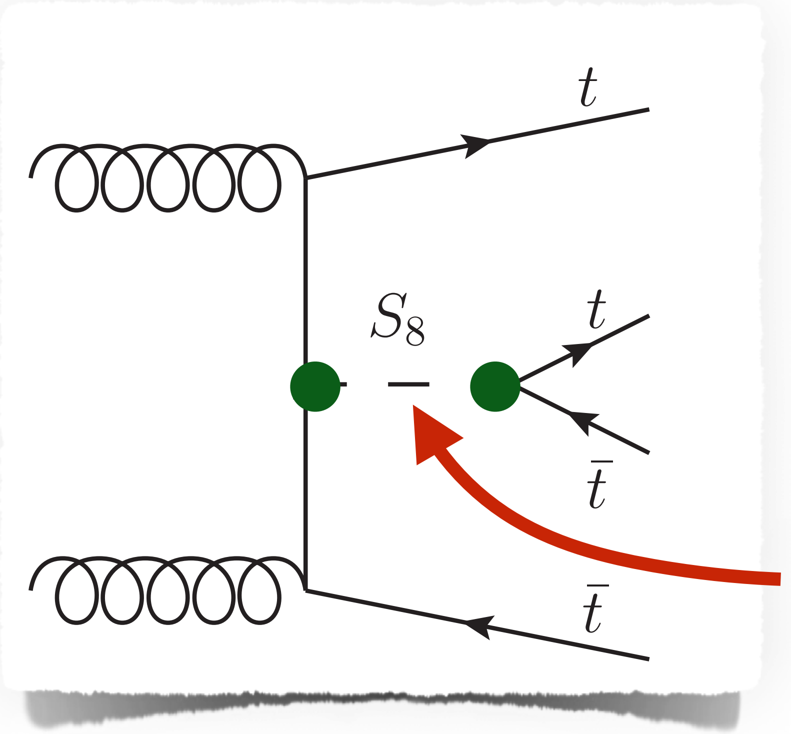
Λ

100 GeV

Example 1 – a heavy top-philic scalar gluon

Four-top production through a heavy scalar gluon

Energy



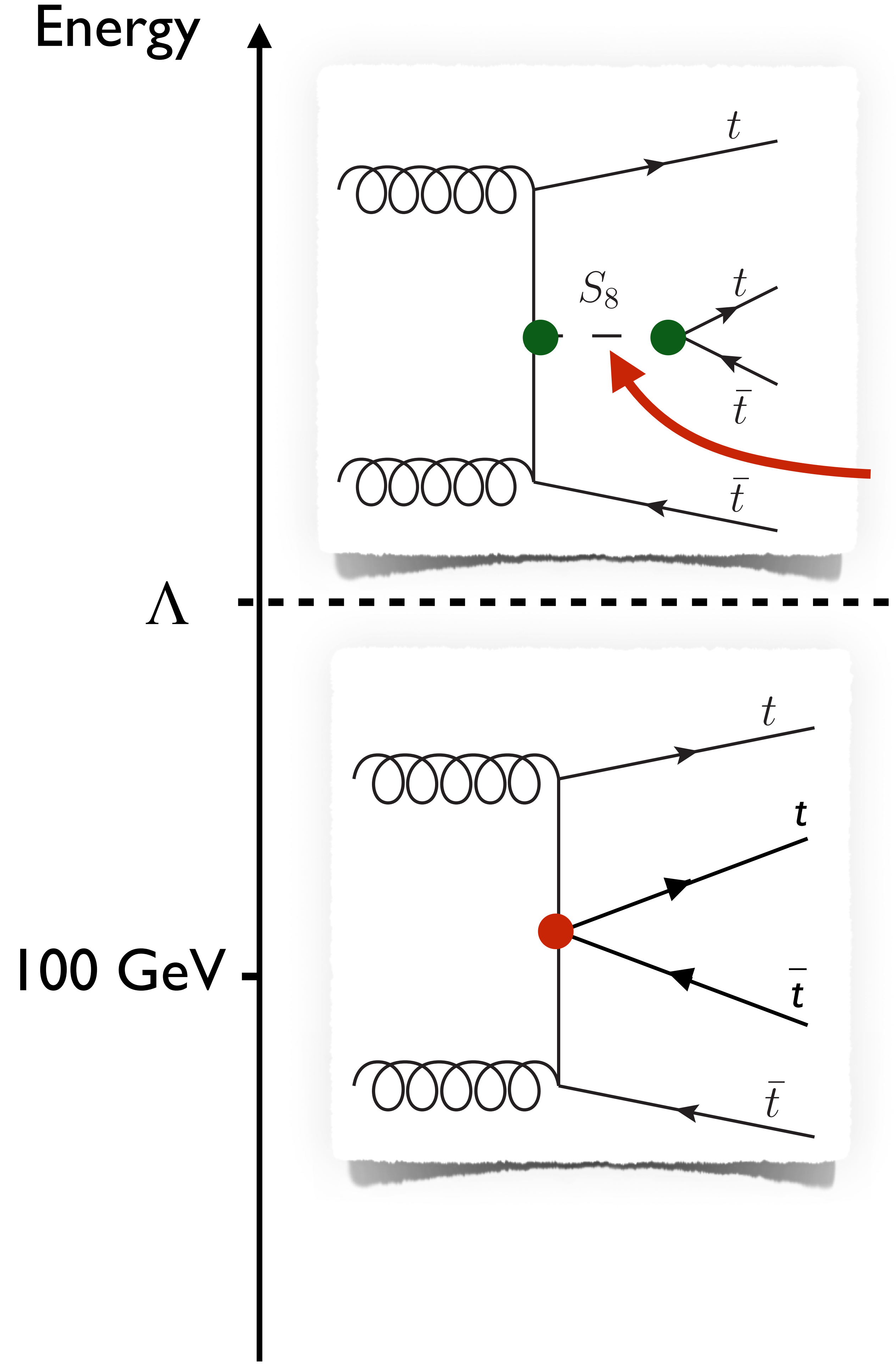
Full theory: $\mathscr{L}_{\text{full}}\left(\varphi_{\text{SM}}, S_8\right)$

$$\frac{y_8^2}{p^2 - M_8^2} = -\frac{y_8^2}{M_8^2}\left[1 + \frac{p^2}{M_8^2} + \ldots\right]$$

Λ

100 GeV

Four-top production through a heavy scalar gluon



Full theory: $\mathscr{L}_{\rm full}\left(\varphi_{\rm SM}, S_8\right)$

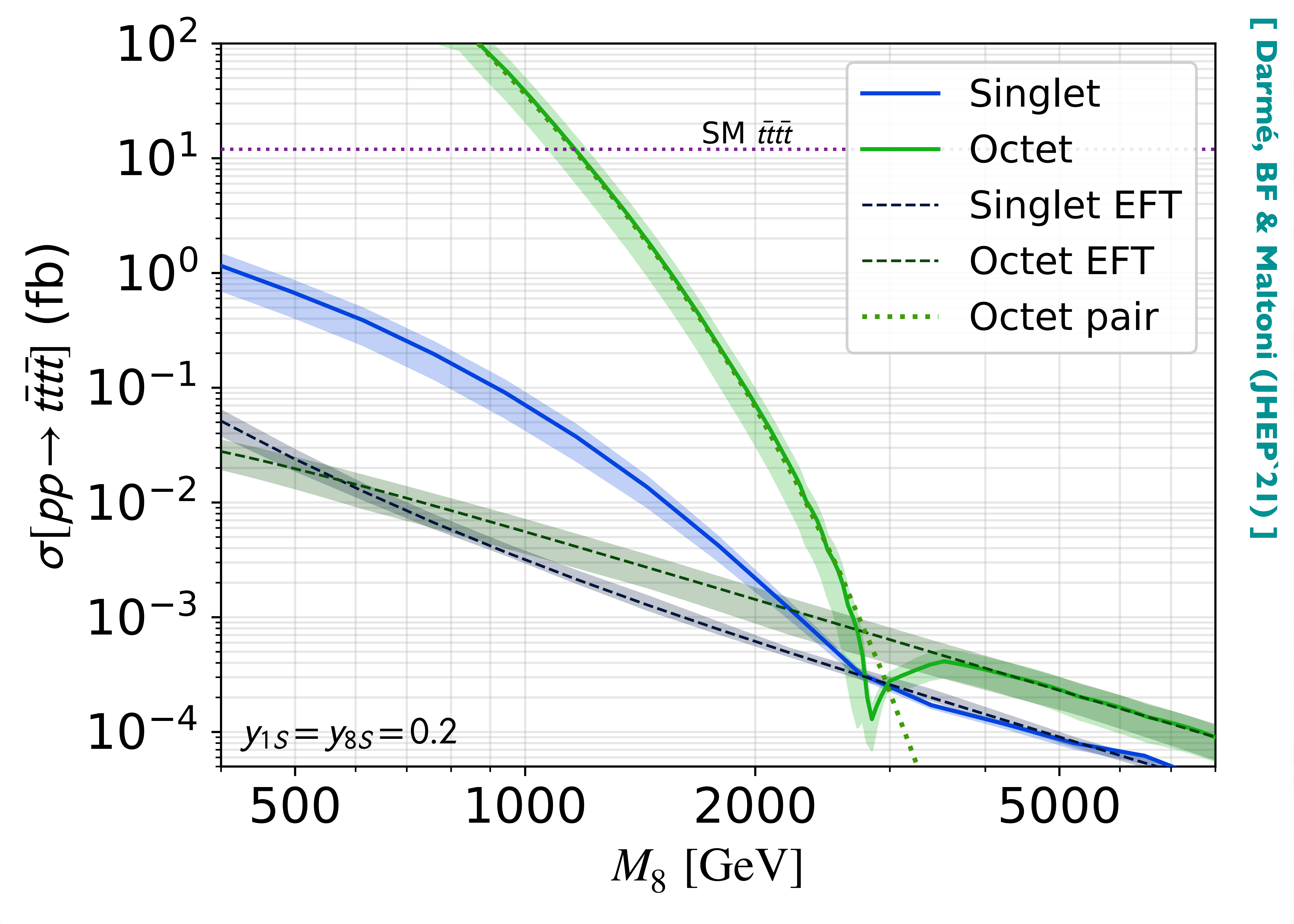
$$\frac{y_8^2}{p^2 - M_8{}^2} = -\frac{y_8^2}{M_8{}^2}\left[1 + \frac{p^2}{M_8{}^2} + \dots\right]$$

Heavy S_8 integrated out

Effective theory: $\mathscr{L}_{\rm eff}\left(\varphi_{\rm SM}\right) = \mathscr{L}_{\rm SM} + \frac{y_8^2}{2M_8{}^2}\,\bar t T_A t\;\bar t T^A t$

EFT and cross sections

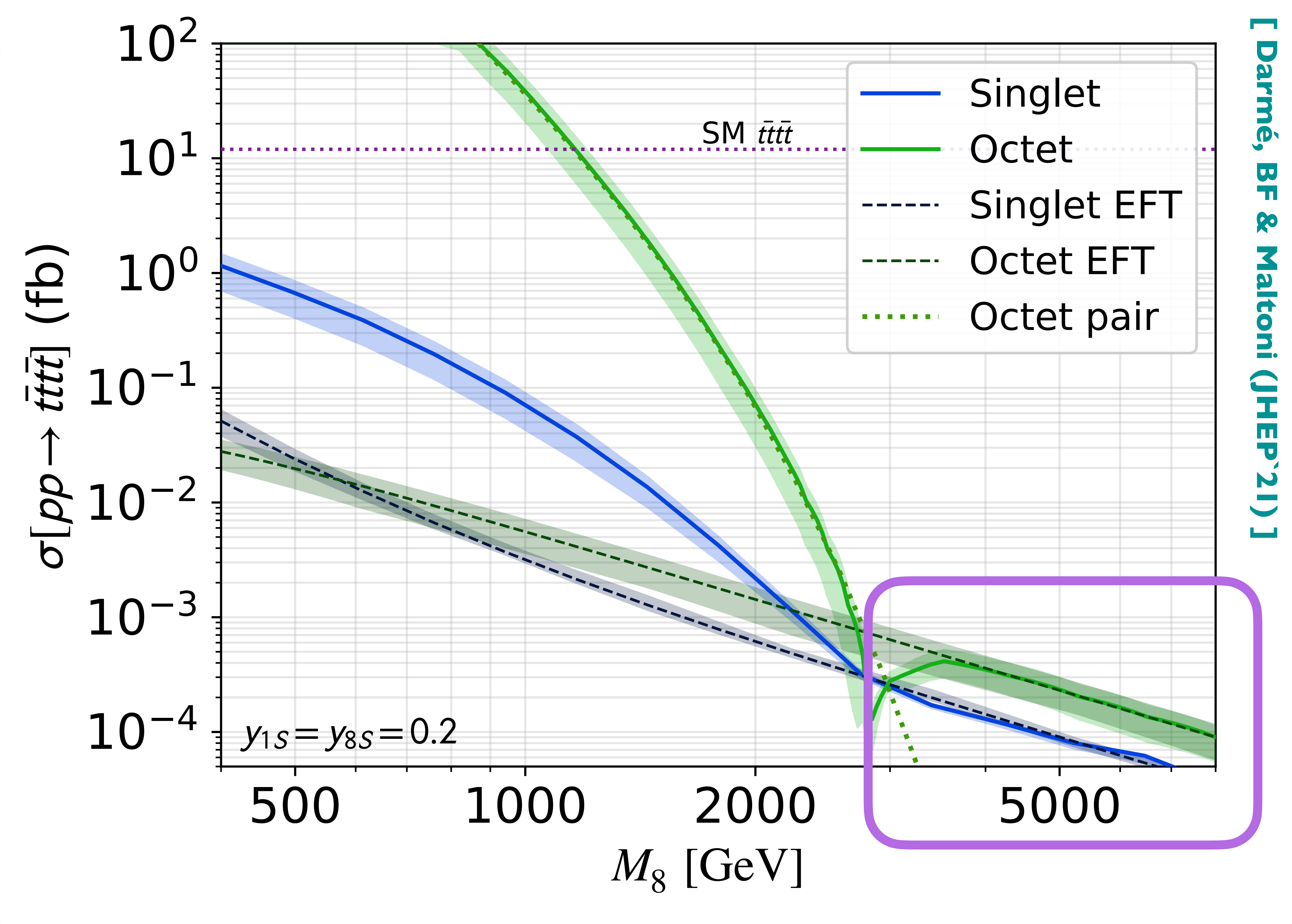
EFT validity from a cross section calculation point of view



[Darmé, BF & Maltoni (JHEP`21)]

- S_8 contributions to four-top production
  - ➙ Light green curve = full theory $\mathscr{L}_{\rm full}(\varphi_{\rm SM}, S_8)$
  - ➙ Dark green curve = EFT $\mathscr{L}_{\rm eff}(\varphi_{\rm SM})$

EFT validity from a cross section calculation point of view



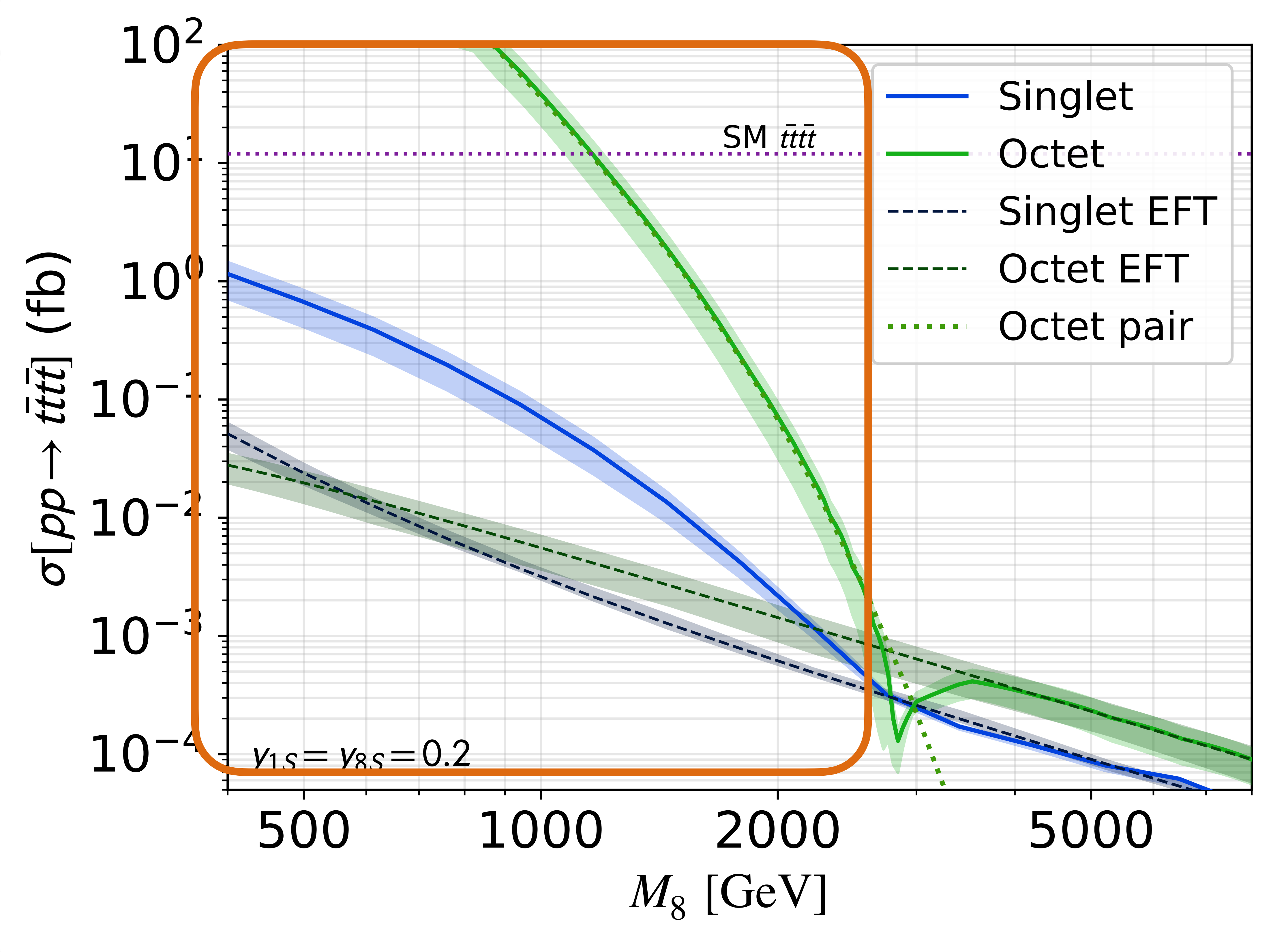
[Darmé, BF & Maltoni (JHEP'21)]

- S_8 contributions to four-top production
  - ➞ Light green curve = full theory $\mathscr{L}_{\rm full}(\varphi_{\rm SM}, S_8)$
  - ➞ Dark green curve = EFT $\mathscr{L}_{\rm eff}(\varphi_{\rm SM})$

- At high mass
  - ➞ Heavy S_8 ↔ sensible EFT
  - ➞ Agreement between predictions

EFT and cross sections

EFT validity from a cross section calculation point of view

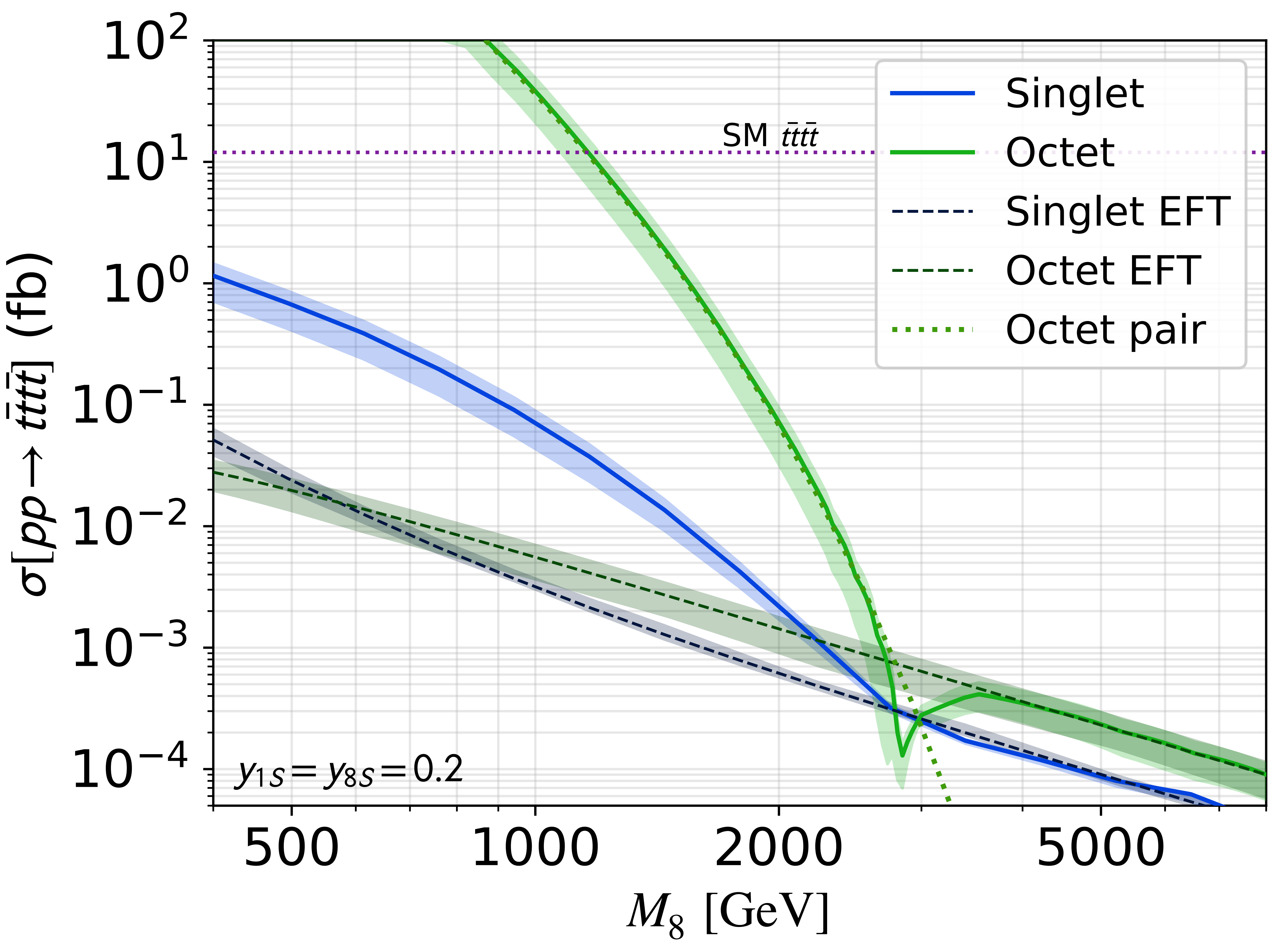


[Darmé, BF & Maltoni (JHEP'21)]

- S_8 contributions to four-top production
  - ➞ Light green curve = full theory $\mathscr{L}_{\rm full}(\varphi_{\rm SM}, S_8)$
  - ➞ Dark green curve = EFT $\mathscr{L}_{\rm eff}(\varphi_{\rm SM})$
- At high mass
  - ➞ Heavy S_8 ↔ sensible EFT
  - ➞ Agreement between predictions
- At low mass
  - ➞ Light(ish) S_8 ↔ EFT invalid (E/M_8 not small)
  - ➞ Disagreement between predictions

EFT and cross sections

EFT validity from a cross section calculation point of view

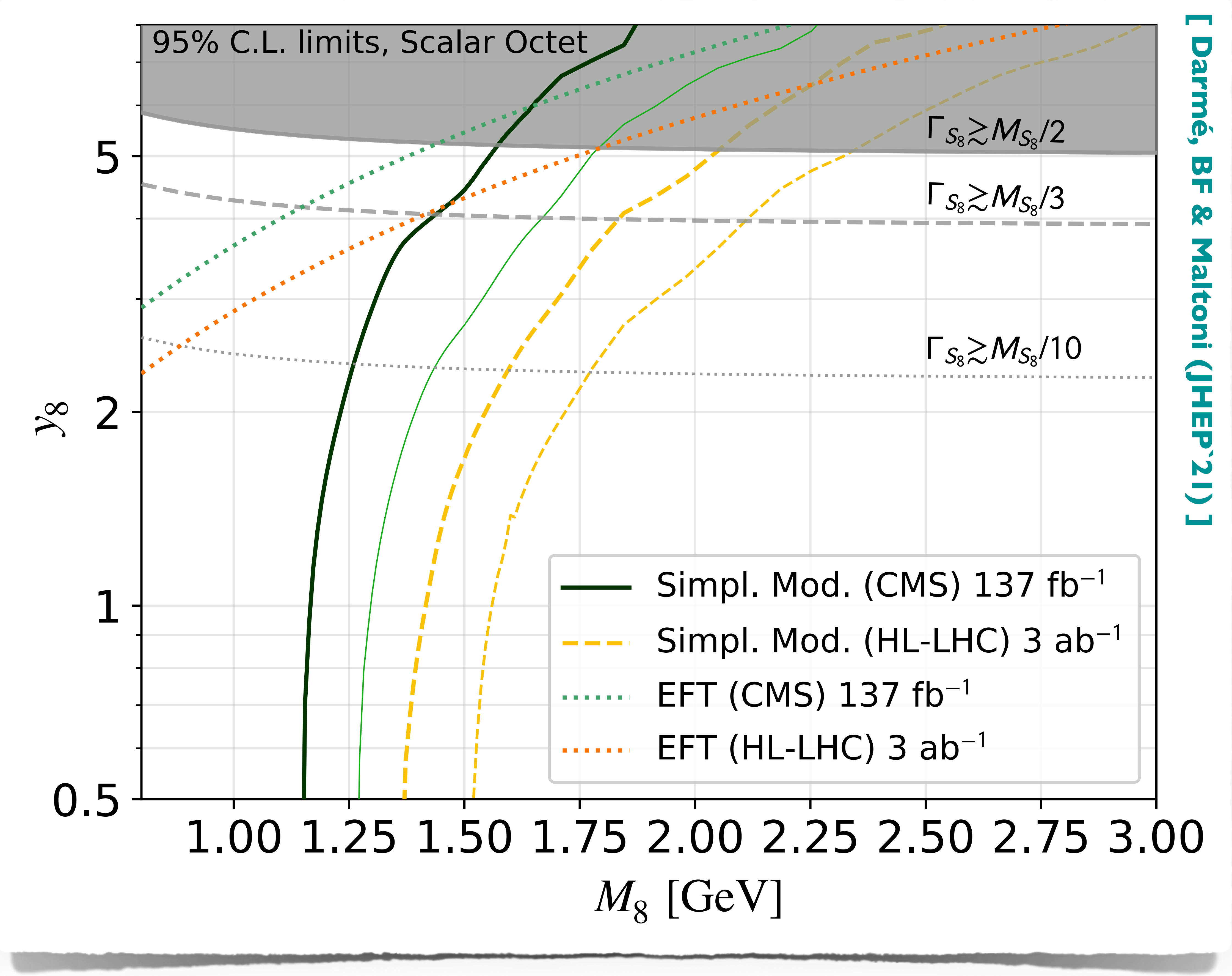


[Darmé, BF & Maltoni (JHEP'21)]

- S_8 contributions to four-top production
  - ➙ Light green curve = full theory $\mathscr{L}_{\rm full}(\varphi_{\rm SM}, S_8)$
  - ➙ Dark green curve = EFT $\mathscr{L}_{\rm eff}(\varphi_{\rm SM})$
- At high mass
  - ➙ Heavy S_8 ↔ sensible EFT
  - ➙ Agreement between predictions
- At low mass
  - ➙ Light(ish) S_8 ↔ EFT invalid (E/M_8 not small)
  - ➙ Disagreement between predictions

Careful when using EFTs
➙ impact on predictions, interpretations, *etc.*

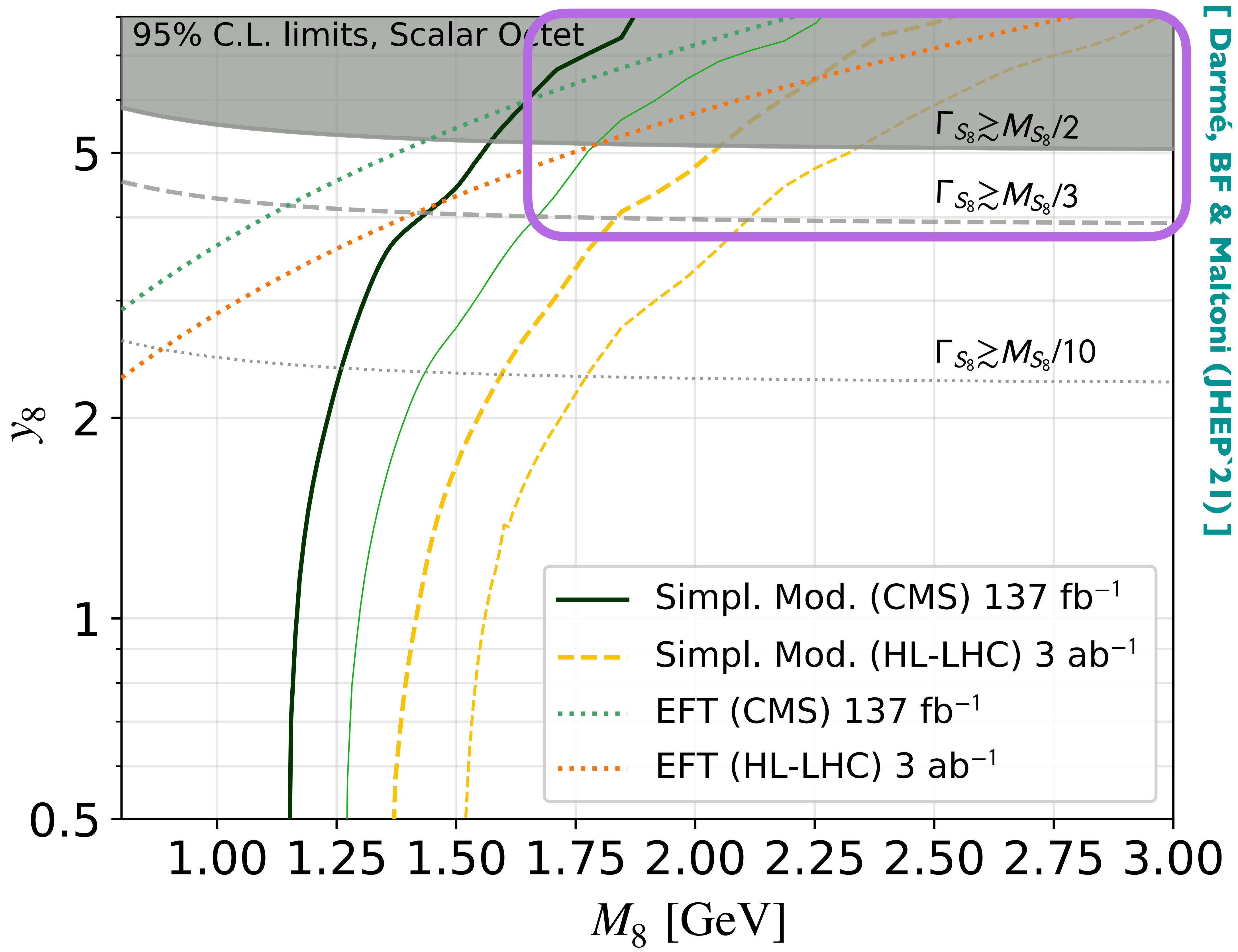
Limit setting in an EFT framework



- LHC Run 2 limits on scalar octets in terms of mass/couplings
  - ➞ Solid green: full theory $\mathscr{L}_{\rm full}\big(\varphi_{\rm SM}, S_8\big)$
  - ➞ Dashed green ≡ EFT $\mathscr{L}_{\rm eff}\big(\varphi_{\rm SM}\big)$
- Orange curves: HL-LHC expectation

EFT and limit setting

Limit setting in an EFT framework



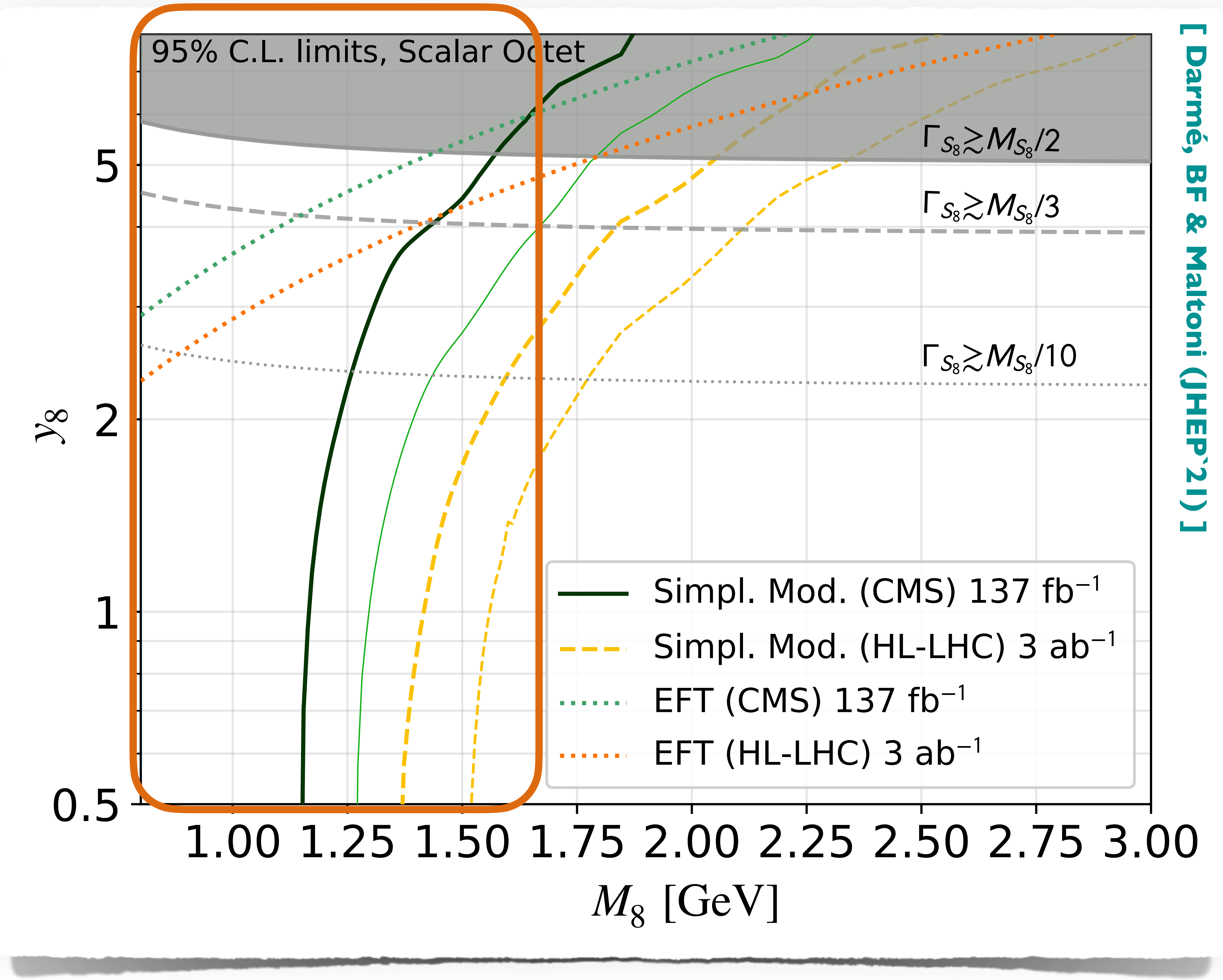
[Darmé, BF & Maltoni (JHEP'21)]

- LHC Run 2 limits on scalar octets in terms of mass/couplings
  - ➙ Solid green: full theory $\mathscr{L}_{\text{full}}(\varphi_{\text{SM}}, S_8)$
  - ➙ Dashed green ≡ EFT $\mathscr{L}_{\text{eff}}(\varphi_{\text{SM}})$
- Orange curves: HL-LHC expectation

- At high mass
  - ➙ Heavy S_8 ⬌ sensible EFT
  - ➙ Agreement between the limits
  - ➙ Other problem: the full theory makes no sense

EFT and limit setting

Limit setting in an EFT framework

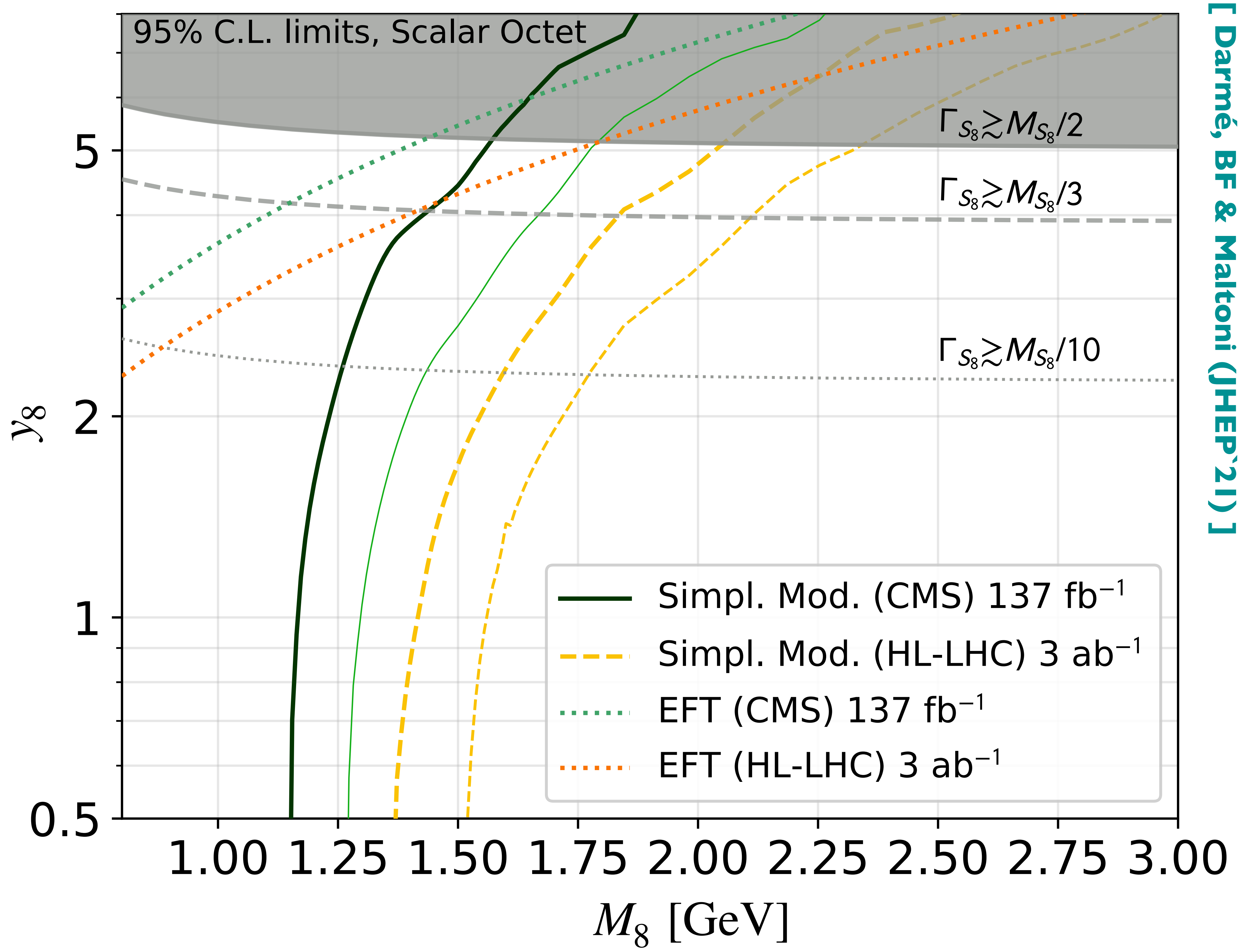


[Darmé, BF & Maltoni (JHEP'21)]

- LHC Run 2 limits on scalar octets in terms of mass/couplings
  - → Solid green: full theory $\mathscr{L}_{\text{full}}(\varphi_{\text{SM}}, S_8)$
  - → Dashed green ≡ EFT $\mathscr{L}_{\text{eff}}(\varphi_{\text{SM}})$
- Orange curves: HL-LHC expectation
- At high mass
  - → Heavy S_8 ↔ sensible EFT
  - → Agreement between the limits
  - → Other problem: the full theory makes no sense
- At low mass
  - → Light(ish) S_8 ↔ EFT invalid (E/M_8 not small)
  - → On-shell production of heavy states
  - → Disagreement between predictions

EFT and limit setting

Limit setting in an EFT framework



[Darmé, BF & Maltoni (JHEP`21)]

- LHC Run 2 limits on scalar octets in terms of mass/couplings
  - ➙ Solid green: full theory $\mathscr{L}_{\rm full}\big(\varphi_{\rm SM}, S_8\big)$
  - ➙ Dashed green ≡ EFT $\mathscr{L}_{\rm eff}\big(\varphi_{\rm SM}\big)$
- Orange curves: HL-LHC expectation
- At high mass
  - ➙ Heavy S_8 ⬌ sensible EFT
  - ➙ Agreement between the limits
  - ➙ Other problem: the full theory makes no sense
- At low mass
  - ➙ Light(ish) S_8 ⬌ EFT invalid (E/M_8 not small)
  - ➙ On-shell production of heavy states
  - ➙ Disagreement between predictions

Crucial to rely on EFTs
only when allowed

Matching EFT
&
UV setups

The Fermi theory of weak interactions

Fermi theory of weak interactions (30 years before the EW theory)

- Excellent description for the scales probed at that time
- Four-fermion interactions

$$\mathcal{L}_{\rm Fermi} = -\frac{G_F}{\sqrt{2}} \Big[\bar\nu_\mu \gamma^\mu \big(1-\gamma_5\big)\mu\Big]\ \Big[\bar\nu_e \gamma_\mu \big(1-\gamma_5\big) e\Big] + {\rm H.c.}$$

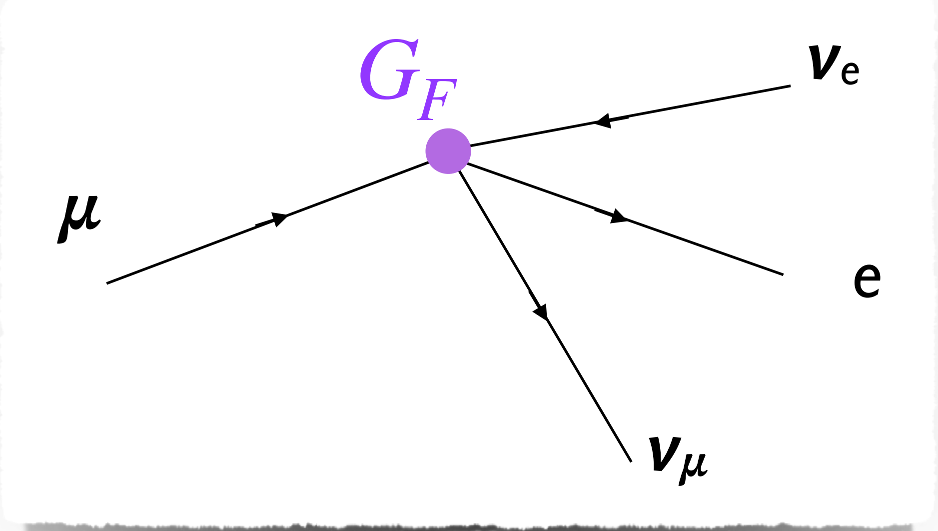
The Fermi theory of weak interactions

Fermi theory of weak interactions (30 years before the EW theory)

- Excellent description for the scales probed at that time
- Four-fermion interactions

$$\mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}}\Big[\bar{\nu}_\mu\gamma^\mu(1-\gamma_5)\mu\Big]\ \Big[\bar{\nu}_e\gamma_\mu(1-\gamma_5)e\Big] + \text{H.c.}$$

Description of beta decays, muon decays, *etc.*



$$G_F \approx 1.167\,10^{-5}\ \text{GeV}^{-2}$$

➞ Information on high-scale physics

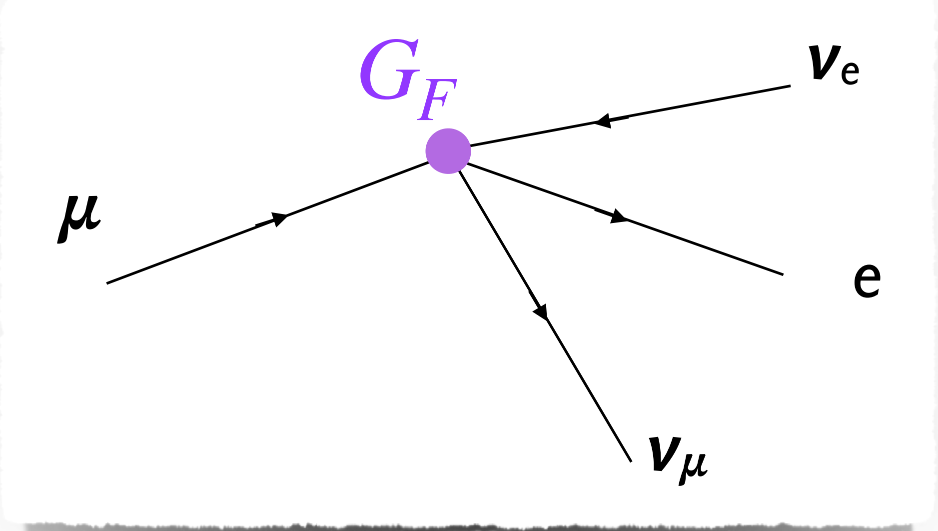
The Fermi theory of weak interactions

Fermi theory of weak interactions (30 years before the EW theory)

- Excellent description for the scales probed at that time
- Four-fermion interactions

$$\mathcal{L}_{\rm Fermi} = -\frac{G_F}{\sqrt{2}}\Big[\bar\nu_\mu\gamma^\mu(1-\gamma_5)\mu\Big]\ \Big[\bar\nu_e\gamma_\mu(1-\gamma_5)e\Big] + {\rm H.c.}$$

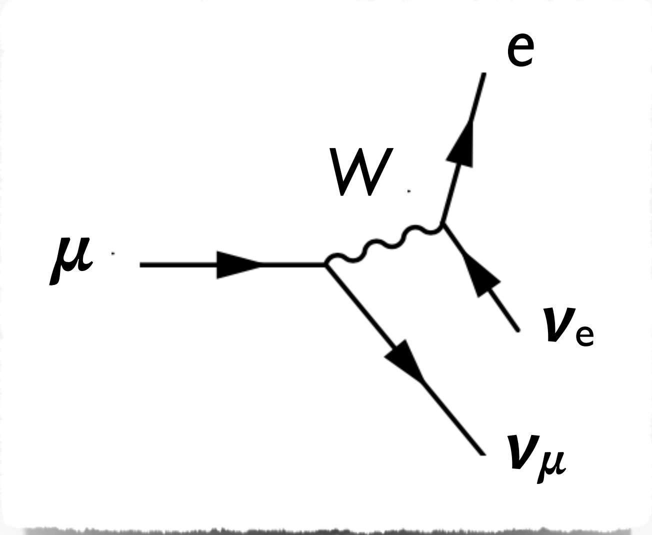
Description of beta decays, muon decays, *etc.*



$G_F \approx 1.167\,10^{-5}\ {\rm GeV}^{-2}$

➞ Information on high-scale physics

High-scale physics: the EW theory



- Mediation through W exchanges
  ➞ Scale separation: m_μ/m_W

Two theories ≡ same physics
➞ Matching

Matching Fermi and EW theories

Fermi theory and the EW theory describe the same physics at low energy

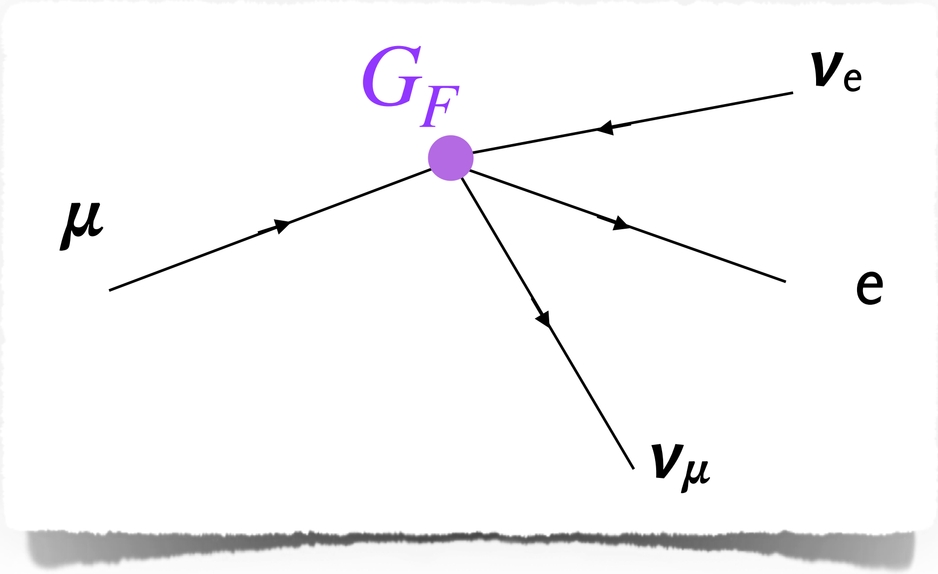
- Matching low-energy amplitudes (for any given process)

Matching Fermi and EW theories

Fermi theory and the EW theory describe the same physics at low energy

- Matching low-energy amplitudes (for any given process)

Muon decay in the Fermi theory



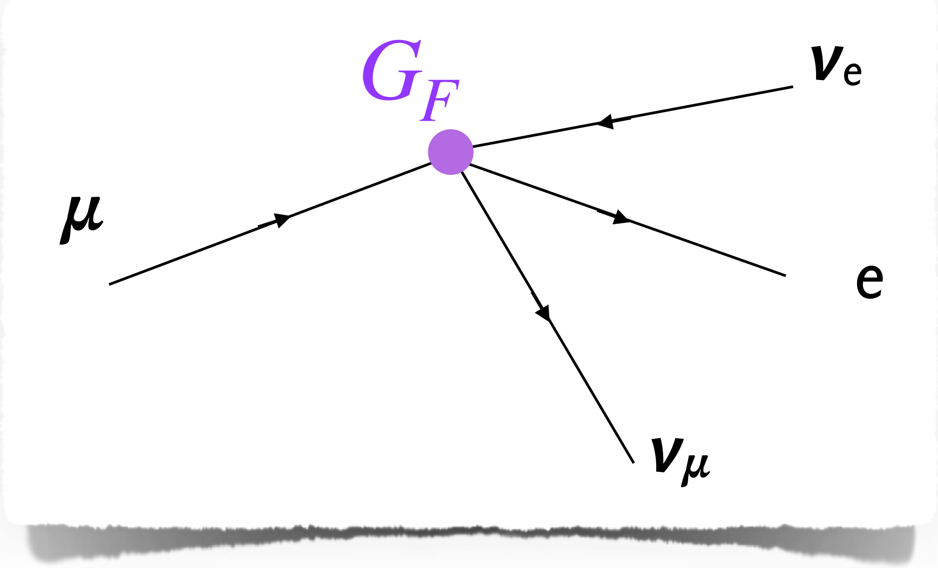
$$iM_{\rm Fermi} = -\frac{iG_F}{\sqrt{2}}\Big[\bar{u}(p_{\nu_\mu})\gamma^\mu(1-\gamma_5)u(p_\mu)\Big]\ \Big[\bar{u}(p_e)\gamma_\mu(1-\gamma_5)v(p_{\nu_e})\Big]$$

Matching Fermi and EW theories

Fermi theory and the EW theory describe the same physics at low energy

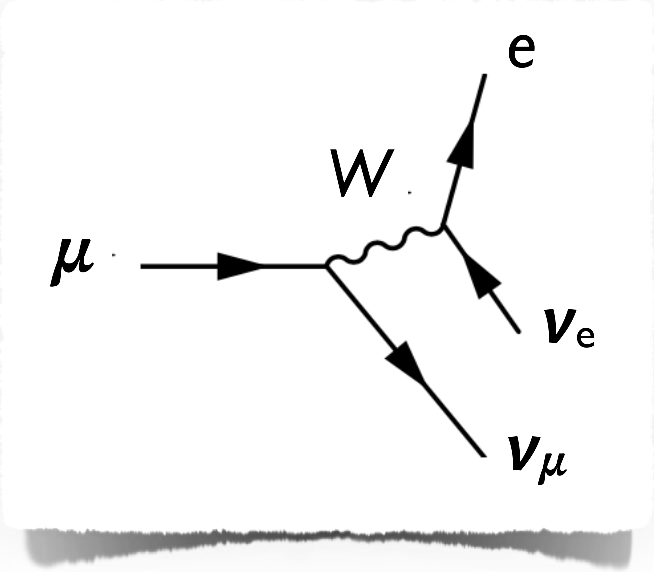
- Matching low-energy amplitudes (for any given process)

Muon decay in the Fermi theory



$$iM_{\rm Fermi} = -\frac{iG_F}{\sqrt{2}}\Big[\bar u(p_{\nu_\mu})\gamma^\mu(1-\gamma_5)u(p_\mu)\Big]\ \Big[\bar u(p_e)\gamma_\mu(1-\gamma_5)v(p_{\nu_e})\Big]$$

The EW theory (at low energy) and muon decay



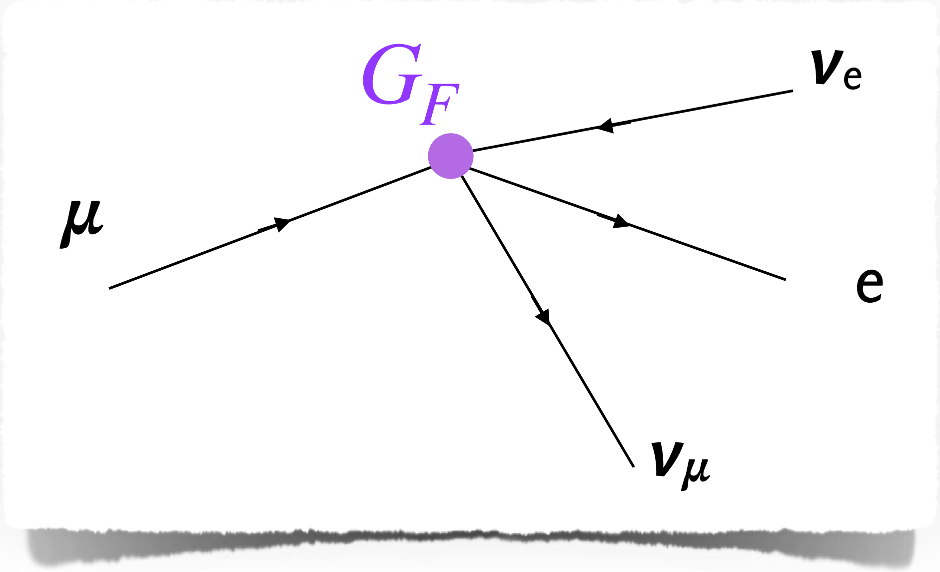
$$iM_{\rm EW} = -\frac{ig^2}{8M_W^2}\ \frac{M_W^2\eta_{\mu\nu}-k_\mu k_\nu}{k^2-M_W^2}\Big[\bar u(p_{\nu_\mu})\gamma^\mu(1-\gamma_5)u(p_\mu)\Big]\Big[\bar u(p_e)\gamma^\nu(1-\gamma_5)v(p_{\nu_e})\Big]$$

Matching Fermi and EW theories

Fermi theory and the EW theory describe the same physics at low energy

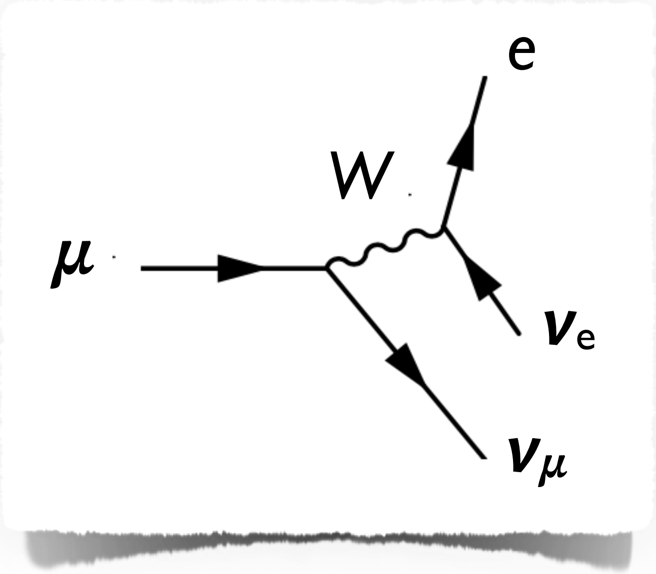
- Matching low-energy amplitudes (for any given process)

Muon decay in the Fermi theory



$$iM_{\rm Fermi} = -\frac{iG_F}{\sqrt{2}}\Big[\bar u(p_{\nu_\mu})\gamma^\mu(1-\gamma_5)u(p_\mu)\Big]\Big[\bar u(p_e)\gamma_\mu(1-\gamma_5)v(p_{\nu_e})\Big]$$

The EW theory (at low energy) and muon decay



$$\begin{aligned} iM_{\rm EW} &= -\frac{ig^2}{8M_W^2}\frac{M_W^2\eta_{\mu\nu}-k_\mu k_\nu}{k^2-M_W^2}\Big[\bar u(p_{\nu_\mu})\gamma^\mu(1-\gamma_5)u(p_\mu)\Big]\Big[\bar u(p_e)\gamma^\nu(1-\gamma_5)v(p_{\nu_e})\Big] \\ &\approx -\frac{ig^2}{8M_W^2}\Big[\bar u(p_{\nu_\mu})\gamma^\mu(1-\gamma_5)u(p_\mu)\Big]\Big[\bar u(p_e)\gamma_\mu(1-\gamma_5)v(p_{\nu_e})\Big] \end{aligned}$$

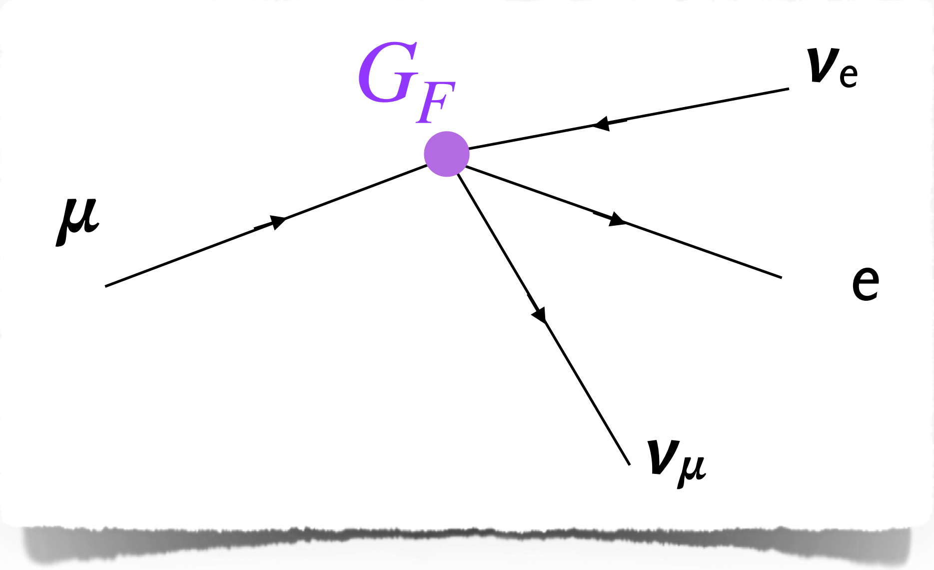
Scale separation
→ $m_\mu \ll m_W$
→ $k^2 \ll m_W^2$

Matching Fermi and EW theories

Fermi theory and the EW theory describe the same physics at low energy

- Matching low-energy amplitudes (for any given process)

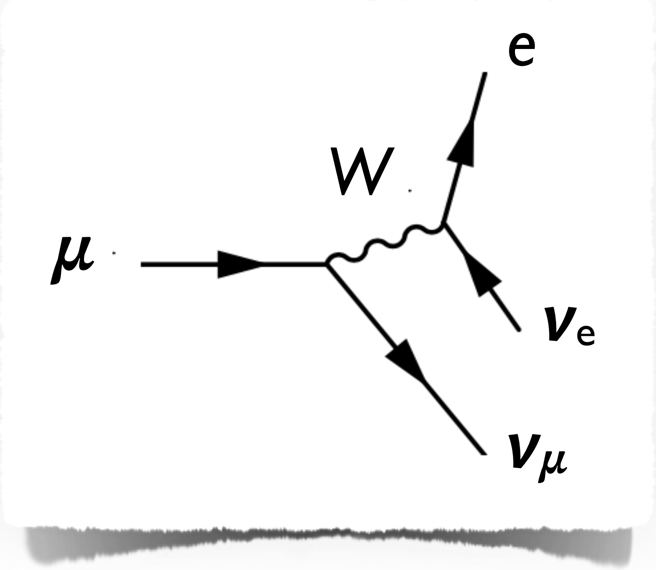
Muon decay in the Fermi theory



$$iM_{\text{Fermi}} = -\frac{iG_F}{\sqrt{2}}\left[\bar{u}(p_{\nu_\mu})\gamma^\mu(1-\gamma_5)u(p_\mu)\right]\left[\bar{u}(p_e)\gamma_\mu(1-\gamma_5)v(p_{\nu_e})\right]$$

The two amplitudes must be equal
$G_F = 1/\left(\sqrt{2}v^2\right)$

The EW theory (at low energy) and muon decay



$$iM_{\text{EW}} = -\frac{ig^2}{8M_W^2}\frac{M_W^2\eta_{\mu\nu} - k_\mu k_\nu}{k^2 - M_W^2}\left[\bar{u}(p_{\nu_\mu})\gamma^\mu(1-\gamma_5)u(p_\mu)\right]\left[\bar{u}(p_e)\gamma^\nu(1-\gamma_5)v(p_{\nu_e})\right]$$
$$\approx -\frac{ig^2}{8M_W^2}\left[\bar{u}(p_{\nu_\mu})\gamma^\mu(1-\gamma_5)u(p_\mu)\right]\left[\bar{u}(p_e)\gamma_\mu(1-\gamma_5)v(p_{\nu_e})\right]$$

Scale separation
→ $m_\mu \ll m_W$
→ $k^2 \ll m_W^2$

A toy example

A light fermion & a heavy scalar

The Yukawa theory of massless fermions

A toy theory with a massless fermion and massive scalar of mass *M*

- M is the heavy mass scale (low energy ≡ $E \ll M$)
- Integrating out the massive field
- Lagrangian:

$$\mathscr{L}_{\rm full}(\psi,\varphi) = i\bar\psi\partial\!\!\!/\psi + \frac{1}{2}(\partial^\mu\varphi)(\partial_\mu\varphi) - \frac{1}{2}M^2\varphi^2 - y\,\bar\psi\psi\varphi$$

massless fermion | massive scalar | Yukawa coupling

Goal: find the EFT at low energy

The Yukawa theory of massless fermions

A toy theory with a massless fermion and massive scalar of mass M

- M is the heavy mass scale (low energy ≡ $E \ll M$)
- Integrating out the massive field
- Lagrangian:

$$\mathscr{L}_{\text{full}}(\psi, \varphi) = i\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}(\partial^\mu\varphi)(\partial_\mu\varphi) - \frac{1}{2}M^2\varphi^2 - y\,\bar{\psi}\psi\varphi$$

massless fermion | massive scalar | Yukawa coupling

Goal: find the EFT at low energy

Matching of the full theory with a dimension-six EFT

- Lagrangian:

$$\mathscr{L}_{\text{EFT}}(\psi) = i\bar{\psi}\partial\!\!\!/\psi + \frac{C}{2M^2}(\bar{\psi}\psi)(\bar{\psi}\psi)$$

massless fermion | Four-fermion interaction

- The coefficient C depends on y and M

The Yukawa theory of massless fermions

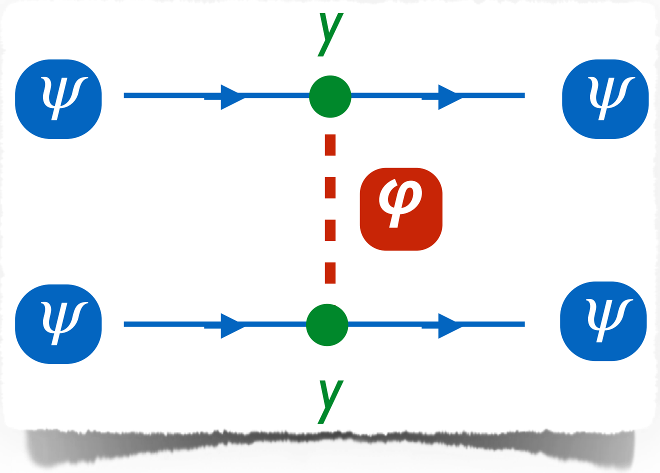
A toy theory with a massless fermion and massive scalar of mass *M*

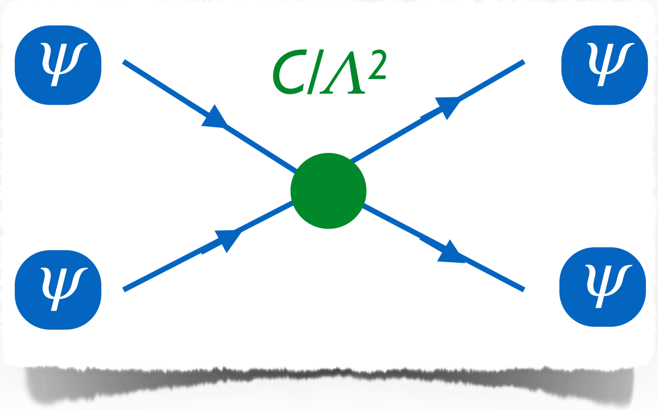
- M is the heavy mass scale (low energy ≡ $E \ll M$)
- Integrating out the massive field
- Lagrangian:

$$\mathscr{L}_{\rm full}(\psi,\varphi) = i\bar\psi\partial\!\!\!/\psi + \frac{1}{2}(\partial^\mu\varphi)(\partial_\mu\varphi) - \frac{1}{2}M^2\varphi^2 - y\,\bar\psi\psi\varphi$$

massless fermion — massive scalar — Yukawa coupling

Goal: find the EFT at low energy





Matching of the full theory with a dimension-six EFT

- Lagrangian:

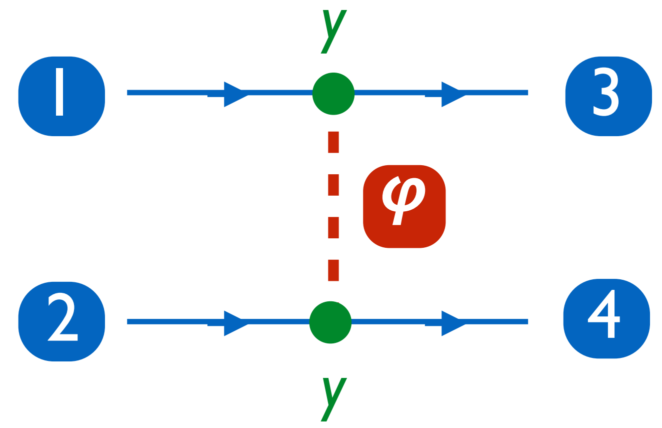
$$\mathscr{L}_{\rm EFT}(\psi) = i\bar\psi\partial\!\!\!/\psi + \frac{C}{2M^2}(\bar\psi\psi)(\bar\psi\psi)$$

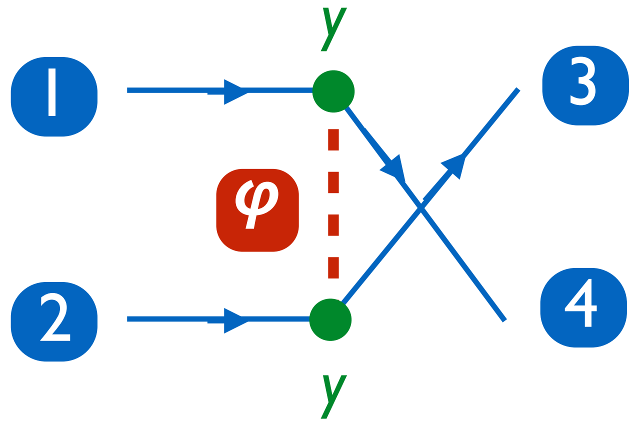
massless fermion — Four-fermion interaction

- The coefficient C depends on y and M

Tree-level matching in action

Amplitude in the full theory at low energy

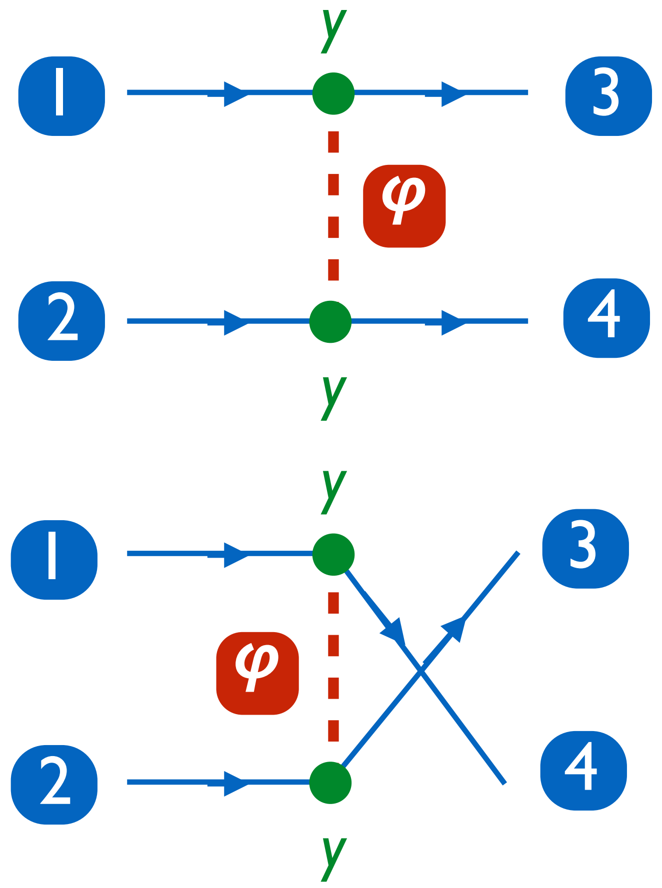




$$iM_{\text{full}} = -\,y^2\,\frac{i}{t-M^2}\,\Big[\bar{u}_3 u_1\ \bar{u}_4 u_2\Big] + y^2\,\frac{i}{u-M^2}\,\Big[\bar{u}_4 u_1\ \bar{u}_3 u_2\Big]$$

Tree-level matching in action

Amplitude in the full theory at low energy



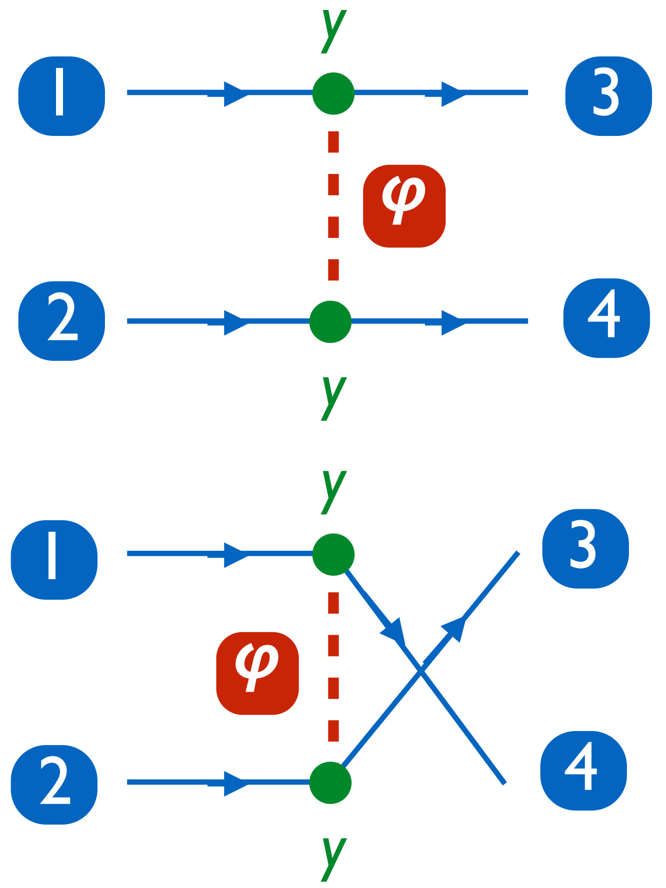
$$iM_{\text{full}} = -y^2 \frac{i}{t-M^2}\left[\bar{u}_3 u_1 \ \bar{u}_4 u_2\right] + y^2 \frac{i}{u-M^2}\left[\bar{u}_4 u_1 \ \bar{u}_3 u_2\right]$$

Scale separation

→ $t, u \ll M^2$

$$\approx \frac{iy^2}{M^2}\left[1+\mathcal{O}(t/M^2)\right]\left[\bar{u}_3 u_1 \ \bar{u}_4 u_2\right] - \frac{iy^2}{M^2}\left[1+\mathcal{O}(u/M^2)\right]\left[\bar{u}_4 u_1 \ \bar{u}_3 u_2\right]$$

Amplitude in the full theory at low energy



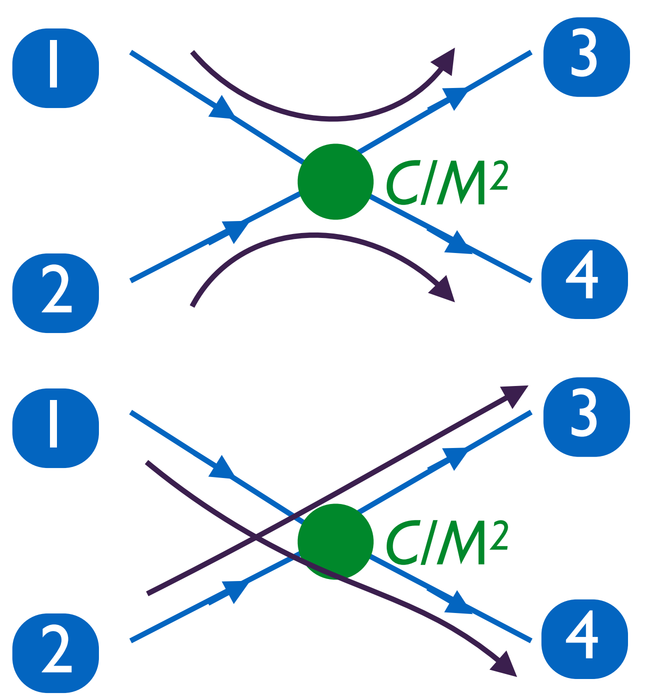
$$iM_{\text{full}} = -y^2 \frac{i}{t-M^2}\left[\bar{u}_3 u_1 \ \bar{u}_4 u_2\right] + y^2 \frac{i}{u-M^2}\left[\bar{u}_4 u_1 \ \bar{u}_3 u_2\right]$$

Scale separation

→ $t, u \ll M^2$

$$\approx \frac{iy^2}{M^2}\left[1+\mathcal{O}(t/M^2)\right]\left[\bar{u}_3 u_1 \ \bar{u}_4 u_2\right] - \frac{iy^2}{M^2}\left[1+\mathcal{O}(u/M^2)\right]\left[\bar{u}_4 u_1 \ \bar{u}_3 u_2\right]$$

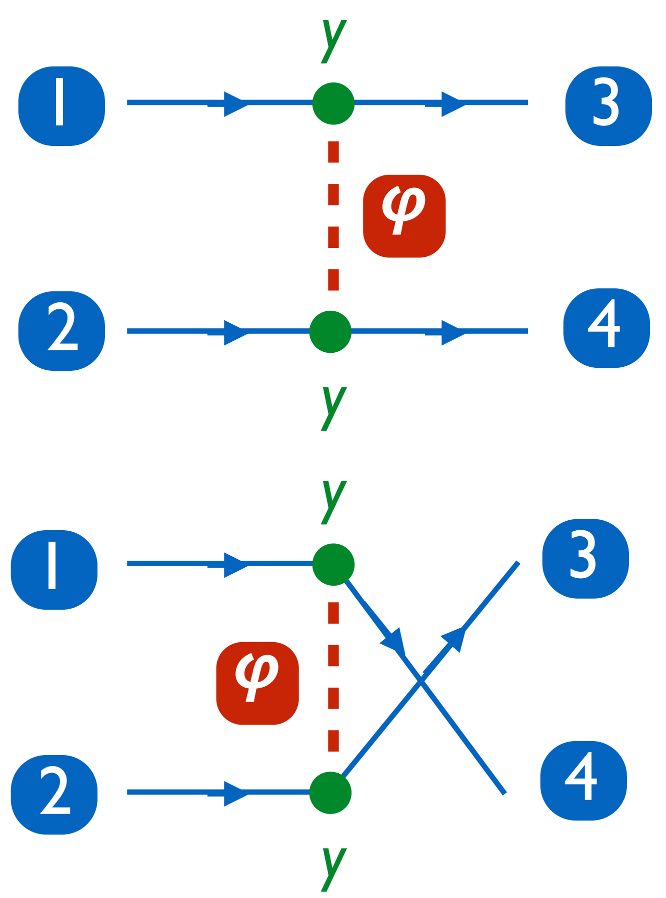
Amplitude in the EFT



$$iM_{\text{EFT}} = \frac{iC}{M^2}\left[\bar{u}_3 u_1 \ \bar{u}_4 u_2\right] - \frac{iC}{M^2}\left[\bar{u}_4 u_1 \ \bar{u}_3 u_2\right]$$

Tree-level matching in action

Amplitude in the full theory at low energy

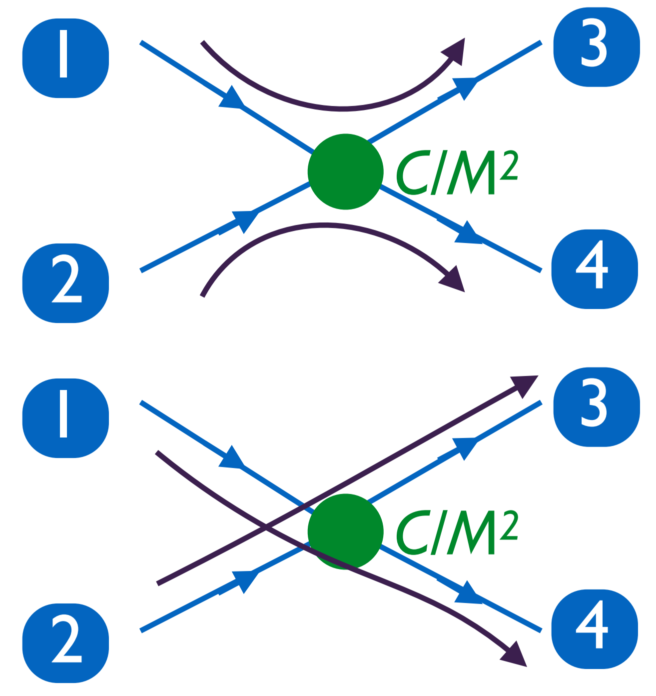


$$iM_{\text{full}} = -y^2 \frac{i}{t-M^2}\left[\bar{u}_3 u_1\ \bar{u}_4 u_2\right] + y^2 \frac{i}{u-M^2}\left[\bar{u}_4 u_1\ \bar{u}_3 u_2\right]$$

Scale separation
→ $t, u \ll M^2$

$$\approx \frac{iy^2}{M^2}\left[1+\mathcal{O}(t/M^2)\right]\left[\bar{u}_3 u_1\ \bar{u}_4 u_2\right] - \frac{iy^2}{M^2}\left[1+\mathcal{O}(u/M^2)\right]\left[\bar{u}_4 u_1\ \bar{u}_3 u_2\right]$$

Amplitude in the EFT



$$iM_{\text{EFT}} = \frac{iC}{M^2}\left[\bar{u}_3 u_1\ \bar{u}_4 u_2\right] - \frac{iC}{M^2}\left[\bar{u}_4 u_1\ \bar{u}_3 u_2\right]$$

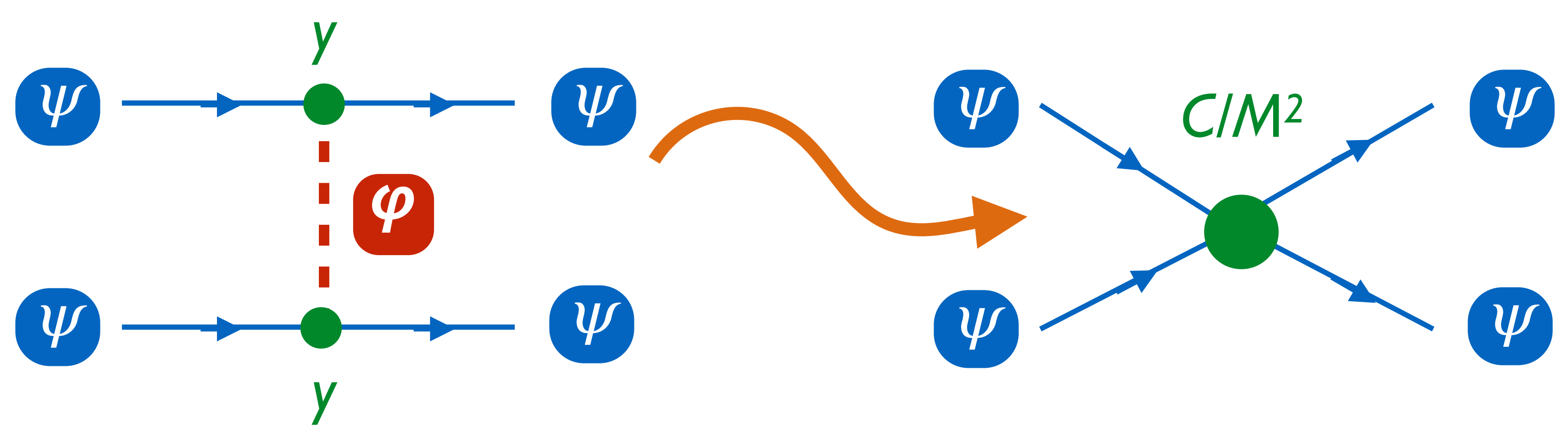
The amplitudes equal if
$C = y^2$

Two equivalent theories

A toy theory with a massless fermion and massive scalar of mass M

$$\mathcal{L}_{\rm full}(\psi,\varphi) = i\bar\psi\partial\!\!\!/\psi + \frac{1}{2}(\partial^\mu\varphi)(\partial_\mu\varphi) - \frac{1}{2}M^2\varphi^2 - y\,\bar\psi\psi\varphi$$

$$\mathcal{L}_{\rm EFT}(\psi) = i\bar\psi\partial\!\!\!/\psi + \frac{y^2}{2M^2}(\bar\psi\psi)(\bar\psi\psi)$$

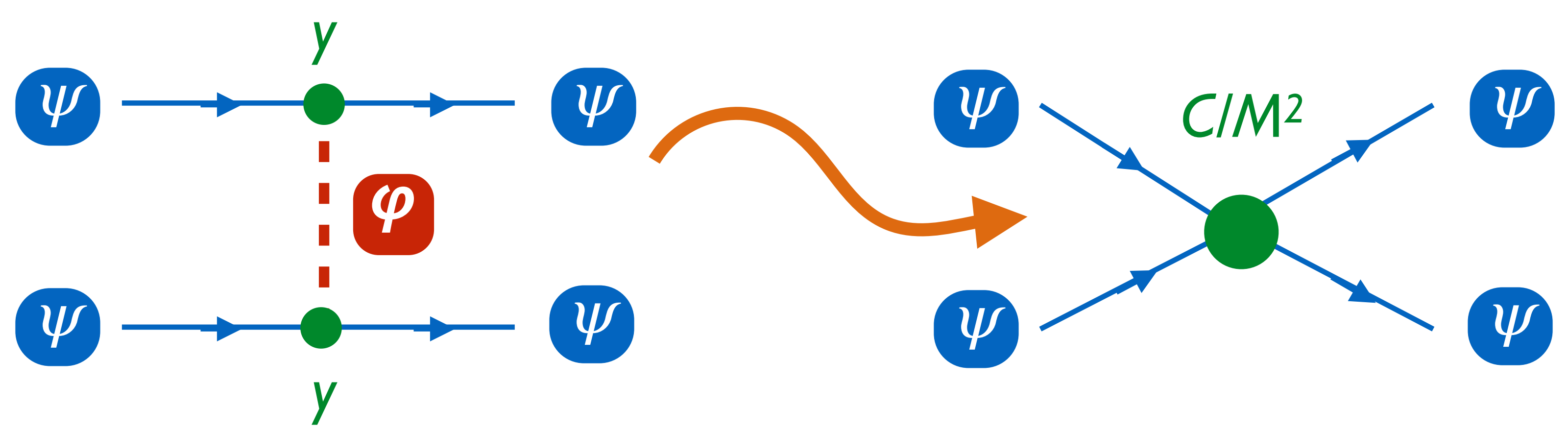


Two equivalent theories

A toy theory with a massless fermion and massive scalar of mass *M*

$$\mathscr{L}_{\rm full}(\psi, \varphi) = i\bar{\psi}\partial\!\!\!/\psi + \frac{1}{2}(\partial^\mu\varphi)(\partial_\mu\varphi) - \frac{1}{2}M^2\varphi^2 - y\,\bar{\psi}\psi\varphi$$

$$\mathscr{L}_{\rm EFT}(\psi) = i\bar{\psi}\partial\!\!\!/\psi + \frac{y^2}{2M^2}(\bar{\psi}\psi)(\bar{\psi}\psi)$$



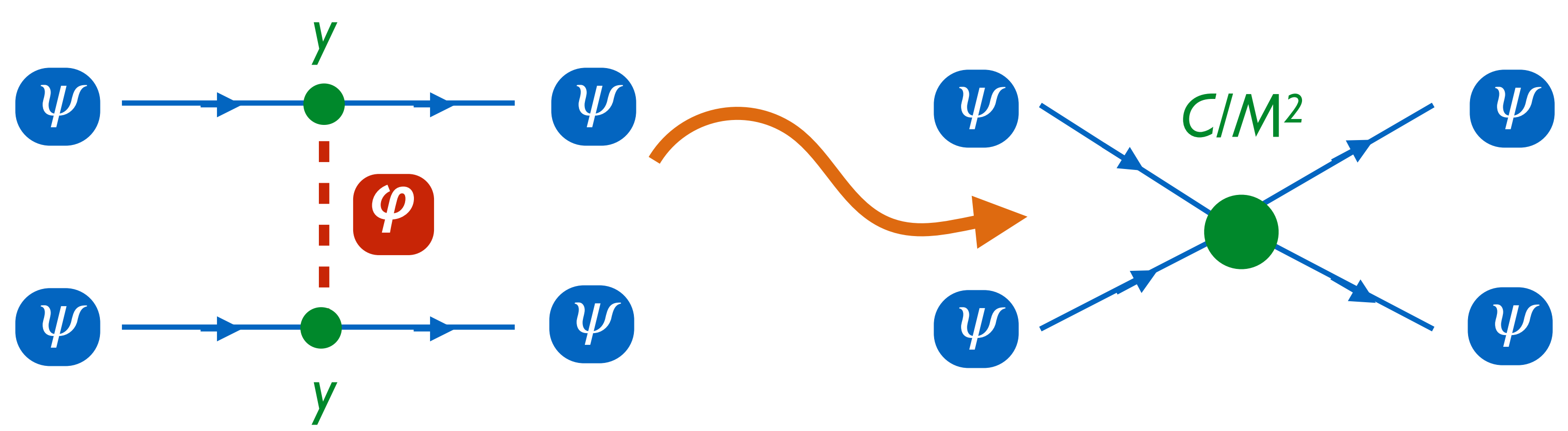
Can we improve the matching procedure? YES!

Two equivalent theories

A toy theory with a massless fermion and massive scalar of mass *M*

$$\mathscr{L}_{\text{full}}(\psi,\varphi) = i\bar\psi\partial\!\!\!/\psi + \frac{1}{2}(\partial^\mu\varphi)(\partial_\mu\varphi) - \frac{1}{2}M^2\varphi^2 - y\,\bar\psi\psi\varphi$$

$$\mathscr{L}_{\text{EFT}}(\psi) = i\bar\psi\partial\!\!\!/\psi + \frac{y^2}{2M^2}(\bar\psi\psi)(\bar\psi\psi)$$



Can we improve the matching procedure? YES!

- Propagator expansion to higher orders (in the full theory)
- At the next order:

$$iM_{\text{full}} \approx \frac{iy^2}{M^2}\left[1 + \frac{t}{M^2} + \mathcal{O}(t^2/M^4)\right]\left[\bar u_3 u_1\ \bar u_4 u_2\right] - \frac{iy^2}{M^2}\left[1 + \frac{u}{M^2} + \mathcal{O}(u^2/M^4)\right]\left[\bar u_4 u_1\ \bar u_3 u_2\right]$$

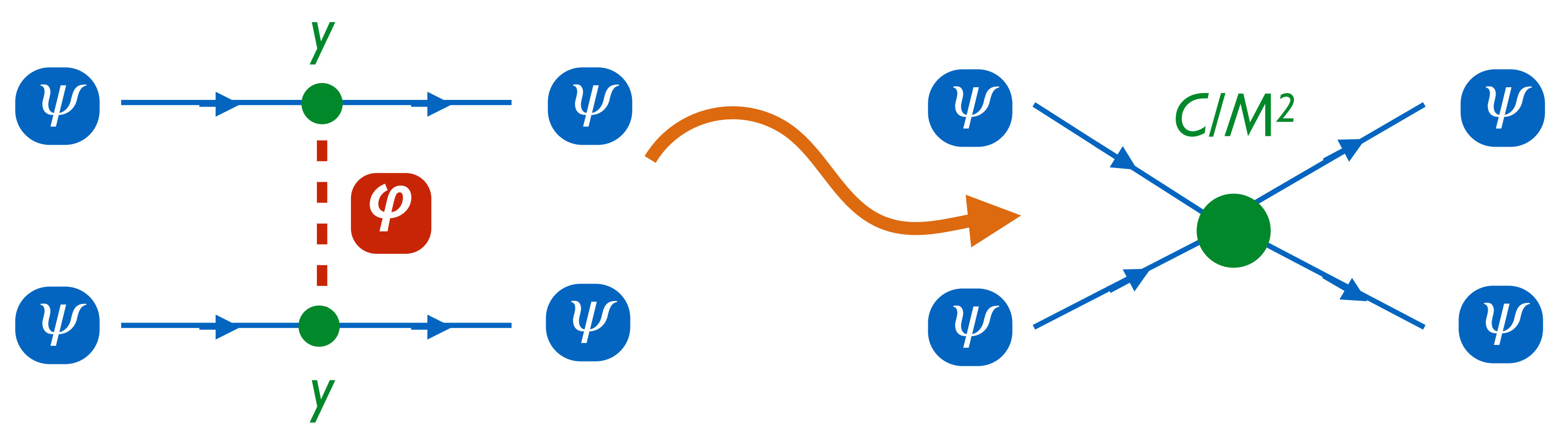
$\mathcal{O}\left(\frac{1}{M^2}\right)$ $\mathcal{O}\left(\frac{1}{M^4}\right)$ $\mathcal{O}\left(\frac{1}{M^2}\right)$ $\mathcal{O}\left(\frac{1}{M^4}\right)$

Two equivalent theories

A toy theory with a massless fermion and massive scalar of mass M

$$\mathscr{L}_{\rm full}(\psi,\varphi) = i\bar\psi\partial\!\!\!/\psi + \frac{1}{2}(\partial^\mu\varphi)(\partial_\mu\varphi) - \frac{1}{2}M^2\varphi^2 - y\,\bar\psi\psi\varphi$$

$$\mathscr{L}_{\rm EFT}(\psi) = i\bar\psi\partial\!\!\!/\psi + \frac{y^2}{2M^2}(\bar\psi\psi)(\bar\psi\psi)$$



Can we improve the matching procedure? YES!

- Propagator expansion to higher orders (in the full theory)
- At the next order:

$$iM_{\rm full} \approx \frac{iy^2}{M^2}\left[1+\frac{t}{M^2}+\mathcal{O}(t^2/M^4)\right]\left[\bar u_3 u_1\ \bar u_4 u_2\right] - \frac{iy^2}{M^2}\left[1+\frac{u}{M^2}+\mathcal{O}(u^2/M^4)\right]\left[\bar u_4 u_1\ \bar u_3 u_2\right]$$

$\mathcal{O}\left(\frac{1}{M^2}\right)$ $\mathcal{O}\left(\frac{1}{M^4}\right)$ $\mathcal{O}\left(\frac{1}{M^2}\right)$ $\mathcal{O}\left(\frac{1}{M^4}\right)$

- Derivatives needed ➙ new effective operator (dimension 8)

$$\mathscr{L}_{\rm EFT}(\psi) = i\bar\psi\partial\!\!\!/\psi + \frac{y^2}{2M^2}(\bar\psi\psi)(\bar\psi\psi) + \frac{C_8}{M^4}(\partial_\mu\bar\psi\partial^\mu\psi)(\bar\psi\psi)$$

Amplitudes equal with $C_8 = y^2$

Improving the matching with QED corrections

The matching can also be improved by going to one loop (or more)

- A QED-invariant full theory ➙ calculation of QED corrections
- ⚠️ Regulator needed (fermion mass term)
- Updated Lagrangian:

$$\mathscr{L}_{\rm full}\big(\psi,\varphi\big) = i\bar\psi\partial\!\!\!/\psi \;-\; \sigma\bar\psi\psi \;+\; \frac{1}{2}\big(D^\mu\varphi\big)\big(D_\mu\varphi\big) \;-\; \frac{1}{2}M^2\varphi^2 \;-\; y\,\bar\psi\psi\varphi$$

massless fermion (with IR regulator) — massive scalar (with QED) — Yukawa coupling

Improving the matching with QED corrections

The matching can also be improved by going to one loop (or more)

- A QED-invariant full theory ➙ calculation of QED corrections
- ⚠️ Regulator needed (fermion mass term)
- Updated Lagrangian:

$$\mathscr{L}_{\rm full}(\psi,\varphi) = i\bar\psi\partial\!\!\!/\psi \;-\; \sigma\bar\psi\psi \;+\; \frac{1}{2}(D^\mu\varphi)(D_\mu\varphi) \;-\; \frac{1}{2}M^2\varphi^2 \;-\; y\;\bar\psi\psi\varphi$$

massless fermion (with IR regulator) — massive scalar (with QED) — Yukawa coupling

A new effective description (different from tree-level)

- Loop-generated operators ➙ UV counterterms from the bare Lagrangian

$$\mathscr{L}_{\rm EFT}(\psi) = i\bar\psi\partial\!\!\!/\psi \;-\; \sigma\bar\psi\psi \;+\; \frac{C_6}{2M^2}\,(\bar\psi\psi)\,(\bar\psi\psi) \;+\; \frac{\hat C_6}{2M^2}\,(\bar\psi\sigma^{\mu\nu}\psi)\,(\bar\psi\sigma_{\mu\nu}\psi)$$

massless fermion (with IR regulator) — Four-fermion interaction — Extra interaction to cancel UV divergences

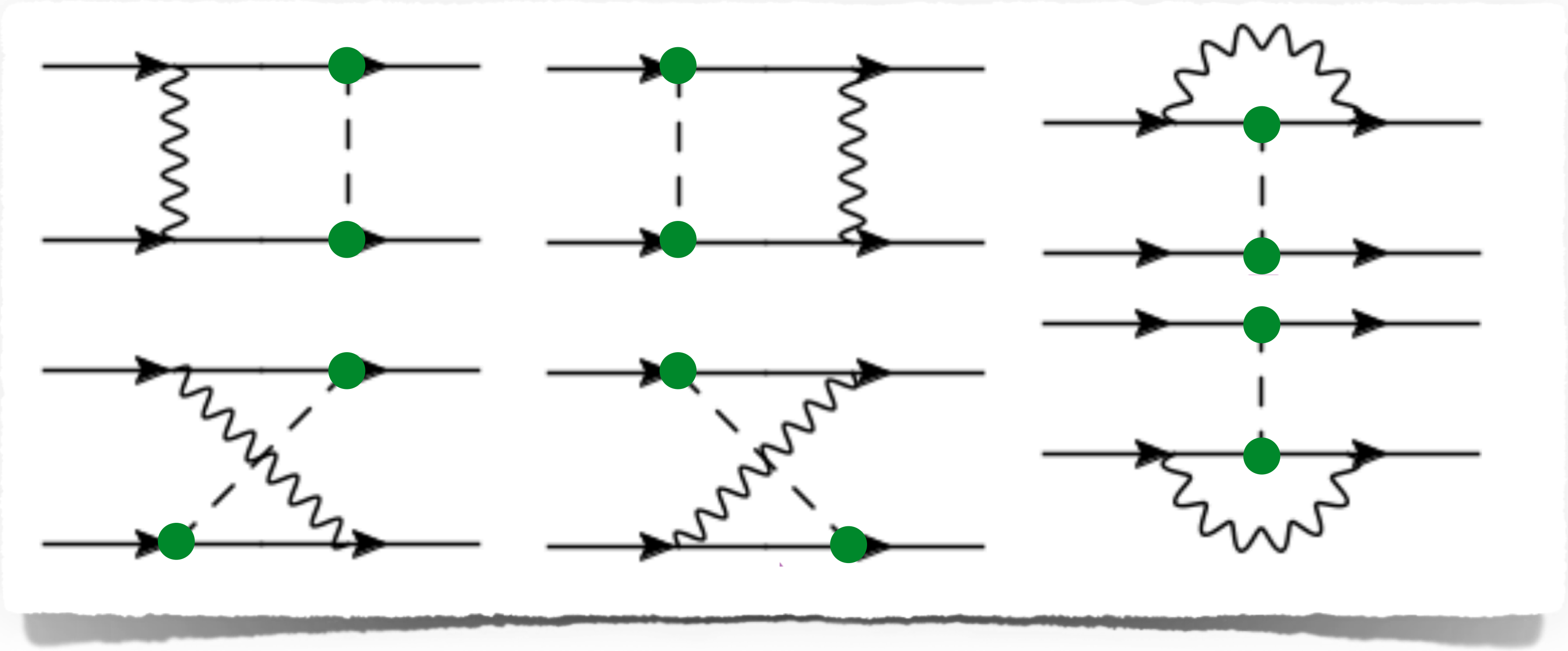
- One-loop corrections to the matching relations

$$C_6 = y^2 + \mathcal{O}(\alpha)$$
$$\hat C_6 = \mathcal{O}(\alpha)$$

One-loop EFT matching

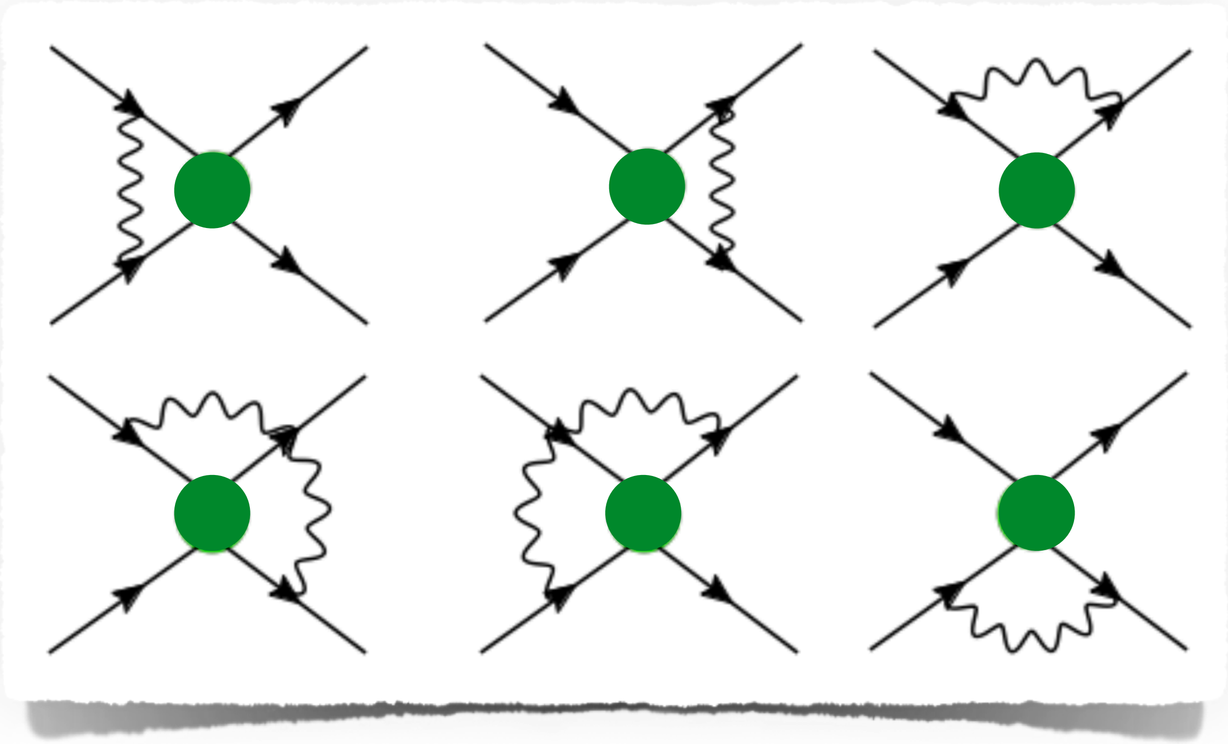
Loop diagrams to be evaluated in both theories

- Full theory, *t*-channel:



- Full theory, *u*-channel similar

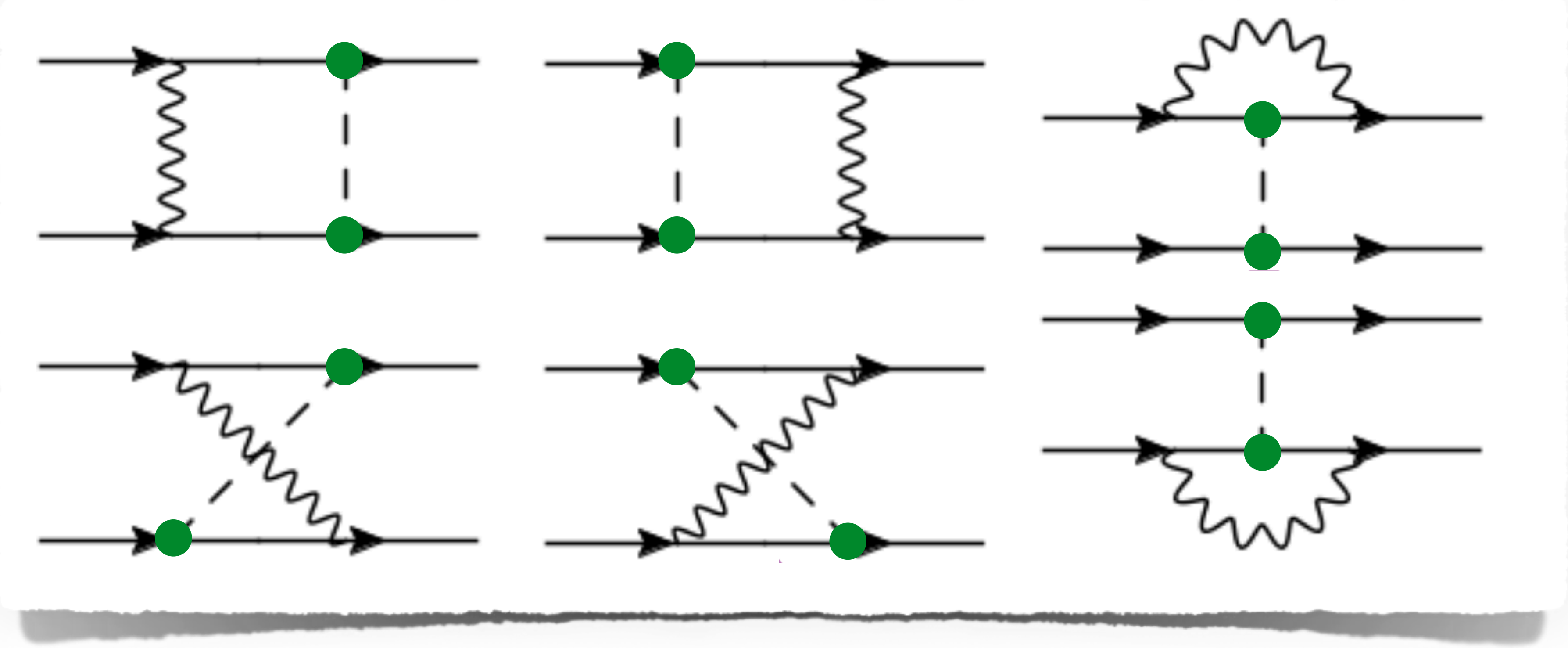
- EFT diagrams



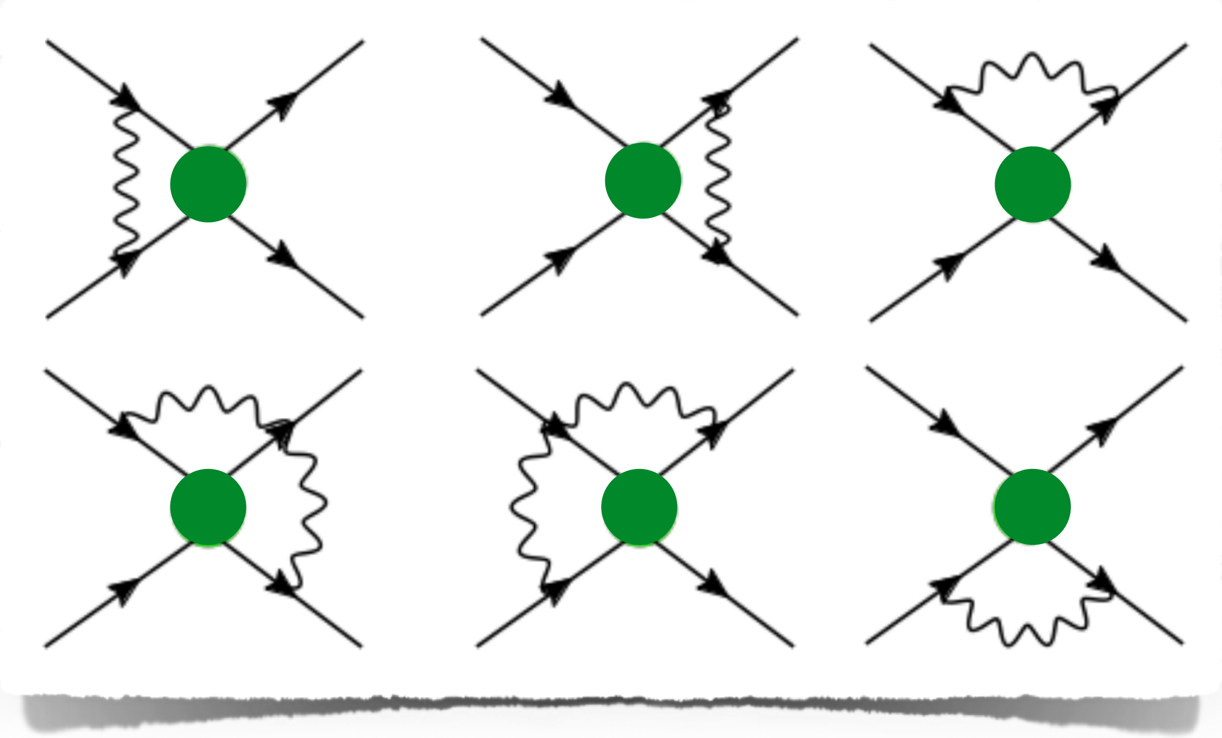
One-loop EFT matching

Loop diagrams to be evaluated in both theories

- Full theory, *t*-channel:



- EFT diagrams



- Full theory, *u*-channel similar

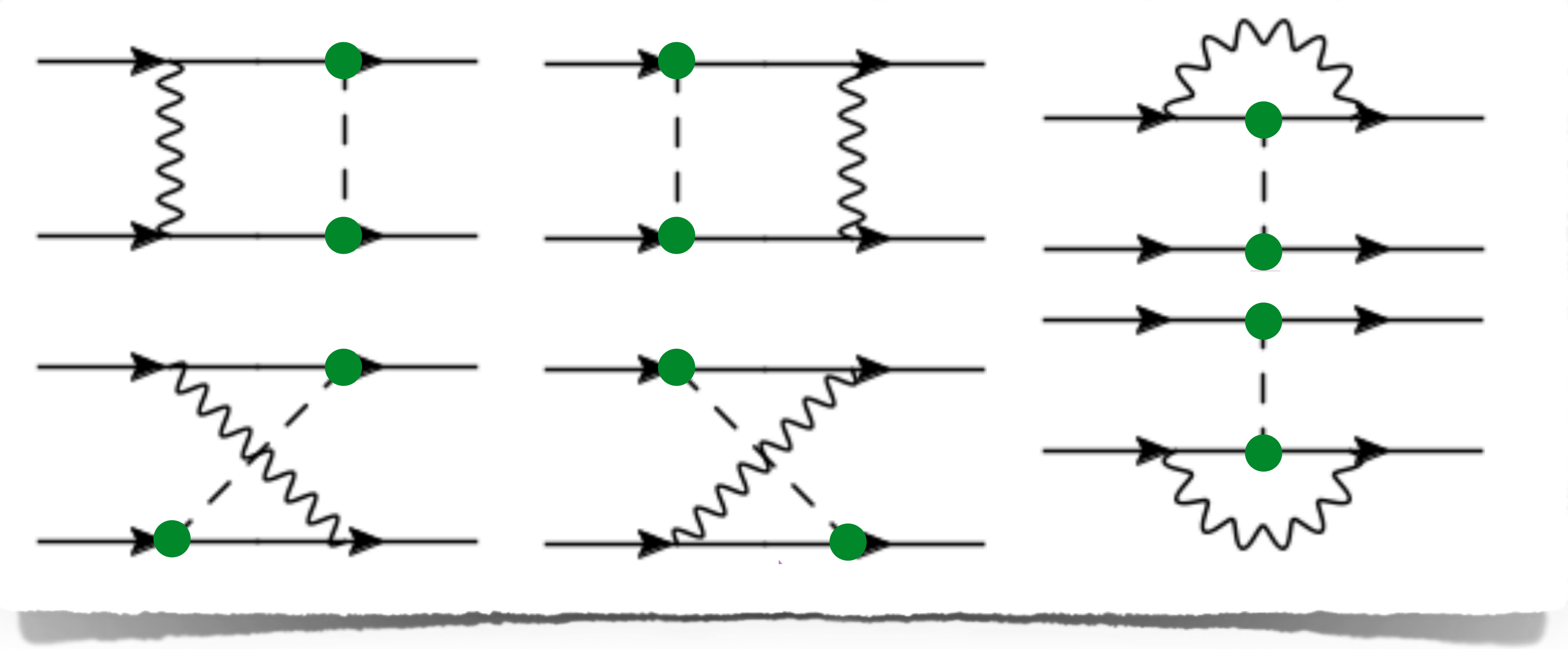
Renormalisation (UV features)

- Need for renormalisation in some scheme (for both theories)
- Different theories in the UV ⬌ UV behaviour possibly different
- Log-dependence on the renormalisation scale ⬌ behaviour at the matching scale

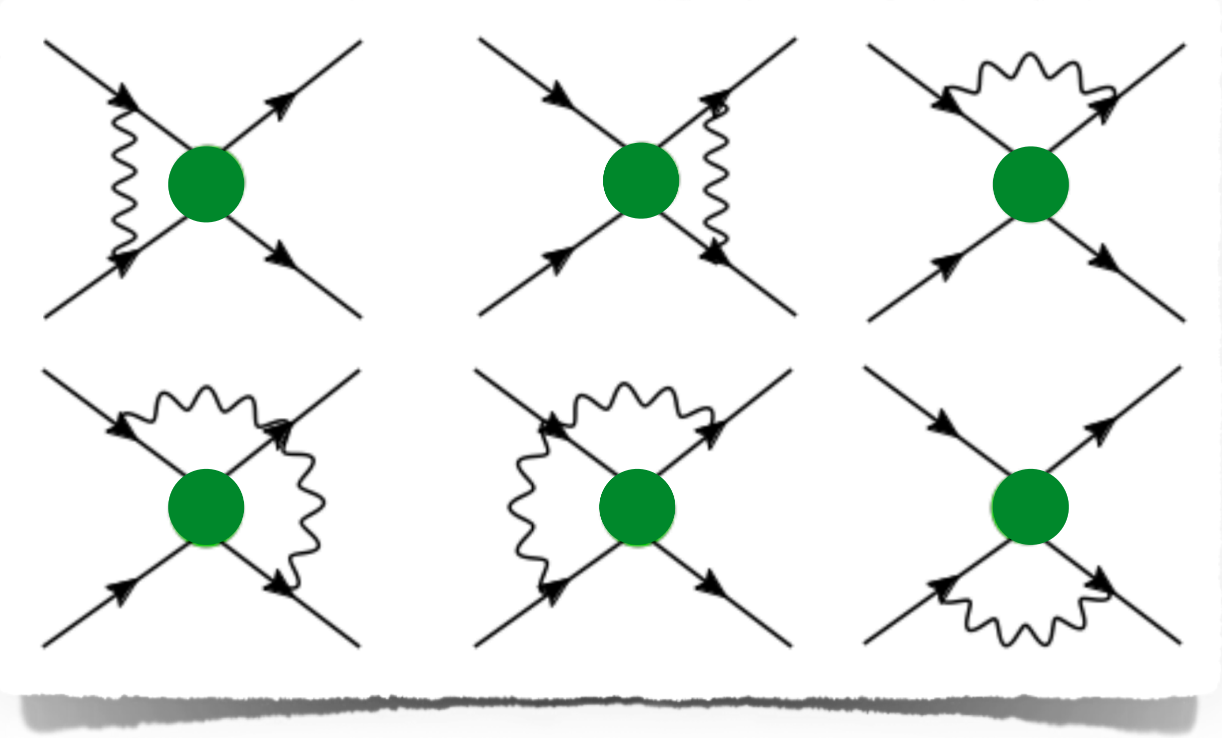
One-loop EFT matching

Loop diagrams to be evaluated in both theories

- Full theory, *t*-channel:



- EFT diagrams



- Full theory, *u*-channel similar

Renormalisation (UV features)

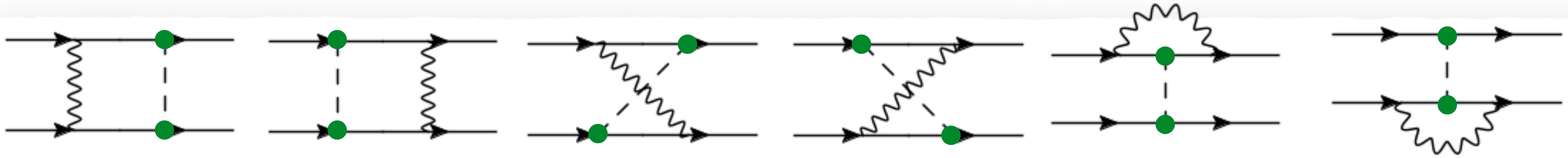
- Need for renormalisation in some scheme (for both theories)
- Different theories in the UV ↔ UV behaviour possibly different
- Log-dependence on the renormalisation scale ↔ behaviour at the matching scale

IR behaviour

- Must be identical (same physics) ↔ poles and finite pieces

One-loop matching – full theory

Full theory in the IR limit



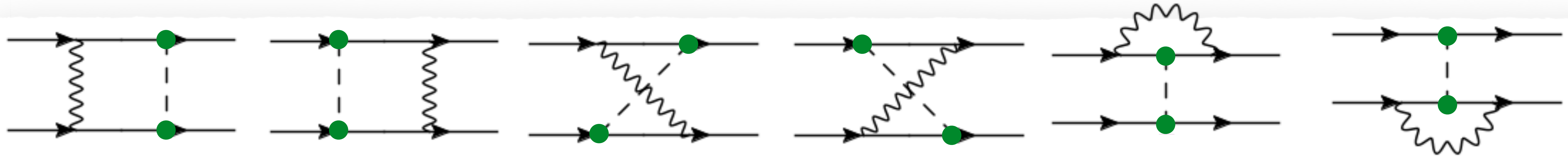
+ *u*-channel

$$iM_{\rm full} \approx \frac{iy^2}{M^2}\Big[u_3u_1\ \bar u_4u_2 - \bar u_4u_1\ \bar u_3u_2\Big]\bigg[1+\frac{\alpha Q_\phi^2}{\pi}\Big(\frac{2}{\varepsilon_{\rm UV}} - 2\log\frac{\sigma^2}{\mu^2} - 1\Big)\bigg]$$

$$+\Big[u_3\sigma^{\mu\nu}u_1\ \bar u_4\sigma_{\mu\nu}u_2 - \bar u_4\sigma^{\mu\nu}u_1\ \bar u_3\sigma_{\mu\nu}u_2\Big]\bigg[\frac{iy^2}{M^2}\ \frac{\alpha Q_\phi^2}{8\pi}\Big(-2\log\frac{\sigma^2}{M^2} - 2\Big)\bigg]$$

One-loop matching – full theory

Full theory in the IR limit



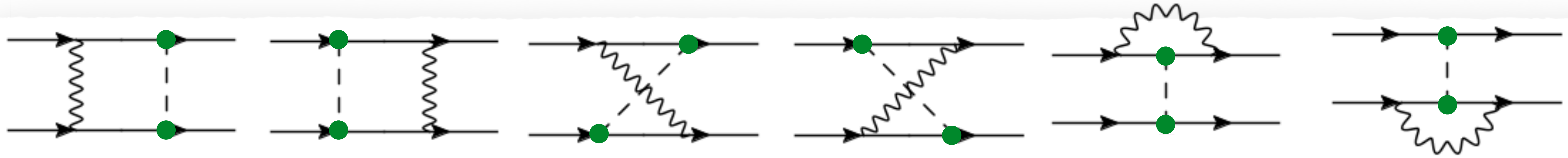
+ u-channel

$$iM_{\rm full} \approx \frac{iy^2}{M^2}\Big[u_3u_1\ \bar u_4u_2 - \bar u_4u_1\ \bar u_3u_2\Big]\bigg[1+\frac{\alpha Q_\phi^2}{\pi}\Big(\frac{2}{\varepsilon_{\rm UV}} - 2\log\frac{\sigma^2}{\mu^2} - 1\Big)\bigg]$$
$$+\Big[u_3\sigma^{\mu\nu}u_1\ \bar u_4\sigma_{\mu\nu}u_2 - \bar u_4\sigma^{\mu\nu}u_1\ \bar u_3\sigma_{\mu\nu}u_2\Big]\bigg[\frac{iy^2}{M^2}\ \frac{\alpha Q_\phi^2}{8\pi}\Big(-2\log\frac{\sigma^2}{M^2} - 2\Big)\bigg]$$

- First line: triangle and tree-level diagrams
  - ➞ Same tensor structure as at leading order
  - ➞ Heavy mass $(1/M^2)$ suppression
  - ➞ IR poles through the regulator to be matched with the EFT
  - ➞ UV poles to be renormalised away (QED corrections to y)

One-loop matching – full theory

Full theory in the IR limit



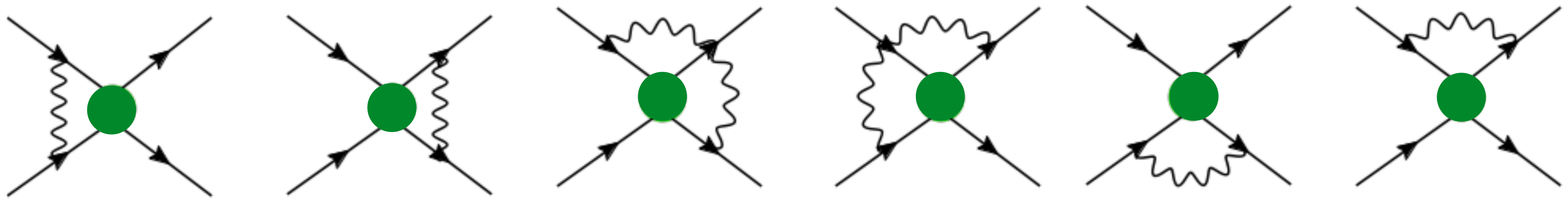
$$iM_{\rm full} \approx \frac{iy^2}{M^2}\Big[u_3u_1\ \bar u_4 u_2 - \bar u_4 u_1\ \bar u_3 u_2\Big]\bigg[1+\frac{\alpha Q_\phi^2}{\pi}\Big(\frac{2}{\varepsilon_{\rm UV}} - 2\log\frac{\sigma^2}{\mu^2} - 1\Big)\bigg]$$

$$+\Big[u_3\sigma^{\mu\nu}u_1\ \bar u_4\sigma_{\mu\nu}u_2 - \bar u_4\sigma^{\mu\nu}u_1\ \bar u_3\sigma_{\mu\nu}u_2\Big]\bigg[\frac{iy^2}{M^2}\ \frac{\alpha Q_\phi^2}{8\pi}\Big(-2\log\frac{\sigma^2}{M^2} - 2\Big)\bigg]$$

- First line: triangle and tree-level diagrams
  - ➞ Same tensor structure as at leading order
  - ➞ Heavy mass $(1/M^2)$ suppression
  - ➞ IR poles through the regulator to be matched with the EFT
  - ➞ UV poles to be renormalised away (QED corrections to *y*)
- Second line: box diagrams
  - ➞ Tensor structure similar to that of the $\hat C_6$ term in the EFT
  - ➞ Heavy mass $(1/M^2)$ suppression
  - ➞ IR poles through the regulator to be matched with the EFT

One loop matching – EFT

EFT predictions

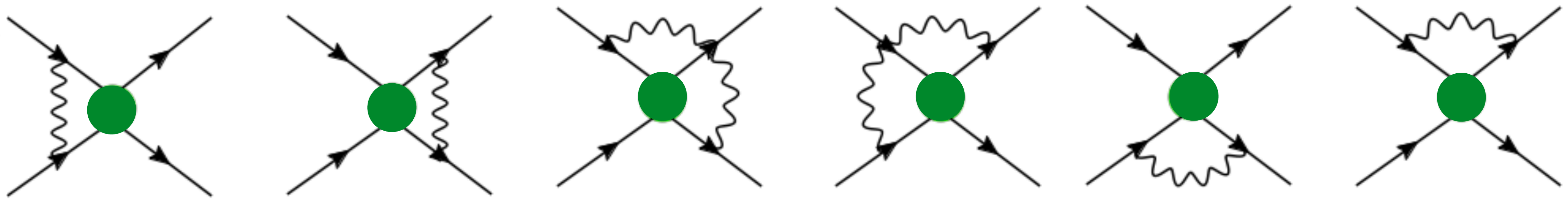


$$iM_{\rm EFT} = \frac{iC_6}{M^2}\Big[u_3u_1\ \bar u_4u_2 - \bar u_4u_1\ \bar u_3u_2\Big]\bigg[1+\frac{\alpha Q_\phi^2}{\pi}\Big(\frac{2}{\varepsilon_{\rm UV}} - 2\log\frac{\sigma^2}{\mu^2} - 1\Big)\bigg]$$

$$+\Big[u_3\sigma^{\mu\nu}u_1\ \bar u_4\sigma_{\mu\nu}u_2 - \bar u_4\sigma^{\mu\nu}u_1\ \bar u_3\sigma_{\mu\nu}u_2\Big]\bigg[\frac{i\hat C_6}{M^2} + \frac{iC_6}{M^2}\ \frac{\alpha Q_\phi^2}{8\pi}\Big(\frac{2}{\varepsilon_{\rm UV}} - 2\log\frac{\sigma^2}{\mu^2} + 1\Big)\bigg]$$

One loop matching – EFT

EFT predictions

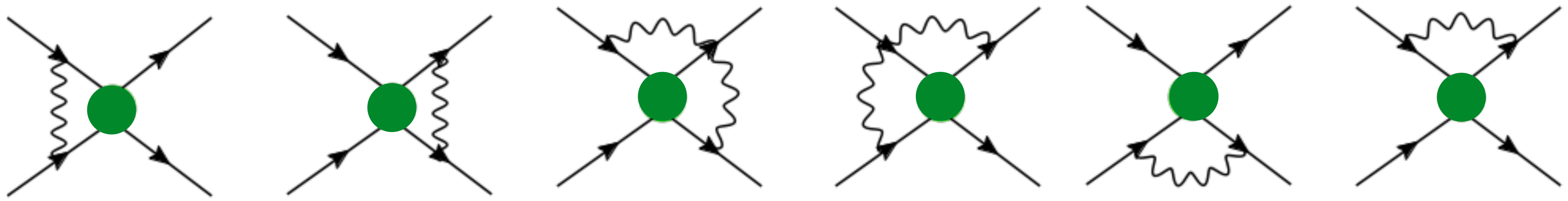


$$iM_{\rm EFT} = \frac{iC_6}{M^2}\Big[u_3u_1\ \bar u_4u_2 - \bar u_4u_1\ \bar u_3u_2\Big]\ \Big[1 + \frac{\alpha Q_\phi^2}{\pi}\Big(\frac{2}{\varepsilon_{\rm UV}} - 2\log\frac{\sigma^2}{\mu^2} - 1\Big)\Big]$$
$$+\Big[u_3\sigma^{\mu\nu}u_1\ \bar u_4\sigma_{\mu\nu}u_2 - \bar u_4\sigma^{\mu\nu}u_1\ \bar u_3\sigma_{\mu\nu}u_2\Big]\ \Big[\frac{i\hat C_6}{M^2} + \frac{iC_6}{M^2}\frac{\alpha Q_\phi^2}{8\pi}\Big(\frac{2}{\varepsilon_{\rm UV}} - 2\log\frac{\sigma^2}{\mu^2} + 1\Big)\Big]$$

- All contributions proportional to C_6/M^2 or $\hat C_6/M^2$
  - ➞ Heavy mass suppression

One loop matching – EFT

EFT predictions



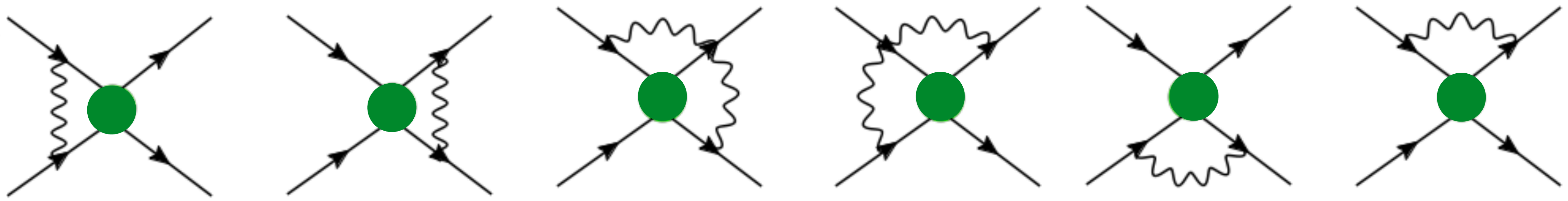
$$iM_{\rm EFT} = \frac{iC_6}{M^2}\Big[u_3 u_1\ \bar u_4 u_2 - \bar u_4 u_1\ \bar u_3 u_2\Big]\Big[1 + \frac{\alpha Q_\phi^2}{\pi}\Big(\frac{2}{\varepsilon_{\rm UV}} - 2\log\frac{\sigma^2}{\mu^2} - 1\Big)\Big]$$
$$+ \Big[u_3\sigma^{\mu\nu}u_1\ \bar u_4\sigma_{\mu\nu}u_2 - \bar u_4\sigma^{\mu\nu}u_1\ \bar u_3\sigma_{\mu\nu}u_2\Big]\Big[\frac{i\hat C_6}{M^2} + \frac{iC_6}{M^2}\,\frac{\alpha Q_\phi^2}{8\pi}\Big(\frac{2}{\varepsilon_{\rm UV}} - 2\log\frac{\sigma^2}{\mu^2} + 1\Big)\Big]$$

- All contributions proportional to C_6/M^2 or $\hat C_6/M^2$
  - ➞ Heavy mass suppression

- First line: structure similar to the tree-level one
  - ➞ Same tensor structure as at leading order
  - ➞ UV/IR poles: similar structure as in the full theory

One loop matching – EFT

EFT predictions



$$iM_{\rm EFT} = \frac{iC_6}{M^2}\Big[u_3u_1\ \bar u_4u_2 - \bar u_4u_1\ \bar u_3u_2\Big]\Big[1+\frac{\alpha Q_\phi^2}{\pi}\Big(\frac{2}{\varepsilon_{\rm UV}} - 2\log\frac{\sigma^2}{\mu^2} - 1\Big)\Big]$$
$$+\Big[u_3\sigma^{\mu\nu}u_1\ \bar u_4\sigma_{\mu\nu}u_2 - \bar u_4\sigma^{\mu\nu}u_1\ \bar u_3\sigma_{\mu\nu}u_2\Big]\Big[\frac{i\hat C_6}{M^2}+\frac{iC_6}{M^2}\frac{\alpha Q_\phi^2}{8\pi}\Big(\frac{2}{\varepsilon_{\rm UV}} - 2\log\frac{\sigma^2}{\mu^2} + 1\Big)\Big]$$

- All contributions proportional to C_6/M^2 or $\hat{C}_6/M^2$
  - ➙ Heavy mass suppression
- First line: structure similar to the tree-level one
  - ➙ Same tensor structure as at leading order
  - ➙ UV/IR poles: similar structure as in the full theory
- Second line: first four diagrams + tree-level $\hat{C}_6$ contributions
  - ➙ New tensor structure with UV poles
    - ⋆ Renormalisation: C_6 corrections to $\hat{C}_6$
    - ⋆ Need for $\hat{C}_6$ in the Lagrangian
    - ⋆ Operator mixing
  - ➙ IR poles (through the regulator)
    - ⋆ Dependence on μ and not M (unlike in the full theory)
    - ⋆ Matching scale to be introduced

One-loop matching in action

Amplitudes in both theories

$$iM_{\rm full} \approx \frac{iy^2}{M^2}\Big[u_3u_1\ \bar u_4u_2 - \bar u_4u_1\ \bar u_3u_2\Big]\left[1+\frac{\alpha Q_\phi^2}{\pi}\Big(\frac{2}{\varepsilon_{\rm UV}} - 2\log\frac{\sigma^2}{\mu^2} - 1\Big)\right]$$

$$+\Big[u_3\sigma^{\mu\nu}u_1\ \bar u_4\sigma_{\mu\nu}u_2 - \bar u_4\sigma^{\mu\nu}u_1\ \bar u_3\sigma_{\mu\nu}u_2\Big]\left[\frac{iy^2}{M^2}\ \frac{\alpha Q_\phi^2}{8\pi}\Big(-2\log\frac{\sigma^2}{M^2} - 2\Big)\right]$$

$$iM_{\rm EFT} = \frac{iC_6}{M^2}\Big[u_3u_1\ \bar u_4u_2 - \bar u_4u_1\ \bar u_3u_2\Big]\left[1+\frac{\alpha Q_\phi^2}{\pi}\Big(\frac{2}{\varepsilon_{\rm UV}} - 2\log\frac{\sigma^2}{\mu^2} - 1\Big)\right]$$

$$+\Big[u_3\sigma^{\mu\nu}u_1\ \bar u_4\sigma_{\mu\nu}u_2 - \bar u_4\sigma^{\mu\nu}u_1\ \bar u_3\sigma_{\mu\nu}u_2\Big]\left[\frac{i\hat C_6}{M^2} + \frac{iC_6}{M^2}\ \frac{\alpha Q_\phi^2}{8\pi}\Big(\frac{2}{\varepsilon_{\rm UV}} - 2\log\frac{\sigma^2}{\mu^2} + 1\Big)\right]$$

One-loop matching in action

Amplitudes in both theories

$$iM_{\text{full}} \approx \frac{iy^2}{M^2}\left[u_3u_1\ \bar{u}_4u_2 - \bar{u}_4u_1\ \bar{u}_3u_2\right]\left[1+\frac{\alpha Q_\phi^2}{\pi}\left(\frac{2}{\varepsilon_{\text{UV}}} - 2\log\frac{\sigma^2}{\mu^2} - 1\right)\right]$$
$$+\left[u_3\sigma^{\mu\nu}u_1\ \bar{u}_4\sigma_{\mu\nu}u_2 - \bar{u}_4\sigma^{\mu\nu}u_1\ \bar{u}_3\sigma_{\mu\nu}u_2\right]\left[\frac{iy^2}{M^2}\ \frac{\alpha Q_\phi^2}{8\pi}\left(-2\log\frac{\sigma^2}{M^2} - 2\right)\right]$$
$$iM_{\text{EFT}} = \frac{iC_6}{M^2}\left[u_3u_1\ \bar{u}_4u_2 - \bar{u}_4u_1\ \bar{u}_3u_2\right]\left[1+\frac{\alpha Q_\phi^2}{\pi}\left(\frac{2}{\varepsilon_{\text{UV}}} - 2\log\frac{\sigma^2}{\mu^2} - 1\right)\right]$$
$$+\left[u_3\sigma^{\mu\nu}u_1\ \bar{u}_4\sigma_{\mu\nu}u_2 - \bar{u}_4\sigma^{\mu\nu}u_1\ \bar{u}_3\sigma_{\mu\nu}u_2\right]\left[\frac{i\hat{C}_6}{M^2} + \frac{iC_6}{M^2}\ \frac{\alpha Q_\phi^2}{8\pi}\left(\frac{2}{\varepsilon_{\text{UV}}} - 2\log\frac{\sigma^2}{\mu^2} + 1\right)\right]$$

- UV behaviour irrelevant
  - ➙ different in the full theory and the EFT
- IR behaviour identical
  - ➙ $\mu = M$ for exact pole cancellation
  - ➙ Definition of the matching scale

One-loop matching in action

Amplitudes in both theories

$$iM_{\rm full} \approx \frac{iy^2}{M^2}\Big[u_3u_1\ \bar u_4 u_2 - \bar u_4 u_1\ \bar u_3 u_2\Big]\bigg[1+\frac{\alpha Q_\phi^2}{\pi}\Big(\frac{2}{\varepsilon_{\rm UV}} - 2\log\frac{\sigma^2}{\mu^2} - 1\Big)\bigg]$$
$$+\Big[u_3\sigma^{\mu\nu}u_1\ \bar u_4\sigma_{\mu\nu}u_2 - \bar u_4\sigma^{\mu\nu}u_1\ \bar u_3\sigma_{\mu\nu}u_2\Big]\bigg[\frac{iy^2}{M^2}\ \frac{\alpha Q_\phi^2}{8\pi}\Big(-2\log\frac{\sigma^2}{M^2} - 2\Big)\bigg]$$

$$iM_{\rm EFT} = \frac{iC_6}{M^2}\Big[u_3u_1\ \bar u_4 u_2 - \bar u_4 u_1\ \bar u_3 u_2\Big]\bigg[1+\frac{\alpha Q_\phi^2}{\pi}\Big(\frac{2}{\varepsilon_{\rm UV}} - 2\log\frac{\sigma^2}{\mu^2} - 1\Big)\bigg]$$
$$+\Big[u_3\sigma^{\mu\nu}u_1\ \bar u_4\sigma_{\mu\nu}u_2 - \bar u_4\sigma^{\mu\nu}u_1\ \bar u_3\sigma_{\mu\nu}u_2\Big]\bigg[\frac{i\hat C_6}{M^2} + \frac{iC_6}{M^2}\ \frac{\alpha Q_\phi^2}{8\pi}\Big(\frac{2}{\varepsilon_{\rm UV}} - 2\log\frac{\sigma^2}{\mu^2} + 1\Big)\bigg]$$

- UV behaviour irrelevant
  - ➙ different in the full theory and the EFT
- IR behaviour identical
  - ➙ $\mu = M$ for exact pole cancellation
  - ➙ Definition of the matching scale

- Same physics in the IR ↔ definitions of C_6 and $\hat C_6$ at the matching scale

$$C_6(M^2) = y^2 + \mathcal{O}(\alpha^2)$$
No loop corrections

$$\hat C_6(M^2) = -\frac{3\alpha Q_\varphi^2}{8\pi}\,y^2 + \mathcal{O}(\alpha^2)$$
LO = one-loop

RG running / operator mixing

Coefficients matched at a high scale M

$$\mathscr{L}_{\rm EFT}(\psi) = i\bar\psi\partial\!\!\!/\psi - \sigma\bar\psi\psi + \frac{C_6}{2M^2}\left(\bar\psi\psi\right)\left(\bar\psi\psi\right) + \frac{\hat C_6}{2M^2}\left(\bar\psi\sigma^{\mu\nu}\psi\right)\left(\bar\psi\sigma_{\mu\nu}\psi\right)$$

$$C_6(M^2) = y^2 + \mathcal{O}(\alpha^2)$$

$$\hat C_6(M^2) = -\frac{3\alpha Q_\varphi^2}{8\pi}\,y^2 + \mathcal{O}(\alpha^2)$$

- Coefficients at low energy, where the EFT is valid!

RG running / operator mixing

Coefficients matched at a high scale M

$$\mathscr{L}_{\rm EFT}(\psi) = i\bar\psi\partial\!\!\!/\psi - \sigma\bar\psi\psi + \frac{C_6}{2M^2}\left(\bar\psi\psi\right)\left(\bar\psi\psi\right) + \frac{\hat C_6}{2M^2}\left(\bar\psi\sigma^{\mu\nu}\psi\right)\left(\bar\psi\sigma_{\mu\nu}\psi\right)$$

$$C_6(M^2) = y^2 + \mathcal{O}(\alpha^2)$$

$$\hat C_6(M^2) = -\frac{3\alpha Q_\varphi^2}{8\pi}\,y^2 + \mathcal{O}(\alpha^2)$$

- Coefficients at low energy, where the EFT is valid!

Anomalous dimension matrix

- Obtained from the counterterms to C_6 and $\hat C_6$
- Operators run and mix

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu}\begin{pmatrix} C_6(\mu) \\ \hat C_6(\mu)\end{pmatrix} = \frac{2\alpha Q_\varphi^2}{\pi}\begin{pmatrix} -\frac{3}{2} & -12 \\ -\frac{1}{4} & \frac{1}{2}\end{pmatrix}\begin{pmatrix} C_6(\mu) \\ \hat C_6(\mu)\end{pmatrix}$$

- Coefficients at the low scale: solution to the RGEs with initial conditions at M

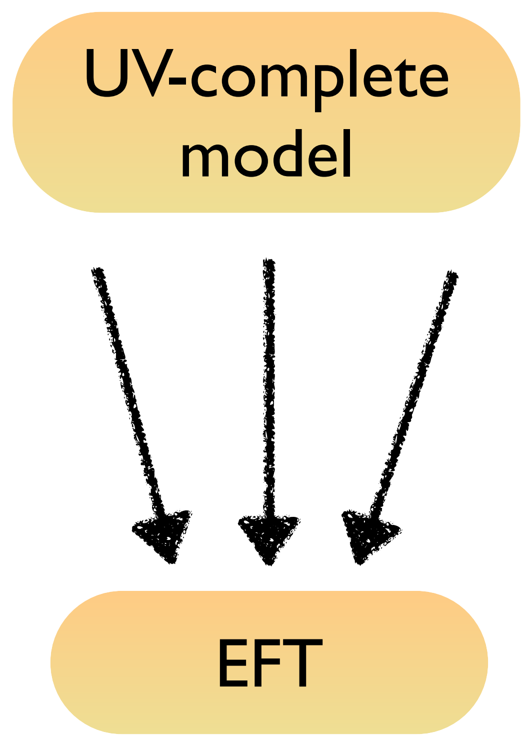
Summary: from the UV to the IR

EFTs as an approximation to physics contexts exhibiting scale separation

- Only low-energy degrees of freedom included
  - ➙ Higher-dimensional operators at low energy

$$\mathcal{L}_{\rm full}\big(\varphi_{\rm light}, \varphi_{\rm heavy}\big) \Rightarrow \mathcal{L}_{\rm full}\big(\varphi_{\rm light}\big) + \mathcal{L}_{\rm eff}\big(\varphi_{\rm light}\big)$$
$$= \mathcal{L}_{\rm full}\big(\varphi_{\rm light}\big) + \sum_i \frac{C_i}{\Lambda^{4-D_i}}\ \mathcal{O}_i^{(D_i)}\big(\varphi_{\rm light}\big)$$

- Coefficients at low energy (where the EFT is valid) needed



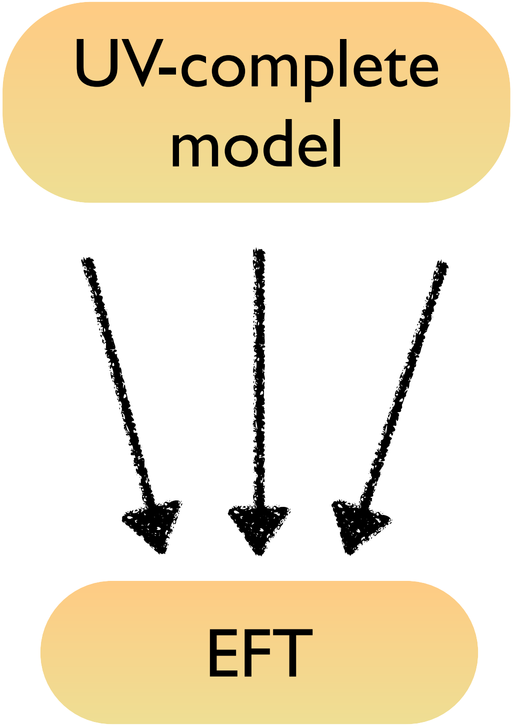
Summary: from the UV to the IR

EFTs as an approximation to physics contexts exhibiting scale separation

- Only low-energy degrees of freedom included
  - ➙ Higher-dimensional operators at low energy

$$\mathcal{L}_{\rm full}\big(\varphi_{\rm light}, \varphi_{\rm heavy}\big) \Rightarrow \mathcal{L}_{\rm full}\big(\varphi_{\rm light}\big) + \mathcal{L}_{\rm eff}\big(\varphi_{\rm light}\big) = \mathcal{L}_{\rm full}\big(\varphi_{\rm light}\big) + \sum_i \frac{C_i}{\Lambda^{4-D_i}}\ \mathcal{O}_i^{(D_i)}\big(\varphi_{\rm light}\big)$$

- Coefficients at low energy (where the EFT is valid) needed



Matching UV and EFT theories

- Knowledge of the full theory ➙ derivation of the EFT operators
  - ➙ improvable (higher orders)
- Top-down analyses (full theory known)
  - ➙ Motivation: simpler calculations in the EFT
  - ➙ Interface relating low-energy observables and high-energy physics
  - ➙ Higher-dimensional operators ⬌ power-like contributions to amplitudes
    - ★ Energy growth in the tails of distributions
- EFT validity: BSM scale higher than any scale involved

EFT as a probe to new physics

Two scenarios for effective hVV interactions

- Lagrangian:

$$\mathscr{L}_{\rm EFT} = \dots \; -g^{(1)}_{hvv}\Big[V_{\mu\nu}V^{\mu\nu}h\Big] - g^{(2)}_{hvv}\Big[V_\nu\partial_\mu V^{\mu\nu}h\Big]$$

- '*Blue*' scenario: both structures
- '*Orange*' scenario: $g^{(1)}_{hvv}$ only

EFT as a probe to new physics

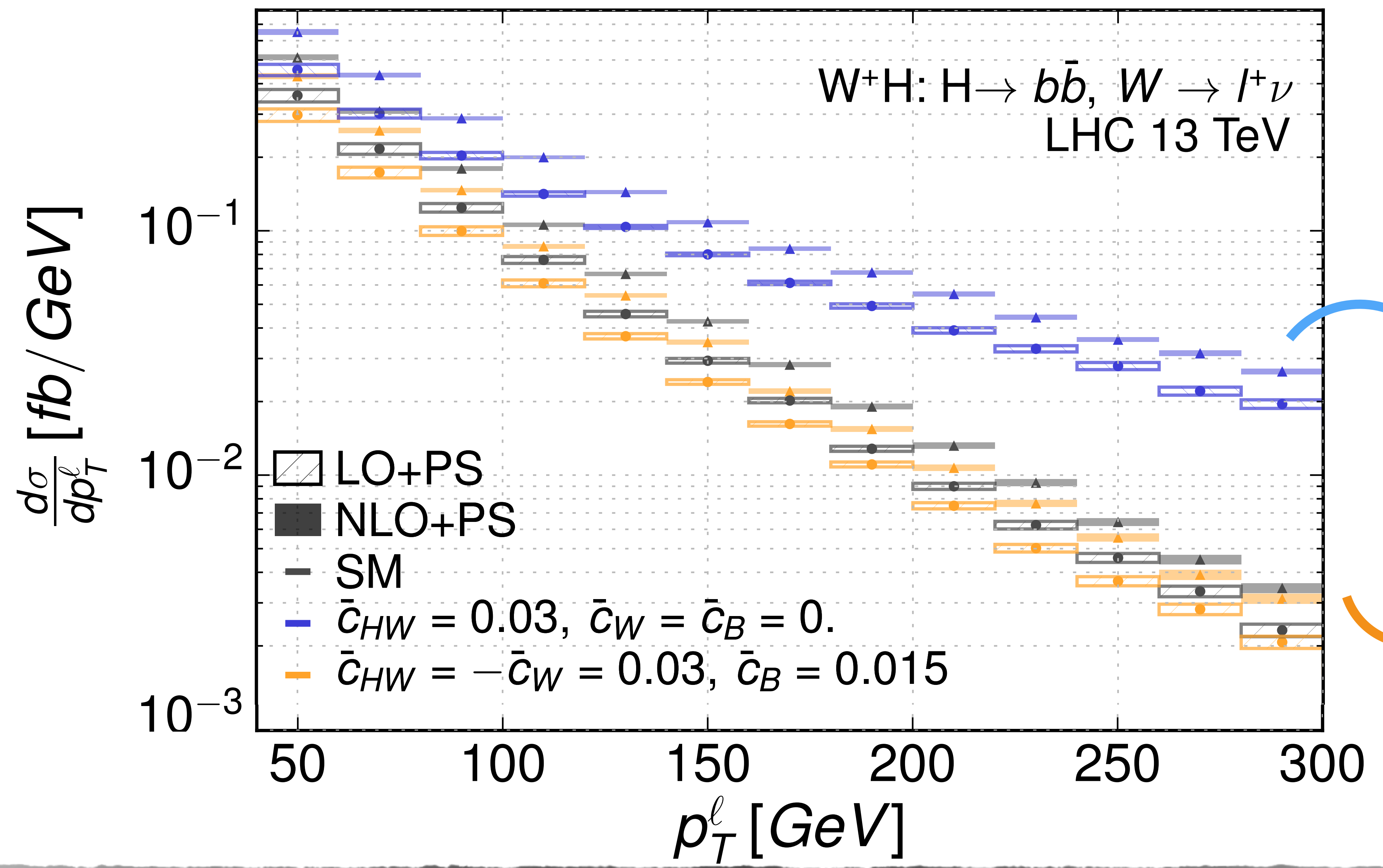
Two scenarios for effective hVV interactions

- Lagrangian:

$$\mathscr{L}_{\rm EFT} = \ldots \; -g^{(1)}_{hvv}\left[V_{\mu\nu}V^{\mu\nu}h\right] - g^{(2)}_{hvv}\left[V_\nu\partial_\mu V^{\mu\nu}h\right]$$

- '*Blue*' scenario: both structures
- '*Orange*' scenario: $g^{(1)}_{hvv}$ only

[Degrande, BF, Mawatari, Mimasu & Sanz (EPJC`17)]



Wh production, leptonic *W* decay

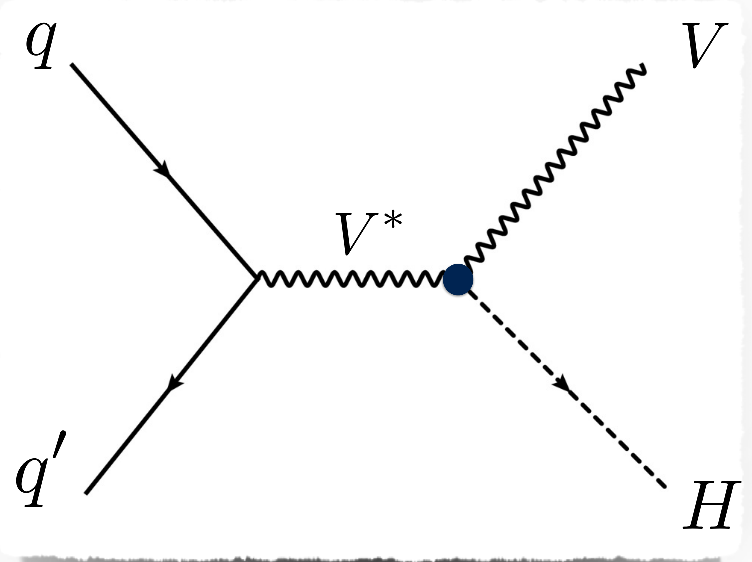
- Extracting BSM constraints from tails of distributions
  - ➞ Exploit a potential energy growth

Large $1/\Lambda^2$ effects in the tail

⚠️ EFT validity

Very close to the SM (black)

🥳 EFT interpretation valid



The SMEFT as a generic BSM framework

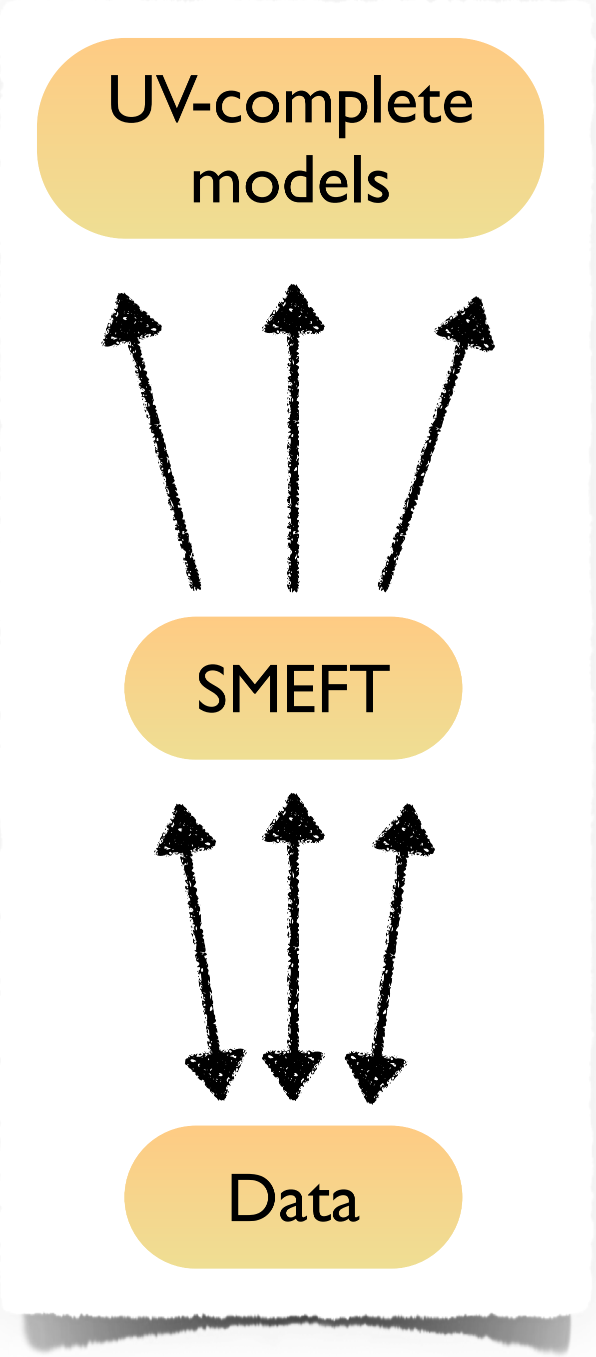
The SMEFT as a new physics framework

The low-energy theory ≡ the SM

- Full theory unknown ➙ SM + higher-dimensional operators in the SM fields

$$\mathscr{L}_{\rm SMEFT}(\varphi_{\rm SM}) = \mathscr{L}_{\rm SM} + \sum_i \frac{c_i^{(5)}}{\Lambda} \mathcal{O}_i^{(5)} + \sum_j \frac{c_j^{(6)}}{\Lambda^2} \mathcal{O}_j^{(6)} + \sum_k \frac{c_k^{(7)}}{\Lambda^3} \mathcal{O}_k^{(7)} + \dots$$

- One coefficient $c_i^{(n)}$ ↔ one free parameter
- The SMEFT applies to all types of heavy new physics
  ➙ The SMEFT as a generic and (more or less) model-independent BSM framework
- Constraints on SMEFT operators once and for all regardless of their origin

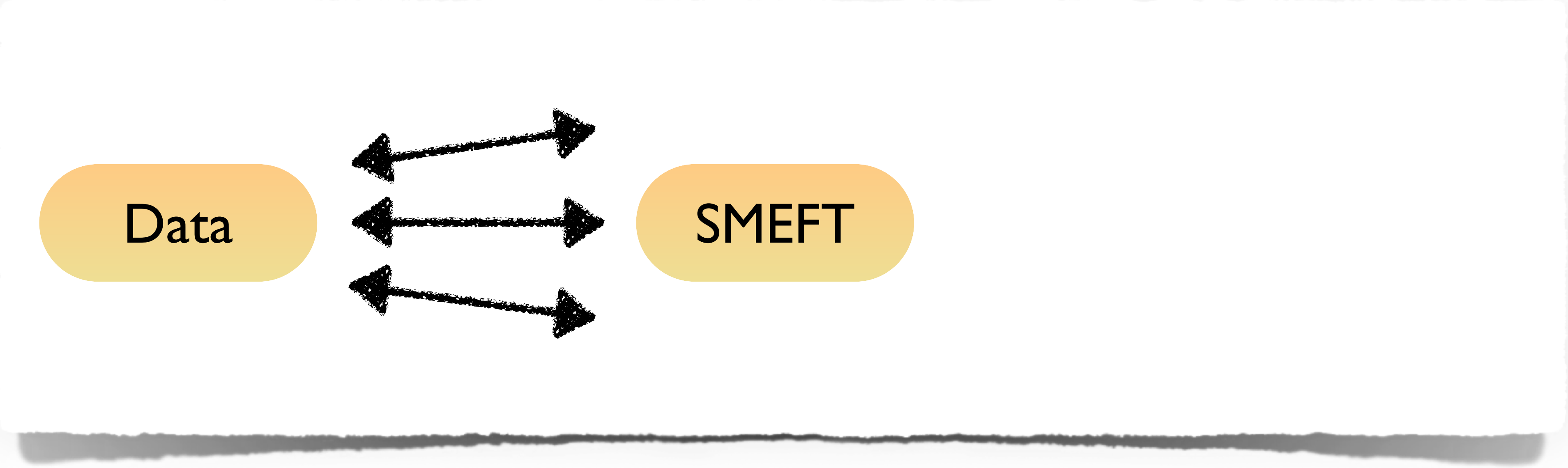


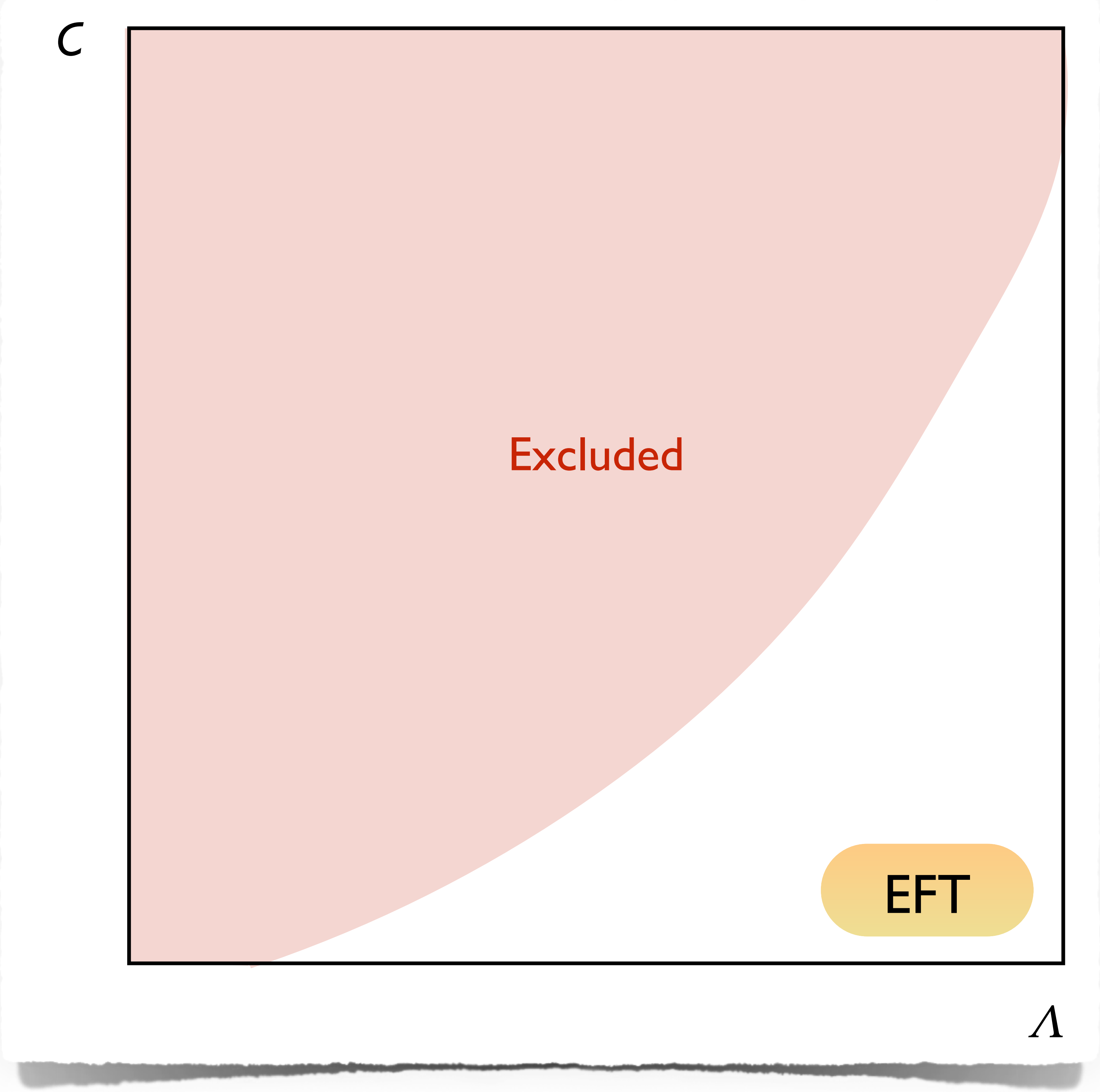
Limits on the SMEFT operators:
constraints on many models at once

Validity of interpretations

A bottom-up approach to new physics

- Constraints from data: C/Λ^2 < some threshold

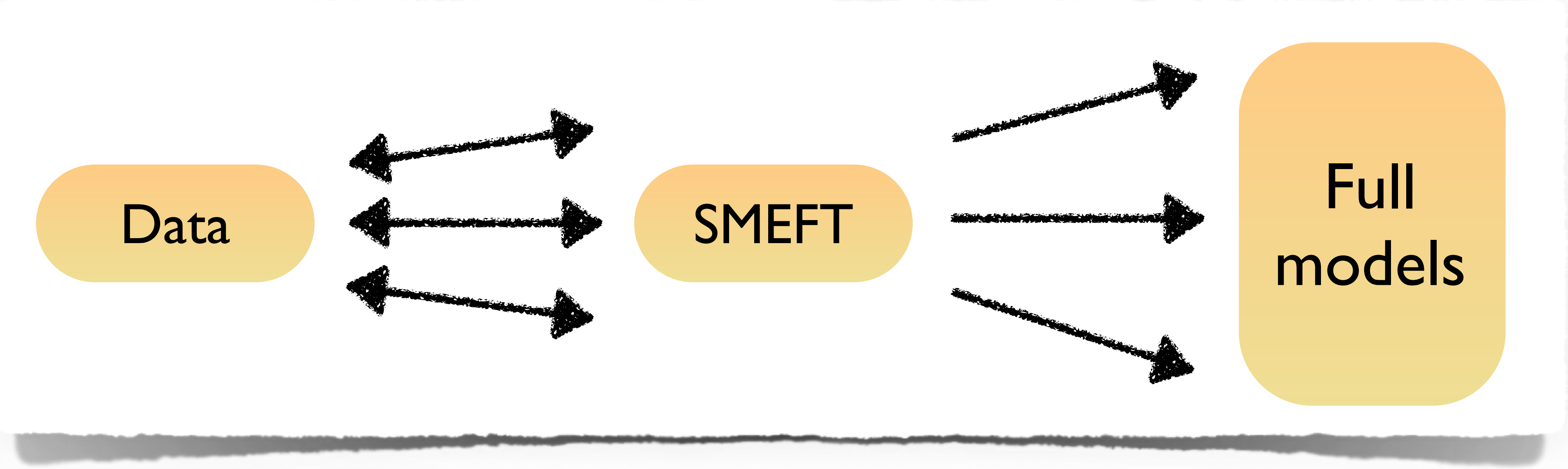


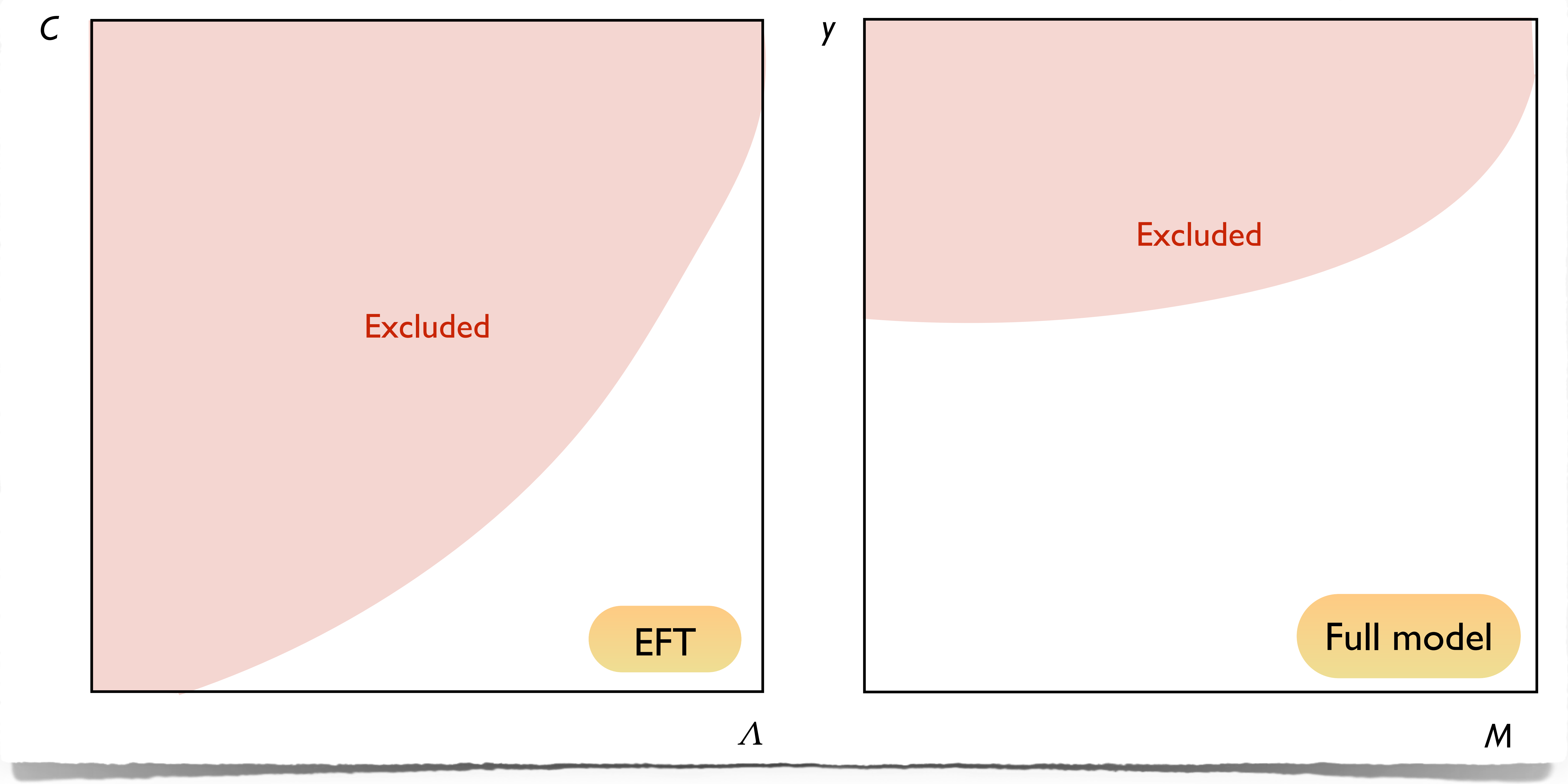


Validity of interpretations

A bottom-up approach to new physics

- Constraints from data: C/Λ^2 < some threshold
- C/Λ^2 related to some parameters y and M of a full theory
- Derivation of constraints on y and M from the EFT bounds on C/Λ^2

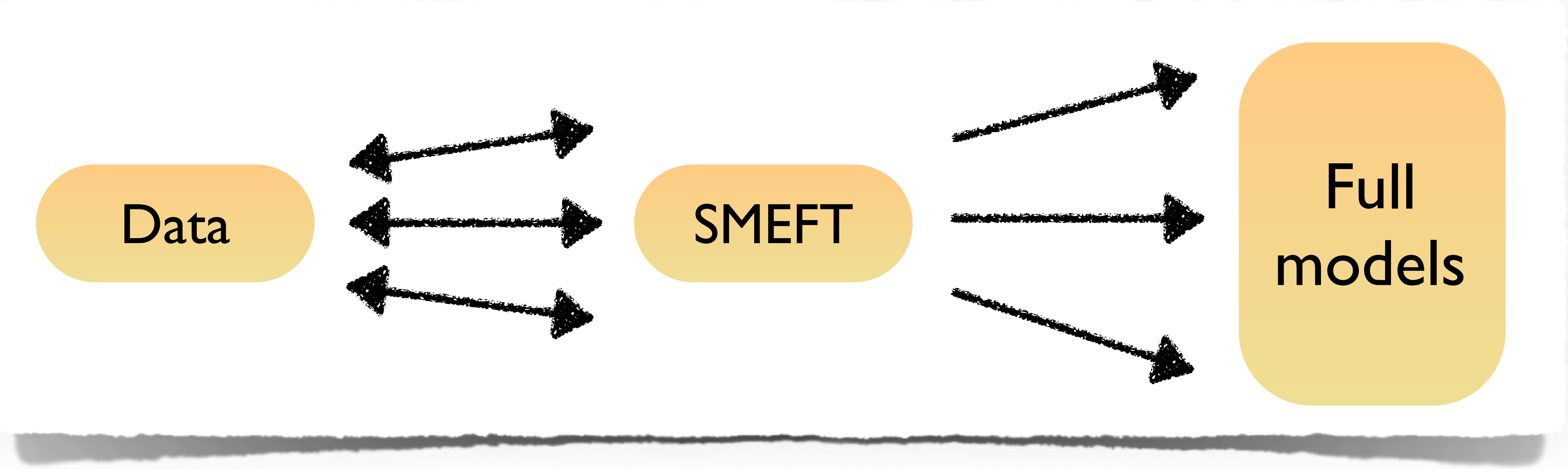


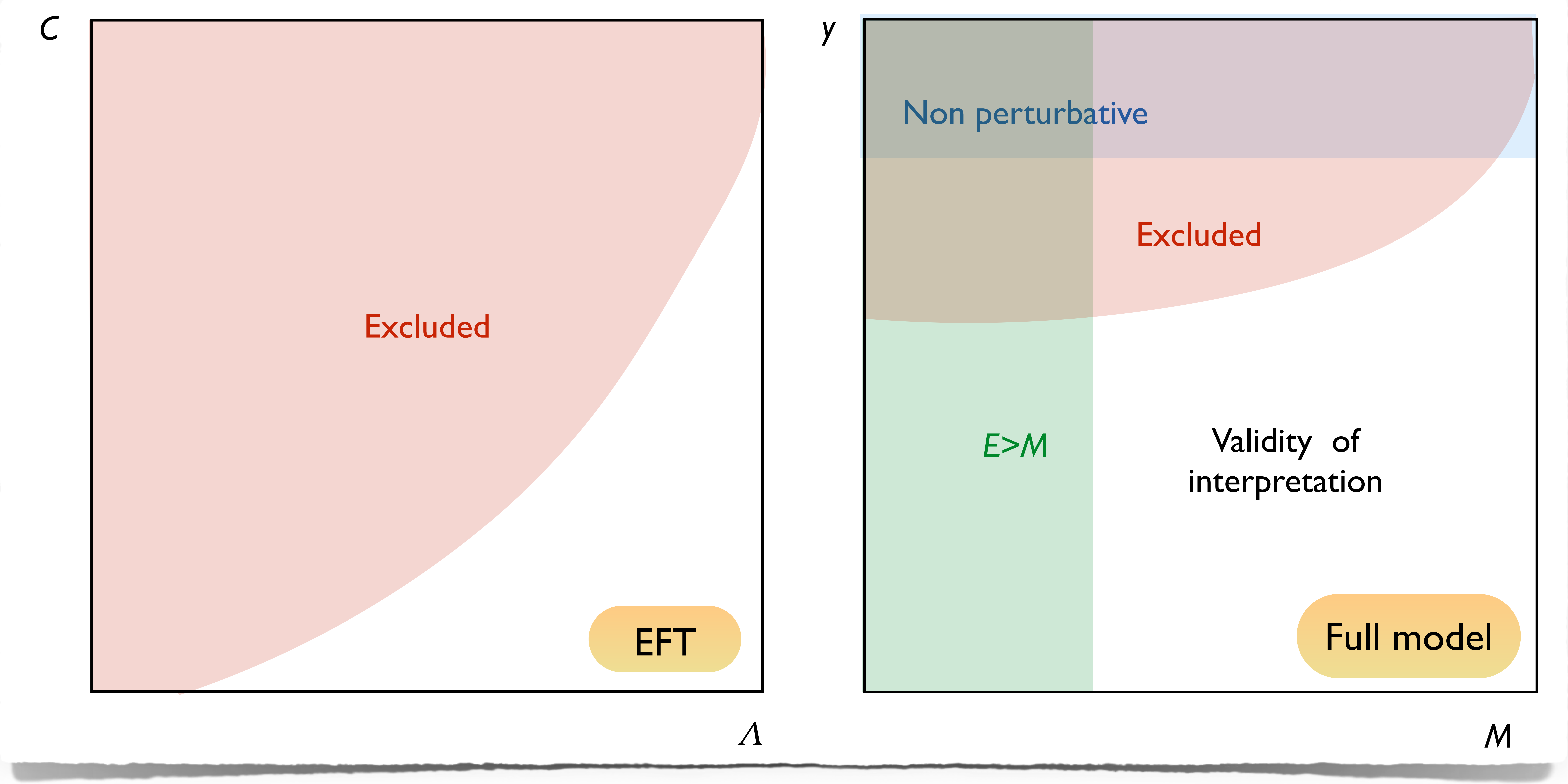


Validity of interpretations

A bottom-up approach to new physics

- Constraints from data: C/Λ^2 < some threshold
- C/Λ^2 related to some parameters y and M of a full theory
- Derivation of constraints on y and M from the EFT bounds on C/Λ^2



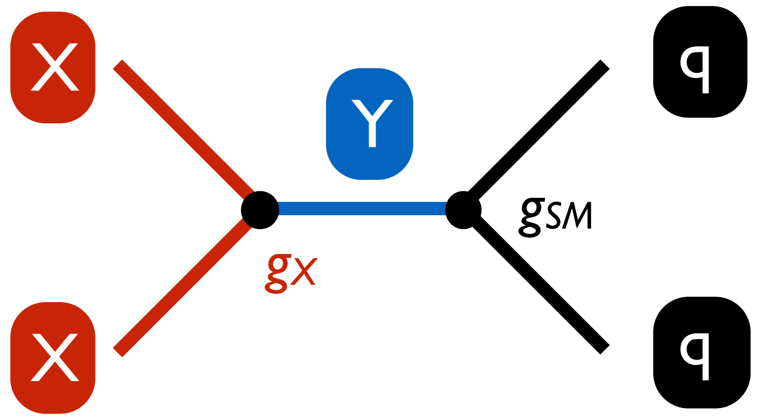


- Perturbativity of the full theory
  - → Loss of predictivity
  - → No matching possible
- EFT validity
  - → $E_{\rm typical} \ll M$ in considered processes
- Restricted domain of validity for the interpretation

The failure of an EFT interpretation – SMEFT + DM

Using an EFT for dark matter at colliders

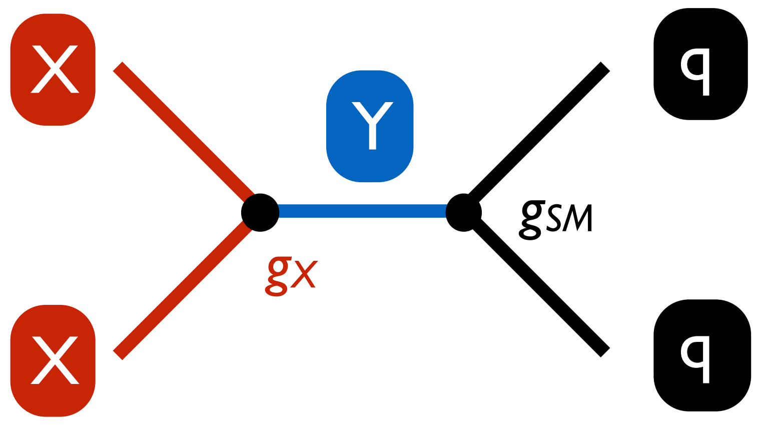
- Integration out of the Y state ➞ relating UV parameters to the EFT scale Λ
- Example: *s*-channel axial-vector mediator *Y* coupling to a dark matter *X*



$$\Rightarrow \quad \frac{C}{\Lambda}\, \bar{X}\gamma^\mu\gamma_5 X\ \bar{q}\gamma_\mu\gamma_5 q \qquad \text{with} \qquad \frac{\Lambda}{C} = \frac{m_Y}{g_X g_{\rm SM}}$$

Using an EFT for dark matter at colliders

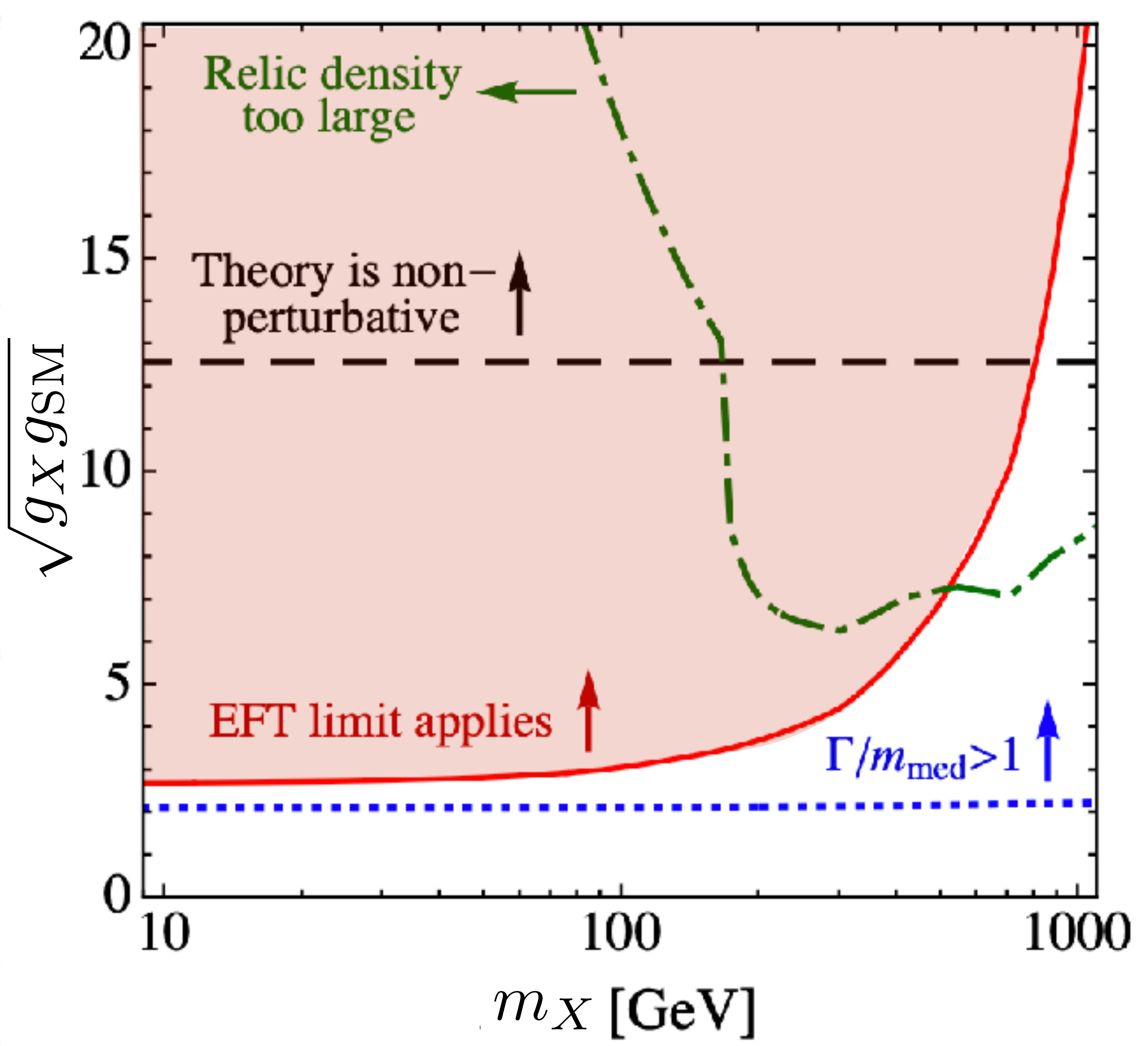
- Integration out of the Y state ➞ relating UV parameters to the EFT scale Λ
- Example: *s*-channel axial-vector mediator Y coupling to a dark matter X



$$\Rightarrow \frac{C}{\Lambda}\, \bar{X}\gamma^\mu\gamma_5 X\ \bar{q}\gamma_\mu\gamma_5 q \qquad \text{with} \qquad \frac{\Lambda}{C} = \frac{m_Y}{g_X g_{\rm SM}}$$

Validation of the EFT interpretation?

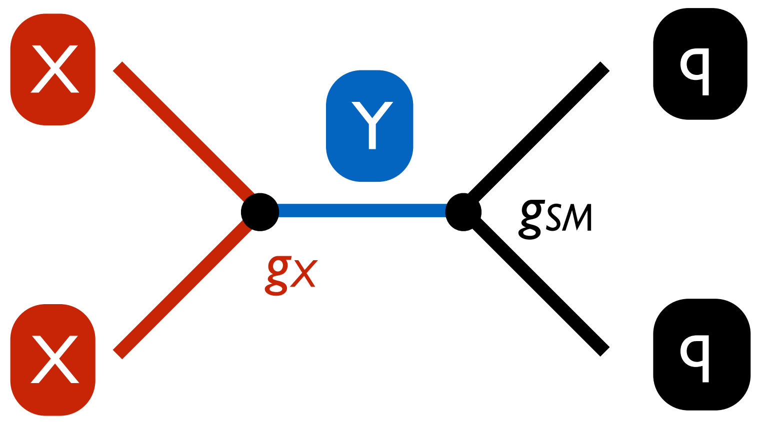
- m_Y is fixed for the EFT to be valid
  ➞ $m_Y \gg$ typical LHC momentum transfer
- Bounds on the coupling from LHC mono-jet searches (red line)



[Buchmueller, Dolan & McCabe (JHEP`14)]

Using an EFT for dark matter at colliders

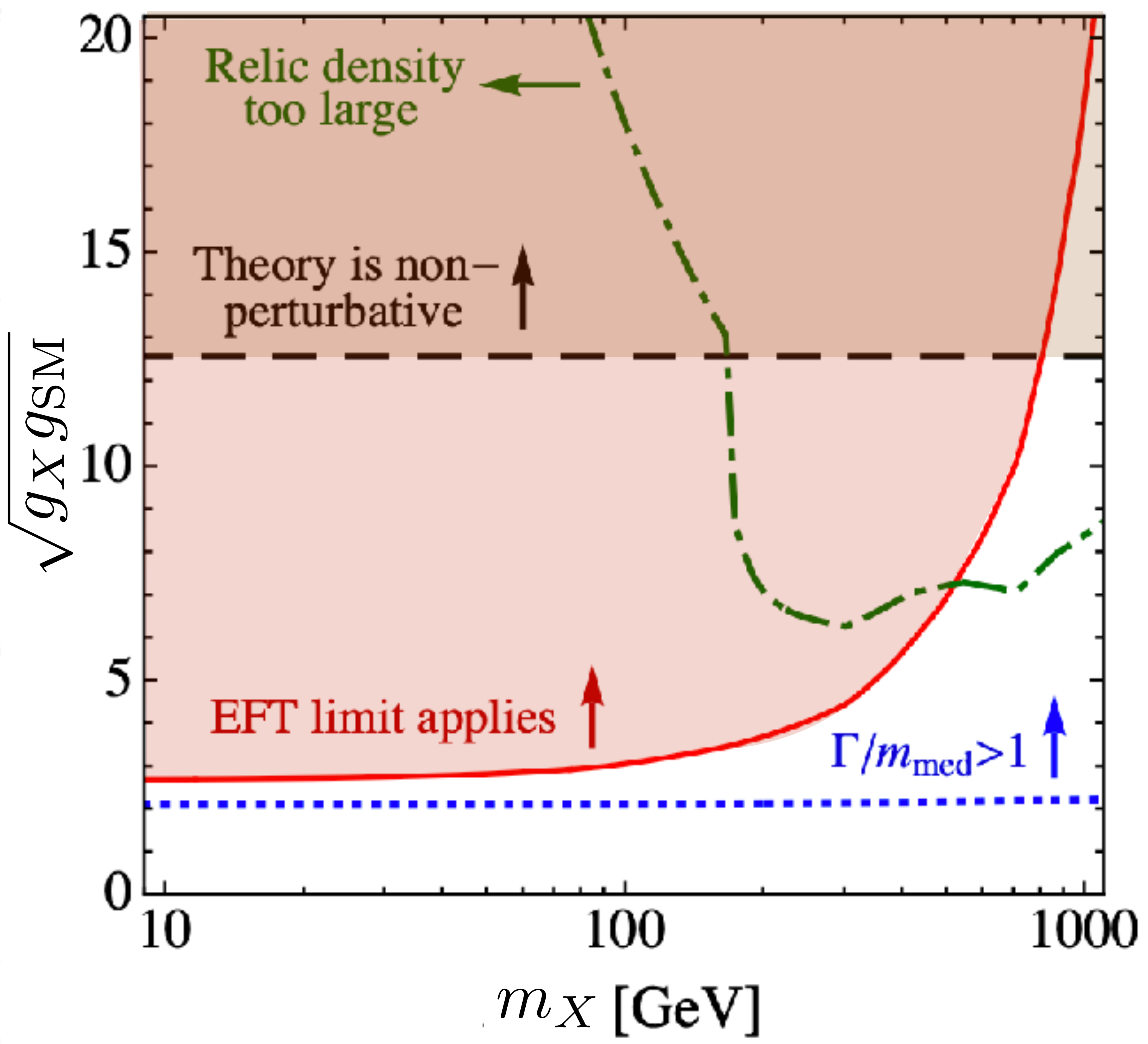
- Integration out of the Y state ➞ relating UV parameters to the EFT scale Λ
- Example: *s*-channel axial-vector mediator Y coupling to a dark matter X



$$\Rightarrow \frac{C}{\Lambda}\ \bar{X}\gamma^\mu\gamma_5 X\ \bar{q}\gamma_\mu\gamma_5 q \qquad \text{with} \qquad \frac{\Lambda}{C} = \frac{m_Y}{g_X g_{SM}}$$

Validation of the EFT interpretation?

- m_Y is fixed for the EFT to be valid
  ➞ $m_Y \gg$ typical LHC momentum transfer
- Bounds on the coupling from LHC mono-jet searches (red line)
- ⚠️ Perturbativity of the full theory
  ➞ Predictions unreliable, *etc.*

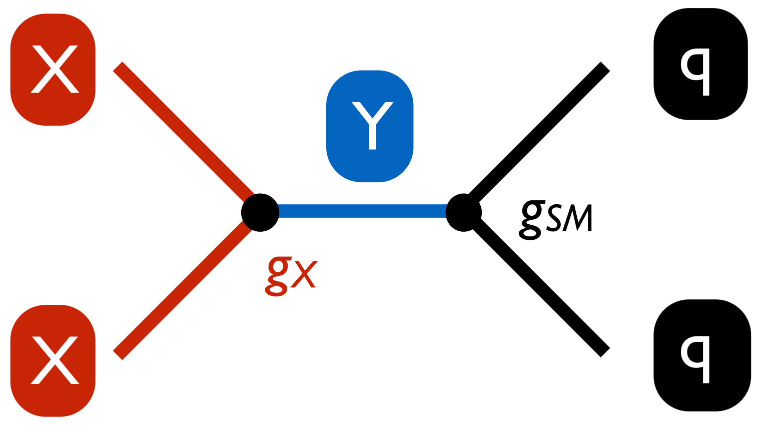


[Buchmueller, Dolan & McCabe (JHEP`14)]

The failure of an EFT interpretation – SMEFT + DM

Using an EFT for dark matter at colliders

- Integration out of the Y state ➞ relating UV parameters to the EFT scale Λ
- Example: *s*-channel axial-vector mediator *Y* coupling to a dark matter *X*

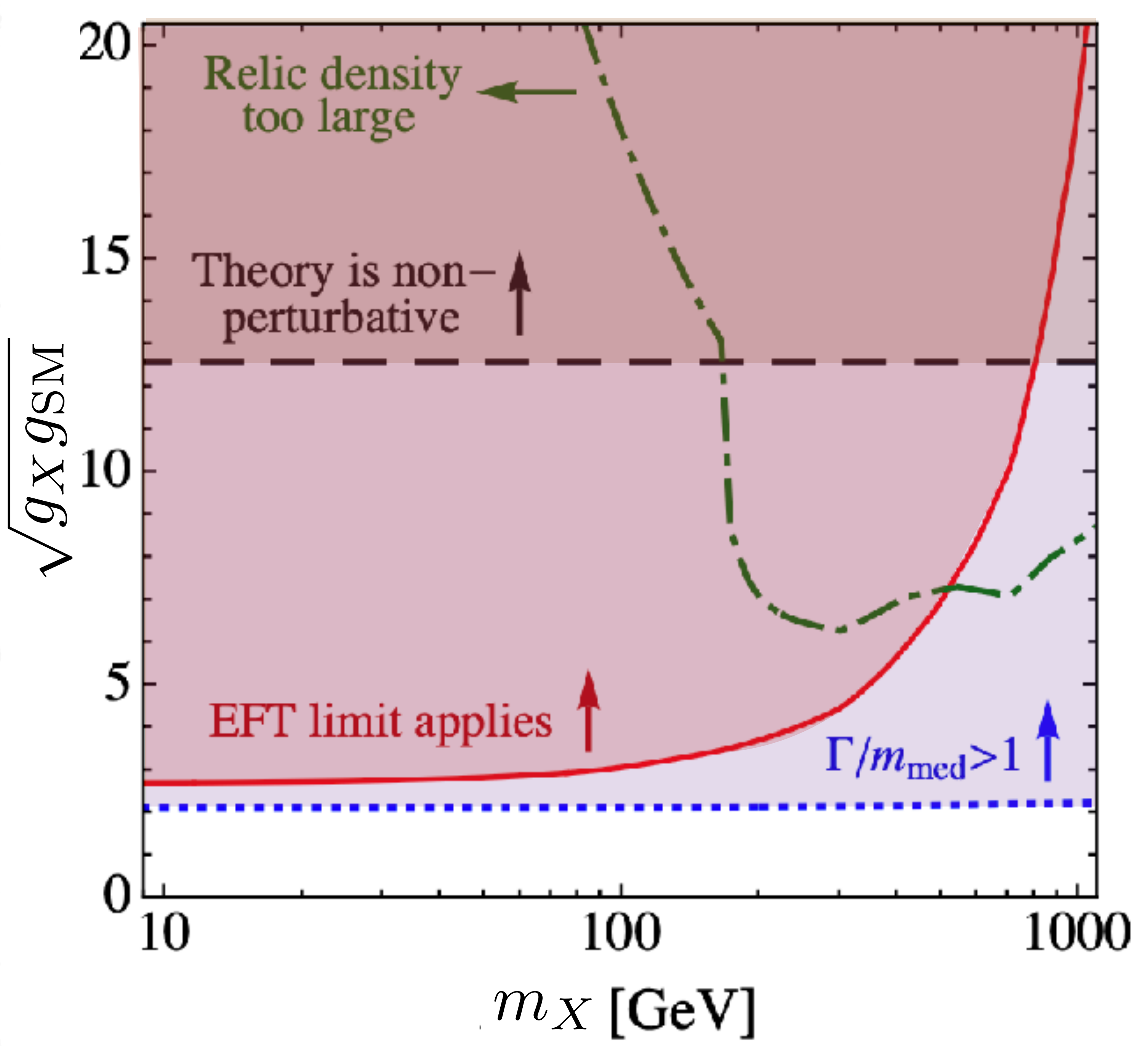


$$\Rightarrow \frac{C}{\Lambda}\ \bar{X}\gamma^\mu\gamma_5 X\ \bar{q}\gamma_\mu\gamma_5 q \qquad \text{with} \qquad \frac{\Lambda}{C} = \frac{m_Y}{g_X\, g_{\rm SM}}$$

Constraints on baroque setups

Validation of the EFT interpretation?

- m_Y is fixed for the EFT to be valid
  ➞ $m_Y \gg$ typical LHC momentum transfer
- Bounds on the coupling from LHC mono-jet searches (red line)
- ⚠️ Perturbativity of the full theory
  ➞ Predictions unreliable, *etc.*
- ⚠️ Mediator width
  ➞ Too large width issues: mediator not even a particle ($\Gamma > m_Y$)
  ➞ Large (not extreme) widths interesting too



[Buchmueller, Dolan & McCabe (JHEP'14)]

The SMEFT as a global BSM framework

SM extended by all operators allowed by the QCDxEW symmetry group

- At a given dimension, e.g. dim-6:

$$\mathscr{L}_{\rm SMEFT}(\varphi_{\rm SM}) = \mathscr{L}_{\rm SM} + \sum_j \frac{c_j^{(6)}}{\Lambda^2} \mathscr{O}_j^{(6)}$$

- All operators included as unknown full theory
  - ➙ Any $c=0$ regenerated through RG running
  - ➙ Cancellations from different operators possible in observables
- Not all operators independent

The SMEFT as a global BSM framework

SM extended by all operators allowed by the QCDxEW symmetry group

- At a given dimension, e.g. dim-6:

$$\mathcal{L}_{\rm SMEFT}(\varphi_{\rm SM}) = \mathcal{L}_{\rm SM} + \sum_j \frac{c_j^{(6)}}{\Lambda^2} \mathcal{O}_j^{(6)}$$

- All operators included as unknown full theory
  - ➙ Any *c=0* regenerated through RG running
  - ➙ Cancellations from different operators possible in observables
- Not all operators independent

A basis of operators

- Removal of redundancies ⬌ set of independent operators
  - ➙ Integration by parts
  - ➙ Fierz identities
  - ➙ Gauge algebra
  - ➙ Field redefinitions and equations of motion
- Dimension-6: 59 operators (76 flavour-universal & 2499 flavour-general)
- S-matrix un-affected [Chisholm (Nucl.Phys.`61)]
- Standard choices: the HISZ, SILH, Warsaw, Higgs bases
- Translations possible

[Hagiwara et al. (PRD`93) ] [ Giudice et al. (JHEP`07)] [Grzadkowski et al. (JHEP`10)]
[Gupta et al. (PRD`15) ] [ Falkowski et al. (EPJC`15)]

BSM effects in the SMEFT

Two classes of effects

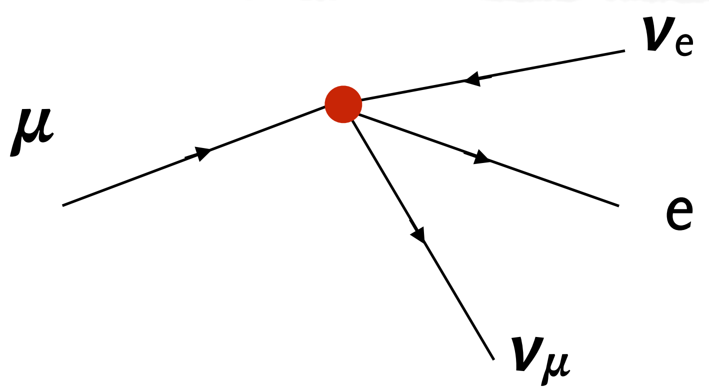
- New Lorentz structures (four-fermion interactions, derivatives)
  ➞ Energy growth in observables
- Modifications of the SM interactions
  ➞ Energy growth from spoliation of unitarity-cancellations

BSM effects in the SMEFT

Two classes of effects

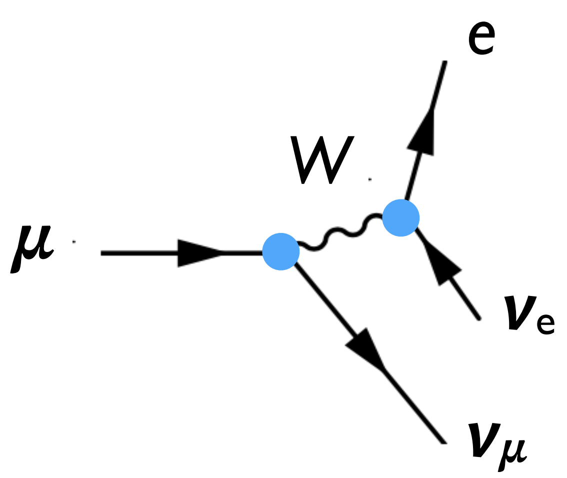
- New Lorentz structures (four-fermion interactions, derivatives)
  ➞ Energy growth in observables
- Modifications of the SM interactions
  ➞ Energy growth from spoliation of unitarity-cancellations

Example: muon decay ➞ modification of the Higgs vev / G_F relation



$$\left[\mathcal{O}_{\ell\ell}\right]^{ijkl} = \left[\bar{\ell}^{(i)}\gamma^\mu\ell^{(j)}\right]\ \left[\bar{\ell}^{(k)}\gamma_\mu\ell^{(l)}\right]$$

$$\left[\mathcal{O}^{(3)}_{\varphi\ell}\right]^{ij} = \left[\varphi^\dagger\frac{\sigma^k}{2}\overleftrightarrow{D}_\mu\varphi\right]\ \left[\bar{\ell}^{(i)}\frac{\sigma^k}{2}\gamma^\mu\ell^{(j)}\right]$$

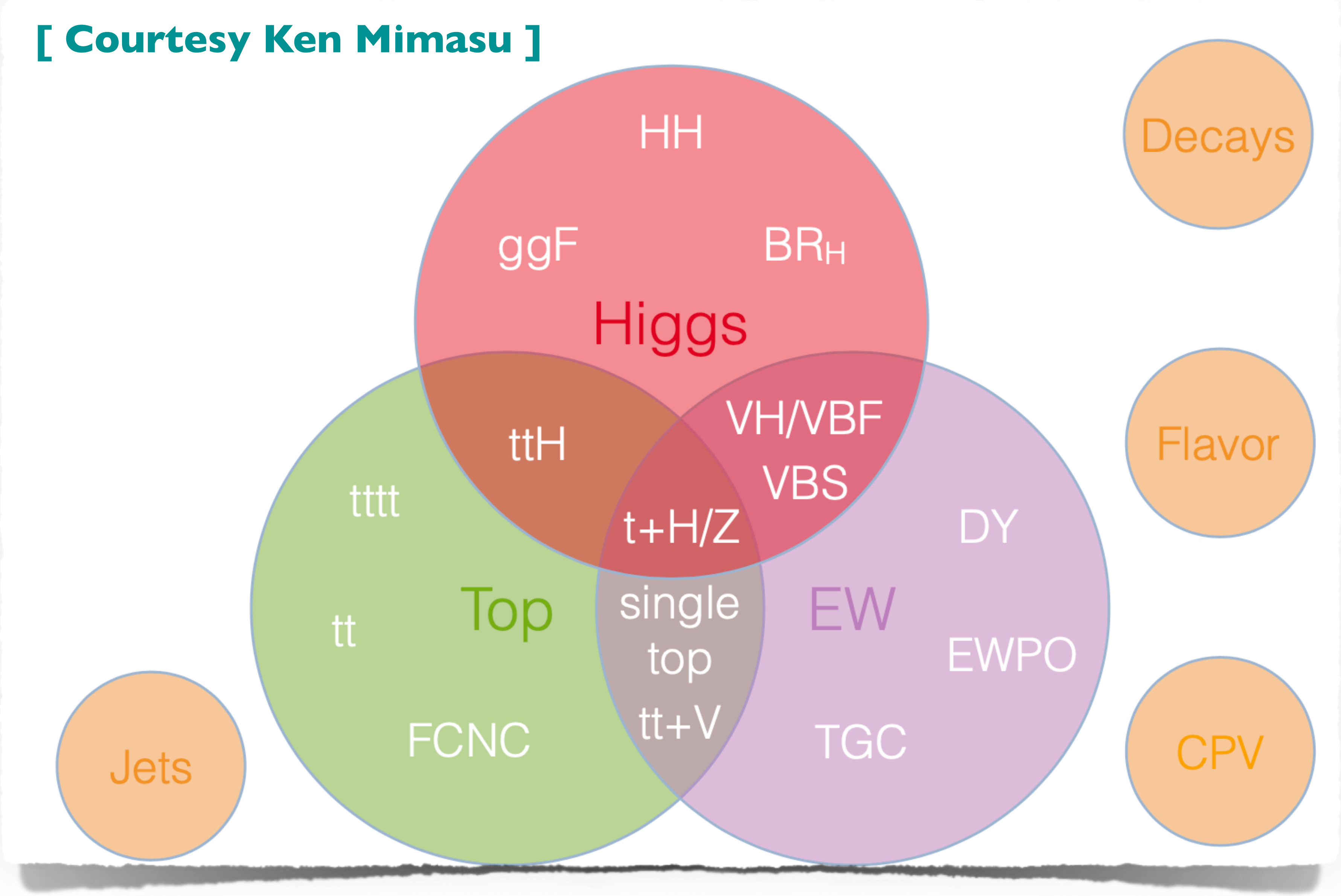


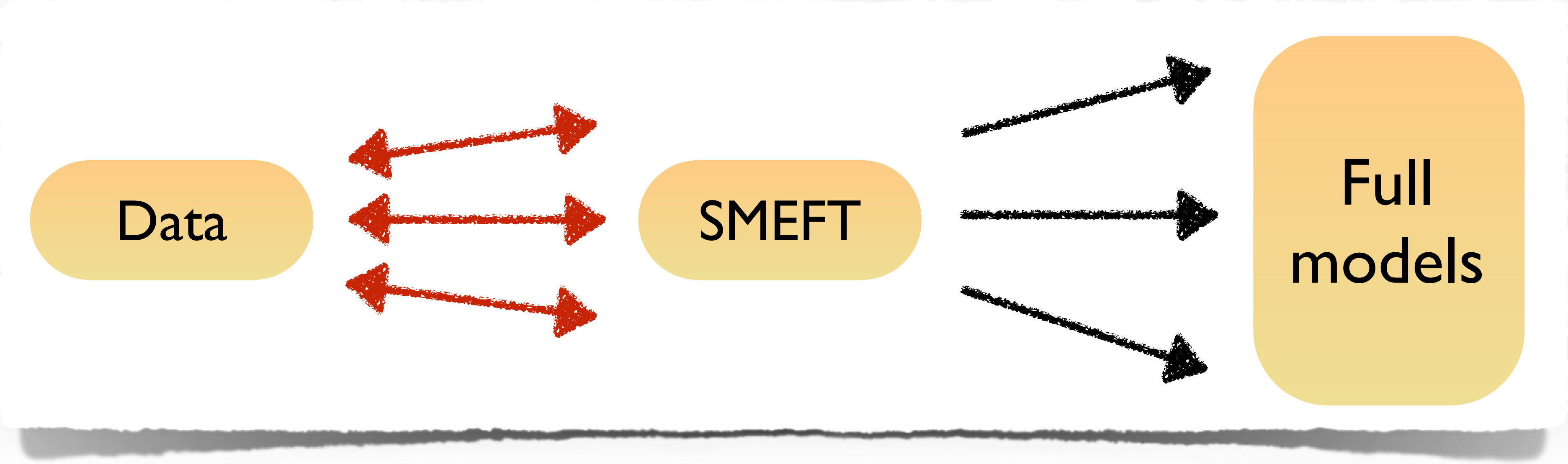
$$\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}\left[1+\frac{v^2}{\Lambda^2}\left[C^{(3)}_{\varphi\ell}\right]^{11}\right]\left[1+\frac{v^2}{\Lambda^2}\left[C^{(3)}_{\varphi\ell}\right]^{22}\right] - \frac{1}{4\Lambda^2}\left[\left[2C_{\ell\ell}\right]^{1212}+\left[C_{\ell\ell}\right]^{1221}+\left[C_{\ell\ell}\right]^{2112}\right] \neq \frac{1}{2v^2}$$

Propagates to all observables depending on v

Constraining the SMEFT

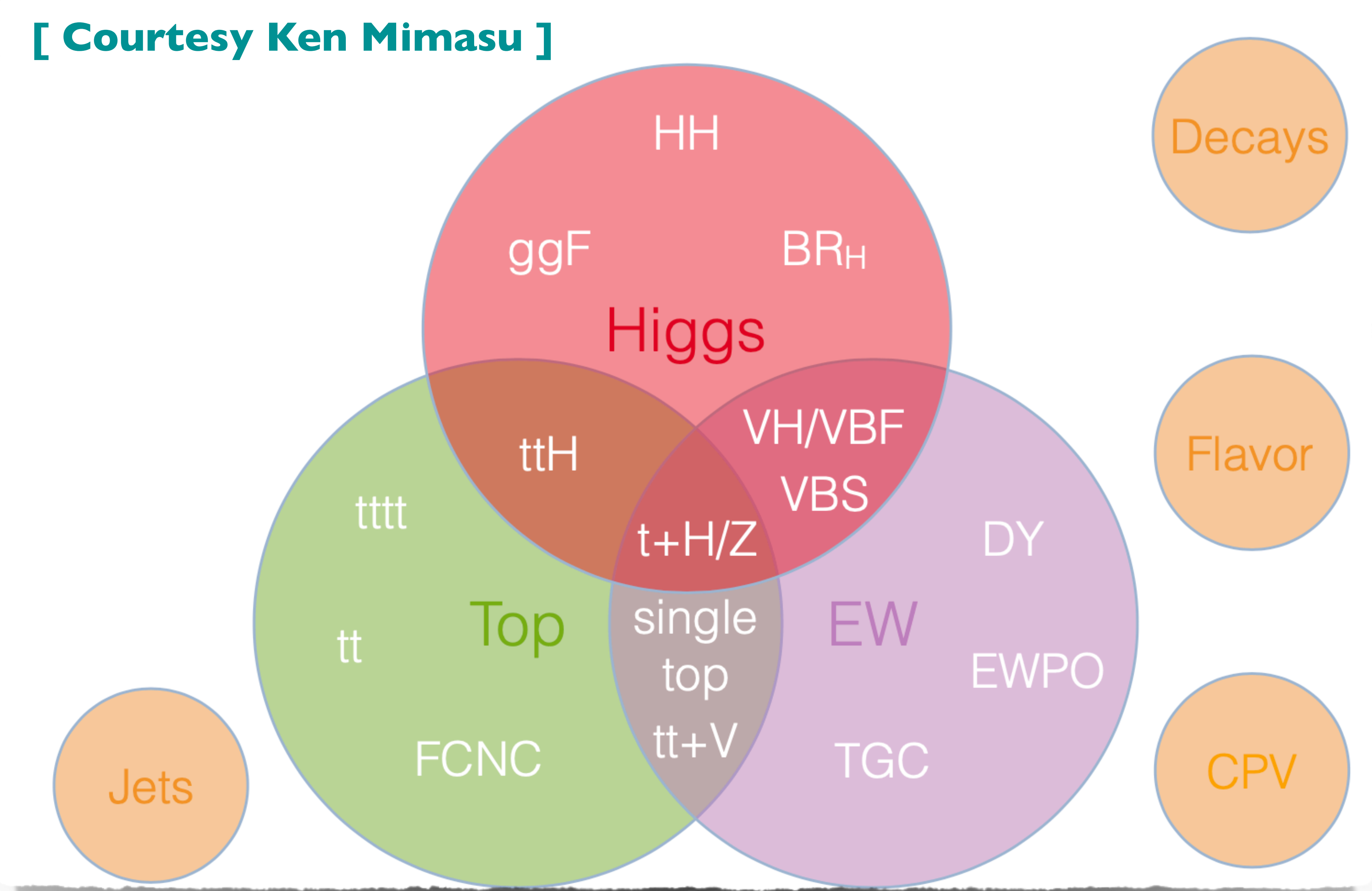
A lot of available data to constrain the SMEFT

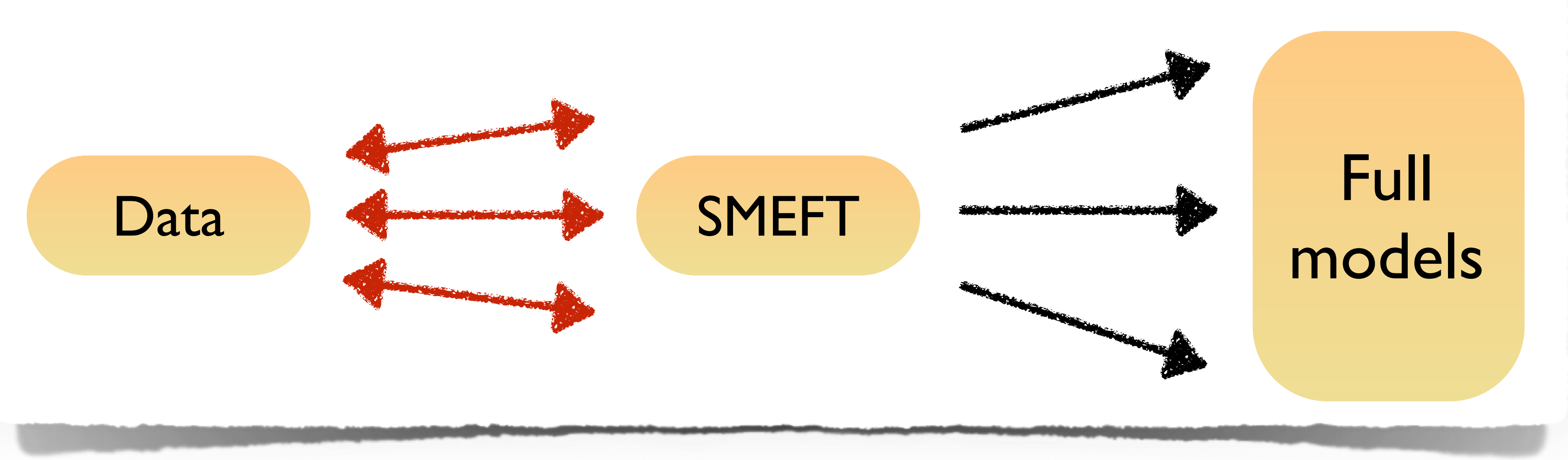




Constraining the SMEFT

A lot of available data to constrain the SMEFT





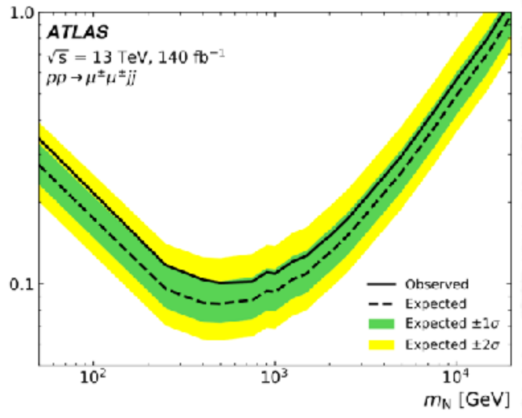
Monte Carlo simulations standard today

- Simulations at NLO (at least QCD) easily achieved

Idea Lagrangian → Simulated collisions → Event analysis

Chain of tools ← SMEFT, Easy validation

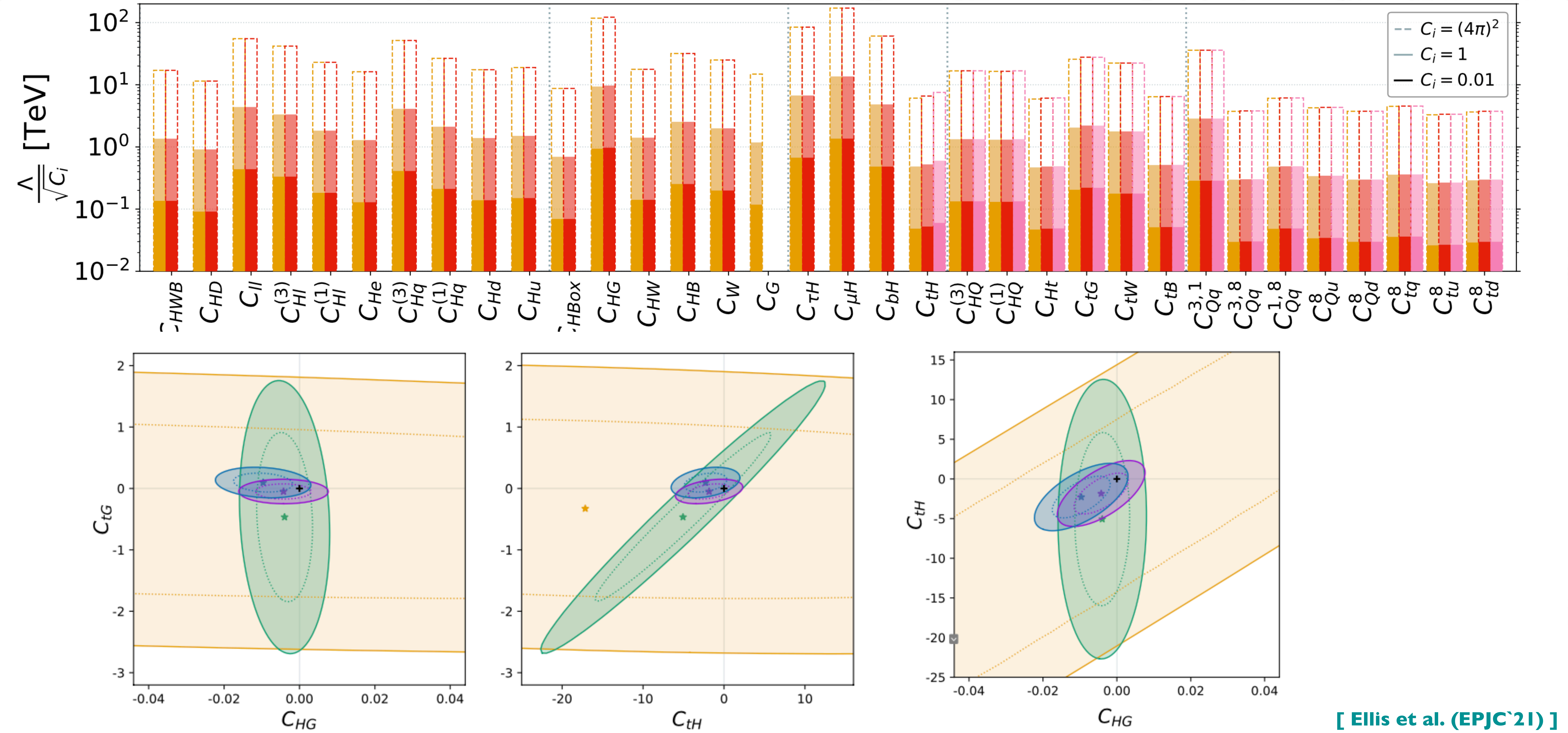
Other tools



A SMEFT global fit

Fit to data [Higgs + top + di-boson + EW precision observables]

- Compatibility with the SM ➞ Λ in the TeV regime



[Ellis et al. (EPJC`21)]

Summary

Summary

EFTs ≡ powerful tools for BSM

- Model-independent searches for heavy new physics
  - ➞ Small deviations from the SM
- Connects UV physics (full model) to low-energy physics (EFT)
  - ➞ Matching, operator running and mixing
  - ➞ Systematically improvable
- Data constraints on EFT operators
  - ➞ Global approach (top, Higgs, EWPO, flavour, *etc.*)
  - ➞ Simultaneous constraints on a large class of BSM setups
- Precision predictions at colliders (not covered in this talk)
  - ➞ SMEFT: SILH & Warsaw bases, topEFTs, *etc.*