

# Beyond the Standard Model physics made easy with FEYNRULES

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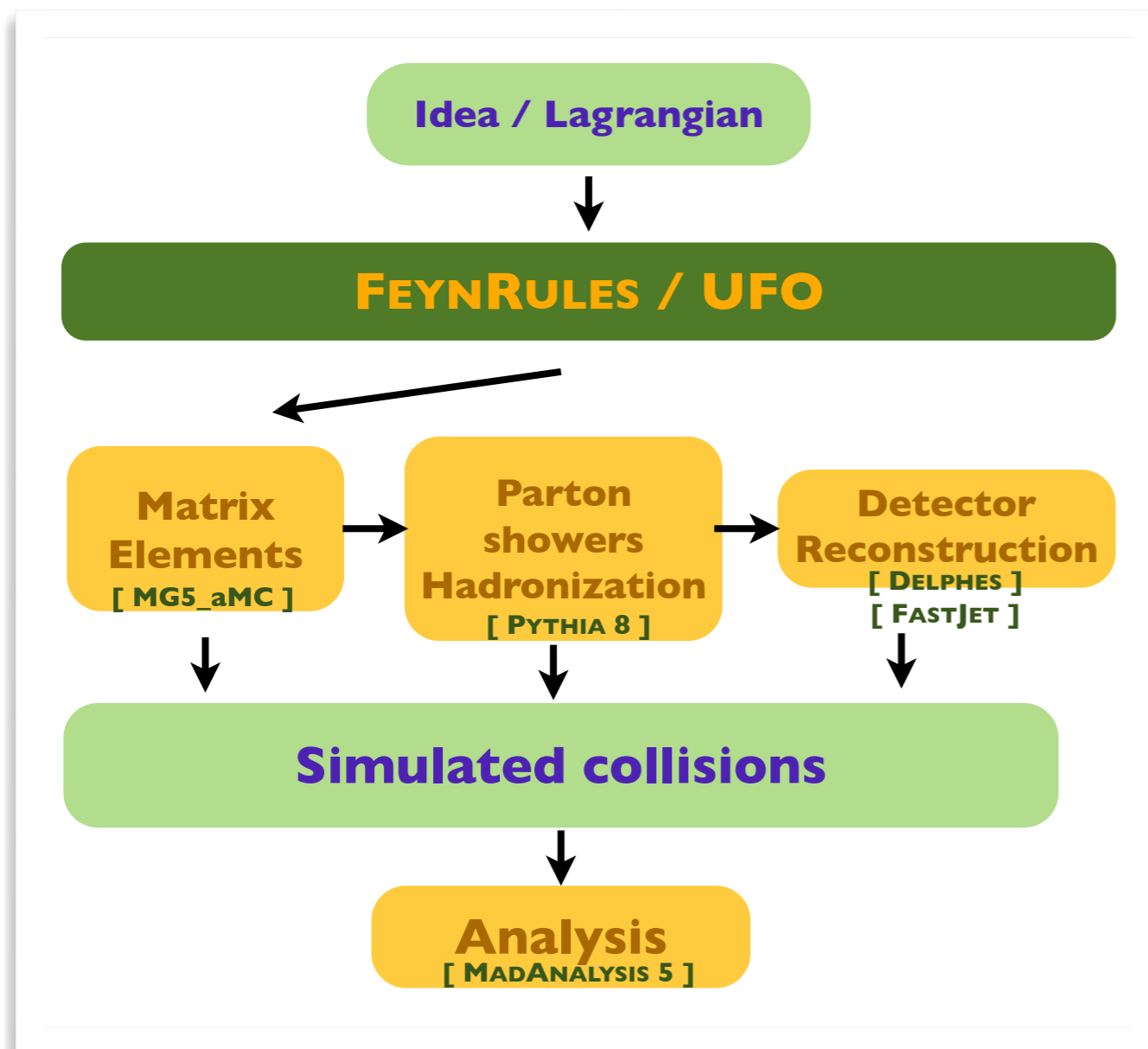
# Outline

1. FEYNRULES in a nutshell
2. Implementing supersymmetric QCD in FEYNRULES
3. Using FEYNRULES with supersymmetric QCD model
4. Summary
5. Appendix: advanced model implementation techniques

# A comprehensive approach to MC simulations

[ Christensen, de Aquino, Degrande, Duhr, BF, Herquet, Maltoni & Schumann (EPJC 11) ]

## Tools connecting an idea to simulated collisions



- Model building
- Hard scattering
  - ★ Feynman diagram and amplitude generation
  - ★ Monte Carlo integration
  - ★ Event generation
- QCD environment
  - ★ Parton showering
  - ★ Hadronisation
  - ★ Underlying event
- Detector simulation
  - ★ Simulation of the detector response
  - ★ Object reconstruction
- Event analysis
  - ★ Signal/background analysis
  - ★ LHC recasting

# FEYNRULES in a nutshell

## What is FEYNRULES?

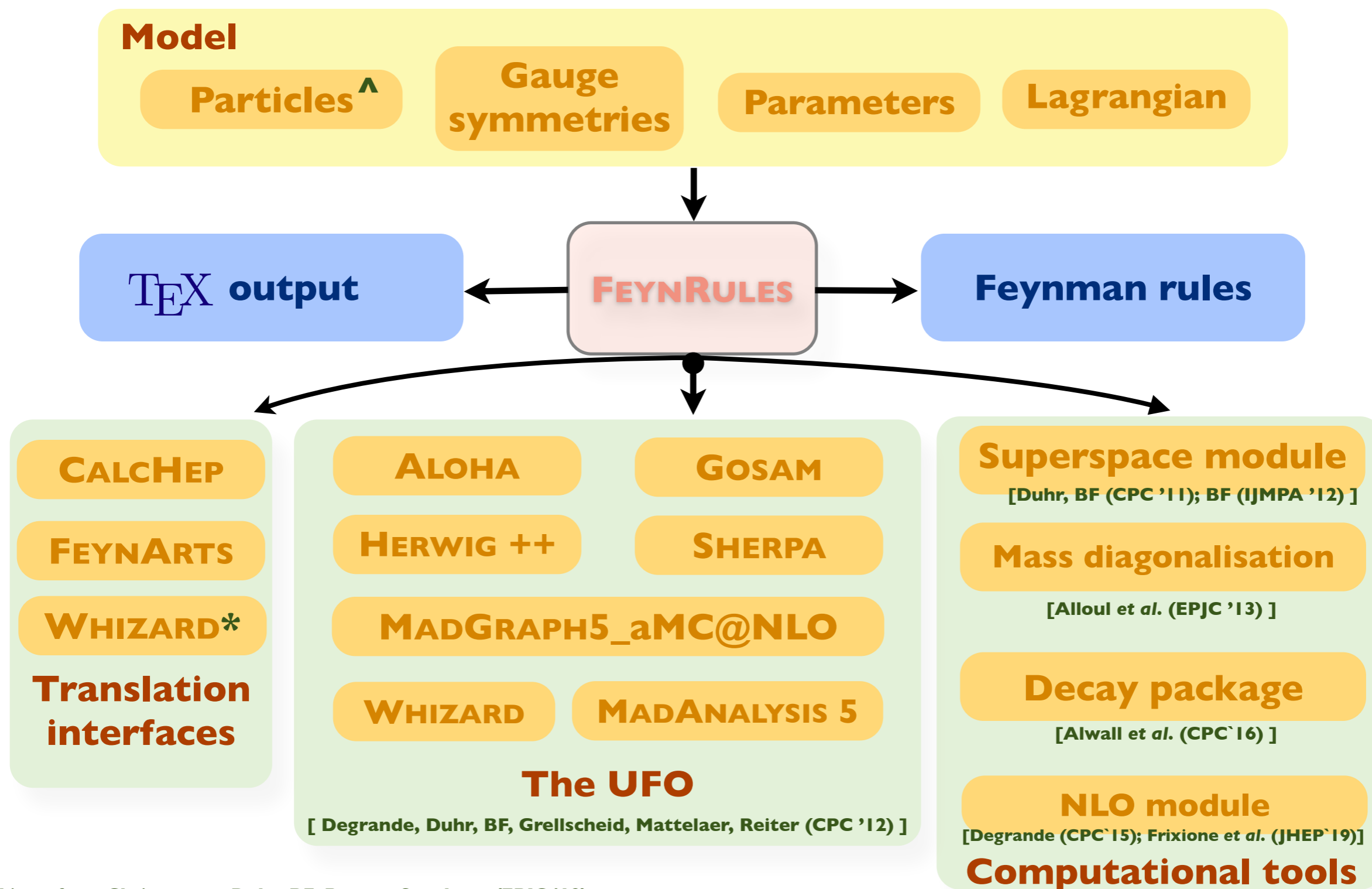
- A framework to **develop new physics models**
- **Automatic export** to Monte Carlo event generators
  - ➡ Facilitate phenomenological investigations of BSM models
  - ➡ Facilitate the confrontation of BSM models to data
- **Validation** of implementations with several Monte Carlo programmes

## Main features

- **MATHEMATICA** package
- Core function: **derives Feynman rules from a Lagrangian**
- **Requirements**: locality, Lorentz and gauge invariance
- **Supported fields**: scalars, (two- and four-component) fermions, vectors (and ghosts), spin-3/2, tensors, superfields

# From FEYNRULES to Monte Carlo tools...

[ Christensen, Duhr (CPC '09); Alloul, Christensen, Degrande, Duhr, BF (CPC'14) ]



\* Whizard interface: Christensen, Duhr, BF, Reuter, Speckner (EPJC '12)

<sup>^</sup> Support for spin 3/2: Christensen, de Aquino, Deutschmann, Duhr, BF, Garcia-Cely, Mattelaer, Mawatari, Oexl, Takaesu (EPJC '13)

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# Supersymmetric QCD: particle content

## Particle content

- **Two matter supermultiplets** in the fundamental representation of  $SU(3)_c$ 
  - ★ One massive Dirac fermion: a **quark**
  - ★ Two massive scalar fields: a left-handed and a right-handed **squark**
- **One  $(SU(3)_c)$  gauge supermultiplet**
  - ★ One massive Majorana fermion: a **gluino**
  - ★ One massless gauge boson: the **gluon**

# (Broken) supersymmetric QCD: the model

The dynamics of the model is embedded in the Lagrangian

$$\begin{aligned}
 \mathcal{L} = & -\frac{1}{4}g_{\mu\nu}g^{\mu\nu} + \frac{i}{2}\bar{g}\not{D}g + D_{\mu}\tilde{q}_L^{\dagger}D^{\mu}\tilde{q}_L + D_{\mu}\tilde{q}_R^{\dagger}D^{\mu}\tilde{q}_R + i\bar{q}\not{D}q \\
 & - m_{\tilde{q}_L}^2\tilde{q}_L^{\dagger}\tilde{q}_L - m_{\tilde{q}_R}^2\tilde{q}_R^{\dagger}\tilde{q}_R - m_q\bar{q}q - \frac{1}{2}m_{\tilde{g}}\bar{g}g \\
 & - \frac{g_s^2}{2}\left[-\tilde{q}_L^{\dagger}T^a\tilde{q}_L + \tilde{q}_R^{\dagger}T^a\tilde{q}_R\right]\left[-\tilde{q}_L^{\dagger}T^a\tilde{q}_L + \tilde{q}_R^{\dagger}T^a\tilde{q}_R\right] \\
 & + \sqrt{2}g_s\left[-\tilde{q}_L^{\dagger}T^a(\bar{g}^a P_L q) + (\bar{q}P_L\tilde{g}^a)T^a\tilde{q}_R + \text{h.c.}\right]
 \end{aligned}$$

*R*

- Kinetic terms for all fields (first line)
- Mass terms for the squarks, quarks and gluino (second line)
- Supersymmetric gauge interactions for all fields (last two lines)



# A FEYNRULES model file

A FEYNRULES model file is an `.fr` file written in MATHEMATICA syntax

## Preamble

- ★ Author information
- ★ Model information
- ★ Index definitions

## Declaration of the gauge group

- ★ Abelian or not
- ★ Representation matrices
- ★ Structure constants
- ★ Coupling constant
- ★ Gauge boson or vector superfield

## Declaration of fields

- ★ Names, spins, PDG codes
- ★ Indices, quantum numbers
- ★ Masses, widths
- ★ Classes and class members

## Declaration of parameters

- ★ External and internal
- ★ Scalar and tensor

A Lagrangian

# Preamble: general information

## An electronic signature for the model implementation

- Important for traceability, documentation, contact with the authors, *etc.*
- Reference publications
- Webpage information

```
M$ModelName = "SUSYQCD";

M$Information = {
  Authors      -> {"Benjamin Fuks"},
  Date         -> "19.11.18",
  Version      -> "1.0",
  Institutions -> {"LPTHE / Sorbonne U."},
  Emails       -> {"fuchs@lpthe.jussieu.fr"}
};
```

# Preamble: indices

## Index dimensions

- Fundamental  $SU(3)_C$  indices for squarks and quarks: *Colour*, dimension 3
- Adjoint  $SU(3)_C$  indices for gluons and gluinos: *Gluon*, dimension 8
- Lorentz and spin indices automatically handled

```
IndexRange[Index[Gluon]] = NoUnfold[Range[8]];
IndexRange[Index[Colour]] = NoUnfold[Range[3]];
```

QCD  $\equiv$  special role in event generators  $\rightarrow$  special names

- *Colour, Sextet and Gluon* for colour indices
- *G* for the gluon field
- *T* for the fundamental representation matrices
- *f* and *d* for the structure constants
- *G/g<sub>s</sub>* and *aS* for the coupling constants

The style of the indices can be specified

- Fundamental indices starting with *m*
- Adjoint indices starting with *a*

```
IndexStyle[Colour, m];
IndexStyle[Gluon, a];
```

# Gauge groups (I)

Each element the group is declared in the  $M\$GaugeGroups$  list

- One group  $\equiv$  one set of MATHEMATICA replacement rules
  - ★ We must only declare  $SU(3)_C$  (name: *SU3C*)

```
M$GaugeGroups = {
  SU3C == {
    Abelian          -> False,
    GaugeBoson       -> G,
    CouplingConstant -> gs,
    StructureConstant -> f,
    Representations  -> { {T, Colour} }
  }
};
```

- One rule  $\equiv$  one group property
  - ★ *Abelian*: abelian or non-abelian group
  - ★ *GaugeBoson*: associated gauge boson
  - ★ *CouplingConstant*, *StructureConstant*: coupling and structure constants
  - ★ *Representations*: list of 2-tuples linking indices to representations

See the manual for more details on gauge groups

# Gauge groups (2)

## Advantages of a proper gauge group declaration

- Writing Lagrangians more easily
  - ★ **Covariant derivatives** (`DC[field, Lorentz index]`)
  - ★ **Field strength tensors** (`FS[field, Lorentz index 1, Lorentz index 2]`)
  - ★ Useful for Lagrangian building in superspace (briefly covered in the last slides)
- Example: gluon and gluino kinetic terms

$$-\frac{1}{4}g_{\mu\nu}g^{\mu\nu} + \frac{i}{2}\bar{g}\not{D}g$$

$$-1/4 \text{ FS}[G, \mu, \nu, a] \text{ FS}[G, \mu, \nu, a] + \\ 1/2 \text{ gobar.Ga}[\mu].\text{DC}[go, \mu]$$

# The gluon field

Each field is an element of the *M\$ClassesDescription* list

- A declaration  $\equiv$  a set of MATHEMATICA replacement rules
- In our SUSY-QCD model, we first declare the  $SU(3)_c$  gauge boson:  $G$

```
M$ClassesDescription = {
  V[1] == {
    ClassName      -> G,
    SelfConjugate  -> True,
    Indices        -> {Index[Gluon]},
    Mass           -> 0,
    Width          -> 0,
    PDG            -> 21
  },
  ...
}
```

- One rule  $\equiv$  one property of the field
  - ★ Vector field  $\rightarrow$  the label is  $V[I]$  (with V, and not F, S, R, T, etc.)
  - ★ **ClassName**: defines a symbol for Lagrangians  $\rightarrow G$
  - ★ **Indices**: the gluon in the adjoint representation
    - $\rightarrow$  tagged as gauge boson of  $SU(3)_c \rightarrow$  *Gluon* internally linked to the adjoint
  - ★ Others: vanishing mass and width, PDG code  $\equiv 21$ , self-conjugate

See the manual for more options for field declarations

# A Majorana gluino field

A second element in the  $M\$ClassesDescription$  list

```
F[1] == {  
  ClassName      -> go,  
  SelfConjugate  -> True,  
  Indices        -> {Index[Gluon]},  
  Mass           -> {Mgo,500},  
  Width          -> {Wgo,10},  
  PDG            -> 1000021  
},
```

- Differences with the gluon field declaration
  - ★ Four-component fermionic field  $\rightarrow$  **F[I]** (starting with an F)
  - ★ **Classname**: two symbols for the Lagrangian  $\rightarrow$  *go* and *gobar*
  - ★ **Mass/width**: two symbols and associated numerical values

See the manual for more details on field declarations

# Weyl/Majorana gluino fields

## Second option: two-component spinors

```
W[1] == {
  ClassName      -> go|w,
  Unphysical     -> True,
  Chirality      -> Left,
  SelfConjugate -> False,
  Indices        -> {Index[Gluon]},
},
```

```
F[1] == {
  ClassName      -> go,
  WeylComponents -> gow,
  SelfConjugate -> True,
  Indices        -> {Index[Gluon]},
  Mass           -> {Mgo,500},
  Width          -> {Wgo,10},
  PDG            -> 1000021
},
```

- **Extra options available**

- ★ Two-component and four-component fermions → **W[I]** (with a W) and **F[I]**
- ★ Weyl fermions **unphysical** → linked to four-component fermions (**WeylComponents**)
- ★ Several symbols defined → *go* and *gobar*; *gow* and *gowbar*
- ★ **Chirality** to specify

See the manual for more details on field declarations



# The (top) quark field

A third element in the *M\$ClassesDescription* list

```
F[2] == {  
  ClassName      -> q,  
  SelfConjugate  -> False,  
  Indices        -> {Index[Colour]},  
  Mass           -> {Mq, 173},  
  Width          -> {Wq, 1.50833649},  
  PDG            -> 6  
},
```

- Nothing special compared to the other fields
  - ★ Fundamental QCD indices specified

# More on quarks: generation indices

## Three generations of up-type quarks

```
F[3] == {
  ClassName      -> uq,
  ClassMembers   -> {u, c, t},
  SelfConjugate  -> False,
  Indices        -> {Index[Gen], Index[Colour]},
  FlavorIndex    -> Gen,
  Mass           -> {Mu, {MU, 2.55*^-3}, {MC, 1.42}, {MT, 173}},
  Width          -> {0, 0, {WT, 1.50833649}},
  PDG            -> {2, 4, 6}
},
```

- Slight differences for a bunch of options (+ new options)
  - ★ **ClassMembers**: all the members of the class
  - ★ A **generation index**: *Gen* (**FlavorIndex** defines the flavour index)
  - ★ **Mass, Width, PDG**: one per class member (+ generic mass symbol)

# Squarks

## Extra elements in the $M\$ClassesDescription$ list

```

S[3] == {
  ClassName      -> sq1,
  SelfConjugate  -> False,
  Indices        -> {Index[Colour]},
  Mass           -> {Msq1,300},
  Width          -> {Wsq1,10},
  PDG            -> 1000006
},
S[4] == {
  ClassName      -> sq2,
  SelfConjugate  -> False,
  Indices        -> {Index[Colour]},
  Mass           -> {Msq2,800},
  Width          -> {Wsq2,2},
  PDG            -> 2000006
}

```

- **Nothing special**

- ★ Scalar field → **S[3]**

- ★ **Classname** → pairs of symbols:  
*sq1, sq2, sq1bar and sq2bar*

- ★ **Mass, width, Indices, etc.** (standard)

Missing:  
squark mixing

# Squark mixing

## Mixing implemented via physical and unphysical fields

```

S[1] == {
  ClassName      -> sqL,
  SelfConjugate  -> False,
  Indices        -> {Index[Colour]},
  Unphysical     -> True,
  Definitions    -> {sqL[c_] -> Cos[theta] sq1[c] - Sin[theta] sq2[c]}
},
S[2] == {
  ClassName      -> sqR,
  SelfConjugate  -> False,
  Indices        -> {Index[Colour]},
  Unphysical     -> True,
  Definitions    -> {sqR[c_] -> Sin[theta] sq1[c] + Cos[theta] sq2[c]}
},

```

- Squark fields mix as

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}$$

- ★ Left and right-handed squarks **unphysical** → removed from the Lagrangian and replaced
- ★ **Definitions** ≡ the replacement rules (inversion of the above relation)
  - Rotations performed **automatically**
  - Lagrangian written in the gauge basis (easier with sqL / sqR)

# Parameters

The model requires to implement three parameters

- Masses and widths handled automatically

$$\begin{aligned}
 \mathcal{L} = & \left[ -\frac{1}{4}g_{\mu\nu}g^{\mu\nu} + \frac{i}{2}\bar{g}\not{D}g + D_\mu\tilde{q}_L^\dagger D^\mu\tilde{q}_L + D_\mu\tilde{q}_R^\dagger D^\mu\tilde{q}_R + i\bar{q}\not{D}q \right. \\
 & - m_{\tilde{q}_L}^2\tilde{q}_L^\dagger\tilde{q}_L - m_{\tilde{q}_R}^2\tilde{q}_R^\dagger\tilde{q}_R - m_q\bar{q}q - \frac{1}{2}m_{\tilde{g}}\bar{g}g \\
 & - \frac{g_s^2}{2} \left[ -\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R \right] \left[ -\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R \right] \\
 & \left. + \sqrt{2}g_s \left[ -\tilde{q}_L^\dagger T^a (\bar{g}^a P_L q) + (\bar{q} P_L \tilde{g}^a) T^a \tilde{q}_R + \text{h.c.} \right] \right]
 \end{aligned}$$

- The strong coupling explicit and implicit
  - $g_s$  needed
  - $\alpha_s$  as required by MC tools
- Squark mixing angle  $\theta$  to add

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}$$

# Parameters: strong coupling

Parameters are declared as elements of the list `M$Parameters`

- One declaration  $\equiv$  one set of MATHEMATICA replacement rules
- Strong coupling:  $\alpha_s$  and  $g_s$  not independent

```
M$Parameters = {
  aS == {
    ParameterType -> External,
    Value         -> 0.1184,
    InteractionOrder -> {QCD, 2}
  },
  gs == {
    ParameterType -> Internal,
    Value         -> Sqrt[4 Pi aS],
    InteractionOrder -> {QCD, 1},
    ParameterName  -> G
  },
}
```

- ★ **Internal** and **External** parameters
  - **External**  $\equiv$  free param. (numerical)
  - **Internal**  $\equiv$  dependent param (formula)
- ★ **InteractionOrder**: specific to some MC
  - more efficient diagram generation
- ★ **ParameterName**: QCD special

See the manual for more details on parameter declarations

# Parameters: squark mixing

## The squark mixing angle

- Nothing special (standard external parameter)

```
theta == {  
  ParameterType  -> External,  
  Value          -> Pi/4  
}
```

## More options (assuming matrix-form mixing)

- Tensor parameters can be implemented too (Indices, Unitary)
- Complex parameters can be implemented too (ComplexParameter)

See the manual for more details on parameter declarations

# Parameter organisation

## Les Houches blocks to organise external parameters

- Les Houches parameter card by MC programmes  
→ BlockName + OrderBlock
- Unspecified: the 'FRBlock' block

## Complete $\alpha_s$ implementation as an example

- Many options (TeX, Description, BlockName, OrderBlock)

```
aS == {  
  ParameterType    -> External,  
  BlockName        -> SMINPUTS,  
  OrderBlock       -> 3,  
  Value            -> 0.1184,  
  InteractionOrder -> {QCD,2},  
  TeX              -> Subscript[\[Alpha],s],  
  Description      -> "Strong coupling constant at the Z pole"  
},
```



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**Getting started**

# Starting a FEYNRULES MATHEMATICA session

## Step 1: loading FEYNRULES into MATHEMATICA

- The FEYNRULES path
- Loading FEYNRULES

```
In[1]:= $FeynRulesPath = SetDirectory["~/Work/tools/FeynRules/branch/feynrules-current"];
<< FeynRules`
- FeynRules -
Version: 2.3.32 ( 12 March 2018).
Authors: A. Alloul, N. Christensen, C. Degrande, C. Duhr, B. Fuks

Please cite:
- Comput.Phys.Commun.185:2250-2300,2014 (arXiv:1310.1921);
- Comput.Phys.Commun.180:1614-1641,2009 (arXiv:0806.4194).

http://feynrules.phys.ucl.ac.be

The FeynRules palette can be opened using the command FRPalette[].
```

## The output:

- Information on FEYNRULES, authors, version numbers, references, etc.

# Loading the model implementation

## Step 2: loading the SUSY-QCD implementation into FEYNRULES

- Moving to the right directory
- Loading the model itself

```
SetDirectory[NotebookDirectory[]];  
LoadModel["susyqcd.fr"];
```

```
This model implementation was created by
```

```
Benjamin Fuks
```

```
Model Version: 1.0
```

```
For more information, type ModelInformation[].
```

- Loading particle classes.
- Loading gauge group classes.
- Loading parameter classes.

```
Model SUSYQCD loaded.
```

### The output:

- Preamble of the model are printed to the screen
- More information from `ModelInformation[]`

## The vector Lagrangian

# The SUSY-QCD vector Lagrangian

The gauge sector of the theory : one gauge supermultiplet

- One massive Majorana fermion: a **gluino**
- One massless gauge boson: the **gluon**

Dynamics embedded in the vector Lagrangian

$$\mathcal{L}_{\text{vector}} = -\frac{1}{4}g_{\mu\nu}g^{\mu\nu} + \frac{i}{2}\bar{\tilde{g}}\not{D}\tilde{g} - \frac{1}{2}m_{\tilde{g}}\bar{\tilde{g}}\tilde{g}$$

- Kinetic terms (both fields)
- Mass terms for the gluino
- Gauge interactions (both fields, through gauge-covariant objects)

# Implementing the vector Lagrangian

Textbook expression:

$$\mathcal{L}_{\text{vector}} = -\frac{1}{4}g_{\mu\nu}g^{\mu\nu} + \frac{i}{2}\bar{g}\not{D}g - \frac{1}{2}m_{\bar{g}}\bar{g}g$$

Strengths of a proper gauge-group implementation

- Shortcut functions (DC, FS, etc.)
- Very compact

```
LVector1 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
            I/2 gobar.Ga[mu].DC[go,mu] -
            1/2 Mgo gobar.go;
```

★ Indices understood (automatic handling)

Indices could be noted explicitly

```
LVector2 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
            I/2 Ga[mu,s1,s2] gobar[s1,a].DC[go[s2,a],mu] -
            1/2 Mgo gobar[s1,a].go[s1,a];
```

# Checks of the implementation

## Textbook expression

$$\mathcal{L}_{\text{vector}} = -\frac{1}{4}g_{\mu\nu}g^{\mu\nu} + \frac{i}{2}\bar{g}\not{D}g - \frac{1}{2}m_{\bar{g}}\bar{g}g$$

## FEYNRULES implementation

```
LVector1 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
            I/2 gobar.Ga[mu].DC[go,mu] -
            1/2 Mgo gobar.go;
```

## Printing the vector Lagrangian

In[6]:= **LVector1**

```
Out[6]= - 1/2 Mgo go . go + 1/2 i go . gamma^mu . (partial_mu [go] - i gs FSU3C^a$658 . go G_mu,a$658) -
         1/4 (-partial_nu [G_mu,a] + partial_mu [G_nu,a] + gs f_a,bb$656,cc$656 G_mu,bb$656 G_nu,cc$656) -
         (-partial_nu [G_mu,a] + partial_mu [G_nu,a] + gs f_a,bb$657,cc$657 G_mu,bb$657 G_nu,cc$657)
```

## The output:

- Covariant derivatives and field strength tensors automatically evaluated
- FSU3C  $\equiv$  structure constants  $\rightarrow$  to be replaced in a second step by  $f_{abc}$



# Expansion of the Lagrangian

All indices restored automatically

In[7]:= `ExpandIndices [LVector1]`

$$\begin{aligned}
 \text{Out[7]} = & -\frac{1}{4} \partial_{\nu} [G_{\mu,a}]^2 + \frac{1}{2} \partial_{\nu} [G_{\mu,a}] \partial_{\mu} [G_{\nu,a}] - \frac{1}{4} \partial_{\mu} [G_{\nu,a}]^2 - \frac{1}{2} M g_0 g_{0_{i1} g_{0_{i2}}} \cdot g_{0_{i1} g_{0_{i2}}} + \\
 & \frac{1}{4} g_s \partial_{\nu} [G_{\mu,a}] f_{a,bb} G_{\mu,bb} G_{\nu,cc} - \frac{1}{4} g_s \partial_{\mu} [G_{\nu,a}] f_{a,bb} G_{\mu,bb} G_{\nu,cc} + \\
 & \frac{1}{4} g_s \partial_{\nu} [G_{\mu,a}] f_{a,bb} G_{\mu,bb} G_{\nu,cc} - \frac{1}{4} g_s \partial_{\mu} [G_{\nu,a}] f_{a,bb} G_{\mu,bb} G_{\nu,cc} - \\
 & \frac{1}{4} g_s^2 f_{a,bb} f_{a,bb} G_{\mu,bb} G_{\mu,bb} G_{\nu,cc} G_{\nu,cc} + \\
 & \frac{1}{4} i g_{0_{i1} g_{0_{i2}}} \cdot \partial_{\mu} [g_{0_{j1} g_{0_{j2}}} \gamma_{i1}^{\mu}] - \\
 & \frac{1}{4} i g_s g_{0_{i1} g_{0_{i2}}} \cdot g_{0_{j1} g_{0_{j2}}} f_{a,bb} G_{\mu,bb} \gamma_{i1}^{\mu} - \\
 & \frac{1}{4} i \partial_{\mu} [g_{0_{j1} g_{0_{j2}}} \cdot g_{0_{i1} g_{0_{i2}}} \gamma_{j1}^{\mu}] + \frac{1}{4} i g_s g_{0_{j1} g_{0_{j2}}} \cdot g_{0_{i1} g_{0_{i2}}} f_{a,bb} G_{\mu,bb} \gamma_{j1}^{\mu}
 \end{aligned}$$

- Kinetic terms (gluon and gluino)
- Mass terms (gluino)
- QCD interactions (with the proper structure constants)

# Checks: Hermiticity

## Textbook expression

$$\mathcal{L}_{\text{vector}} = -\frac{1}{4}g_{\mu\nu}g^{\mu\nu} + \frac{i}{2}\bar{g}\not{D}g - \frac{1}{2}m_{\tilde{g}}\bar{g}g$$

## FEYNRULES implementation

```
LVector1 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
            I/2 gobar.Ga[mu].DC[go,mu] -
            1/2 Mgo gobar.go;
```

## The Lagrangian must be Hermitian

- Feynman rules of the Lagrangian minus its Hermitian conjugate  $\rightarrow \mathcal{L} - \mathcal{L}^\dagger$

```
In[8]:= CheckHermiticity[LVector1];
```

Checking for hermiticity by calculating the Feynman rules contained in L-HC[L].

If the lagrangian is hermitian, then the number of vertices should be zero.

**Starting Feynman rule calculation.**

Expanding the Lagrangian...

No vertices found.

0 vertices obtained.

The lagrangian is hermitian.

# Checks: normalisation

## Textbook expression

$$\mathcal{L}_{\text{vector}} = -\frac{1}{4}g_{\mu\nu}g^{\mu\nu} + \frac{i}{2}\bar{g}\not{D}g - \frac{1}{2}m_{\tilde{g}}\bar{g}g$$

## FEYNRULES implementation

```
LVector1 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
            I/2 gobar.Ga[mu].DC[go,mu] -
            1/2 Mgo gobar.go;
```

## Kinetic and mass terms canonically normalised

- Normalisation of the quadratic terms (kinetic and mass terms)
- Absence of non-diagonal quadratic terms

```
In[9]:= CheckKineticTermNormalisation[LVector1];
Neglecting all terms with more than 2 particles.
All kinetic terms are diagonal.
All kinetic terms are correctly normalized.
```

```
In[11]:= CheckDiagonalMassTerms[LVector1];
All mass terms are diagonal.
```

- Other checks: *CheckDiagonalQuadraticTerms*, *CheckDiagonalKineticTerms*

# Checks: the mass spectrum

## FEYNRULES gluino implementation

```
F[1] == {
  ClassName      -> go,
  WeylComponents -> gow,
  SelfConjugate  -> True,
  Indices        -> {Index[Gluon]},
  Mass           -> {Mgo,500},
  Width          -> {Wgo,10},
  PDG            -> 1000021
},
```

## FEYNRULES Lagrangian implementation

```
LVector1 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
            I/2 gobar.Ga[mu].DC[go,mu] -
            1/2 Mgo gobar.go;
```

## Consistent mass information (numerically)

- Masses extracted from the Lagrangian
- Masses fixed in particle declarations

```
In[12]:= CheckMassSpectrum[LVector1]
```

```
Neglecting all terms with more than 2 particles.
```

```
All mass terms are diagonal.
```

```
Getting mass spectrum.
```

```
Checking for less than 0.1% agreement with model file values.
```

```
Out[12]//TableForm=
```

| Particle | Analytic value | Numerical value | Model-file value |
|----------|----------------|-----------------|------------------|
| go       | Mgo            | 500.            | 500.             |

# Feynman rules

Extract all  $N$ -point interactions from the Lagrangian (with  $N > 2$ )

```
In[14]:= FeynmanRules[LVector1, ScreenOutput -> True];
```

```
Starting Feynman rule calculation.
```

```
Expanding the Lagrangian...
```

```
Collecting the different structures that enter the vertex.
```

```
3 possible non-zero vertices have been found -> starting the computation: 3 / 3.
```

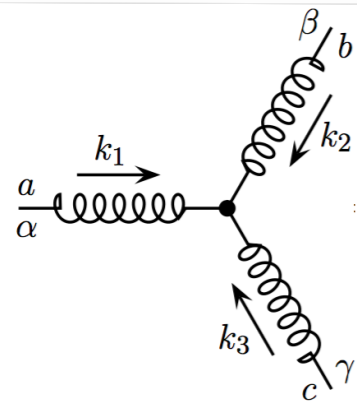
```
3 vertices obtained.
```

```
(* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *)
```

- **Three vertices** found in the Lagrangian
  - ★ One triple and quartic gluon interactions
  - ★ One gluino-gluon interaction

# The triple gluon vertex

## Literature



$$g_s f_{abc} \left[ (k_1 - k_2)^\gamma \eta^{\alpha\beta} + (k_3 - k_1)^\beta \eta^{\gamma\alpha} + (k_2 - k_3)^\alpha \eta^{\beta\gamma} \right]$$

[Incoming momenta]

## FEYNRULES

Vertex 1

Particle 1 : Vector , G

Particle 2 : Vector , G

Particle 3 : Vector , G

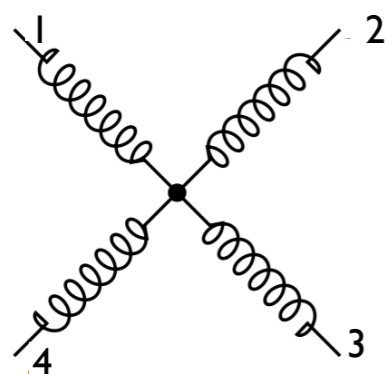
Vertex:

$$g_s f_{a_1, a_2, a_3} p_1^{\mu_3} \eta_{\mu_1, \mu_2} - g_s f_{a_1, a_2, a_3} p_2^{\mu_3} \eta_{\mu_1, \mu_2} - g_s f_{a_1, a_2, a_3} p_1^{\mu_2} \eta_{\mu_1, \mu_3} + \\ g_s f_{a_1, a_2, a_3} p_3^{\mu_2} \eta_{\mu_1, \mu_3} + g_s f_{a_1, a_2, a_3} p_2^{\mu_1} \eta_{\mu_2, \mu_3} - g_s f_{a_1, a_2, a_3} p_3^{\mu_1} \eta_{\mu_2, \mu_3}$$

- Colour: index  $a_i$  related to the  $i^{\text{th}}$  particle ( $a$  = style for adjoint  $SU(3)_C$  indices)
- Spin: index  $\mu_i$  = Lorentz index of the  $i^{\text{th}}$  (vector) particle

# The quartic gluon vertex

## Literature



$$\begin{aligned}
 & ig_s^2 f^{a_1 a_2 b} f^{b a_3 a_4} (\eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3} - \eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4}) \\
 & + ig_s^2 f^{a_1 a_3 b} f^{b a_2 a_4} (\eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3} - \eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4}) \\
 & + ig_s^2 f^{a_1 a_4 b} f^{b a_2 a_3} (\eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4} - \eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4})
 \end{aligned}$$

## FEYNRULES

Vertex 2

Particle 1 : Vector , G

Particle 2 : Vector , G

Particle 3 : Vector , G

Particle 4 : Vector , G

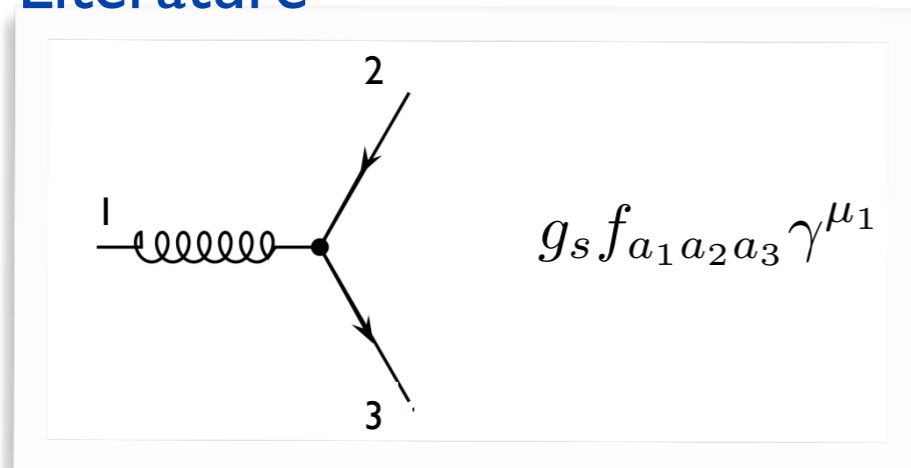
Vertex:

$$\begin{aligned}
 & i g_s^2 ( f_{a_1, a_3, \text{Gluon}\$1} f_{a_2, a_4, \text{Gluon}\$1} \eta_{\mu_1, \mu_4} \eta_{\mu_2, \mu_3} + f_{a_1, a_2, \text{Gluon}\$1} f_{a_3, a_4, \text{Gluon}\$1} \eta_{\mu_1, \mu_4} \eta_{\mu_2, \mu_3} + \\
 & f_{a_1, a_4, \text{Gluon}\$1} f_{a_2, a_3, \text{Gluon}\$1} \eta_{\mu_1, \mu_3} \eta_{\mu_2, \mu_4} - f_{a_1, a_2, \text{Gluon}\$1} f_{a_3, a_4, \text{Gluon}\$1} \eta_{\mu_1, \mu_3} \eta_{\mu_2, \mu_4} - \\
 & f_{a_1, a_4, \text{Gluon}\$1} f_{a_2, a_3, \text{Gluon}\$1} \eta_{\mu_1, \mu_2} \eta_{\mu_3, \mu_4} - f_{a_1, a_3, \text{Gluon}\$1} f_{a_2, a_4, \text{Gluon}\$1} \eta_{\mu_1, \mu_2} \eta_{\mu_3, \mu_4} )
 \end{aligned}$$

- Colour:  $a_i$  related to the  $i^{\text{th}}$  particle (**Gluon\$I summed over**)
- Spin:  $\mu_i$  related to the  $i^{\text{th}}$  (vector) particle

# Gluon and gluino interactions

## Literature



## FEYNRULES

Vertex 3

Particle 1 : Majorana , go

Particle 2 : Majorana , go

Particle 3 : Vector , G

Vertex:

$g_s f_{a_1, a_2, a_3} \gamma_{s_1, s_2}^{\mu_3}$

- Colour:  $a_i$  related to the  $i^{\text{th}}$  particle
- Spin:  $\mu_i$  related to the  $i^{\text{th}}$  vector;  $s_i$  related to the  $i^{\text{th}}$  fermion



# The FeynmanRules function

The function `FeynmanRules` has many options

- Restriction on the interactions to display: `MaxParticles`, `MaxCanonicalDimension`, etc.
- Selection of specific particles: `Free`, `Contains`, etc.
- `ScreenOutput`: displaying the vertices to the screen or not
- `FlavorExpand`: perform a flavour expansion (otherwise classes used)

See the manual for more details on the function `FeynmanRules`

## The matter Lagrangian

# The SUSY-QCD matter sector

## Matter fields

- **Two matter supermultiplets** in the fundamental representation of  $SU(3)_c$ 
  - ★ One massive Dirac fermion: a **quark**
  - ★ Two mixing massive scalar fields: two **squark**
  - ★ Gauge coupling to the  **$SU(3)_c$  gauge supermultiplet**

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}$$

## Lagrangian:

$$\begin{aligned} \mathcal{L}_{\text{matter}} = & D_\mu \tilde{q}_L^\dagger D^\mu \tilde{q}_L + D_\mu \tilde{q}_R^\dagger D^\mu \tilde{q}_R + i\bar{q}\not{D}q - m_{\tilde{q}_i}^2 \tilde{q}_i^\dagger \tilde{q}_i - m_q \bar{q}q \\ & - \frac{g_s^2}{2} \left[ -\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R \right] \left[ -\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R \right] \\ & + \sqrt{2}g_s \left[ -\tilde{q}_L^\dagger T^a (\bar{g}^a P_L q) + (\bar{q} P_L \tilde{g}^a) T^a \tilde{q}_R \right] + \text{h.c.} \end{aligned}$$

- **Mixed use of mass / gauge bases** (makes it easier to implement)
- Kinetic terms (first three terms) and D-terms (second line) in the gauge basis
- Mass terms (end of first line) in the mass basis
- SUSY gauge quark-squark-gluino interactions (fourth line) in the gauge basis

# Kinetic and gauge interactions

Kinetic and gauge interactions, as well as mass terms

$$\mathcal{L}_{\text{matter}} = D_{\mu} \tilde{q}_L^{\dagger} D^{\mu} \tilde{q}_L + D_{\mu} \tilde{q}_R^{\dagger} D^{\mu} \tilde{q}_R + i \bar{q} \not{D} q - m_{\tilde{q}_i}^2 \tilde{q}_i^{\dagger} \tilde{q}_i - m_q \bar{q} q$$

The implementation in FEYNRULES follows the textbook expression

- Shortcut functions (DC)
- Very compact implementation
- Repeated indices summed

```
Lkin := DC[sqLbar[cc],mu] DC[sqL[cc],mu] +
        DC[sqRbar[cc],mu] DC[sqR[cc],mu] +
        I qbar.Ga[mu].DC[q,mu] -
        Mq qbar.q -
        Msq1^2 sq1bar[cc] sq1[cc] -
        Msq2^2 sq2bar[cc] sq2[cc];
```

**Gauge eigenstates are used**

**Mass eigenstates are used**

# Kinetic term Feynman rules

## Feynman rules

```
In[18]:= Simplify[FeynmanRules[Lkin]] // MatrixForm
```

Starting Feynman rule calculation.

Expanding the Lagrangian...

Collecting the different structures that enter the vertex.

9 possible non-zero vertices have been found -> starting the computation: 9 / 9.

5 vertices obtained.

```
Out[18]/MatrixForm=
```

$$\left( \begin{array}{l} \{ \{G, 1\}, \{sq1, 2\}, \{sq1^\dagger, 3\} \} \quad i \, g_s \, (p_2^{\mu 1} - p_3^{\mu 1}) \, T_{m_3, m_2}^{a_1} \\ \{ \{G, 1\}, \{sq2, 2\}, \{sq2^\dagger, 3\} \} \quad i \, g_s \, (p_2^{\mu 1} - p_3^{\mu 1}) \, T_{m_3, m_2}^{a_1} \\ \{ \{G, 1\}, \{G, 2\}, \{sq1, 3\}, \{sq1^\dagger, 4\} \} \quad i \, g_s^2 \, \eta_{\mu_1, \mu_2} \, (T_{m_4, \text{Colour}\$1}^{a_1} T_{\text{Colour}\$1, m_3}^{a_2} + T_{\text{Colour}\$1, m_3}^{a_1} T_{m_4, \text{Colour}\$1}^{a_2}) \\ \{ \{G, 1\}, \{G, 2\}, \{sq2, 3\}, \{sq2^\dagger, 4\} \} \quad i \, g_s^2 \, \eta_{\mu_1, \mu_2} \, (T_{m_4, \text{Colour}\$1}^{a_1} T_{\text{Colour}\$1, m_3}^{a_2} + T_{\text{Colour}\$1, m_3}^{a_1} T_{m_4, \text{Colour}\$1}^{a_2}) \\ \{ \{ \bar{q}, 1 \}, \{q, 2\}, \{G, 3\} \} \quad i \, g_s \, \gamma_{s_1, s_2}^{\mu_3} \, T_{m_1, m_2}^{a_3} \end{array} \right)$$

- Field rotations have been performed automatically
  - ★ No more gauge eigenstates (sqL, sqR)
- All squark and quark QCD interactions derived
  - ★ The 3-point qqg interaction
  - ★ Two 3-point squark-gluon interactions
  - ★ Two 4-point squark-gluon interactions

# D-terms

## D-terms (in the gauge eigenbasis)

$$\mathcal{L}_{\text{matter}} = -\frac{g_s^2}{2} \left[ -\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R \right] \left[ -\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R \right]$$

The FEYNRULES implementation follows the textbook expression

```
LD := -1/2 gs^2 *
      (sqRbar[cc1] T[a,cc1,cc2] sqR[cc2] - sqLbar[cc1] T[a,cc1,cc2] sqL[cc2]) *
      (sqRbar[cc3] T[a,cc3,cc4] sqR[cc4] - sqLbar[cc3] T[a,cc3,cc4] sqL[cc4]);
```

- Repeated indices ( $cc1, cc2, cc3, cc4$ ) summed
- **A single index can only be used twice**

# D-term Feynman rules

## D-terms (in the gauge eigenbasis)

$$\mathcal{L}_{\text{matter}} = -\frac{g_s^2}{2} \left[ -\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R \right] \left[ -\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R \right]$$

## Feynman rules computation

```
Simplify[FeynmanRules[LD]] // MatrixForm
```

$$\begin{pmatrix} \{\{sq1, 1\}, \{sq1, 2\}, \{sq1^\dagger, 3\}, \{sq1^\dagger, 4\}\} & -i g_s^2 \text{Cos}[2 \text{theta}]^2 (T_{m_3, m_2}^{\text{Gluon} \$1} T_{m_4, m_1}^{\text{Gluon} \$1} + T_{m_3, m_1}^{\text{Gluon} \$1} T_{m_4, m_2}^{\text{Gluon} \$1}) \\ \{\{sq1, 1\}, \{sq1^\dagger, 2\}, \{sq1^\dagger, 3\}, \{sq2, 4\}\} & \frac{1}{2} i g_s^2 \text{Sin}[4 \text{theta}] (T_{m_2, m_4}^{\text{Gluon} \$1} T_{m_3, m_1}^{\text{Gluon} \$1} + T_{m_2, m_1}^{\text{Gluon} \$1} T_{m_3, m_4}^{\text{Gluon} \$1}) \\ \{\{sq1^\dagger, 1\}, \{sq1^\dagger, 2\}, \{sq2, 3\}, \{sq2, 4\}\} & -i g_s^2 \text{Sin}[2 \text{theta}]^2 (T_{m_1, m_4}^{\text{Gluon} \$1} T_{m_2, m_3}^{\text{Gluon} \$1} + T_{m_1, m_3}^{\text{Gluon} \$1} T_{m_2, m_4}^{\text{Gluon} \$1}) \\ \{\{sq1, 1\}, \{sq1, 2\}, \{sq1^\dagger, 3\}, \{sq2^\dagger, 4\}\} & \frac{1}{2} i g_s^2 \text{Sin}[4 \text{theta}] (T_{m_3, m_2}^{\text{Gluon} \$1} T_{m_4, m_1}^{\text{Gluon} \$1} + T_{m_3, m_1}^{\text{Gluon} \$1} T_{m_4, m_2}^{\text{Gluon} \$1}) \\ \{\{sq1, 1\}, \{sq1^\dagger, 2\}, \{sq2, 3\}, \{sq2^\dagger, 4\}\} & -i g_s^2 (\text{Sin}[2 \text{theta}]^2 T_{m_2, m_3}^{\text{Gluon} \$1} T_{m_4, m_1}^{\text{Gluon} \$1} - \text{Cos}[2 \text{theta}]^2 T_{m_2, m_1}^{\text{Gluon} \$1} T_{m_4, m_3}^{\text{Gluon} \$1}) \\ \{\{sq1^\dagger, 1\}, \{sq2, 2\}, \{sq2, 3\}, \{sq2^\dagger, 4\}\} & -\frac{1}{2} i g_s^2 \text{Sin}[4 \text{theta}] (T_{m_1, m_3}^{\text{Gluon} \$1} T_{m_4, m_2}^{\text{Gluon} \$1} + T_{m_1, m_2}^{\text{Gluon} \$1} T_{m_4, m_3}^{\text{Gluon} \$1}) \\ \{\{sq1, 1\}, \{sq1, 2\}, \{sq2^\dagger, 3\}, \{sq2^\dagger, 4\}\} & -i g_s^2 \text{Sin}[2 \text{theta}]^2 (T_{m_3, m_2}^{\text{Gluon} \$1} T_{m_4, m_1}^{\text{Gluon} \$1} + T_{m_3, m_1}^{\text{Gluon} \$1} T_{m_4, m_2}^{\text{Gluon} \$1}) \\ \{\{sq1, 1\}, \{sq2, 2\}, \{sq2^\dagger, 3\}, \{sq2^\dagger, 4\}\} & -\frac{1}{2} i g_s^2 \text{Sin}[4 \text{theta}] (T_{m_3, m_2}^{\text{Gluon} \$1} T_{m_4, m_1}^{\text{Gluon} \$1} + T_{m_3, m_1}^{\text{Gluon} \$1} T_{m_4, m_2}^{\text{Gluon} \$1}) \\ \{\{sq2, 1\}, \{sq2, 2\}, \{sq2^\dagger, 3\}, \{sq2^\dagger, 4\}\} & -i g_s^2 \text{Cos}[2 \text{theta}]^2 (T_{m_3, m_2}^{\text{Gluon} \$1} T_{m_4, m_1}^{\text{Gluon} \$1} + T_{m_3, m_1}^{\text{Gluon} \$1} T_{m_4, m_2}^{\text{Gluon} \$1}) \end{pmatrix}$$

- All nine vertices automatically derived from the (very compact) Lagrangian

# SUSY-gauge squark-quark-gluino couplings

The SUSY gauge gluino-quark-squark interactions (in the gauge basis)

$$\mathcal{L}_{\text{matter}} = \sqrt{2}g_s \left[ -\tilde{q}_L^\dagger T^a (\tilde{g}^a P_L q) + (\bar{q} P_L \tilde{g}^a) T^a \tilde{q}_R \right] + \text{h.c.}$$

The FEYNRULES again following the textbook expression

```
Lgosqq := Sqrt[2] gs ProjM[s1,s2] * (
  - sqLbar[cc1] T[a,cc1,cc2] gobar[s1,a].q[s2,cc2] +
  qbar[s1,cc1].go[s2,a] T[a,cc1,cc2] sqR[cc2]);
```

- **All indices explicit** (scalars not included in fermion chains)
- *ProjM*  $\equiv$  left-handed chirality projector (*ProjP* right-handed)
- The dot connects the elements of a fermion chain
  - ★ Remark: *ProjM*(s1,s2) is a scalar object



# Feynman rules from the D-terms

The SUSY gauge gluino-quark-squark interactions (in the gauge basis)

$$\mathcal{L}_{\text{matter}} = \sqrt{2}g_s \left[ -\tilde{q}_L^\dagger T^a (\tilde{g}^a P_L q) + (\bar{q} P_L \tilde{g}^a) T^a \tilde{q}_R \right] + \text{h.c.}$$

## Feynman rules computation

- Including the Hermitian conjugate pieces with HC

```
In[19]:= Simplify[FeynmanRules[Lgosqq + HC[Lgosqq]]] // MatrixForm
```

**Starting Feynman rule calculation.**

Expanding the Lagrangian...

Collecting the different structures that enter the vertex.

4 possible non-zero vertices have been found -> starting the computation: 4 / 4.

4 vertices obtained.

```
Out[19]//MatrixForm=
```

$$\left( \begin{array}{l} \left\{ \left\{ \bar{q}, 1 \right\}, \left\{ g_0, 2 \right\}, \left\{ sq1, 3 \right\} \right\} \quad -i \sqrt{2} g_s \left( \text{Cos}[\text{theta}] P_{+s_1, s_2} - P_{-s_1, s_2} \text{Sin}[\text{theta}] \right) T_{m_1, m_3}^{a2} \\ \left\{ \left\{ g_0, 1 \right\}, \left\{ q, 2 \right\}, \left\{ sq1^\dagger, 3 \right\} \right\} \quad -i \sqrt{2} g_s \left( \text{Cos}[\text{theta}] P_{-s_1, s_2} - P_{+s_1, s_2} \text{Sin}[\text{theta}] \right) T_{m_3, m_2}^{a1} \\ \left\{ \left\{ \bar{q}, 1 \right\}, \left\{ g_0, 2 \right\}, \left\{ sq2, 3 \right\} \right\} \quad i \sqrt{2} g_s \left( \text{Cos}[\text{theta}] P_{-s_1, s_2} + P_{+s_1, s_2} \text{Sin}[\text{theta}] \right) T_{m_1, m_3}^{a2} \\ \left\{ \left\{ g_0, 1 \right\}, \left\{ q, 2 \right\}, \left\{ sq2^\dagger, 3 \right\} \right\} \quad i \sqrt{2} g_s \left( \text{Cos}[\text{theta}] P_{+s_1, s_2} + P_{-s_1, s_2} \text{Sin}[\text{theta}] \right) T_{m_3, m_2}^{a1} \end{array} \right)$$

**To phenomenology**

# From FEYNRULES to phenomenology

The SUSY-QCD model has been implemented into FEYNRULES

```
LMatter := Lkin + LD + Lgosqq + HC[Lgosqq];  
LSUSYQCD := LVector1 + LMatter;
```

The Feynman rules have been extracted (and checked)

Ready to export the model to MC tools

- CALCHEP / COMPHEP
- FEYNARTS / FORMCALC
- UFO: MADGRAPH5\_AMC@NLO / SHERPA / HERWIG++ / WHIZARD

```
WriteCHOutput[{LVector1, LMatter}];  
WriteFeynArtsOutput[{LVector1, LMatter}];  
WriteUFO[{LVector1, LMatter}];
```

# Limitations and fineprints

## Particle / parameter names

- **The strong interaction has a special role**
  - ★ Name for the gluon field, the coupling constant, etc.
  - ★ Which parameter is internal/external
  - ★ The numerical value of  $\alpha_s$  at the Z-pole
- **For some generators, the electroweak interaction has also a special role**
  - ★ Name for the Fermi coupling, the Z-boson mass
  - ★ Which parameter is external/internal
  - ★ At which scale must the numerical values be given

## Colour structures: up to octets in FEYNRULES, not in MC generators

- **Interfaces discard non-supported vertices**

## Lorentz structures/spins: up to spin-2 in FEYNRULES, not in MC generators

- **Interfaces discard non-supported vertices**

# Outline

1. FEYNRULES in a nutshell
2. Implementing supersymmetric QCD in FEYNRULES
3. Using FEYNRULES with supersymmetric QCD model
- 4. Summary**
5. Appendix: advanced model implementation techniques

# Advanced techniques

- More about UFOs
- Extension / restriction of existing models
- The superspace module of FEYNRULES
- Mass diagonalisation
- Two-body decays
- Next-to-leading order module

# Summary

## The quest for new physics at the LHC

- Relies on **event generators** for background and signal modelling
- FEYNRULES facilitates the implementation of new physics models

## FEYNRULES: <http://feynrules.irmp.ucl.ac.be>

- **Straightforward implementation** of new physics model in Monte Carlo tools
  - ★ Interfaces to many programs
- FEYNRULES shipped with its **own computational modules**
  - ★ A superspace module
  - ★ A decay package
  - ★ A mass diagonalisation module (ASPERGE)
  - ★ An NLO module

Try it on with your favorite model!

# Outline

1. FEYNRULES in a nutshell
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# Advanced techniques for FEYNRULES implementation

- ◆ More about the UFO
- ◆ Extension / restriction of existing models
- ◆ The superspace module of FEYNRULES
- ◆ Mass diagonalization
- ◆ Two-body decays
- ◆ Next-to-leading order module

## The UFO

# A step further: the Universal FEYNRULES Output

## ◆ The UFO in a nutshell

[ Degrande, Duhr, BF, Grellscheid, Mattelaer, Reiter (CPC '12) ]  
[ Degrande, Duhr, BF, Hirschi, Mattelaer, Shao et al. (in prep.) ]

- ❖ UFO  $\equiv$  Universal FEYNRULES output
  - ★ **Universal** as not tied to any specific Monte Carlo program
- ❖ Consists of a set of **PYTHON files** to be linked to any code
- ❖ It contains **all the model information**
  - ★ Allows the models to contain **generic color and Lorentz** structures
- ❖ Can be employed for next-to-leading order calculations

## ◆ The UFO is now a standard and used by many other programs

ALOHA

GOSAM

HERWIG ++

MADANALYSIS 5

SHERPA

MADGRAPH5\_aMC@NLO

WHIZARD

LANHEP

SARAH

# The UFO in practice

## ◆ The UFO is a set of PYTHON files

- ❖ Factorization of the information: particles, interactions, propagation, parameters, NLO, etc.

## ◆ Example

```
[fuchs@Benjamins-MacBook-Pro-3 ~/Work/tools/FeynRules/trunk/models/SUSYQCD_UFO$] ls
CT_couplings.py      SUSYQCD_UFO.log      couplings.py          object_library.py     propagators.py
CT_parameters.py     __init__.py          function_library.py  parameters.py          vertices.py
CT_vertices.py       coupling_orders.py   lorentz.py           particles.py           write_param_card.py
[fuchs@Benjamins-MacBook-Pro-3 ~/Work/tools/FeynRules/trunk/models/SUSYQCD_UFO$]
```

NLO

Interactions

Particles

Parameters

Propagators

# Particles

## ◆ Particles are stored in the `particles.py` file

- ❖ Instances of the particle class
- ❖ Attributes: particle spin, color representation, mass, width, PDG code, etc.
- ❖ Antiparticles automatically derived

```
G = Particle(pdg_code = 21,
             name = 'G',
             antiname = 'G',
             spin = 3,
             color = 8,
             mass = Param.ZERO,
             width = Param.ZERO,
             texname = 'G',
             antitexname = 'G',
             charge = 0)
```

```
go = Particle(pdg_code = 1000021,
              name = 'go',
              antiname = 'go',
              spin = 2,
              color = 8,
              mass = Param.Mgo,
              width = Param.Wgo,
              texname = 'go',
              antitexname = 'go',
              charge = 0)
```

```
sq1 = Particle(pdg_code = 1000006,
               name = 'sq1',
               antiname = 'sq1~',
               spin = 1,
               color = 3,
               mass = Param.Msq1,
               width = Param.Wsq1,
               texname = 'sq1',
               antitexname = 'sq1~',
               charge = 0)
```

```
sq1__tilde__ = sq1.anti()
```

```
sq2 = Particle(pdg_code = 2000006,
               name = 'sq2',
               antiname = 'sq2~',
               spin = 1,
               color = 3,
               mass = Param.Msq2,
               width = Param.Wsq2,
               texname = 'sq2',
               antitexname = 'sq2~',
               charge = 0)
```

```
sq2__tilde__ = sq2.anti()
```

```
q = Particle(pdg_code = 6,
             name = 'q',
             antiname = 'q~',
             spin = 2,
             color = 3,
             mass = Param.Mq,
             width = Param.Wq,
             texname = 'q',
             antitexname = 'q~',
             charge = 0)
```

```
q__tilde__ = q.anti()
```

# Parameters

## ◆ Parameters are stored in the `parameters.py` file

- ❖ Instances of the parameter class
- ❖ External parameters are organized following a Les Houches-like structure (blocks and counters)
- ❖ PYTHON-compliant formula for the internal parameters

```
aS = Parameter(name = 'aS',
               nature = 'external',
               type = 'real',
               value = 0.1184,
               texname = '\\alpha_s',
               lhablock = 'SMINPUTS',
               lhacode = [ 3 ])

G = Parameter(name = 'G',
              nature = 'internal',
              type = 'real',
              value = '2*cmath.sqrt(aS)*cmath.sqrt(cmath.pi)',
              texname = 'G')
```

```
Mgo = Parameter(name = 'Mgo',
                nature = 'external',
                type = 'real',
                value = 500,
                texname = '\\text{Mgo}',
                lhablock = 'MASS',
                lhacode = [ 1000021 ])

Wq = Parameter(name = 'Wq',
               nature = 'external',
               type = 'real',
               value = 1.50833649,
               texname = '\\text{Wq}',
               lhablock = 'DECAY',
               lhacode = [ 6 ])
```

# Interactions: generalities

◆ Vertices decomposed in a **spin x color** basis (coupling strengths  $\equiv$  coordinates)

♣ Example: the quartic gluon vertex can be written as

$$\begin{aligned}
 & ig_s^2 f^{a_1 a_2 b} f^{b a_3 a_4} (\eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3} - \eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4}) \\
 & + ig_s^2 f^{a_1 a_3 b} f^{b a_2 a_4} (\eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3} - \eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4}) \\
 & + ig_s^2 f^{a_1 a_4 b} f^{b a_2 a_3} (\eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4} - \eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4})
 \end{aligned}
 \Rightarrow
 \begin{aligned}
 & (f^{a_1 a_2 b} f^{b a_3 a_4}, f^{a_1 a_3 b} f^{b a_2 a_4}, f^{a_1 a_4 b} f^{b a_2 a_3}) \\
 & \times \begin{pmatrix} ig_s^2 & 0 & 0 \\ 0 & ig_s^2 & 0 \\ 0 & 0 & ig_s^2 \end{pmatrix} \begin{pmatrix} \eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3} - \eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4} \\ \eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3} - \eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4} \\ \eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4} - \eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4} \end{pmatrix}
 \end{aligned}$$

- ★ 3 elements for the color basis
- ★ 3 elements for the spin (Lorentz structure) basis
- ★ 9 coordinates (6 are zero)

♣ Several files are used for the storage of the information

# Example: the quartic gluon vertex

## ◆ General information in vertex.py

```
V_2 = Vertex(name = 'V_2',
             particles = [ P.G, P.G, P.G, P.G ],
             color = [ 'f(-1,1,2)*f(3,4,-1)', 'f(-1,1,3)*f(2,4,-1)', 'f(-1,1,4)*f(2,3,-1)' ],
             lorentz = [ L.VVVV1, L.VVVV2, L.VVVV3 ],
             couplings = {(1,1):C.GC_4, (0,0):C.GC_4, (2,2):C.GC_4})
```

- ★ **lorentz**  $\equiv$  spin basis  
(in lorentz.py; common to all vertices)
- ★ **color**  $\equiv$  color basis
- ★ **couplings**  $\equiv$  coordinates  
(in couplings.py; common to all vertices)

$$\left( f^{a_1 a_2 b} f^{b a_3 a_4}, f^{a_1 a_3 b} f^{b a_2 a_4}, f^{a_1 a_4 b} f^{b a_2 a_3} \right) \times \begin{pmatrix} ig_s^2 & 0 & 0 \\ 0 & ig_s^2 & 0 \\ 0 & 0 & ig_s^2 \end{pmatrix} \begin{pmatrix} \eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3} - \eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4} \\ \eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3} - \eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4} \\ \eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4} - \eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4} \end{pmatrix}$$

## ◆ Lorentz structures: straightforward implementations in lorentz.py

```
VVVV1 = Lorentz(name = 'VVVV1',
                spins = [ 3, 3, 3, 3 ],
                structure = 'Metric(1,4)*Metric(2,3) - Metric(1,3)*Metric(2,4)')
```

## ◆ Couplings: straightforward implementations in couplings.py

```
GC_4 = Coupling(name = 'GC_4',
                value = 'complex(0,1)*G**2',
                order = {'QCD':2})
```

Coupling orders: for selecting diagrams





## Extending models

# Merging and extending models (I)

## ◆ Many BSM models of interest are simple extensions of another model

### ♣ FEYNRULES allows one to start from a given model

- ★ Add new particles, parameters, Lagrangian terms
- ★ Modify existing particles, parameters, Lagrangian terms
- ★ Remove some particles, parameters, Lagrangian terms

## ◆ Simplified models

### ♣ Special cases very relevant for LHC physics: Simplified Model Spectra

- ★ The Standard Model + one or two new particles
- ★ Often inspired by the MSSM or dark matter models
- ★ Ex.: the SM + lightest stop and neutralino + relevant subset of MSSM interactions

# Merging and extending models (2)

## ◆ Merged FEYNRULES model contains two .fr files

- ✿ The parent model implementation
- ✿ One extra file with the modifications
- ✿ They must be loaded together (the parent model first)

```
LoadModel["SM.fr", "stops.fr"];|
```

★ No need to re-implement what is common (gauge groups, etc.)

## ◆ One can start from the models available on the FEYNRULES database

<http://feynrules.irmp.ucl.ac.be>

# The FEYNRULES model database

## ◆ $O(100)$ models are available online

### ✿ Simple extensions of the Standard Model

- ★ Simplified model spectra
- ★ Four generation models
- ★ Vector-like quarks
- ★ Two-Higgs-Doublet Models, Hidden Abelian Higgs
- ★ *etc.*

### ✿ Supersymmetric models

- ★ MSSM with and without R-parity
- ★ The NMSSM
- ★ R-symmetric supersymmetric models
- ★ Left-right supersymmetric models

### ✿ Extra-dimensional models

- ★ Universal extra-dimensions
- ★ Large extra-dimensions
- ★ Heidi, Minimal Higgsless models
- ★ Randall-Sundrum

### ✿ Strongly coupled and effective field theories

- ★ Technicolor
- ★ Models with dimension-six and dimension-eight operators
- ★ *etc.*

# Restricting model implementations

- ◆ Many BSM models of interest are subset of other models
  - ❖ Equivalent to the parent model, with some parameters set to 0 or 1
    - ★ Example 1: the massless version of a model (massless light quarks in the SM)
    - ★ Example 2: a mixing matrix set to the identity (no-CKM matrix in the SM)
  - ❖ FEYNRULES allows one to start from a given model
    - ★ Write the restrictions under the form of a list of MATHEMATICA replacement rules
      - `M$Restrictions`
    - ★ Read them into FEYNRULES
    - ★ Apply them before the computation of any Feynman rule

- ◆ The output Feynman rules (and thus Monte Carlo model files)
  - ❖ Are free from the restricted parameters
  - ❖ Smaller files, more efficiency at the MC level
    - ★ Ex.: the MSSM has more than 10000 vertices; its flavor-conserving version ~1000

# Example of restrictions

## ◆ One practical example: the Standard Model without CKM-mixing

```
M$Restrictions = {  
    CKM[i_,i_] -> 1,  
    CKM[i_?NumericQ, j_?NumericQ] :> 0 /; (i != j),  
    cabi -> 0  
}
```

```
LoadRestriction["DiagonalCKM.rst"]
```

## Superspace module

**See the manual for more details on the superspace module**

# Superfield declaration

- ◆ A module dedicated to calculations in superspace
  - ♣ Superfield declaration and links to the component fields

## A left-handed squark superfield

```
CSF[1] == {  
  ClassName      -> QL,  
  Chirality      -> Left,  
  Weyl          -> qLw,  
  Scalar        -> sqL,  
  Indices       -> {Index[Colour]}  
}
```

- ★ Component fields to be declared too
- ★ Vector superfields can be linked to gauge groups



# Supersymmetric model implementation

## ◆ Supersymmetric model implementation

- ❖ Declaration of the model **gauge group**
- ❖ Declaration of all **fields and superfields**
- ❖ Declaration of all model **parameters**
- ❖ **Writing the Lagrangian (in a simplified way)**

## ◆ Supersymmetric Lagrangian in superspace are very compact

$$\mathcal{L} = \Phi^\dagger e^{-2gV} \Phi|_{\theta^2\bar{\theta}^2} + \frac{1}{16g^2\tau_{\mathcal{R}}} \text{Tr}(W^\alpha W_\alpha)|_{\theta^2} + \frac{1}{16g^2\tau_{\mathcal{R}}} \text{Tr}(\bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}})|_{\bar{\theta}^2} \\ + W(\Phi)|_{\theta^2} + W^*(\Phi^\dagger)|_{\bar{\theta}^2} + \mathcal{L}_{\text{soft}}$$

- ❖ First line: kinetic and gauge interaction terms for all fields
  - **Model independent**  $\Rightarrow$  can be automated
- ❖ Second line: superpotential and supersymmetry breaking Lagrangian
  - **Model dependent**  $\Rightarrow$  to be provided
- ❖ Series expansion in terms of component fields
- ❖ Automatic **derivation of supersymmetric Lagrangians**
- ❖ Solving the equations of motion of the unphysical fields

# Implementing supersymmetric Lagrangians

## ◆ Supersymmetric Lagrangian in superspace are very compact

$$\mathcal{L} = \Phi^\dagger e^{-2gV} \Phi|_{\theta^2\bar{\theta}^2} + \frac{1}{16g^2\tau_{\mathcal{R}}} \text{Tr}(W^\alpha W_\alpha)|_{\theta^2} + \frac{1}{16g^2\tau_{\mathcal{R}}} \text{Tr}(\bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}})|_{\bar{\theta}^2} \\ + W(\Phi)|_{\theta^2} + W^*(\Phi^\dagger)|_{\bar{\theta}^2} + \mathcal{L}_{\text{soft}}$$

- ❖ First line: kinetic and gauge interaction terms for all fields
  - ★ Model independent  $\Rightarrow$  can be automated (**CSFKineticTerms** / **VSFKineticTerms**)
  - ★ Dedicated methods to access the components of a superfield (**Theta2Component**, etc.)
- ❖ Second line: superpotential and supersymmetry-breaking terms
  - ★ Model dependent  $\Rightarrow$  to be provided by the user (*SuperW* and *LSoft*)

```
Lag = Theta2Thetabar2Component[ CSFKineticTerms[] ] +
      Theta2Component[ VSFKineticTerms[] ] +
      Thetabar2Component[ VSFKineticTerms[] ];
```

```
Lag2 = LSoft + Theta2Component[ SuperW ] + Thetabar2Component[ SuperW ];
```

# Implementing supersymmetric Lagrangians

## ◆ The implemented Lagrangian above must be post-processed

```
Lag = Theta2Thetabar2Component[ CSFKineticTerms[] ] +  
      Theta2Component[ VSFKineticTerms[] ] +  
      Thetabar2Component[ VSFKineticTerms[] ];
```

```
Lag2 = LSoft + Theta2Component[ SuperW ] + Thetabar2Component[ SuperW ];
```

- ♣ Solving the **equation of motion** for the auxiliary fields
- ♣ Inserting the solution back into the Lagrangian
  - ★ **Automated** (**SolveEqMotionF** and **SolveEqMotionD**)
- ♣ Replacement of **Weyl spinors** in terms of Majorana and Dirac spinors
  - ★ **Automated** (**WeylToDirac**)
- ♣ Rotation to the mass basis
  - ★ **Standard FEYNRULES function** (**ExpandIndices**)

## Mass matrix diagonalization

See the manual for more details on the **ASPERGE** module

# Mass matrices and their diagonalization

## ◆ The problematics of the mass matrices

- ❖ Lagrangians are usually easily written in the gauge basis
- ❖ The included mass matrices are thus in general non-diagonal
  - diagonalization required
- ❖ The gauge basis must be rotated to the mass basis where the mass matrices are diagonal
- ❖ This diagonalization cannot in general be achieved analytically

## ◆ The ASPERGE package of FEYNRULES

- ❖ A module allowing one to extract the mass matrices from the Lagrangian
- ❖ A generator of C++ code ➤ numerical diagonalization of all mass matrices (the generated code can be used in a standalone way)
- ❖ See: Alloul, D'Hondt, De Causmaecker, BF, Rausch de Traubenberg [ EPJC 73 (2013) 2325 ]

# Example of the Z-boson/photon mixing

## ◆ Example: the Z-boson and photon in the Standard Model

- ❖ Each mixing is declared as a set of replacement rules (in `MixingsDescription`)
- ❖ Each rule represent a property of the mixing relation

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = U_w \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} \Rightarrow$$

```
Mix["AZ"] == {
  MassBasis      -> {A, Z},
  GaugeBasis     -> {B, Wi[3]},
  MixingMatrix   -> UW,
  BlockName      -> WEAKMIX
}
```

- ❖ ASPERGE can compute the mass matrices:
- ❖ ASPERGE can generate its standalone C++ version

```
ComputeMassMatrix[Lag];
WriteASperGe[Lag];
```

## Decays

**See the manual for more details on the decay module**

# Two-body decays

- ◆ **The problematics of the decay widths and branching ratios**
  - ♣ Some MC tools need decay tables (widths and branching ratios) to decay particles
  - ♣ Widths and branching ratios are not independent quantities
    - **need to be calculated**
  - ♣ Some Monte Carlo tools compute these quantities on the fly
    - **the procedure is repeated each time it is needed**
  - ♣ FEYNRULES offers a way to include analytical information on the two-body decay



# Two-body decays

## ◆ Two-body decays in general

- ♣ Two-body decays can be directly read **from three-point vertices** ( $\mathcal{V}$ )

$$\Gamma_{1 \rightarrow 2} = \frac{1}{2|M|S} \int d\Phi_N |\mathcal{M}_{1 \rightarrow 2}|^2 = \frac{\sqrt{\lambda(M^2, m_1^2, m_2^2)}}{16 \pi S |M|^3} \mathcal{V}_{l_1 l_2 l_3}^{a_1 a_2 a_3} \mathcal{P}_1^{l_1 l'_1} \mathcal{P}_2^{l_2 l'_2} \mathcal{P}_3^{l_3 l'_3} (\mathcal{V}^*)_{l'_1 l'_2 l'_3}^{a_1 a_2 a_3}$$

- ★ Partial width for the decay of a particle of mass  $M$  to two particles of masses  $m_1$  and  $m_2$
- ★ Includes a symmetry factor  $S$  and  $\mathcal{P}$  denotes the polarization tensor of each particle

## ◆ The decay module of FEYNRULES

- ♣ FEYNRULES makes use of MATHEMATICA to compute all partial widths of the model
  - ★ Ignores open and closed channels ➤ **benchmark independent**
  - ★ **The information is exported to the UFO** (used, e.g, by MADWIDTH)

# Running the decay module of FEYNRULES

## ◆ Automatic decay width computations

- ❖ All two-body decay widths can be easily computed from the Lagrangian

```
verts      = FeynmanRules[Lag];  
vertsexp   = FlavorExpansion[verts];  
results    = ComputeWidths[vertsexp];
```

- ❖ Many functions available for analytical calculations
  - **PartialWidth, TotWidth, BranchingRatio**
- ❖ The numerical value provided for the particle widths can be updated accordingly
  - **UpdateWidths**
- ❖ See: Alwall, Duhr, BF, Mattelaer, Öztürk, Shen [ CPC (2015) ]

# Snippet of the UFO output

## ◆ The information is (by default) employed by the UFO interface

- ❖ Can be turned of: (**AddDecays** → **False**)
- ❖ The UFO contains an extra file *decays.py*
- ❖ This file can be used by MC codes
- ❖ Example of the Standard Model UFO: the top quark

```
Decay_t = Decay(name = 'Decay_t',
               particle = P.t,
               partial_widths = {(P.W__plus__, P.d): '((MT**2 - MW**2)*((3*CKM3x1*ee**2*MT**2*complexconjugate(CKM3x1))/(2.*sw**2) +
(3*CKM3x1*ee**2*MT**4*complexconjugate(CKM3x1))/(2.*MW**2*sw**2) - (3*CKM3x1*ee**2*MW**2*complexconjugate(CKM3x1))/sw**2))/
(96.*cmath.pi*abs(MT)**3)',
                               (P.W__plus__, P.s): '((MT**2 - MW**2)*((3*CKM3x2*ee**2*MT**2*complexconjugate(CKM3x2))/(2.*sw**2) +
(3*CKM3x2*ee**2*MT**4*complexconjugate(CKM3x2))/(2.*MW**2*sw**2) - (3*CKM3x2*ee**2*MW**2*complexconjugate(CKM3x2))/sw**2))/
(96.*cmath.pi*abs(MT)**3)',
                               (P.W__plus__, P.b): '((3*CKM3x3*ee**2*MB**2*complexconjugate(CKM3x3))/(2.*sw**2) +
(3*CKM3x3*ee**2*MT**2*complexconjugate(CKM3x3))/(2.*sw**2) + (3*CKM3x3*ee**2*MB**4*complexconjugate(CKM3x3))/(2.*MW**2*sw**2) -
(3*CKM3x3*ee**2*MB**2*MT**2*complexconjugate(CKM3x3))/(MW**2*sw**2) + (3*CKM3x3*ee**2*MT**4*complexconjugate(CKM3x3))/(2.*MW**2*sw**2) -
(3*CKM3x3*ee**2*MW**2*complexconjugate(CKM3x3))/sw**2)*cmath.sqrt(MB**4 - 2*MB**2*MT**2 + MT**4 - 2*MB**2*MW**2 - 2*MT**2*MW**2 +
MW**4))/(96.*cmath.pi*abs(MT)**3)')})
```

**NLO**

# Higher-order corrections (in QCD)

- ◆ NLO calculations matched to parton shower (for BSM) are automated
  - ♣ Model-dependent parts of calculations (on top of the tree-level information)
    - ★ Counterterms
    - ★ Finite pieces of the loop-integrals
  - ♣ Model independent contributions
    - ★ Subtraction of the divergences
    - ★ Matching to the parton showers

**UFO @ NLO**

[ Degrande, Duhr, BF, Hirschi, Mattelaer, Shao et al. (in prep.) ]

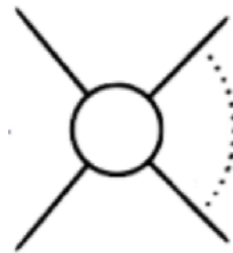
# Recap' on NLO calculations

## ◆ Contributions to an NLO result in QCD

♣ Three ingredients: the Born, virtual loop and real emission contributions

$$\sigma_{NLO} = \int d^4\Phi_n \mathcal{B} + \int d^4\Phi_n \int_{\text{loop}} d^d\ell \mathcal{V} + \int d^4\Phi_{n+1} \mathcal{R}$$

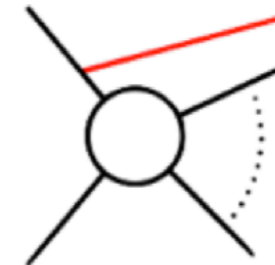
Born



Virtuals: one extra power of  $\alpha_s$  and divergent



Reals: one extra power of  $\alpha_s$  and divergent



**Extra information is needed**

# Virtual contributions

## ◆ Loop diagram calculations

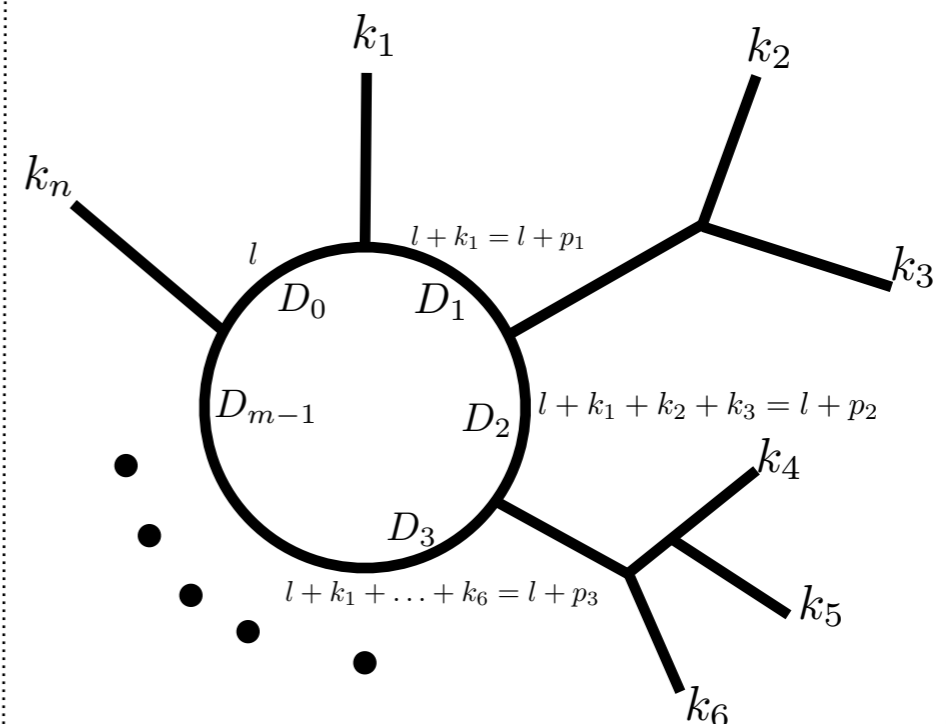
- ✿ Calculations to be done in  $d=4-2\epsilon$  dimensions
  - ★ Divergences made explicit ( $1/\epsilon^2$ ,  $1/\epsilon$ )

- ✿ Rewriting loop integrals with **scalar integrals**

$$\int d^d \ell \frac{N(\ell)}{D_0 D_1 \cdots D_{m-1}} = \sum a_i \int d^d \ell \frac{1}{D_{i_0} D_{i_1} \cdots}$$

- ★ Involves integrals with **up to four denominators**
  - The decomposition basis is finite
  - **Can be computed once and for all**
- ★ The reduction is the process-dependent part

$m$ -point diagram with  $n$  external momenta



# The rational terms ( $R_1$ and $R_2$ )

◆ The loop momentum lives in a  $d$ -dimensional space

♣ Reduction to be done in  $d$  dimensions

$$\int d^d \ell \frac{N(\ell, \tilde{\ell})}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}} \quad \text{with } \bar{\ell} = \ell + \tilde{\ell}$$

D-dim
4-dim
(-2ε)-dim

♣ Numerical methods works in 4 dimensions: need to be compensated!

◆ The  $R_1$  terms originates from the denominators

♣ Connected to the internal propagators

◆ The  $R_2$  terms originates from the numerator

♣ Can be seen as extra diagrams with special Feynman rules



# R<sub>1</sub> terms

◆ The R<sub>1</sub> terms originates from the denominators

$$\frac{1}{\bar{D}} = \frac{1}{D} \left( 1 - \frac{\tilde{\ell}^2}{\bar{D}} \right)$$

♣ These extra pieces can be calculated **generically** (3 integrals in total)

$$\int d^d \bar{\ell} \frac{\tilde{\ell}^2}{\bar{D}_i \bar{D}_j} = -\frac{i\pi^2}{2} \left[ m_i^2 + m_j^2 - \frac{(p_i - p_j)^2}{2} \right] + \mathcal{O}(\varepsilon)$$

$$\int d^d \bar{\ell} \frac{\tilde{\ell}^2}{\bar{D}_i \bar{D}_j \bar{D}_k} = -\frac{i\pi^2}{2} + \mathcal{O}(\varepsilon)$$

$$\int d^d \bar{\ell} \frac{\tilde{\ell}^2}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_l} = -\frac{i\pi^2}{6} + \mathcal{O}(\varepsilon)$$

♣ The denominator structure is already known at the reduction time

♣ The R<sub>1</sub> coefficients are extracted during the reduction

# R<sub>2</sub> terms

◆ The R<sub>2</sub> terms originates from the numerator

$$\bar{N}(\bar{\ell}) = \underset{\substack{\downarrow \\ \text{D-dim}}}{N}(\bar{\ell}) + \underset{\substack{\downarrow \\ \text{4-dim}}}{N}(\bar{\ell}) + \underset{\substack{\downarrow \\ \text{(-2}\varepsilon\text{)-dim}}}{\tilde{N}}(\tilde{\ell}, \ell, \varepsilon) \quad \Rightarrow \quad R_2 \equiv \lim_{\varepsilon \rightarrow 0} \frac{1}{(2\pi)^4} \int d^d \bar{\ell} \frac{\tilde{N}(\tilde{\ell}, \ell, \varepsilon)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

❖ Practically, we isolate the epsilon part

❖ There is only a finite set of loops for which it does not vanish

◆ They can be re-expressed in terms of R<sub>2</sub> Feynman rules

$$\propto \int d^d \bar{\ell} \frac{\tilde{\ell}^2}{\bar{D}_i \bar{D}_j \bar{D}_k} = -\frac{i\pi^2}{2} + \mathcal{O}(\varepsilon) \Rightarrow \text{Diagram} \quad -i \frac{\alpha}{2\pi} e \gamma^\mu$$

# R<sub>2</sub> Feynman rules

## ◆ The R<sub>2</sub> are process dependent and model-dependent (like Feynman rules)

- ❖ In a renormalizable theory, there is a finite number of them
- ❖ They can be derived from the sole knowledge of the bare Lagrangian

[ Ossala, Papadopoulos, Pittau (JHEP'08) ]

## ◆ The R<sub>2</sub> calculation can be automated and performed once and for all

- ❖ Development of the NLOCT package (extension of FEYNRULES)
- ❖ Computation, for any model, of all R<sub>2</sub> and UV counterterms
  - ★ In the on-shell and MSbar schemes
- ❖ Inclusion of the output in the UFO

[ Degrande (CPC'15) ]

# Automated NLO simulations with MG5\_AMC

## ◆ A comprehensive approach to Monte Carlo simulations at the NLO in QCD

