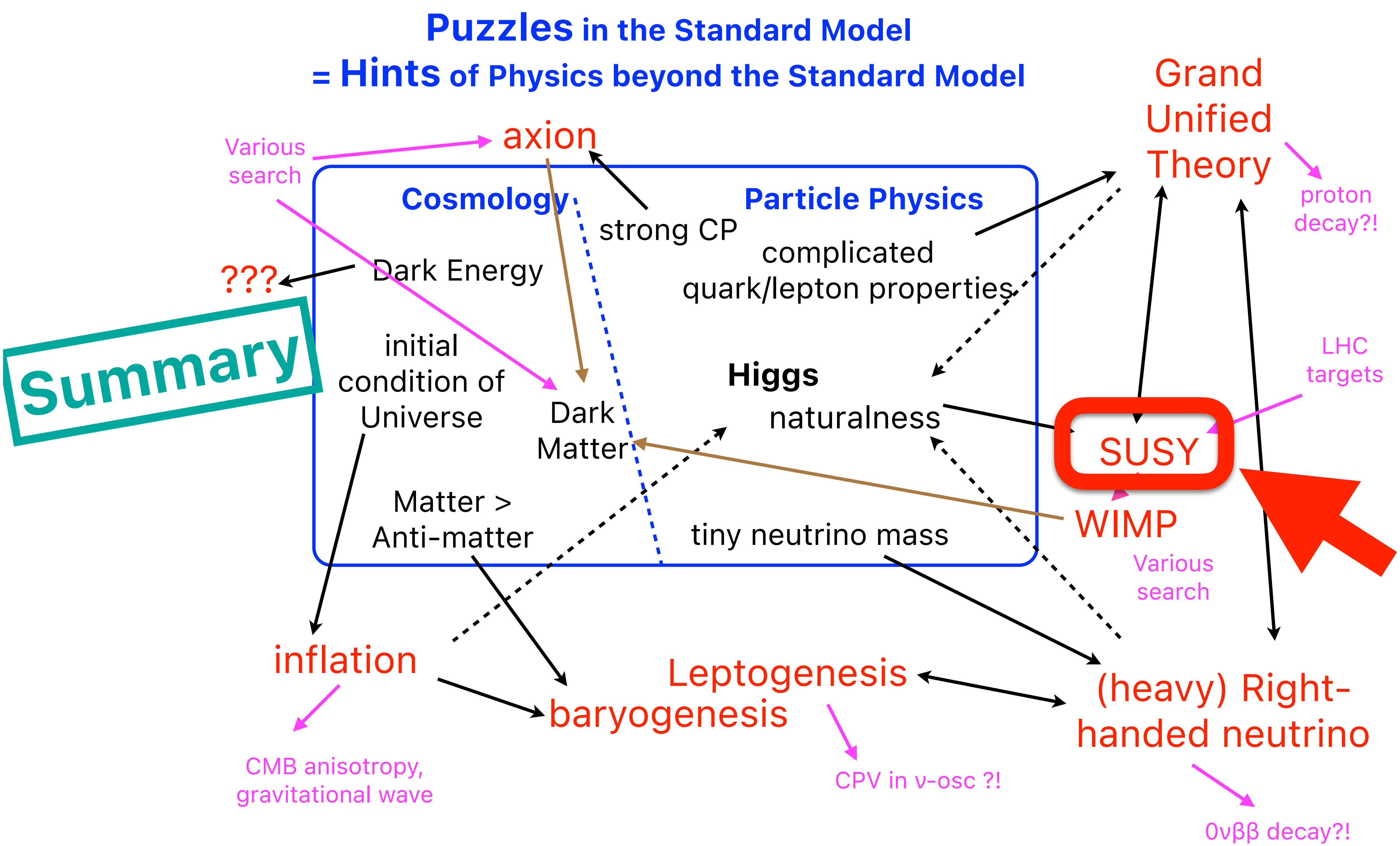
20225-03-01 SUSY informal lecture, Koichi Hamaguchi

@The 4th International Iwate Collider School (ICS2025)

Iwate, Feb.24-Mar.1, 2025.



PLAN

G.0. naturalness

§G. SUSY:

G.1. motivations

G.2. supersymmetry

G.3. MSSM (minimal SUSY Standard Model)

G.4. MSSM Lagrangian

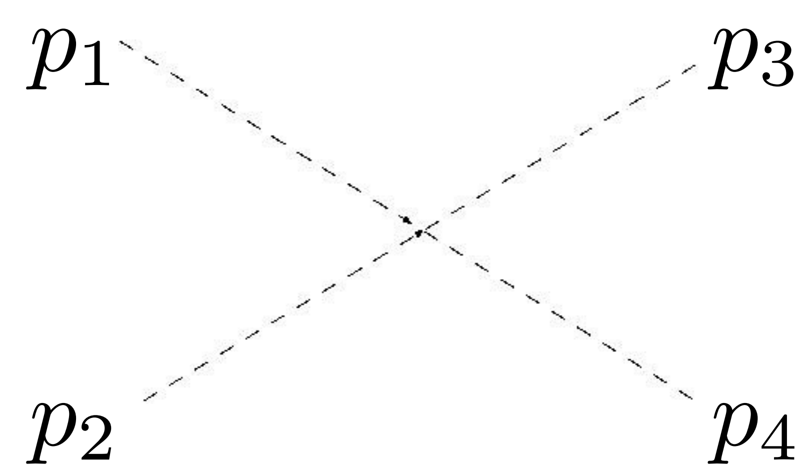
G.5. SUSY after Higgs discovery

(i) renormalization

This part is based on A.Zee's textbook, Chap. III
& a lecture note by R.Kitano (for HEP spring school, May 2013, Biwako, Japan)

Consider a 2-body scattering in a scalar ϕ^4 theory.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$



The **tree level** amplitude is

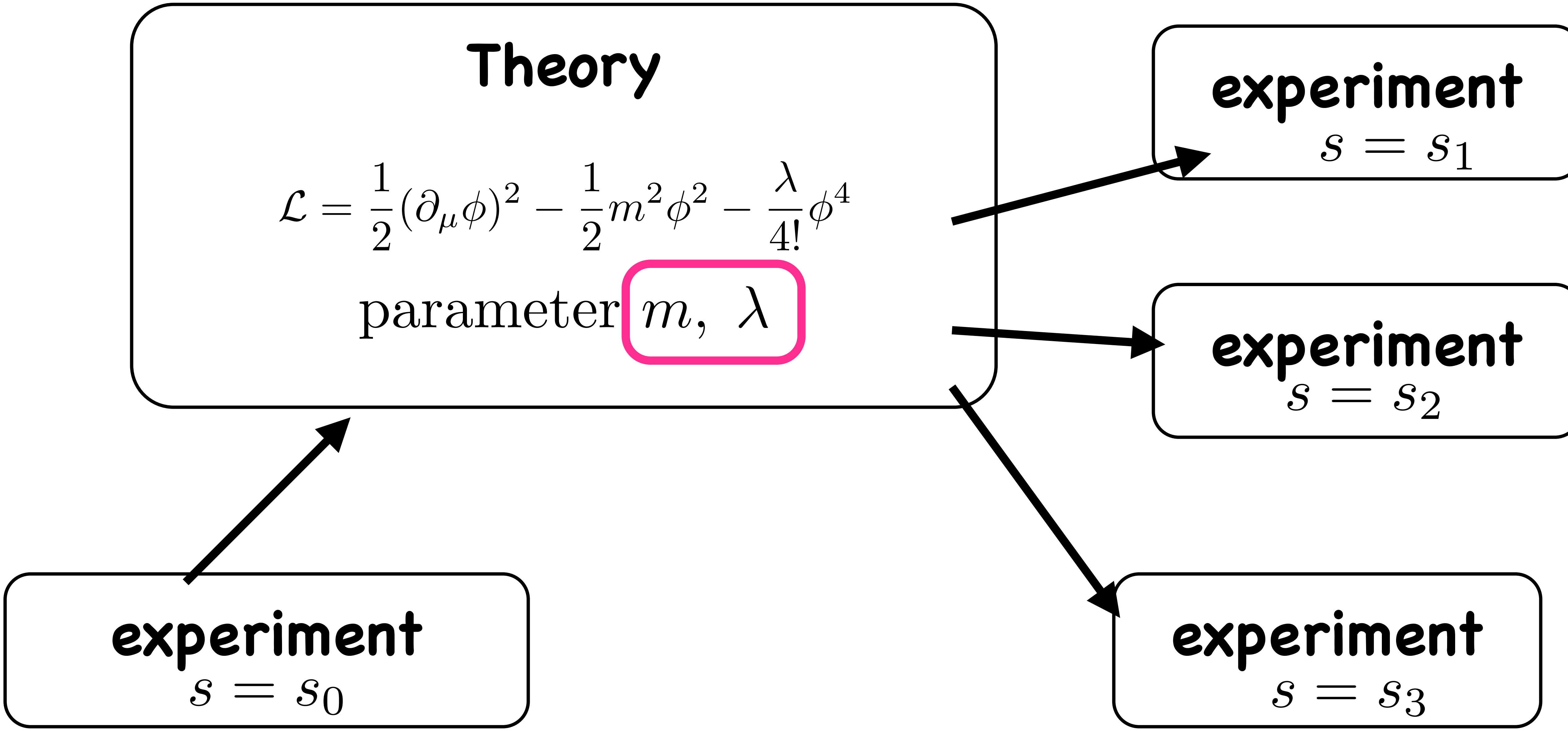
$$\mathcal{M} = -\lambda + \mathcal{O}(\lambda^2)$$

The differential cross section is

$$d\sigma = \frac{1}{16\pi} \cdot \frac{d\Omega}{4\pi} \cdot \frac{1}{s} \underbrace{\cdot|\mathcal{M}|^2,}_{\lambda^2 + \mathcal{O}(\lambda^3)} \qquad s=(p_1+p_2)^2$$

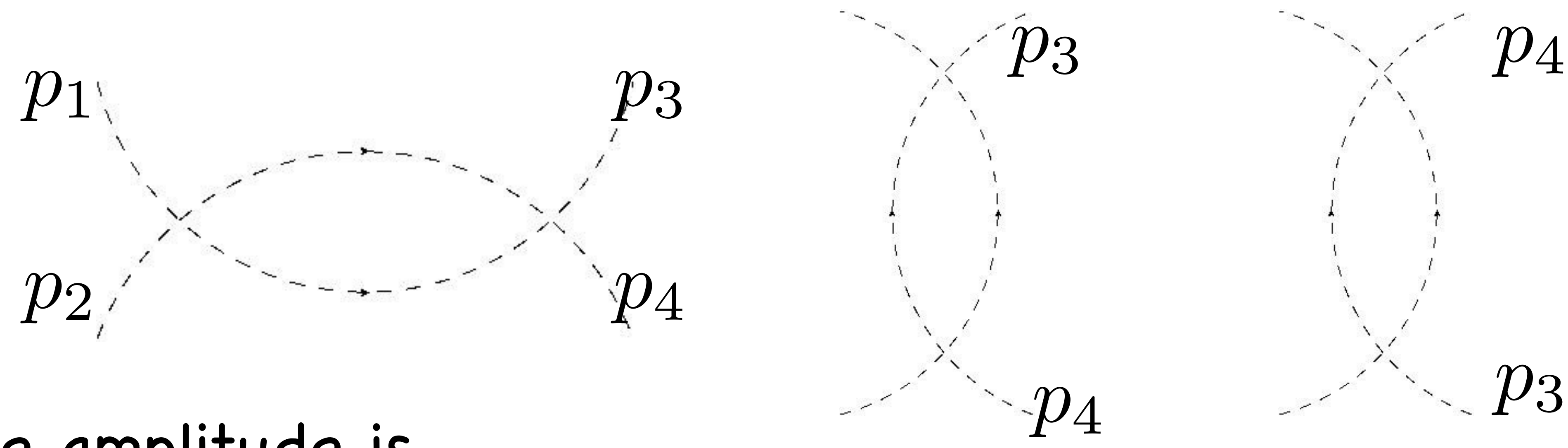
By measuring scattering cross section at e.g., $s = s_0$, we can fix λ.

(i) renormalization



(i) renormalization

Now let's see the next order in perturbation theory.



The amplitude is

$$\mathcal{M} = -\lambda + \lambda^2 \cdot \frac{-i}{2} \int \frac{d^4k}{(2\pi)^4} \cdot \frac{1}{k^2 - m^2 + i\epsilon} \cdot \frac{1}{(k + p_1 + p_2)^2 - m^2 + i\epsilon} + \mathcal{O}(\lambda^3) + (s \to t, u)$$

Now the integral **diverges**!

$$\mathcal{M} \sim \int^{\infty} \frac{d^4k}{k^4} \sim \int^{\infty} \frac{dk}{k} \sim \log(\infty)$$

But that's OK.

Suppose that the theory is valid only up to a scale Λ,

and cut off the momentum integration. $$\int^{\infty} dk \longrightarrow \int^{\Lambda} dk$$

(regularization)

(i) renormalization

Then the amplitude becomes (neglecting the mass, for simplicity),

$$\mathcal{M} = -\lambda + C\lambda^2 \left[\log\left(\frac{\Lambda^2}{s}\right) + \log\left(\frac{\Lambda^2}{t}\right) + \log\left(\frac{\Lambda^2}{u}\right) \right] + \mathcal{O}(\lambda^3) \qquad C = 1/32\pi^2$$

LHS can be measured (by scattering at $s = s_0$, for instance).

RHS depends on the artificial cut-off, Λ.

Is that OK? Can this theory still make a prediction?

No problem. We can still compare between experiments.

Theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!}\phi^4$$

experiment
$s = s_1$

experiment
$s = s_2$

(i) renormalization

Then the amplitude becomes (neglecting the mass, for simplicity),

$$\mathcal{M} = -\lambda + C\lambda^2 \left[\log\left(\frac{\Lambda^2}{s}\right) + \log\left(\frac{\Lambda^2}{t}\right) + \log\left(\frac{\Lambda^2}{u}\right)\right] + \mathcal{O}(\lambda^3) \qquad C = 1/32\pi^2$$

LHS can be measured (by scattering at $s = s_0$, for instance).

RHS depends on the artificial cut-off, Λ.

Is that OK? Can this theory still make a prediction?

No problem. We can still compare between experiments.

$$\text{exp.1:}\ \mathcal{M}(s_1, t_1, u_1) = -\lambda + C\lambda^2 \left[\log\left(\frac{\Lambda^2}{s_1}\right) + \log\left(\frac{\Lambda^2}{t_1}\right) + \log\left(\frac{\Lambda^2}{u_1}\right)\right] + \mathcal{O}(\lambda^3)$$

$$\text{exp.2:}\ \mathcal{M}(s_2, t_2, u_2) = -\lambda + C\lambda^2 \left[\log\left(\frac{\Lambda^2}{s_2}\right) + \log\left(\frac{\Lambda^2}{t_2}\right) + \log\left(\frac{\Lambda^2}{u_2}\right)\right] + \mathcal{O}(\lambda^3)$$

In each eqs, RHS depends on the artificial cut-off Λ.

But if we subtract,...

(i) renormalization

$$\text{exp.1: } \mathcal{M}(s_1,t_1,u_1) = -\lambda + C\lambda^2\left[\log\left(\frac{\Lambda^2}{s_1}\right)+\log\left(\frac{\Lambda^2}{t_1}\right)+\log\left(\frac{\Lambda^2}{u_1}\right)\right]+\mathcal{O}(\lambda^3)$$

$$\text{exp.2: } \mathcal{M}(s_2,t_2,u_2) = -\lambda + C\lambda^2\left[\log\left(\frac{\Lambda^2}{s_2}\right)+\log\left(\frac{\Lambda^2}{t_2}\right)+\log\left(\frac{\Lambda^2}{u_2}\right)\right]+\mathcal{O}(\lambda^3)$$

In each eqs, RHS depends on the artificial cut-off Λ.

But if we subtract,...

$$\begin{aligned}\mathcal{M}(s_2,t_2,u_2) &= \mathcal{M}(s_1,t_1,u_1) + C\lambda^2\left[\log\left(\frac{s_1}{s_2}\right)+\log\left(\frac{t_1}{t_2}\right)+\log\left(\frac{u_1}{u_2}\right)\right]+\mathcal{O}(\lambda^3)\\ &= \mathcal{M}(s_1,t_1,u_1) + C\mathcal{M}(s_1,t_1,u_1)^2\left[\log\left(\frac{s_1}{s_2}\right)+\log\left(\frac{t_1}{t_2}\right)+\log\left(\frac{u_1}{u_2}\right)\right]+\mathcal{O}(\mathcal{M}(s_1,t_1,u_1)^3)\end{aligned}$$

The exp.2 observable is completely determined by the exp.1 observable.
Dependences on the cut-off Λ and λ disappear!

Though the intermediate calculation involves an artificial cut-off Λ,

the final relation between exp.1 and exp.2 is independent of Λ.

This is the **"renormalization".**

(i) renormalization

$$\mathcal{M}(s_2,t_2,u_2) = \mathcal{M}(s_1,t_1,u_1) + C\lambda^2\left[\log\left(\frac{s_1}{s_2}\right) + \log\left(\frac{t_1}{t_2}\right) + \log\left(\frac{u_1}{u_2}\right)\right] + \mathcal{O}(\lambda^3)$$

$$= \mathcal{M}(s_1,t_1,u_1) + C\mathcal{M}(s_1,t_1,u_1)^2\left[\log\left(\frac{s_1}{s_2}\right) + \log\left(\frac{t_1}{t_2}\right) + \log\left(\frac{u_1}{u_2}\right)\right] + \mathcal{O}(\mathcal{M}(s_1,t_1,u_1)^3)$$

OK, we can now compare experimentally measurable quantities.
But what is the coupling λ then?

Recall

$$\mathcal{M}(s_0,t_0,u_0) = -\lambda + C\lambda^2\left[\log\left(\frac{\Lambda^2}{s_0}\right) + \log\left(\frac{\Lambda^2}{t_0}\right) + \log\left(\frac{\Lambda^2}{u_0}\right)\right] + \mathcal{O}(\lambda^3)$$

Thus

$$\lambda = -\mathcal{M}(s_0,t_0,u_0) + C\lambda^2\left[\log\left(\frac{\Lambda^2}{s_0}\right) + \log\left(\frac{\Lambda^2}{t_0}\right) + \log\left(\frac{\Lambda^2}{u_0}\right)\right] + \mathcal{O}(\lambda^3)$$

$$= -\mathcal{M}(s_0,t_0,u_0) + C\mathcal{M}(s_0,t_0,u_0)^2\left[\log\left(\frac{\Lambda^2}{s_0}\right) + \log\left(\frac{\Lambda^2}{t_0}\right) + \log\left(\frac{\Lambda^2}{u_0}\right)\right] + \mathcal{O}(\mathcal{M}^3)$$

Fine. λ is now expressed in terms of observable $\mathcal{M}(s_0,t_0,u_0)$.

But it depends on Λ!

$$\boxed{\lambda = \lambda(\Lambda)}$$

(i) renormalization

But it depends on Λ!

$$\boxed{\lambda = \lambda(\Lambda)}$$

No problem. Physics does not depend on Λ.

The combination $\big(\Lambda, \lambda(\Lambda)\big)$ determines the observable.
and the observables are independent of Λ.

By this requirement we can also obtain a differential equation for $\lambda(\Lambda)$.

$$\mathcal{M}(s_0, t_0, u_0) = -\lambda(\Lambda) + C\lambda(\Lambda)^2 \left[\log\left(\frac{\Lambda^2}{s_0}\right) + \log\left(\frac{\Lambda^2}{t_0}\right) + \log\left(\frac{\Lambda^2}{u_0}\right)\right] + \mathcal{O}(\lambda^3)$$

$$\to 0 = \frac{d\mathcal{M}(s_0, t_0, u_0)}{d\log\Lambda} = -\frac{d\lambda}{d\log\Lambda} + 6C\lambda^2 + 2C\lambda\frac{d\lambda}{d\log\Lambda}\left[\log\left(\frac{\Lambda^2}{s_0}\right) + \cdots\right] + \mathcal{O}(\lambda^3)$$

$$\to \boxed{\frac{d\lambda}{d\log\Lambda} = 6C\lambda^2 + \mathcal{O}(\lambda^3)}$$

Renormalization Group Equation.

As far as this is satisfied, observables are independent of Λ.

(i) renormalization

We regularized the divergent integral by a momentum cut-off Λ.

There are other regularizations.

Dimensional regularization:

We don't discuss what it is, but the basic idea is the same.

- There is an artificial parameter μ with a mass dimension one.
- The combination $\big(\mu, \lambda(\mu)\big)$ determines the observable, and the observables are independent of μ.

- The μ-dependence is given by the Renormalization Group Equation.

$$\frac{d\lambda(\mu)}{d\log\mu} = 6C\lambda^2 + \mathcal{O}(\lambda^3)$$

(i) renormalization

We regularized the divergent integral by a momentum cut-off Λ.

There are other regularizations.

Dimensional regularization:

REMARK: Although the observable is independent of μ,

it is better to use μ close to the energy scale of your interest

when you calculate by perturbation theory.

$$\mathcal{M}(s,\cdots) = -\lambda(\mu) + C\lambda(\mu)^2 \left[\log\left(\frac{\mu^2}{s}\right) + \cdots\right] + \mathcal{O}(\lambda^3)$$

When $\mu \sim$ s, this log factor is small

--> better convergence.

e.g., For hard processes at LHC, $\alpha_S(\mu)$ with μ >> 1 GeV is used.

PLAN

G.0. renormalization and naturalness

(i) renormalization

done

$\boxed{\text{(ii) naturalness}}$

§G. SUSY:

G.1. motivations

G.2. supersymmetry

G.3. MSSM (minimal SUSY Standard Model)

G.4. MSSM Lagrangian

G.5. SUSY after Higgs discovery

naturalness

Now let's discuss the **naturalness** for the Higgs mass parameter.

$$V(H) = -m^2|H|^2 + \lambda_H|H|^4.$$

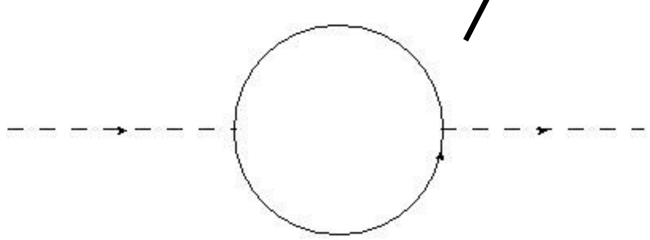
Consider with the **cut-off regularization.**

The correction to the mass parameter is

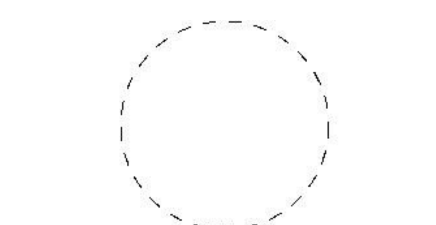
$$\delta m^2 = -\frac{3}{8\pi^2}\left(y_t^2 - \lambda_H - \frac{3}{8}g^2 - \frac{3}{8}g'^2 + \cdots\right)\Lambda^2$$

NOTE:

quadratic dependence (not logarithmic)



top loop



Higgs loop

gauge boson loops

naturalness

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NOTE:
quadratic dependence
(not logarithmic)

Let's consider the largest top contribution.
The corrected mass parameter is then...

$$m^2 = m^2(\Lambda) - \frac{3}{8\pi^2}y_t^2\Lambda^2 + \cdots$$

naturalness

$$m^2 = m^2(\Lambda) - \frac{3}{8\pi^2} y_t^2 \Lambda^2 + \cdots$$

m^2: (~100 GeV)^2

$m^2(\Lambda)$: (~100000…… GeV)^2 + (~100 GeV)^2

$\frac{3}{8\pi^2} y_t^2 \Lambda^2$: (~100000…… GeV)^2

depending on the cut-off

If $m^2(\Lambda)$ is a fundamental parameter,

(for instance, if our space-time becomes somehow latticed at very small scale Λ^{-1})

this is unnatural.

$\boxed{\textbf{naturalness problem}}$

naturalness

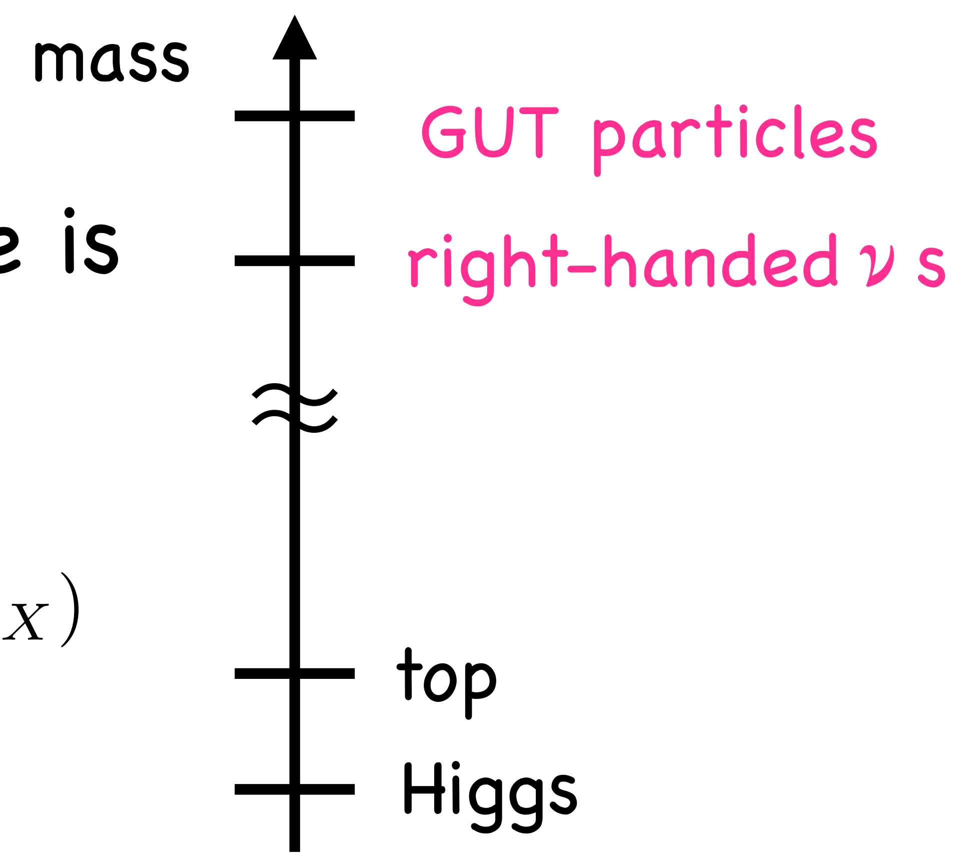
$$m^2 = m^2(\Lambda) - \frac{3}{8\pi^2} y_t^2 \Lambda^2 + \cdots$$

naturalness problem

Remark

Λ^2 term may be an artifact of cut-off regularization.
For instance, it doesn't exist in dimensional regularization.

But even if the Λ^2 term is absent,
a large correction exists as far as there is
a **heavy particles** coupled to Higgs.



$$m^2 = m^2(\mu) + C_X m_X^2 \log\left(\frac{\mu^2}{m_X^2}\right) + \cdots \quad (\text{for } \mu > m_X)$$

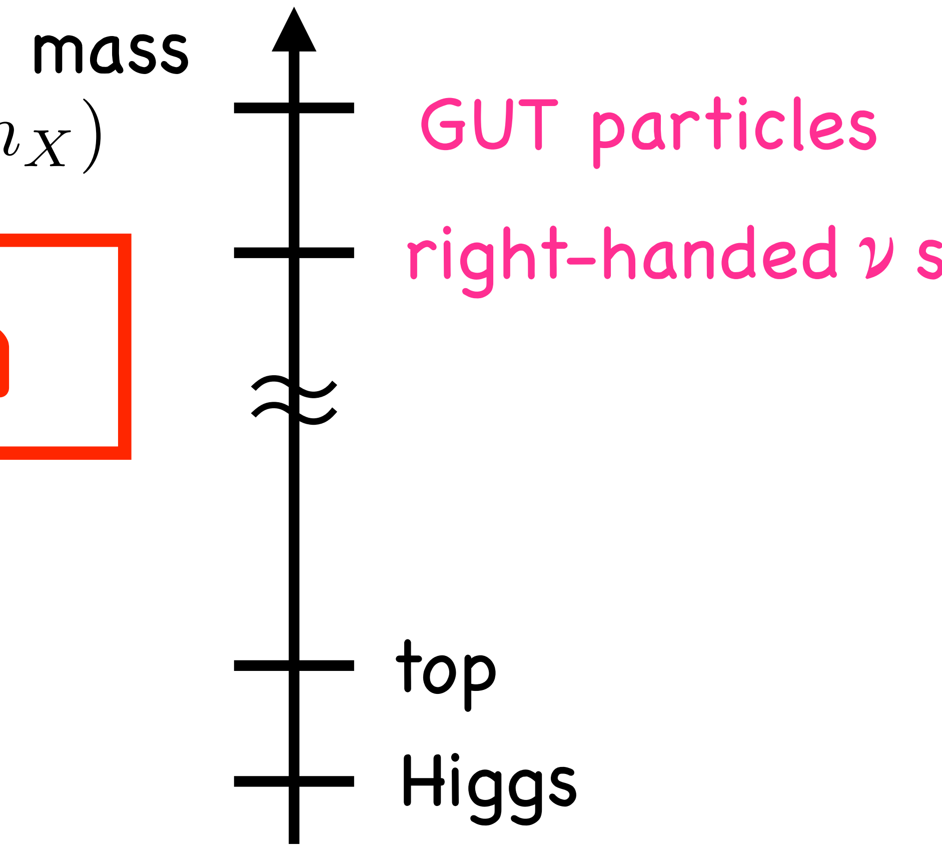
X's coupling to Higgs

X's mass

[See e.g., 1303.7244, 1402.2658]

naturalness

$$m^2 = m^2(\mu) + C_X m_X^2 \log\left(\frac{\mu^2}{m_X^2}\right) + \cdots \quad (\text{for } \mu > m_X)$$



naturalness problem

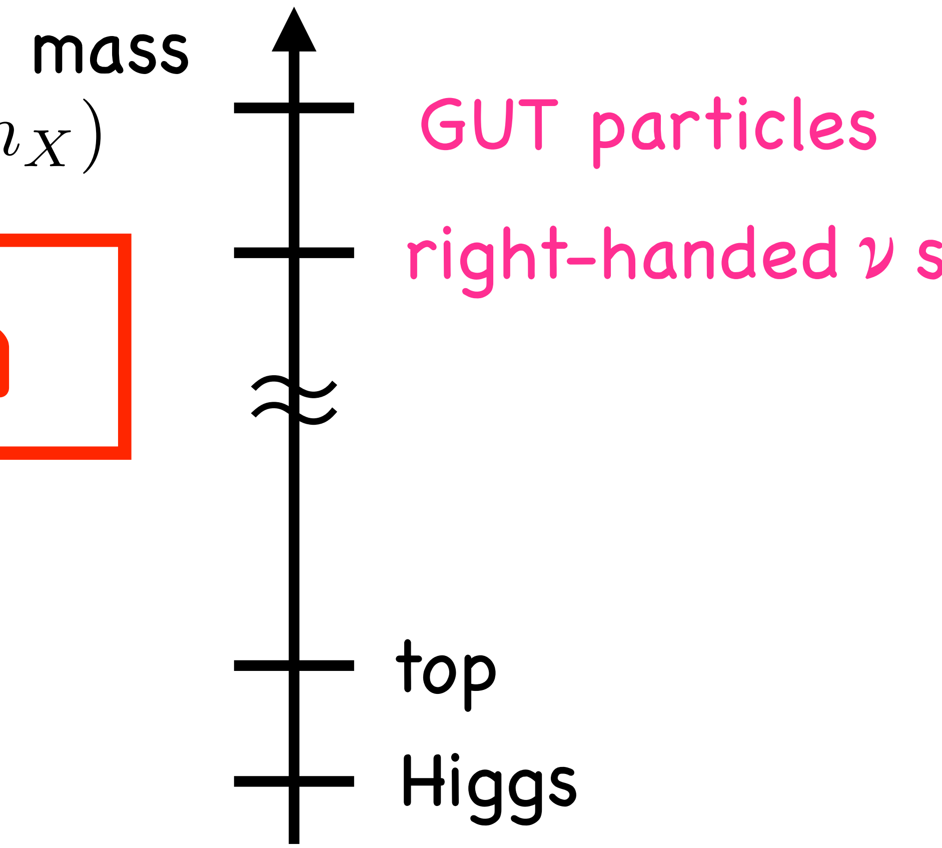
If we think m(μ) at μ > m_X is more fundamental than m(μ) at weak scale μ,

(For instance, in QCD, $m_{u,d}$ at $\mu > \Lambda_{QCD}$ seems more fundamental than the pion masses.....)

this is unnatural.

naturalness

$$m^2 = m^2(\mu) + C_X m_X^2 \log\left(\frac{\mu^2}{m_X^2}\right) + \cdots \quad (\text{for } \mu > m_X)$$



naturalness problem

solutions

- Don't mind.

  There is no problem in experimental observables.
  (Don't listen too much to theorists…… 😜.)

- Landscape + anthropic principle

  We live in a fine-tuned vacuum because otherwise we cannot live.

- No such heavy particles

  or 4d perturbative QFT breaks down anyway before that.

- cancellation among loop corrections
  - SUSY
  - little Higgs (top correction canceled.)
  - . . .

PLAN

G.0. naturalness

§G. SUSY:

G.1. motivations

G.2. supersymmetry

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reference: hep-ph/9709356 by S.P.Martin.

G. Supersymmetry

Fermion <-> Boson

Standard Model		spin		
quark	q	1/2 <-> 0	squark	$\widetilde{q}$
lepton	ℓ	1/2 <-> 0	slepton	$\widetilde{\ell}$
Higgs	H	0 <-> 1/2	higgsino	$\widetilde{h}$
gauge bosons γ, Z, W, g		1 <-> 1/2	gaugino $\widetilde{B}, \widetilde{W}, \widetilde{g}$	

MSSM (minimal SUSY Standard Model)

PLAN

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G. Supersymmetry

§ G.1. motivations

(i) naturalness of Higgs boson mass

(ii) coupling unification

(iii) · · ·

G. Supersymmetry

§ G.1. motivations

(i) naturalness of Higgs boson mass

In terms of cut-off regularization,

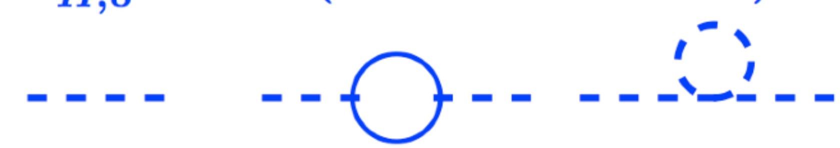
fine-tuning problem

$$m_H^2 = m_{H,0}^2 + \Lambda^2 \qquad (\Lambda \gg m_H)$$

(fine tuning like 1.0000000000000001 - 1)

→ solved by the supersymmetry !

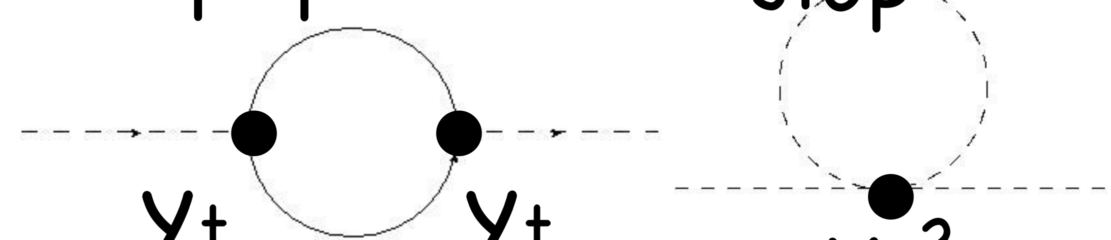
$$m_H^2 = m_{H,0}^2 + (\Lambda^2 - \Lambda^2)$$



fermion boson

top quark stop

For instance,...



$y_t^2 \Lambda^2 \quad - y_t^2 \Lambda^2$

In terms of dim. reg. + heavy particle X,

$\delta m^2(\mu) \sim (m_X [\text{scalar}]^2 - m_X [\text{fermions}]^2) \log (\mu / m_X)$

G. Supersymmetry

§ G.1. motivation

(ii) coupling unification

- gauge field kinetic term of GUT (SU(5))

$$\frac{1}{g_{\rm GUT}^2}\sum_{a=1}^{24} F^a_{\mu\nu}F^{a\mu\nu} \qquad \left(F_{\mu\nu}F^{\mu\nu} \to \frac{1}{g^2}F_{\mu\nu}F^{\mu\nu} \text{ by field redefinition } A_\mu \to \frac{1}{g}A_\mu \right)$$

$$= \frac{1}{g_3^2}G^a_{\mu\nu}G^{a\mu\nu} + \frac{1}{g_2^2}W^a_{\mu\nu}W^{a\mu\nu} + \frac{1}{g_1^2}B_{\mu\nu}B^{\mu\nu} + (X, Y \text{ gauge bosons})$$

$$\boxed{\Longrightarrow \quad g_1 = g_2 = g_3 = g_{\rm GUT} \quad @ \quad \mu = \text{unification scale} \;\; (= \text{the scale where GUT is broken})}$$

- running gauge coupling

R.G.eq $$\frac{d}{d\log\mu}\alpha_i^{-1} = -\frac{1}{2\pi}b_i \qquad \text{(1-loop)}$$

G. Supersymmetry

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- running gauge coupling

R.G.eq
$$\frac{d}{d\log\mu}\alpha_i^{-1} = -\frac{1}{2\pi}b_i \qquad \text{(1-loop)}$$

	INPUT $\alpha_i(m_Z)$	$b_i^{\rm SM}$	$b_i^{\rm MSSM}$
SU(3)	$\simeq 0.118$	-7	-3
SU(2)	$\dfrac{\alpha}{\sin^2\theta_W} \simeq \dfrac{1/128}{0.23}$	$-19/6$	$+1$
U(1)	$\alpha_1 = \left(\dfrac{5}{3}\right)\alpha_Y = \left(\dfrac{5}{3}\right)\dfrac{\alpha}{\cos^2\theta_W}$	$+41/10$	$+33/5$

$g_1 = \sqrt{\frac{5}{3}}g_Y, \quad b_1 = \frac{3}{5}b_Y$

G. Supersymmetry

§ G.1. motivation

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- running gauge coupling

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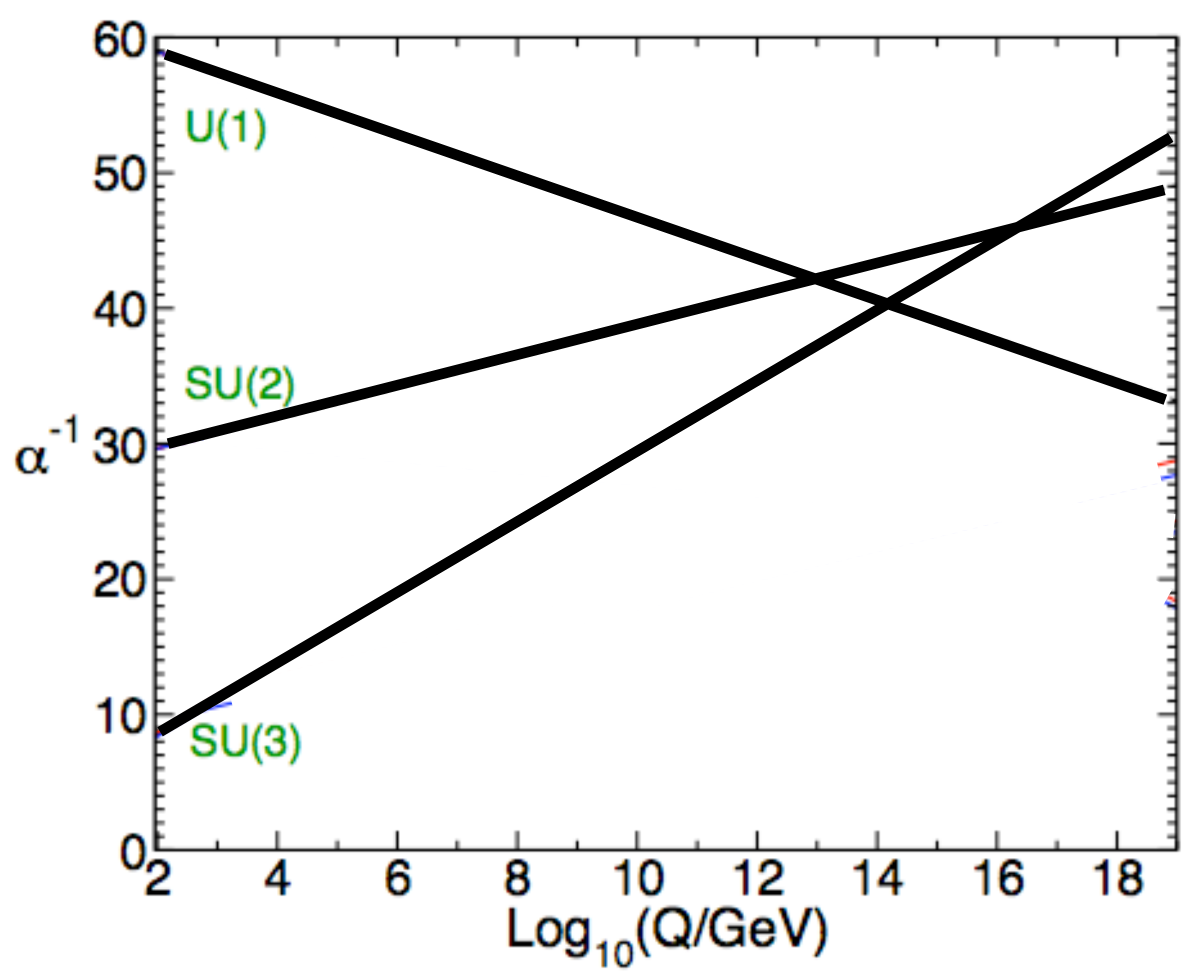
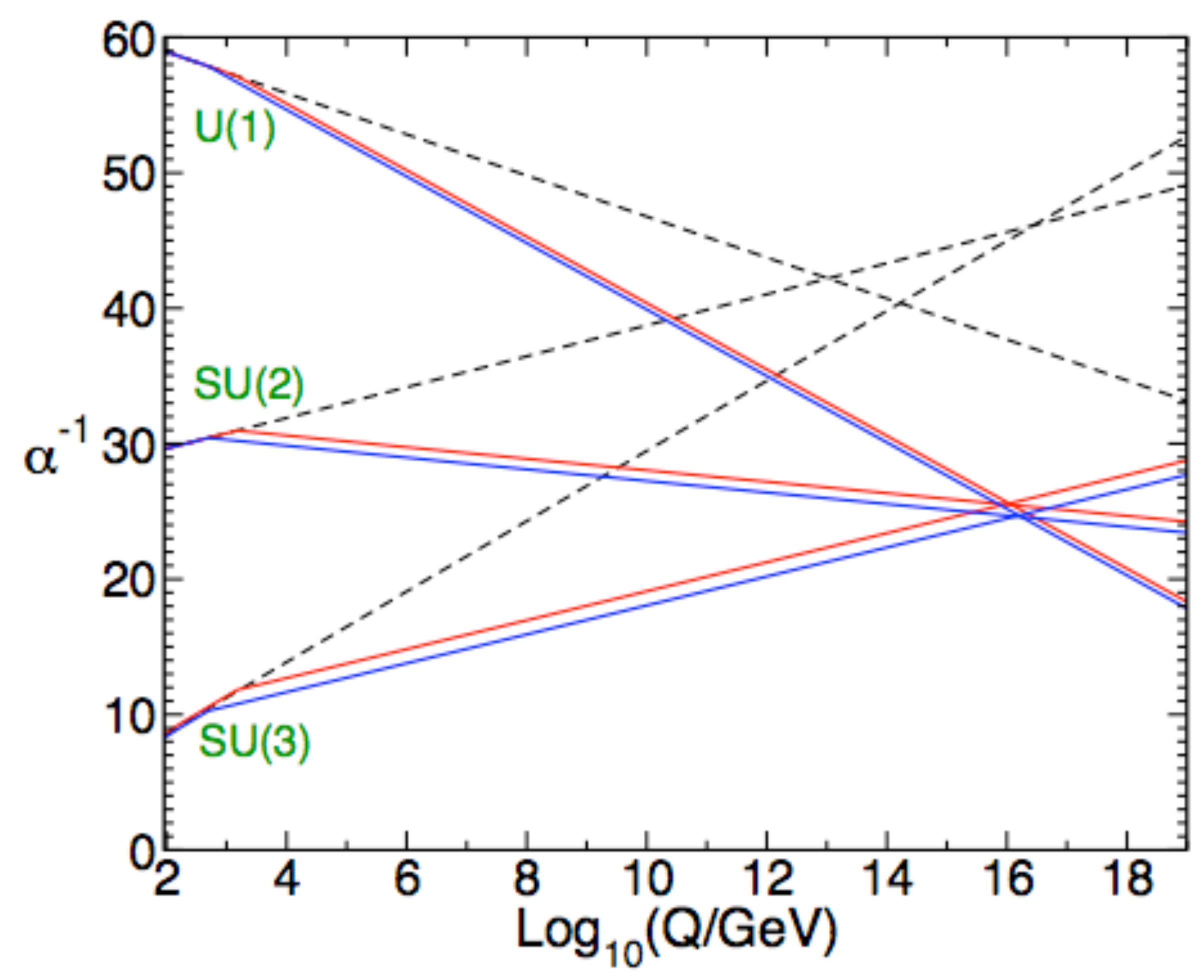
Standard Model



Standard Model + SUSY



PLAN

G.0. naturalness

done

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G. Supersymmetry

§ G.2. SUSY

(i) simplest model (in 4-dim)

$$\underset{\substack{\text{complex}\\ \text{scalar}}}{\phi} \quad \longleftrightarrow \quad \underset{\substack{\text{2-component}\\ \text{Weyl fermion}}}{\psi}$$

$$\underline{\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi}$$

free massless

(
notataion

$\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi = \psi^\dagger_{\dot{\alpha}} \left(\bar{\sigma}^\mu\right)^{\dot{\alpha}\alpha} \partial_\mu \psi_\alpha$ (sum is taken as ${}_{\dot{\alpha}}{}^{\dot{\alpha}}$ and ${}^{\alpha}{}_{\alpha}$)

$$\sigma^\mu = \left(\mathbf{1}, \sigma^i\right), \quad \bar{\sigma}^\mu = \left(\mathbf{1}, -\sigma^i\right), \quad \sigma^i = \left(\begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \begin{pmatrix} & -i \\ i & \end{pmatrix}, \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}\right)$$

$$g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$$

$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta, \quad \psi^\dagger_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\psi^{\dagger\dot{\beta}}, \quad \epsilon^{12} = -\epsilon^{21} = \epsilon_{21} = -\epsilon_{12} = 1, \quad \epsilon^{ii} = \epsilon_{ii} = 0$$
)

(i) simplest model (in 4-dim)

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi$$

What kind of symmetry does it have ?

example: U(1) symmetry (scalar sector)

$$\phi \to e^{i\alpha} \phi$$

or

$$\delta\phi = i\alpha\phi$$

Lagrangian is invariant under this U(1) transformation.

$$\begin{aligned} \delta\mathcal{L} &= \partial_\mu(\delta\phi^*)\partial_\mu\phi + \partial_\mu\phi^*\partial_\mu(\delta\phi) \\ &= \partial_\mu(-i\alpha\phi^*)\partial_\mu\phi + \partial_\mu\phi^*\partial_\mu(i\alpha\phi) \quad = 0 \end{aligned}$$

(i) simplest model (in 4-dim)

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi$$

What kind of symmetry does it have ?

SUSY transformation **Fermion <-> Boson**

(i) simplest model (in 4-dim)

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi$$

What kind of symmetry does it have ?

SUSY transformation $\boxed{\text{Fermion <-> Boson}}$

$$\begin{cases} \delta\phi & = \chi^\alpha \psi_\alpha \\ \delta\psi_\alpha & = -i(\sigma^\mu \chi^\dagger)_\alpha \partial_\mu \phi \end{cases}$$

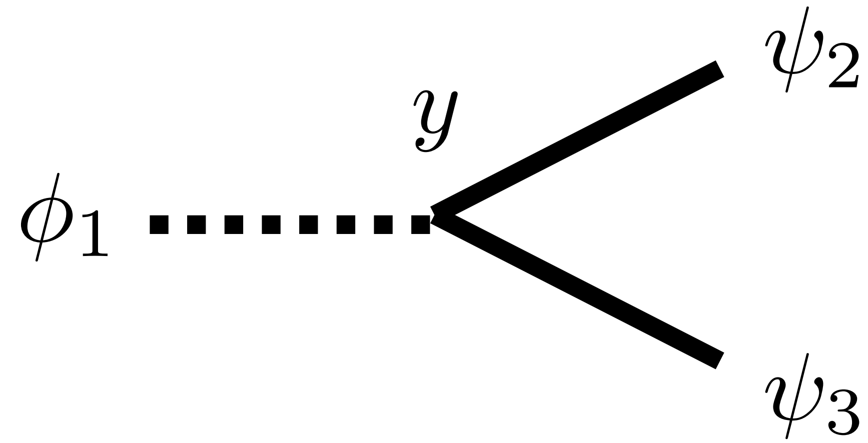
χ_α SUSY transformation parameter.
- 2-component
- anti-commuting (fermionic)

Then,...

$$\begin{aligned} \delta\mathcal{L} &= \delta\left(\partial_\mu \phi^* \partial^\mu \phi\right) + \delta\left(i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi\right) \\ &= \partial_\mu(\delta\phi^*)\partial^\mu\phi + \partial_\mu\phi^*\partial^\mu(\delta\phi) + i(\delta\psi^\dagger)\bar{\sigma}^\mu\partial_\mu\psi + i\psi^\dagger\bar{\sigma}^\mu\partial_\mu(\delta\psi) \\ &= \partial_\mu(\chi^\dagger\psi^\dagger)\partial^\mu\phi + \partial_\mu\phi^*\partial^\mu(\chi\psi) + \ \ldots \\ &= 0 \end{aligned}$$

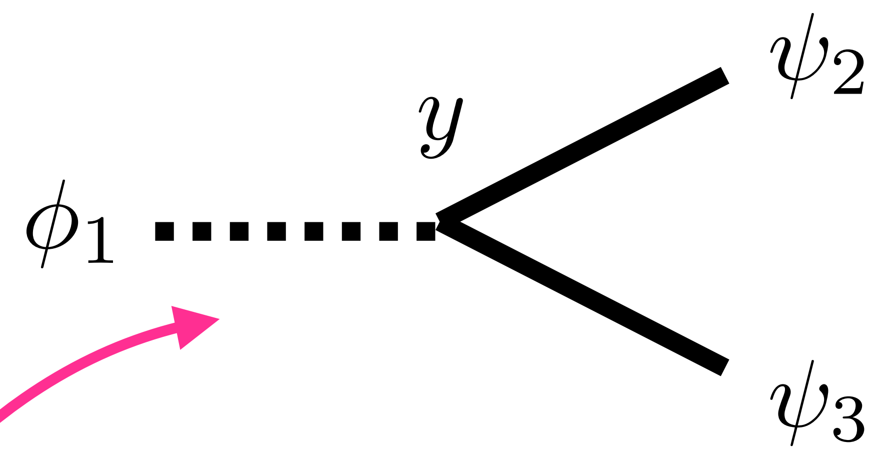
(ii) interaction

$$\mathcal{L}_{\rm int} = -y \cdot \phi_1 \psi_2 \psi_3$$



(ii) interaction

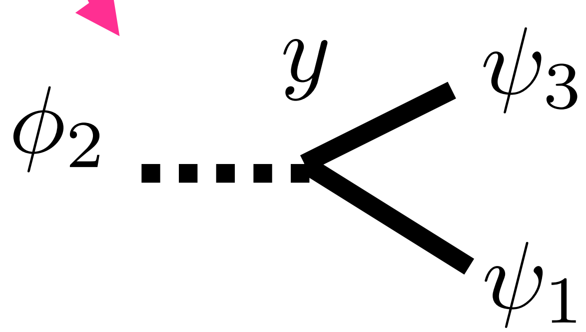
$$\mathcal{L}_{\rm int} = -y \cdot \phi_1 \psi_2 \psi_3$$

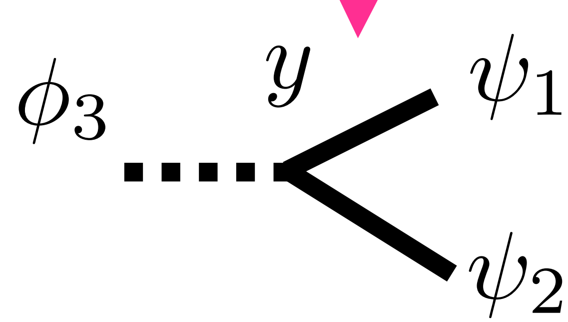


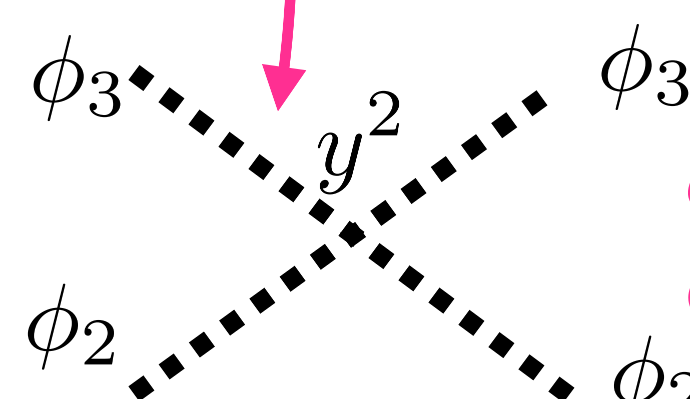
"supersymmetrize"

$$\begin{aligned}\mathcal{L}_{int} = &- y \cdot \phi_1 \psi_2 \psi_3 \qquad - |y|^2 |\phi_2|^2 |\phi_3|^2 \\ &- y \cdot \phi_2 \psi_3 \psi_1 \qquad - |y|^2 |\phi_3|^2 |\phi_1|^2 \\ &- y \cdot \phi_3 \psi_1 \psi_2 \qquad - |y|^2 |\phi_1|^2 |\phi_2|^2\end{aligned}$$

With all these terms, it is invariant under SUSY transformation.



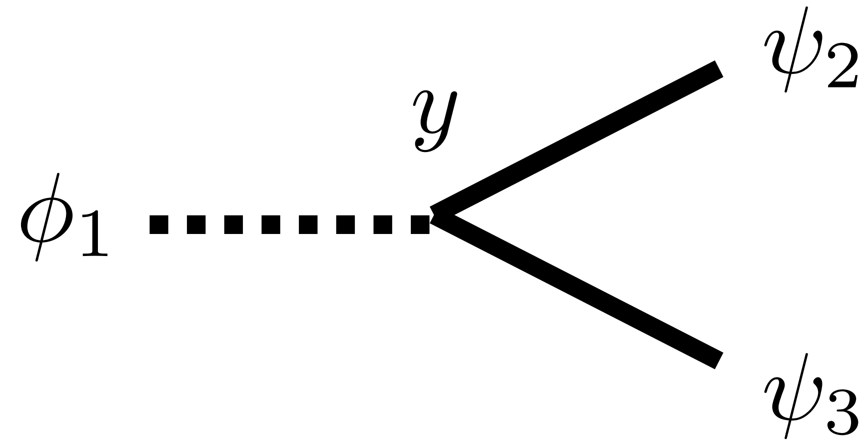




all the same coupling

(ii) interaction

$$\mathcal{L}_{\rm int} = -y \cdot \phi_1 \psi_2 \psi_3$$



"supersymmetrize"

$$\begin{aligned}\mathcal{L}_{int} = & - y \cdot \phi_1 \psi_2 \psi_3 & - |y|^2 |\phi_2|^2 |\phi_3|^2 \\ & - y \cdot \phi_2 \psi_3 \psi_1 & - |y|^2 |\phi_3|^2 |\phi_1|^2 \\ & - y \cdot \phi_3 \psi_1 \psi_2 & - |y|^2 |\phi_1|^2 |\phi_2|^2\end{aligned}$$

With all these terms, it is invariant under SUSY transformation.

$$\boxed{W = y \cdot \Phi_1 \Phi_2 \Phi_3}$$

superpotential

$$\mathcal{L}_{\rm int} = -\frac{\partial^2 W(\phi)}{\partial \phi_i \partial \phi_j} \psi_i \psi_j - \left| \frac{\partial W}{\partial \phi_i} \right|^2$$

$$\Phi_i \sim (\phi_i, \psi_i)$$

superfield: contains boson-fermion pair

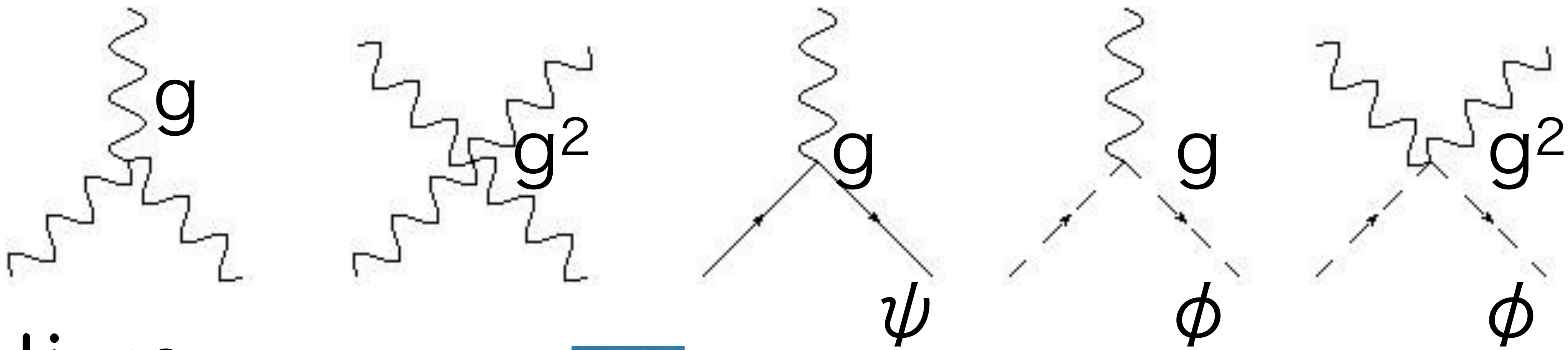
(iii) mass term

$$
\begin{aligned}
W &= M\Phi_1\Phi_2 \\
\to \mathcal{L} &= -\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\psi_i\psi_j - \left|\frac{\partial W}{\partial\phi_i}\right|^2 \\
&= -M\psi_1\psi_2 - |M|^2|\phi_1|^2 - |M|^2|\phi_2|^2
\end{aligned}
$$

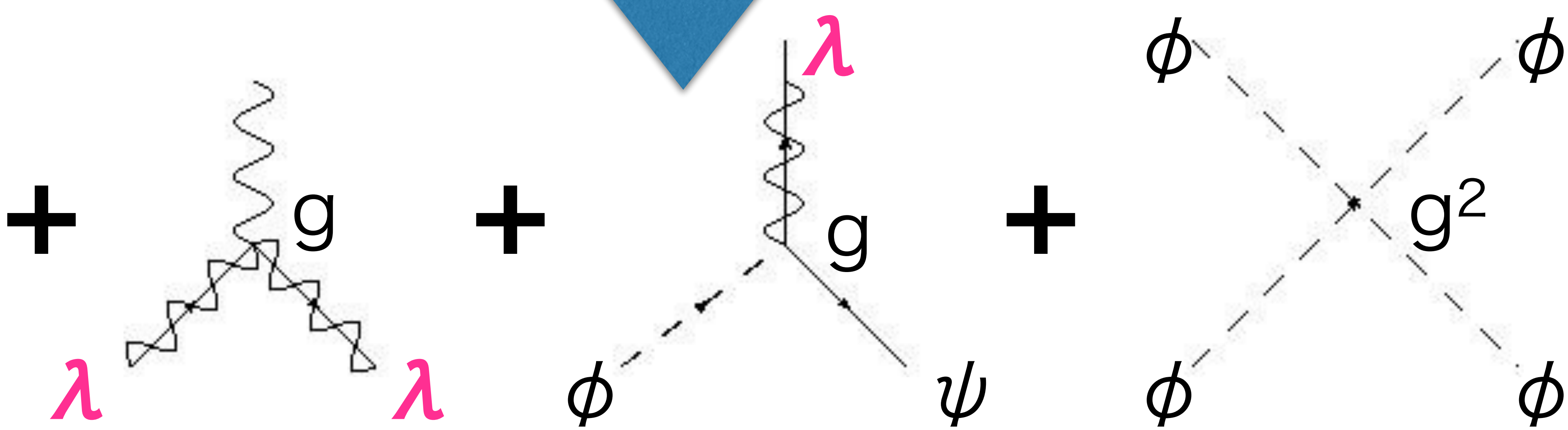
fermion mass = boson mass

(iv) gauge sector

gauge interactions



"supersymmetrize"



gaugino
(= fermionic partner of gauge boson)

With all these terms, the Lagrangian is invariant under SUSY transformation.

(v) summary

ϕ (scalar) $\longleftrightarrow$ ψ (fermion)

A_μ (gauge) $\longleftrightarrow$ λ (fermion)

They have the same masses, and the same couplings.

PLAN

G.0. naturalness

§G. SUSY:

done

G.1. motivations

G.2. supersymmetry

G.3. MSSM (minimal SUSY Standard Model)

G.4. MSSM Lagrangian

G.5. SUSY after Higgs discovery

reference: hep-ph/9709356 by S.P.Martin.

§ G.3. MSSM

Standard Model

		spin		
quark	q	1/2 <-> 0	squark	$\widetilde{q}$
lepton	ℓ	1/2 <-> 0	slepton	$\widetilde{\ell}$
Higgs	H	0 <-> 1/2	higgsino	$\widetilde{h}$
gauge bosons γ, Z, W, g		1 <-> 1/2	gaugino $\widetilde{B}, \widetilde{W}, \widetilde{g}$	

MSSM (minimal SUSY Standard Model)

§ G.3. MSSM

More precisely.........

spin

Standard Model		spin		
quark	q	1/2 <-> 0	squark	$\widetilde{q}$
lepton	ℓ	1/2 <-> 0	slepton	$\widetilde{\ell}$
Higgs	$\cancel{H}$	0 <-> 1/2	higgsino	$\widetilde{h}$
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MSSM (minimal SUSY Standard Model)

§ G.3. MSSM

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MSSM (minimal SUSY Standard Model)

§ G.3. MSSM

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MSSM (minimal SUSY Standard Model)

comment 1: Why 2 Higgs ?

reason 1. Yukawa coupling.

$$\mathcal{L}^{\rm SM} = -y_u H^* \bar{u} Q - y_d H \bar{d} Q$$

conjugate

but in SUSY,

$$W = y_u \cancel{H^* Q u^c} + y_d H Q d^c$$

$W(\phi)$ must be a function of ϕ, not , ϕ^*

$$W = y_u H_u Q u^c + y_d H_d Q d^c$$

2 Higgs

§ G.3. MSSM

		spin	
quark	q	1/2 <-> 0	squark $\widetilde{q}$
lepton	ℓ	1/2 <-> 0	slepton $\widetilde{\ell}$
Higgs	H_u, H_d	0 <-> 1/2	higgsino $\widetilde{h}$
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MSSM (minimal SUSY Standard Model)

comment 1: Why 2 Higgs ?

reason 2. Anomaly

$$1 \text{ Higgs}: \quad H \Longleftrightarrow \widetilde{h} = (1,2)_{1/2} \quad \text{gauge anomaly}$$

$$2 \text{ Higgs}: \quad H_u \Longleftrightarrow \widetilde{h_u} = (1,2)_{1/2} \quad \text{anomaly}$$

$$H_d \Longleftrightarrow \widetilde{h_d} = (1,2)_{-1/2} \quad \text{cancellation}$$

§ G.3. MSSM

		spin		
quark	q	1/2 <-> 0	squark	$\widetilde{q}$
lepton	ℓ	1/2 <-> 0	slepton	$\widetilde{\ell}$
Higgs	H_u, H_d	0 <-> 1/2	higgsino	$\widetilde{h}$
gauge bosons γ, Z, W, g		1 <-> 1/2	gaugino $\widetilde{B}, \widetilde{W}, \widetilde{g}$	

MSSM (minimal SUSY Standard Model)

comment 2:

mass eigenstates

neutral fermions $\widetilde{h_u^0}, \widetilde{h_d^0}, \widetilde{B}, \widetilde{W}^0 \Longrightarrow \widetilde{\chi}^0_{1,2,3,4}$ neutralinos

charged fermions $\widetilde{h_u^+}, \widetilde{h_d^-}, \widetilde{W}^\pm \Longrightarrow \widetilde{\chi}^\pm_{1,2}$ charginos

§ G.3. MSSM

		spin		
quark	q	1/2 <-> 0	squark	$\widetilde{q}$
lepton	ℓ	1/2 <-> 0	slepton	$\widetilde{\ell}$
Higgs	H_u, H_d	0 <-> 1/2	higgsino	$\widetilde{h}$
gauge bosons γ, Z, W, g		1 <-> 1/2	gaugino	$\widetilde{B}, \widetilde{W}, \widetilde{g}$

MSSM (minimal SUSY Standard Model)

§ G.3. MSSM

More precisely,...

	Standard Model			spin	SUSY partner			$(SU(3), SU(2))_{U(1)}$
Q_i	$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\frac{1}{2} \longleftrightarrow 0$	$\begin{pmatrix} \widetilde{u} \\ \widetilde{d} \end{pmatrix}_L$	$\begin{pmatrix} \widetilde{c} \\ \widetilde{s} \end{pmatrix}_L$	$\begin{pmatrix} \widetilde{t} \\ \widetilde{b} \end{pmatrix}_L$	$(3, 2)_{1/6}$
$\bar{u}_i$	$u_R^\dagger$	$c_R^\dagger$	$t_R^\dagger$		$\widetilde{u}_R^*$	$\widetilde{c}_R^*$	$\widetilde{t}_R^*$	$(\bar{3}, 1)_{-2/3}$
$\bar{d}_i$	$d_R^\dagger$	$s_R^\dagger$	$b_R^\dagger$		$\widetilde{d}_R^*$	$\widetilde{s}_R^*$	$\widetilde{b}_R^*$	$(\bar{3}, 1)_{1/3}$
L_i	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\frac{1}{2} \longleftrightarrow 0$	$\begin{pmatrix} \widetilde{\nu}_e \\ \widetilde{e} \end{pmatrix}_L$	$\begin{pmatrix} \widetilde{\nu}_\mu \\ \widetilde{\mu} \end{pmatrix}_L$	$\begin{pmatrix} \widetilde{\nu}_\tau \\ \widetilde{\tau} \end{pmatrix}_L$	$(1, 2)_{-1/2}$
$\bar{e}_i$	$e_R^\dagger$	$\mu_R^\dagger$	$\tau_R^\dagger$		$\widetilde{e}_R^*$	$\widetilde{\mu}_R^*$	$\widetilde{\tau}_R^*$	$(1, 1)_1$
Higgs		$\begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}_L$		$0 \longleftrightarrow \frac{1}{2}$		$\begin{pmatrix} \widetilde{h}_u^+ \\ \widetilde{h}_u^0 \end{pmatrix}$		$(1, 2)_{+1/2}$
		$\begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}_L$				$\begin{pmatrix} \widetilde{h}_d^0 \\ \widetilde{h}_d^- \end{pmatrix}$		$(1, 2)_{-1/2}$
gauge	γ		B	$1 \longleftrightarrow \frac{1}{2}$		$\widetilde{B}$		$(1, 1)_0$
	Z		W^0			$\widetilde{W}^0$		$(1, 3)_0$
		$W^\pm$				$\widetilde{W}^\pm$		
		g				$\widetilde{g}$		$(8, 1)_0$
		graviton e_μ^a		$2 \longleftrightarrow \frac{3}{2}$		gravitino ψ_μ^α		

PLAN

G.0. naturalness

§G. SUSY:

G.1. motivations

G.2. supersymmetry

G.3. MSSM (minimal SUSY Standard Model)

done

G.4. MSSM Lagrangian

G.5. SUSY after Higgs discovery

reference: hep-ph/9709356 by S.P.Martin.

§ G.4. MSSM Lagrangian

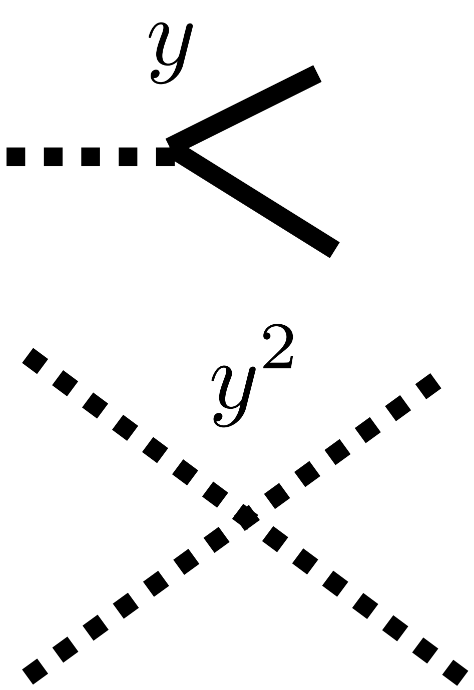
$$\mathcal{L}_{\mathrm{MSSM}} = \mathcal{L}_{\mathrm{SM}\to\mathrm{SUSY}} + \mathcal{L}_{\mathrm{soft}\ \cancel{\mathrm{SUSY}}}$$

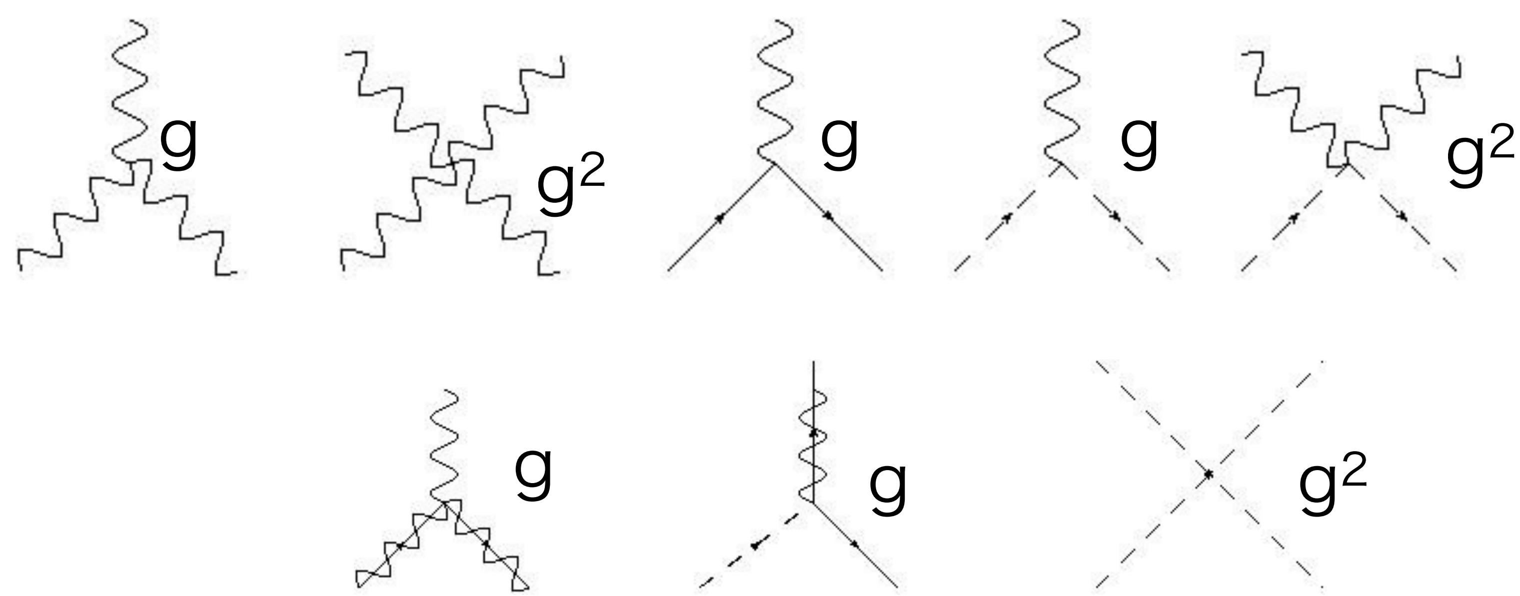
§ G.4. MSSM Lagrangian

$$\mathcal{L}_{\mathrm{MSSM}} = \mathcal{L}_{\mathrm{SM}\to\mathrm{SUSY}} + \mathcal{L}_{\mathrm{soft}\ \cancel{\mathrm{SUSY}}}$$

(i) $\mathcal{L}_{\mathrm{SM}\to\mathrm{SUSY}}$

$$= \mathcal{L}_{\text{from superpotential}} + \mathcal{L}_{\text{from gauge interactions}}$$





no new free parameter
compared to SM.

§ G.4. MSSM Lagrangian

$$\mathcal{L}_{\mathrm{MSSM}} = \mathcal{L}_{\mathrm{SM}\to\mathrm{SUSY}} + \mathcal{L}_{\mathrm{soft}\ \cancel{\mathrm{SUSY}}}$$

$$\textbf{(i)}\quad \mathcal{L}_{\mathrm{SM}\to\mathrm{SUSY}}$$

$$= \mathcal{L}_{\text{from superpotential}} + \mathcal{L}_{\text{from gauge interactions}}$$

$$W_{\mathrm{MSSM}} = \underbrace{(y_u)_{ij} H_u Q_i u^c_j + (y_d)_{ij} H_d Q_i d^c_j + (y_e)_{ij} H_d Q_i e^c_i}_{\text{corresponding to SM Yukawa}} + \underbrace{\mu H_u H_d}_{\text{“}\mu\text{-term”}}$$

§ G.4. MSSM Lagrangian

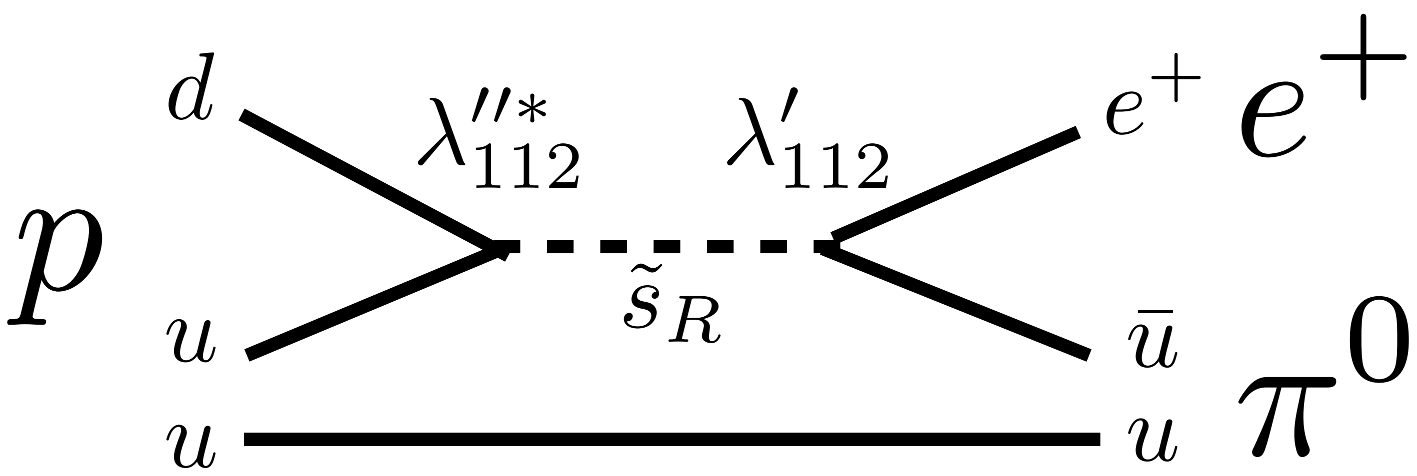
$$W_{\mathrm{MSSM}} = \underbrace{(y_u)_{ij} H_u Q_i u_j^c + (y_d)_{ij} H_d Q_i d_j^c + (y_e)_{ij} H_d Q_i e_i^c}_{\text{corresponding to SM Yukawa}} + \underbrace{\mu H_u H_d}_{\text{“}\mu\text{-term”}}$$

NOTE:

There are other renormalizable terms allowed by gauge invariance.

$$W_{\mathrm{RpV}} = \underbrace{\frac{1}{2}\lambda^{ijk} L_i L_j e_k^c + \lambda'^{ijk} L_i Q_j d_k^c + \mu_i' L_i H_u}_{L\text{-violating}} + \underbrace{\frac{1}{2}\lambda''^{ijk} u_i^c d_j^c d_k^c}_{B\text{-violating}}$$

But they mediate a very rapid proton decay!



exp. bound (super-K)

$$\tau(p \to e^+ \pi^0) > 1.3 \times 10^{34} \text{years}$$

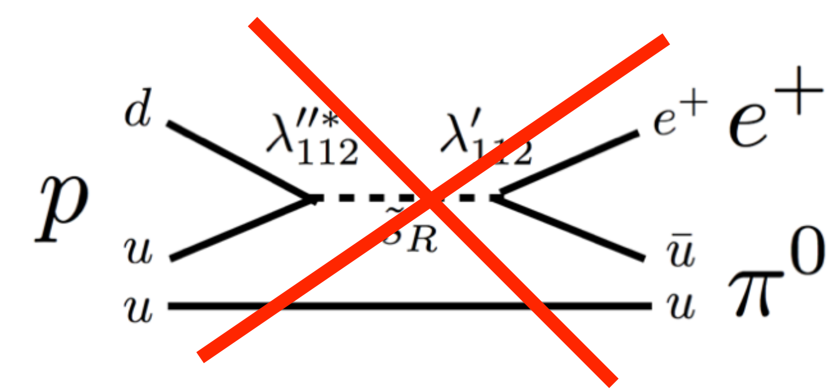
$$\to |\lambda'_{11k}\lambda''_{11k}| \lesssim 10^{-29} \left(\frac{m_{\tilde{d}_i}}{1\ \mathrm{TeV}}\right)^2$$

§ G.4. MSSM Lagrangian

$$W_{\mathrm{MSSM}} = \underbrace{(y_u)_{ij} H_u Q_i u_j^c + (y_d)_{ij} H_d Q_i d_j^c + (y_e)_{ij} H_d Q_i e_i^c}_{\text{corresponding to SM Yukawa}} + \underbrace{\mu H_u H_d}_{\text{“}\mu\text{-term”}}$$

$$\cancel{W_{\mathrm{RpV}} = \underbrace{\frac{1}{2}\lambda^{ijk} L_i L_j e_k^c + \lambda'^{ijk} L_i Q_j d_k^c + \mu_i' L_i H_u}_{L\text{-violating}} + \underbrace{\frac{1}{2}\lambda''^{ijk} u_i^c d_j^c d_k^c}_{B\text{-violating}}}$$

There is a parity symmetry which forbids W_{RpV} and allows W_{MSSM} .



R-parity

SM particles —> even (+)
SUSY particles —> odd (-)

rapid proton decay is forbidden.

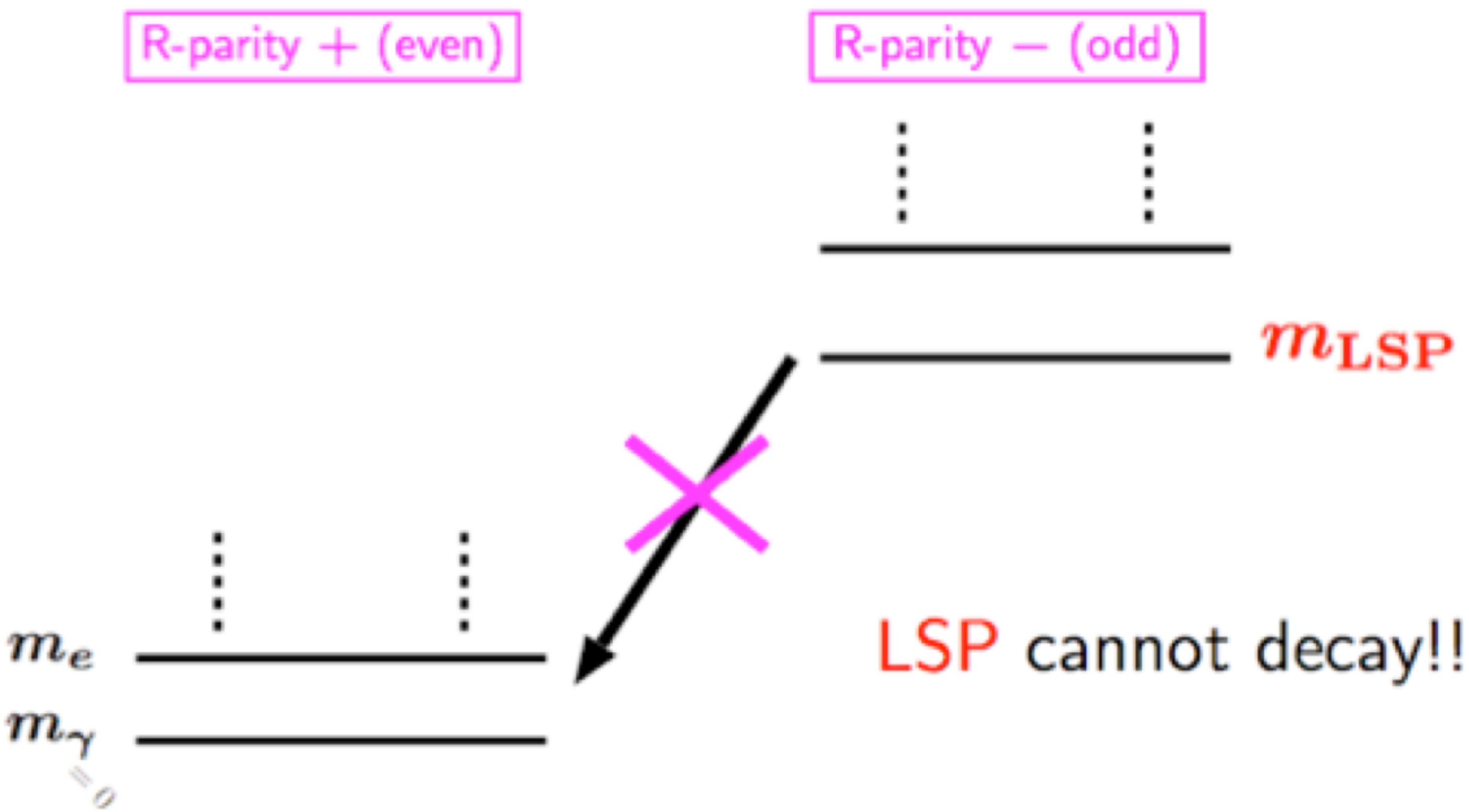
R-parity

SM particles —> even (+)

SUSY particles —> odd (-)

—> Lightest SUSY Particle (**LSP**) becomes stable.

—> Dark Matter candidate!



R-parity

SM particles —> even (+)

SUSY particles —> odd (-)

—> Lightest SUSY Particle (**LSP**) becomes stable.

—> **Dark Matter candidate!**

$$\text{squarks}: \begin{pmatrix} \widetilde{u_L} \\ \widetilde{d_L} \end{pmatrix}_i \quad \begin{matrix} \widetilde{u_{Ri}} \\ \widetilde{d_{Ri}} \end{matrix} \qquad \text{sleptons}: \begin{pmatrix} \widetilde{\nu_L} \\ \widetilde{e_L} \end{pmatrix}_i \quad \widetilde{e_{Ri}}$$

$$\text{gauginos and higgssinos}: \widetilde{\chi_i^0}, \quad \widetilde{\chi_i^\pm}, \quad \tilde{g}$$

$$\text{gravitino}: \tilde{G}$$

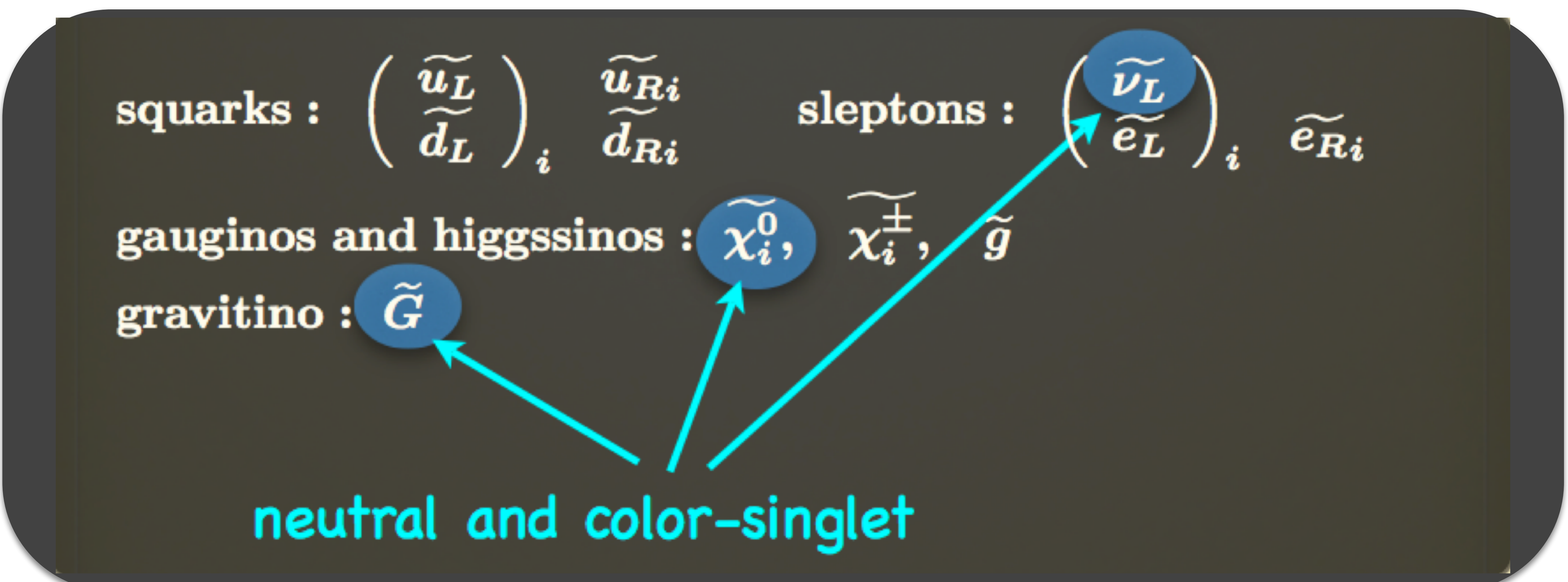
R-parity

SM particles —> even (+)

SUSY particles —> odd (-)

—> Lightest SUSY Particle (**LSP**) becomes stable.

—> **Dark Matter candidate!**



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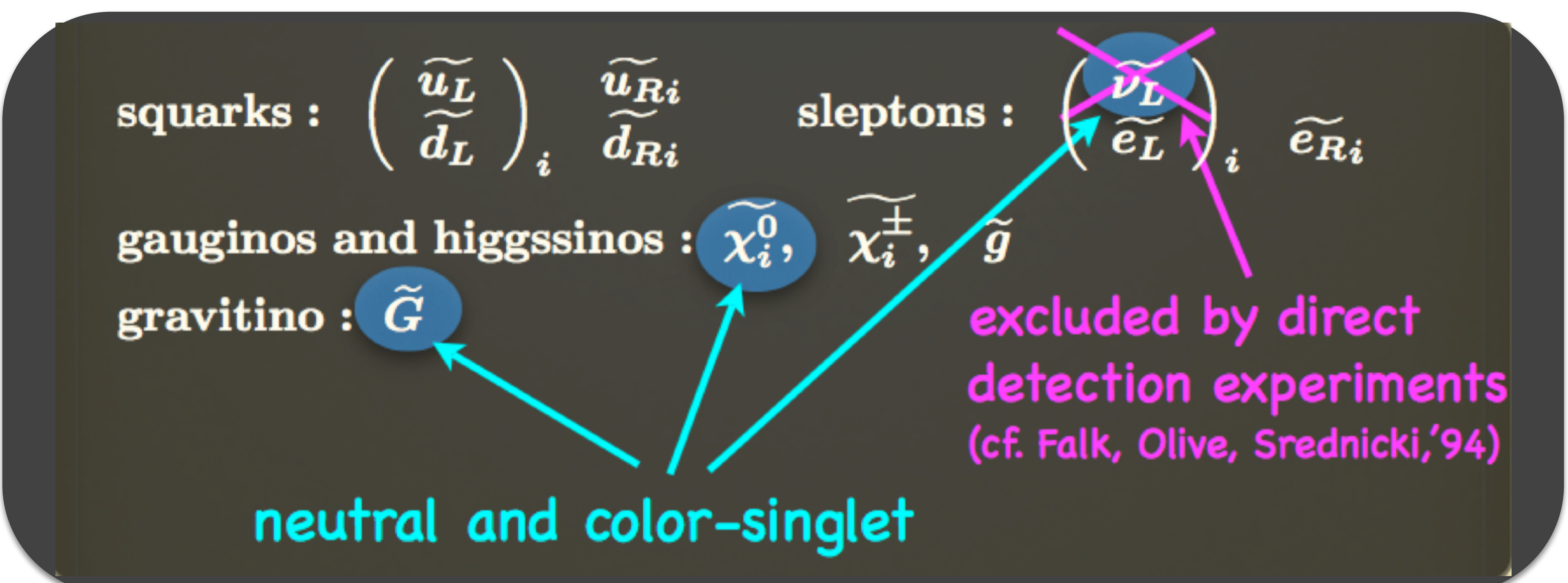


R-parity

SM particles —> even (+)
SUSY particles —> odd (-)

—> Lightest SUSY Particle (**LSP**) becomes stable.
—> Dark Matter candidate!

LSP DM candidates within MSSM (+ supergravity):

- **neutralino**
- **gravitino**

§ G.4. MSSM Lagrangian

$$\mathcal{L}_{\mathrm{MSSM}} = \mathcal{L}_{\mathrm{SM}\to\mathrm{SUSY}} + \mathcal{L}_{\mathrm{soft}\ \cancel{\mathrm{SUSY}}}$$

$$\textbf{(ii)}\quad \mathcal{L}_{\mathrm{soft}\ \cancel{\mathrm{SUSY}}}$$

Supersymmetry must be
a (spontaneously) broken symmetry

electron ~~**selectron (scalar)**~~

511 keV <—> ~~**511 keV**~~

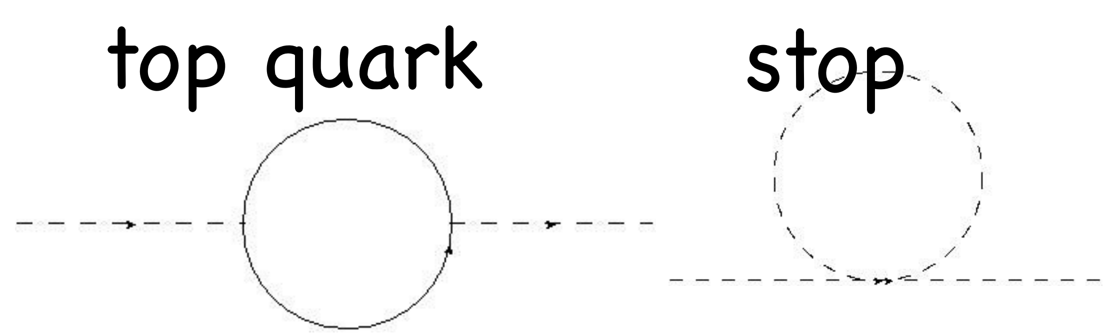
§ G.4. MSSM Lagrangian

$$\mathcal{L}_{\rm MSSM} = \mathcal{L}_{\rm SM \to SUSY} + \mathcal{L}_{\rm soft\ \cancel{SUSY}}$$

$$\textbf{(ii)} \quad \mathcal{L}_{\boxed{\rm soft}\ \cancel{\rm SUSY}}$$

SUSY must broken only by **parameters with mass dim. > 0.**

$\delta\, m_H^2 \sim$



hard breaking $\quad y_{top} \neq y_{stop}\,, \longrightarrow \delta\, m_H^2 \sim (y_{stop}^2 - y_{top}^2)\,\Lambda^2$ 😩

soft breaking $\quad m_{top} \neq m_{stop}\,, \longrightarrow \delta\, m_H^2 \sim (m_{stop}^2 - m_{top}^2)\,\log \Lambda$ 😐

§ G.4. MSSM Lagrangian

$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SM}\to\text{SUSY}} + \mathcal{L}_{\text{soft } \cancel{\text{SUSY}}}$$

$$\textbf{(ii)} \quad \mathcal{L}_{\boxed{\text{soft}}\ \cancel{\text{SUSY}}}$$

SUSY must broken only by **parameters with mass dim. > 0.**

$$\begin{aligned}
\mathcal{L}_{\text{soft}}^{\text{MSSM}} \;=\; & -\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + \text{c.c.}\right) \\
& -\left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d + \text{c.c.}\right) \\
& -\widetilde{Q}^\dagger\,\mathbf{m^2_Q}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m^2_L}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m^2_{\overline{u}}}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m^2_{\overline{d}}}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m^2_{\overline{e}}}\,\widetilde{\overline{e}}^\dagger \\
& - m^2_{H_u}H_u^*H_u - m^2_{H_d}H_d^*H_d - (bH_uH_d + \text{c.c.})\,.
\end{aligned}$$

gaugino masses

A-terms
(3x3 matrices.)

Higgs soft terms

squark and slepton masses
(3x3 matrices.)

§ G.4. MSSM Lagrangian

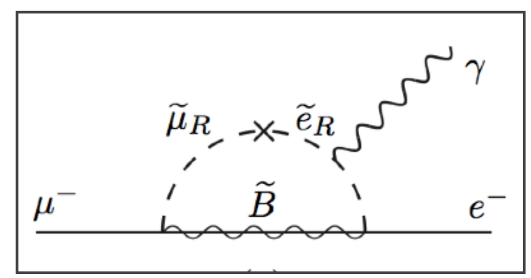
$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SM}\to\text{SUSY}} + \mathcal{L}_{\text{soft } \cancel{\text{SUSY}}}$$

(ii) $\mathcal{L}_{\text{soft } \cancel{\text{SUSY}}}$

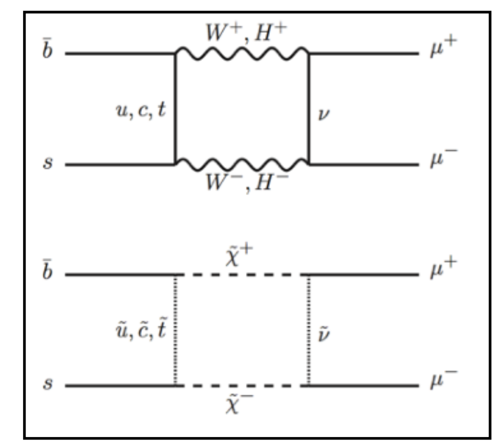
$$\begin{aligned}
\mathcal{L}^{\text{MSSM}}_{\text{soft}} = & -\frac{1}{2}\left(M_3 \widetilde{g}\widetilde{g} + M_2 \widetilde{W}\widetilde{W} + M_1 \widetilde{B}\widetilde{B} + \text{c.c.}\right) \\
& - \left(\widetilde{\bar{u}}\, \mathbf{a_u}\, \widetilde{Q} H_u - \widetilde{\bar{d}}\, \mathbf{a_d}\, \widetilde{Q} H_d - \widetilde{\bar{e}}\, \mathbf{a_e}\, \widetilde{L} H_d + \text{c.c.}\right) \\
& - \widetilde{Q}^\dagger\, \mathbf{m^2_Q}\, \widetilde{Q} - \widetilde{L}^\dagger\, \mathbf{m^2_L}\, \widetilde{L} - \widetilde{\bar{u}}\, \mathbf{m^2_{\bar{u}}}\, \widetilde{\bar{u}}^\dagger - \widetilde{\bar{d}}\, \mathbf{m^2_{\bar{d}}}\, \widetilde{\bar{d}}^\dagger - \widetilde{\bar{e}}\, \mathbf{m^2_{\bar{e}}}\, \widetilde{\bar{e}}^\dagger \\
& - m^2_{H_u} H^*_u H_u - m^2_{H_d} H^*_d H_d - (b H_u H_d + \text{c.c.})\,.
\end{aligned}$$

This part contains a variety of interesting SUSY phenomenologies……

- SUSY particle masses,
- Higgs sector (tree level mass, loop corrections…),
- **Flavor Changing Neutral Current (FCNC) and CP-violation,**
- SUSY breaking mechanism and its mediations, model-building,
  (Gravity mediation, Gauge mediation, Anomaly mediation,……)
- **Collider physics,**
- **Dark Matter**
- ……

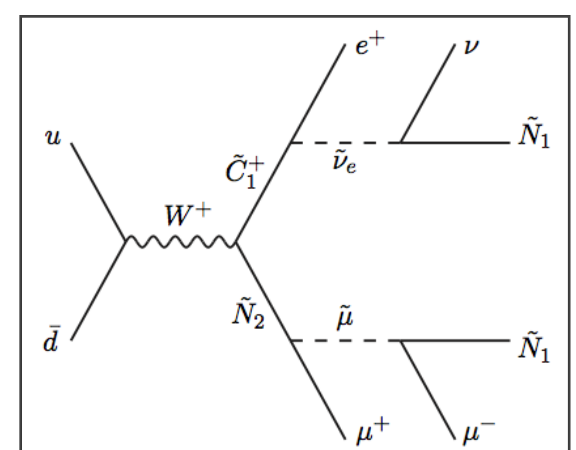


$\mu^- \to e^- \gamma$



various B-decays

But I skip the details here.
For a review, see,e.g., hep-ph/9709356 by S.P.Martin.



collider signals…

PLAN

G.0. naturalness

§G. SUSY:

G.1. motivations

G.2. supersymmetry

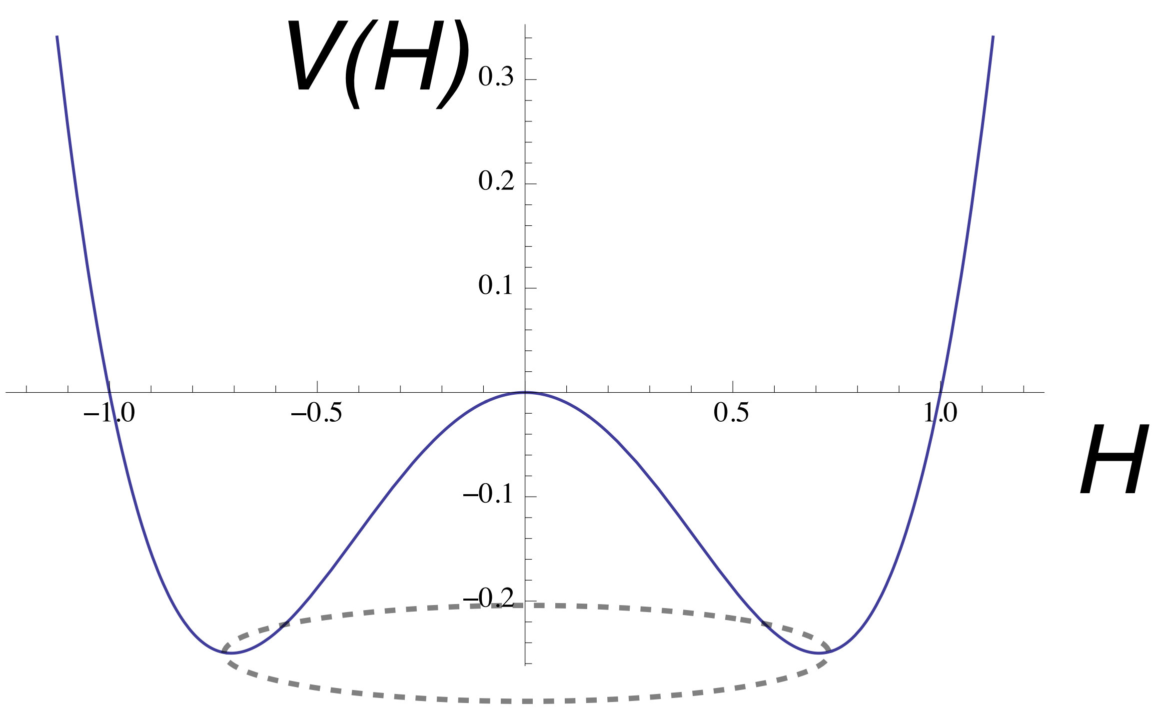
G.3. MSSM (minimal SUSY Standard Model)

G.4. MSSM Lagrangian

G.5. SUSY after Higgs discovery

125 GeV Higgs

$$V(H) = -m^2(H^\dagger H) + \lambda_\mathrm{H}(H^\dagger H)^2$$



125 GeV Higgs

$$V(H) = -m^2 (H^\dagger H) + \lambda_{\mathrm{H}} (H^\dagger H)^2$$

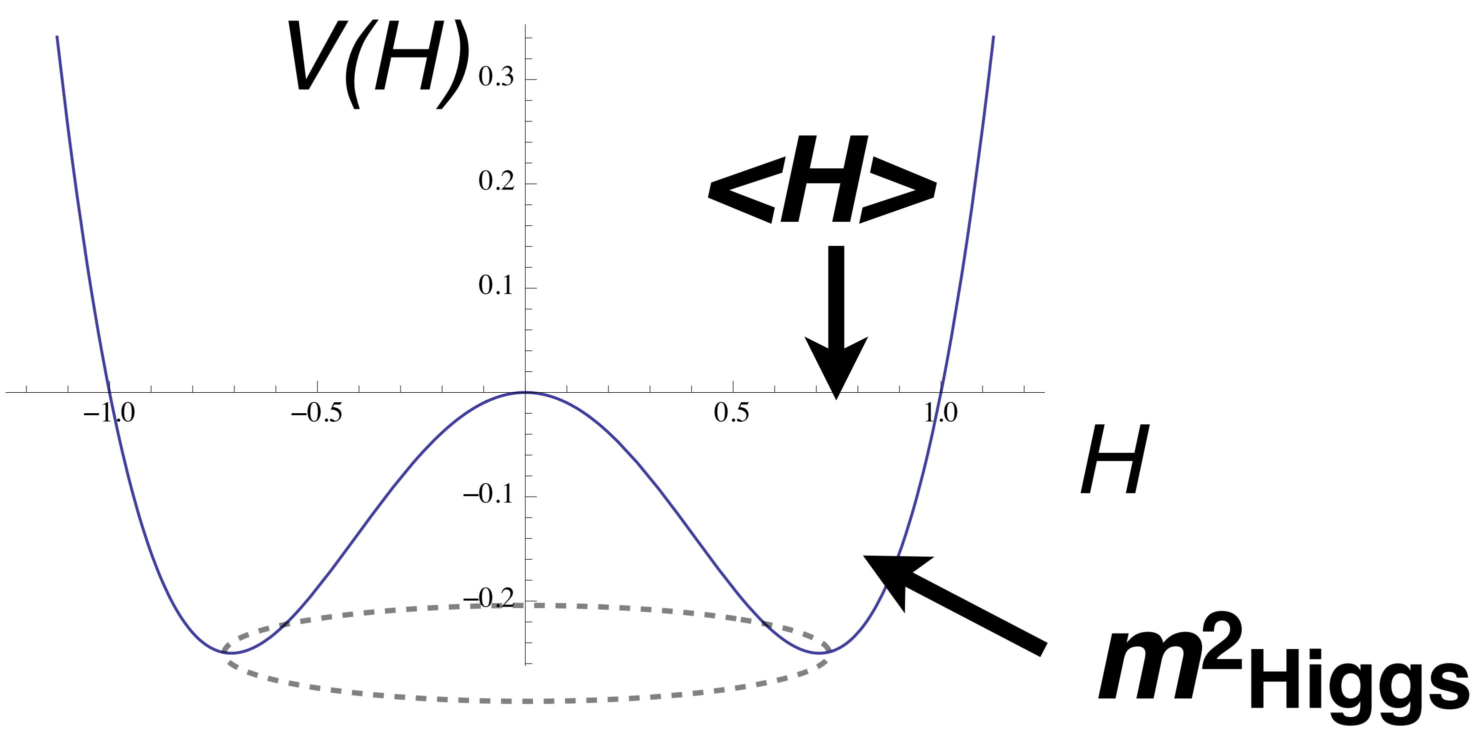
We knew...

Now we also know

$$\rightarrow \begin{cases} \langle H \rangle^2 = \dfrac{m^2}{2\,\lambda_{\mathrm{H}}} = \dfrac{1}{2\sqrt{2}\,G_F} \simeq (174\ \mathrm{GeV})^2 \\ m^2_{\mathrm{Higgs}} = 2\,m^2 \simeq (\mathbf{125\ GeV})^2 \end{cases}$$

Fermi constant

$G_F \simeq 1.17 \times 10^{-5}\mathrm{GeV}^{-2}$



125 GeV Higgs

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Now we also know

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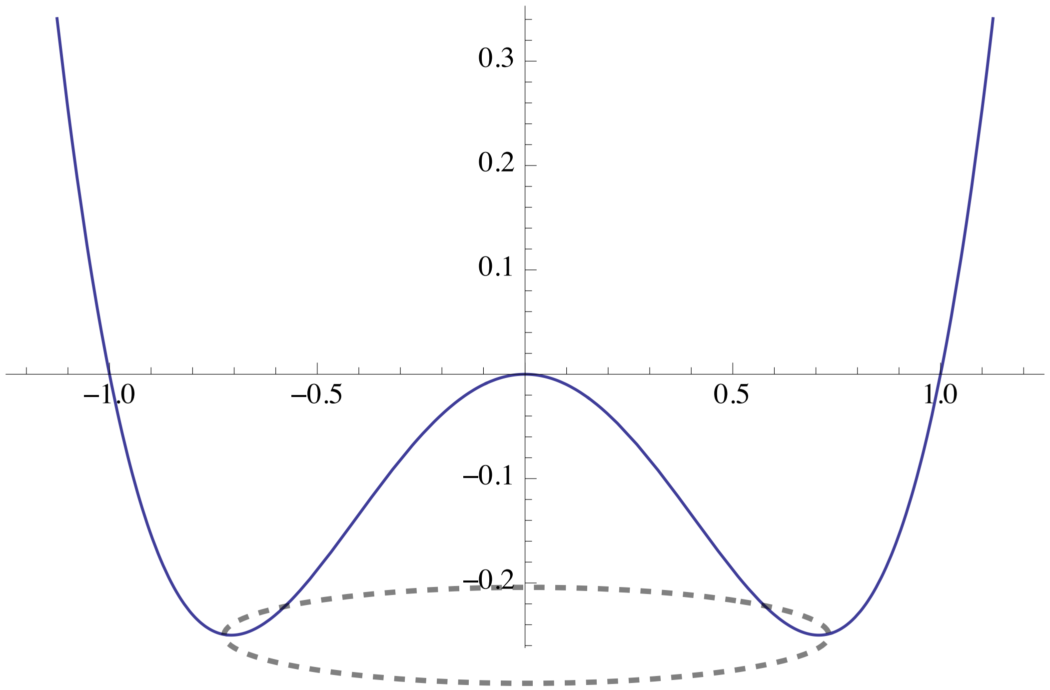


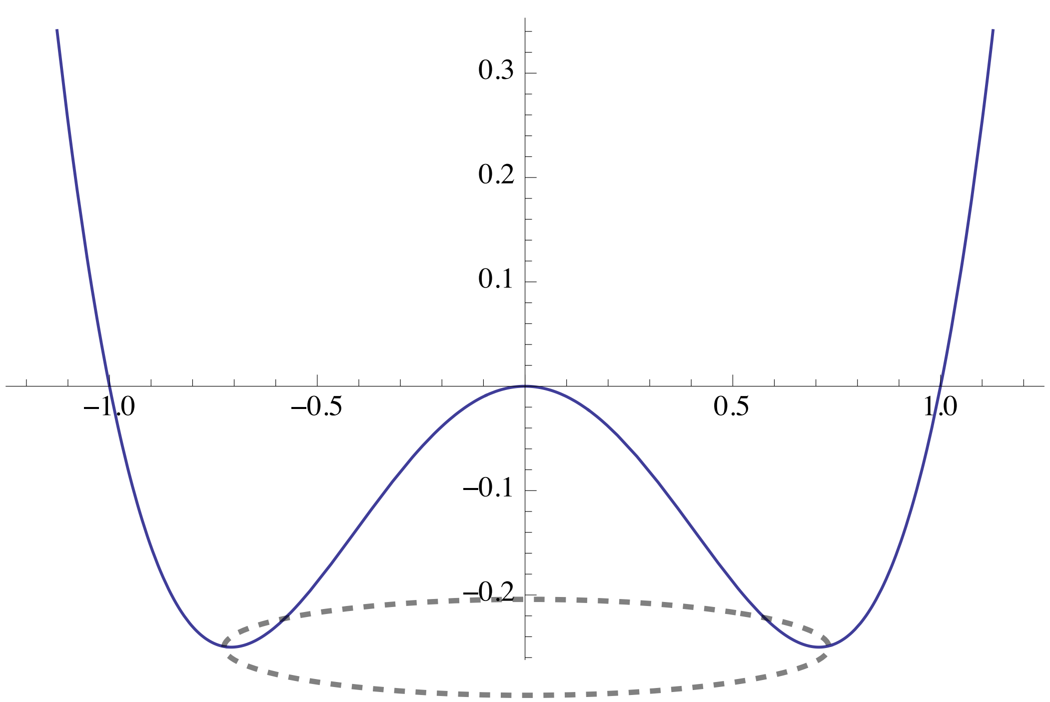
$$\to \begin{cases} m^2 = \dfrac{m^2_{\mathrm{Higgs}}}{2} \simeq (\mathbf{89\ GeV})^2 \\ \lambda_H = \dfrac{m^2_{\mathrm{Higgs}}}{4\langle H \rangle^2} \simeq \mathbf{0.13} \end{cases}$$

125 GeV Higgs

$$V(H) = -m^2 (H^\dagger H) + \lambda_{\rm H} (H^\dagger H)^2$$

$$\mathbf{(89\ GeV)^2} \qquad\qquad \mathbf{0.13}$$

completely determined !



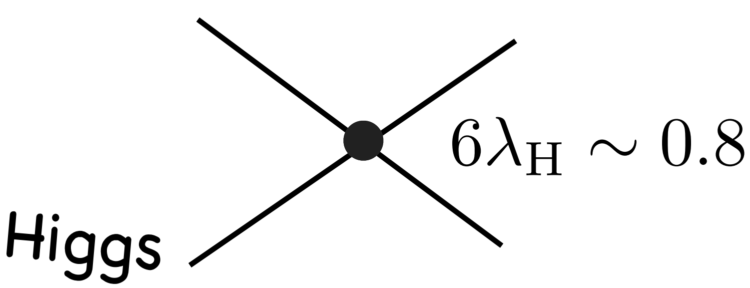
$$\rightarrow \begin{cases} m^2 = \dfrac{m^2_{\rm Higgs}}{2} \simeq \mathbf{(89\ GeV)^2} \\ \lambda_{\rm H} = \dfrac{m^2_{\rm Higgs}}{4\langle H\rangle^2} \simeq \mathbf{0.13} \end{cases}$$

125 GeV Higgs

$$V(H) = -m^2(H^\dagger H) + \lambda_H (H^\dagger H)^2$$

$$(89\ \mathrm{GeV})^2 \qquad\qquad 0.13$$

It seems... Higgs sector is also described by weakly coupled, perturbative QFT.
(at least no sign of strong interaction etc, so far...)



Imlications for BSM (in my opinion.....)

This is compatible with....

- GUT and coupling unification in perturbative QFT.
- heavy right-handed neutrinos (Seesaw + Leptogenesis)
- Supersymmetry

Supersymmetry

boson $\Leftrightarrow$ fermion

Supersymmetry (SUSY)

	spin		spin	
quarks q	$\frac{1}{2}$	$\longleftrightarrow$	0	squarks $\tilde{q}$
leptons ℓ	$\frac{1}{2}$	$\longleftrightarrow$	0	sleptons $\tilde{\ell}$
gauge bosons A_μ	1	$\longleftrightarrow$	$\frac{1}{2}$	gauginos λ
Higgs bosons H	0	$\longleftrightarrow$	$\frac{1}{2}$	higgsinos $\tilde{h}$

Standard Model

naturalness

fine-tuning problem

$$m_H^2 = m_{H,0}^2 + \Lambda^2 \qquad (\Lambda \gg m_H)$$

(fine tuning like 1.0000000000000001 - 1)

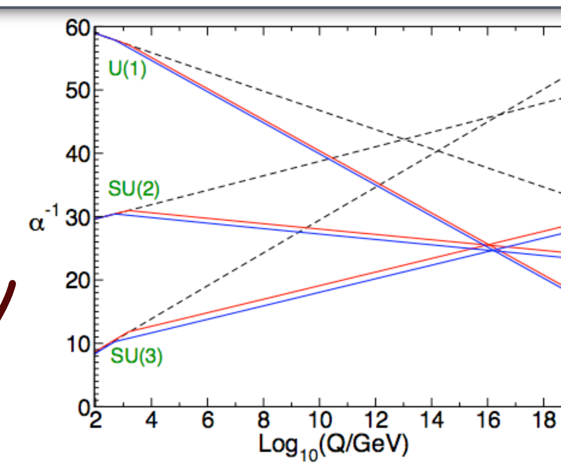
$\rightarrow$ solved by the supersymmetry !

$$m_H^2 = m_{H,0}^2 + (\ \Lambda^2 - \Lambda^2\ )$$

fermion boson

coupling unification

Grand Unified Theory



without SUSY

with SUSY

Dark Matter = Lightest SUSY particle

OK, then,....

What's the implications of 125 GeV Higgs for **Supersymmetry (SUSY)** ??

125 GeV Higgs and SUSY

$$V(H) = -m^2 (H^\dagger H) + \lambda_{\mathrm{H}} (H^\dagger H)^2$$

$m^2 = (\mathbf{89\ GeV})^2$ $\qquad \lambda_{\mathrm{H}} = \mathbf{0.13}$

in SUSY...

$$= \lambda_H^{\mathrm{tree}} + \delta\lambda_H^{\mathrm{loop}}$$

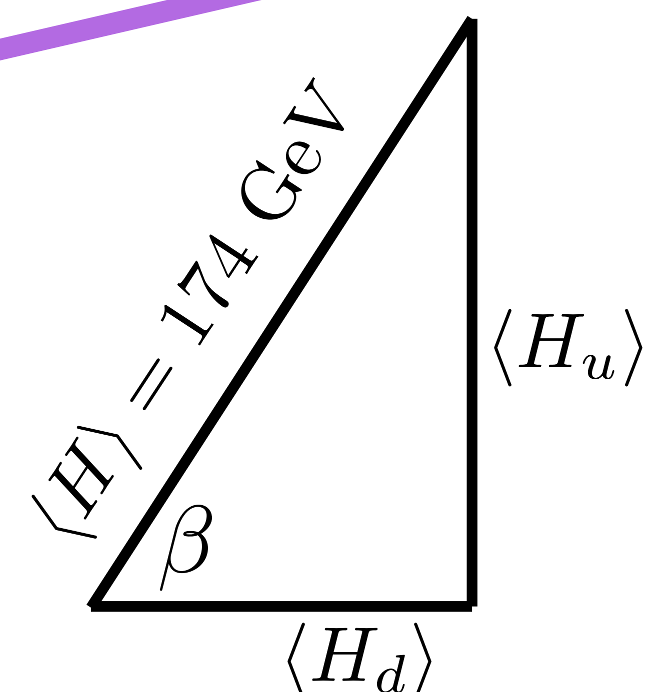
$$\lambda_H^{\mathrm{tree}} = \frac{g^2 \cos^2 2\beta}{8 \cos^2 \theta_W} \simeq \mathbf{0.069} \cos^2 2\beta$$

parameters
in Standard Model
(known)

too small...

What is β??

$$\tan\beta = \frac{\langle H_u \rangle}{\langle H_d \rangle}$$



125 GeV Higgs and SUSY

$$V(H) = -m^2 (H^\dagger H) + \lambda_{\rm H} (H^\dagger H)^2$$

$(89~\mathbf{GeV})^2$

0.13

in SUSY…

$$= \lambda_H^{\rm tree} + \delta\lambda_H^{\rm loop}$$

$$\frac{g^2 \cos^2 2\beta}{8 \cos^2 \theta_W} \simeq \mathbf{0.069} \cos^2 2\beta$$

$$\frac{3 y_t^4}{16\pi^2} \left(\log\left(\frac{m_{\rm stop}^2}{m_t^2} \right) + \alpha^2 - \frac{\alpha^4}{12} \right) + \cdots$$

for large $\tan\beta$. $(\alpha \simeq A_t / m_{\rm stop})$

…requires **heavy stop**

and/or **large A-term**

125 GeV Higgs and SUSY

$$V(H) = -m^2 (H^\dagger H) + \lambda_H (H^\dagger H)^2$$

$(89\ \mathrm{GeV})^2$ 0.13

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$$\frac{g^2 \cos^2 2\beta}{8 \cos^2 \theta_W} \simeq 0.069 \cos^2 2\beta$$

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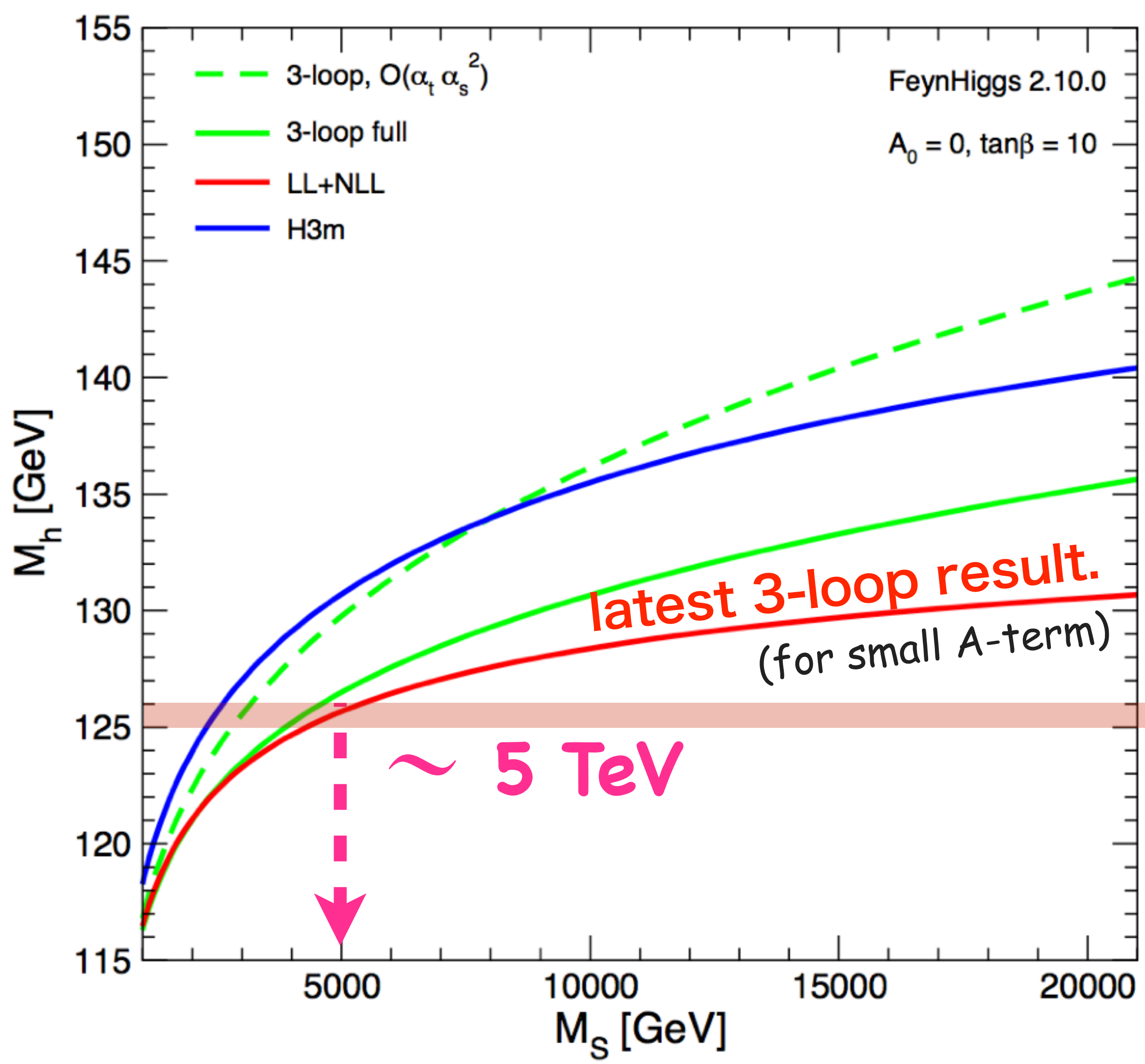

1404.0186 Hahn et.al.

...requires **heavy stop** and/or **large A-term**

125 GeV Higgs and SUSY

$$V(H) = -m^2 (H^\dagger H) + \lambda_H (H^\dagger H)^2$$

$(89\ \mathrm{GeV})^2$ 0.13

$$= \lambda_H^{\text{tree}} + \delta\lambda_H^{\text{loop}}$$

on the other hand

$$-m^2 \simeq |\mu|^2 + m_{H_u}^{2\,(\text{tree})} + \delta m_{H_u}^{2\,(\text{loop})}$$

up to $\mathcal{O}\left(\frac{1}{\tan^2\beta}\right)$

Higgsino mass

soft mass for up-type Higgs

$$\frac{g^2\cos^2 2\beta}{8\cos^2\theta_W} \simeq \mathbf{0.069}\ \cos^2 2\beta$$

$$\frac{3y_t^4}{16\pi^2}\left(\log\left(\frac{m_{\text{stop}}^2}{m_t^2}\right) + \alpha^2 - \frac{\alpha^4}{12}\right) + \cdots$$

for large $\tan\beta$. $(\alpha \simeq A_t/m_{\text{stop}})$

...requires **heavy stop** and/or **large A-term**

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$$V(H) = -m^2 (H^\dagger H) + \lambda_{\mathrm{H}} (H^\dagger H)^2$$

$(\mathbf{89\ GeV})^2$

$\mathbf{0.13}$

$$= \lambda_H^{\text{tree}} + \delta\lambda_H^{\text{loop}}$$

on the other hand

$$-m^2 \simeq |\mu|^2 + m_{H_u}^{2\,(\text{tree})} + \delta m_{H_u}^{2\,(\text{loop})}$$

$$\delta m_{H_u}^{2\,(\text{loop})} \sim \frac{-3y_t^2}{8\pi^2}\left(m^2_{\widetilde{t}_L} + m^2_{\widetilde{t}_R} + |A_t|^2\right) \log\left(\frac{M_{\text{mess}}}{m_{\widetilde{t}}}\right) + \cdots$$

tension !!

requires **Light stop and small A-term** to avoid a fine-tuning.

$$\frac{g^2 \cos^2 2\beta}{8 \cos^2 \theta_W} \simeq \mathbf{0.069} \cos^2 2\beta$$

$$\frac{3y_t^4}{8\pi^2}\left(\log\left(\frac{m^2_{\text{stop}}}{m_t^2}\right) + \alpha^2 - \frac{\alpha^4}{12}\right) + \cdots$$

for large $\tan\beta$. ($\alpha \simeq A_t/m_{\text{stop}}$)

...requires **heavy stop** and/or **large A-term**

125 GeV Higgs and SUSY

Fine-tuning worse than 1% seems unavoidable in MSSM.

(MSSM =Minimal SUSY Standard Model)

What does it imply ??

1. **No SUSY ?**

2. (It's anyway fine-tuned, then....)

   Very heavy SUSY ? (10-100 TeV, or even higher...)

3. (still.....)

   O(0.1-1) TeV SUSY ? (fine-tuned, but less than 2 and 3...)

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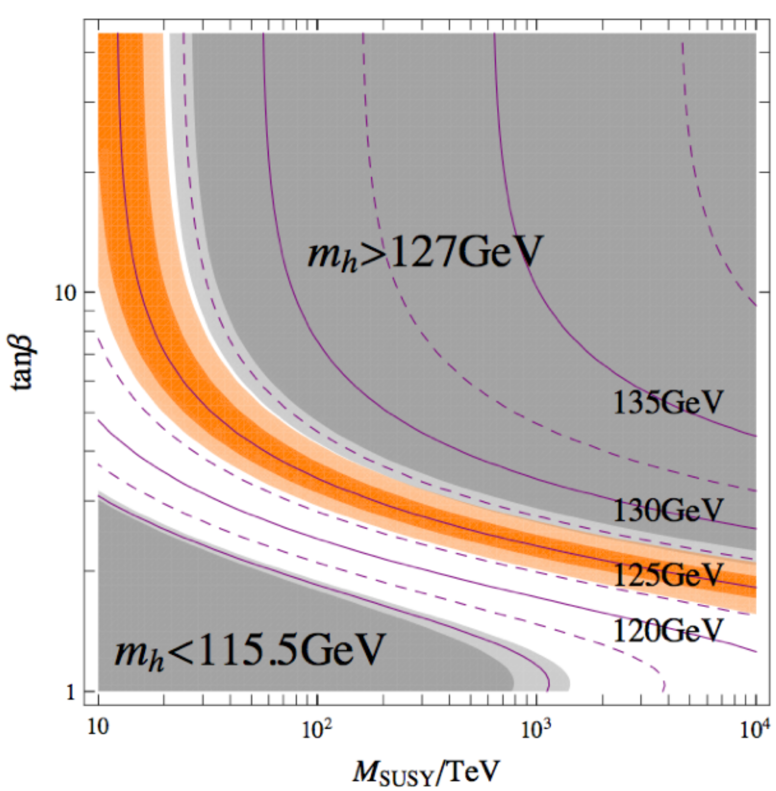
125 GeV Higgs and SUSY

(It's anyway fine-tuned, then....)

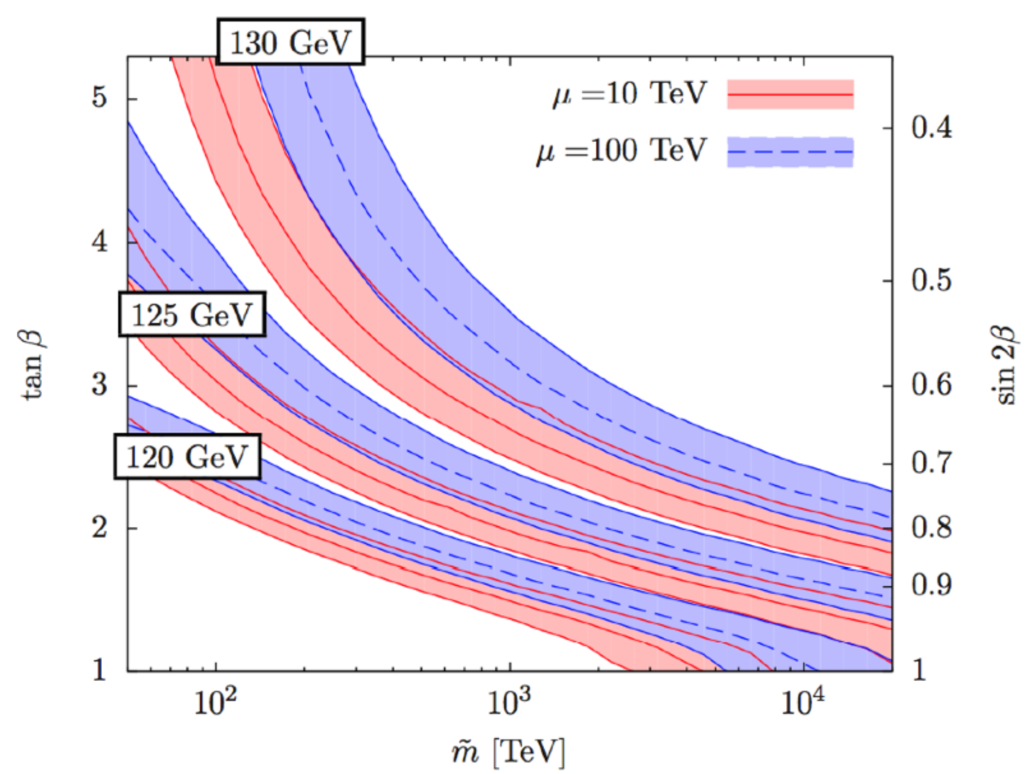
Very heavy SUSY

$$
\begin{aligned}
m_H^2 &= 4\lambda_H \langle H\rangle^2 \\
\rightarrow \lambda_H &\simeq 0.13 \\
&= \underbrace{\lambda_H^{\rm tree}}_{0.07\cos^2 2\beta} + \underbrace{\delta\lambda_H^{\rm loop}}_{\sim \log(m_{\rm stop}^2)}
\end{aligned}
$$

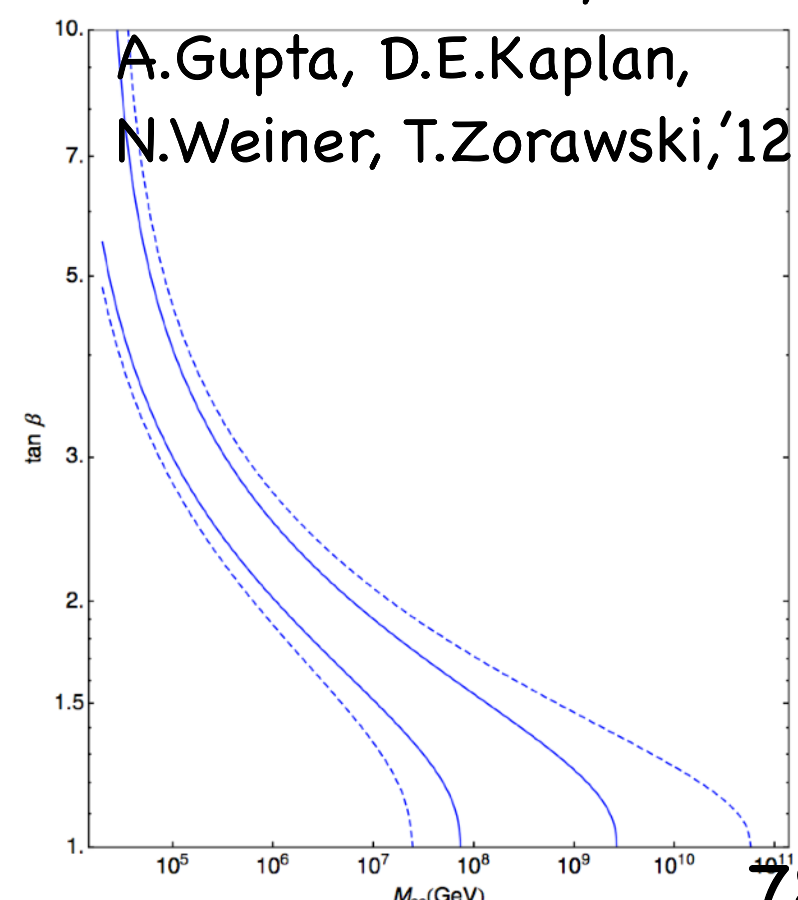
Ibe, Matsumoto, Yanagida,'12



L.Hall, Y.Nomura, S.Shirai '12



N.Arkani-Hamed, A.Gupta, D.E.Kaplan, N.Weiner, T.Zorawski,'12

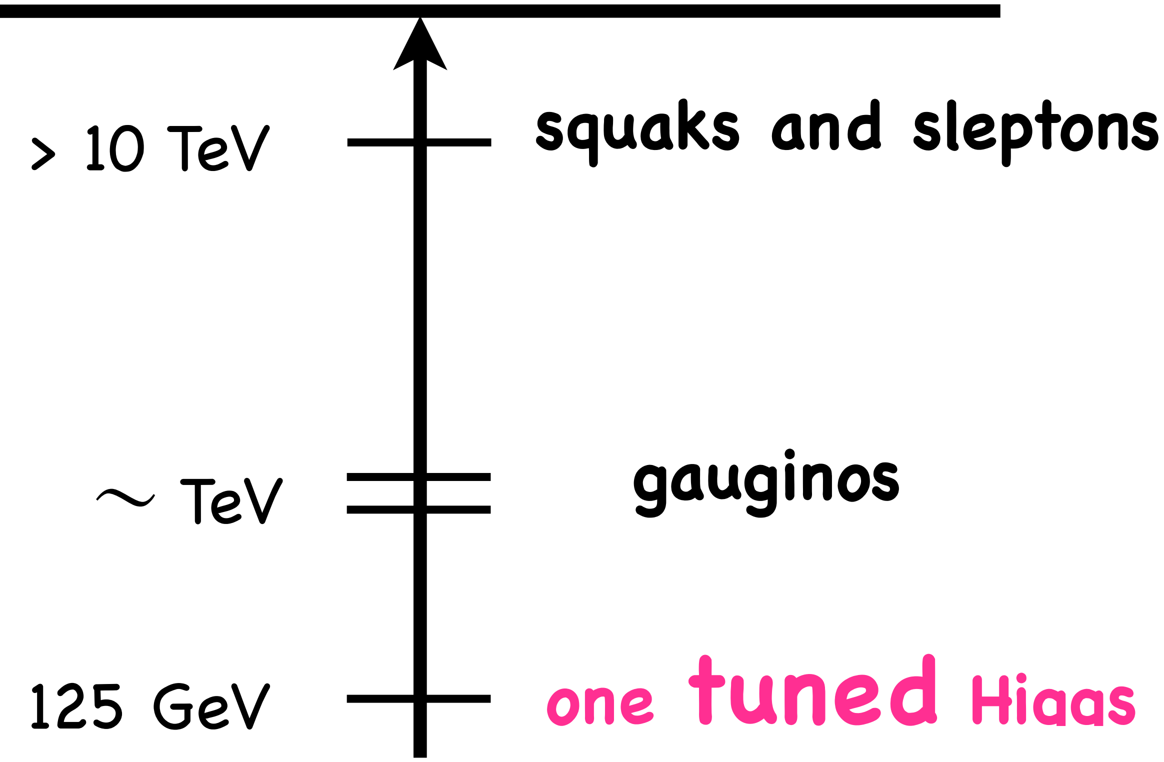


125 GeV Higgs and SUSY

(It's anyway fine-tuned, then....)

Very heavy SUSY

- consistent with 125 GeV Higgs
- No cosmological gravitino problem
- Coupling Unification is OK
- Dark Matter is also OK



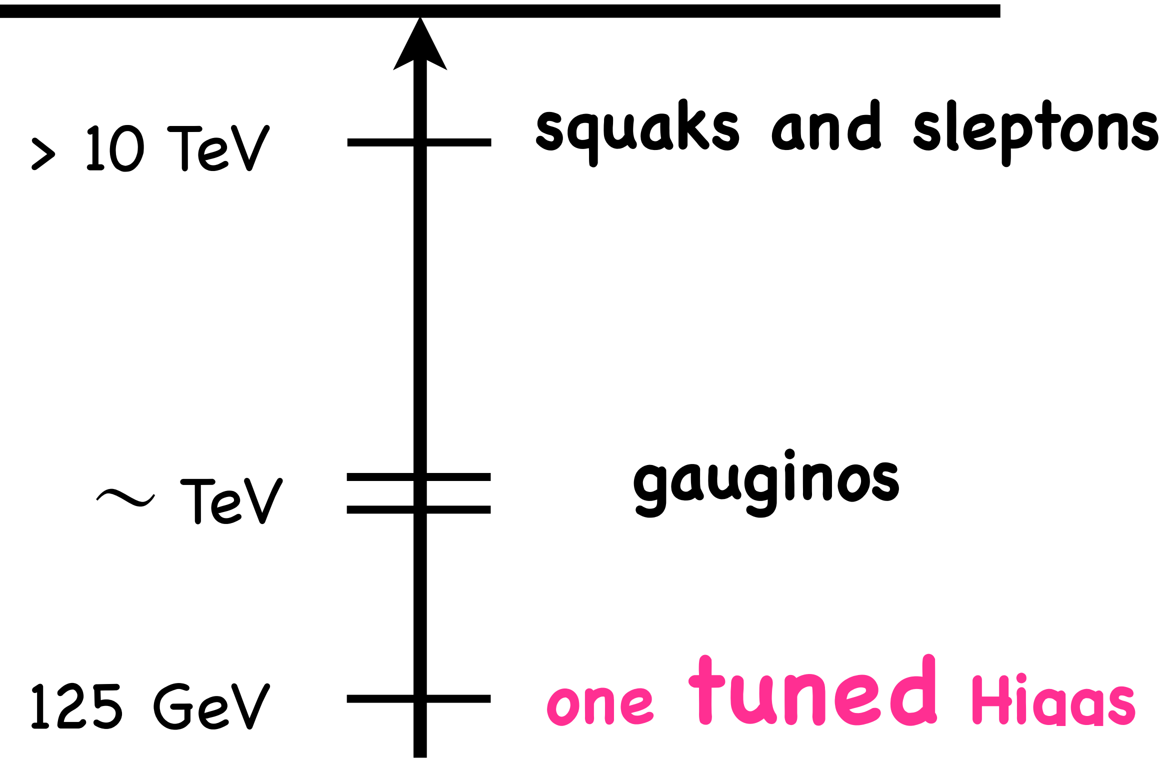
Many many works..... (too many to list all...)

Ibe,Yanagida'11, Ibe,Matsumoto,Yanagida'12,
Bhattacherjee,Feldstein,Ibe,Matsumoto,Yanagida'12,
Hall,Nomura'11, Hall,Nomura,Shirai'12,
Giudice,Strumia'11, Arvanitaki,Craig,Dimopoulos,Villadoro'12
Arkani-Hamed,Gupta,Kaplan,Weiner,Zorawski'12, Ibanez,Valenzuela'13,
Jeong,Shimosuka,Yamaguchi'11, Hisano,Ishiwata,Nagata'12, Sato,Shirai,Tobioka'12,
Moroi,Nagai'13, McKeen,Pospelov,Ritz'13,
Hisano,Kuwahara,Nagata'13, Hisano,Kobayashi,Kuwahara,Nagata'13, etc etc.....

125 GeV Higgs and SUSY

(It's anyway fine-tuned, then....)

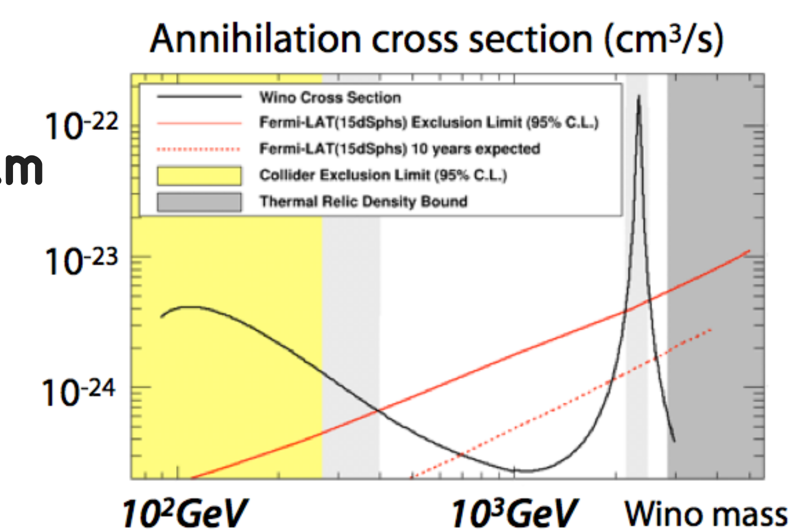
Very heavy SUSY



Typical DM = Wino DM
(AMSB)

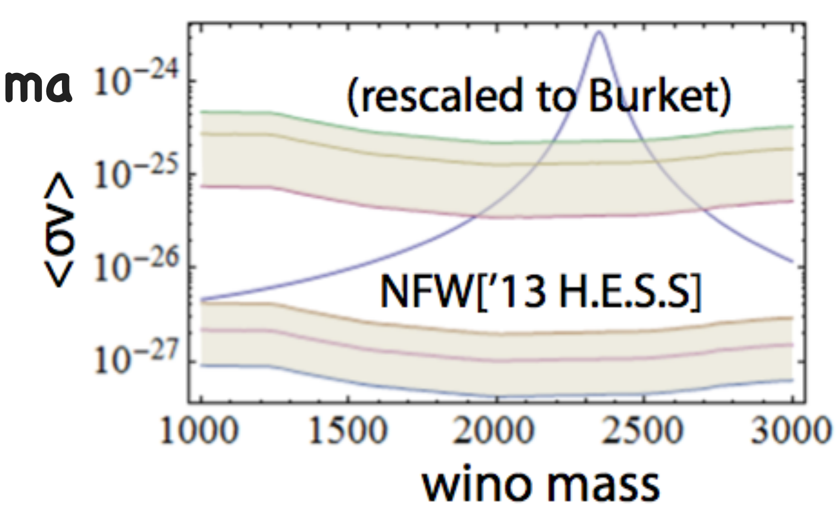
- if thermal relic,... 2.7 TeV (>> LHC reach) (Hisano,Matsumoto,Nagai,Saito,Senami'07)
- if non-thermal, it can be lighter.
- indirect DM signal expected !

gamma continuum



[Fermi-LAT:1310.0828 (Translated by K.Ichikawa)]

gamma line



[Figure by S.Matsumoto]

Figures from talk by M.Ibe at KIAS workshop, October 2014.

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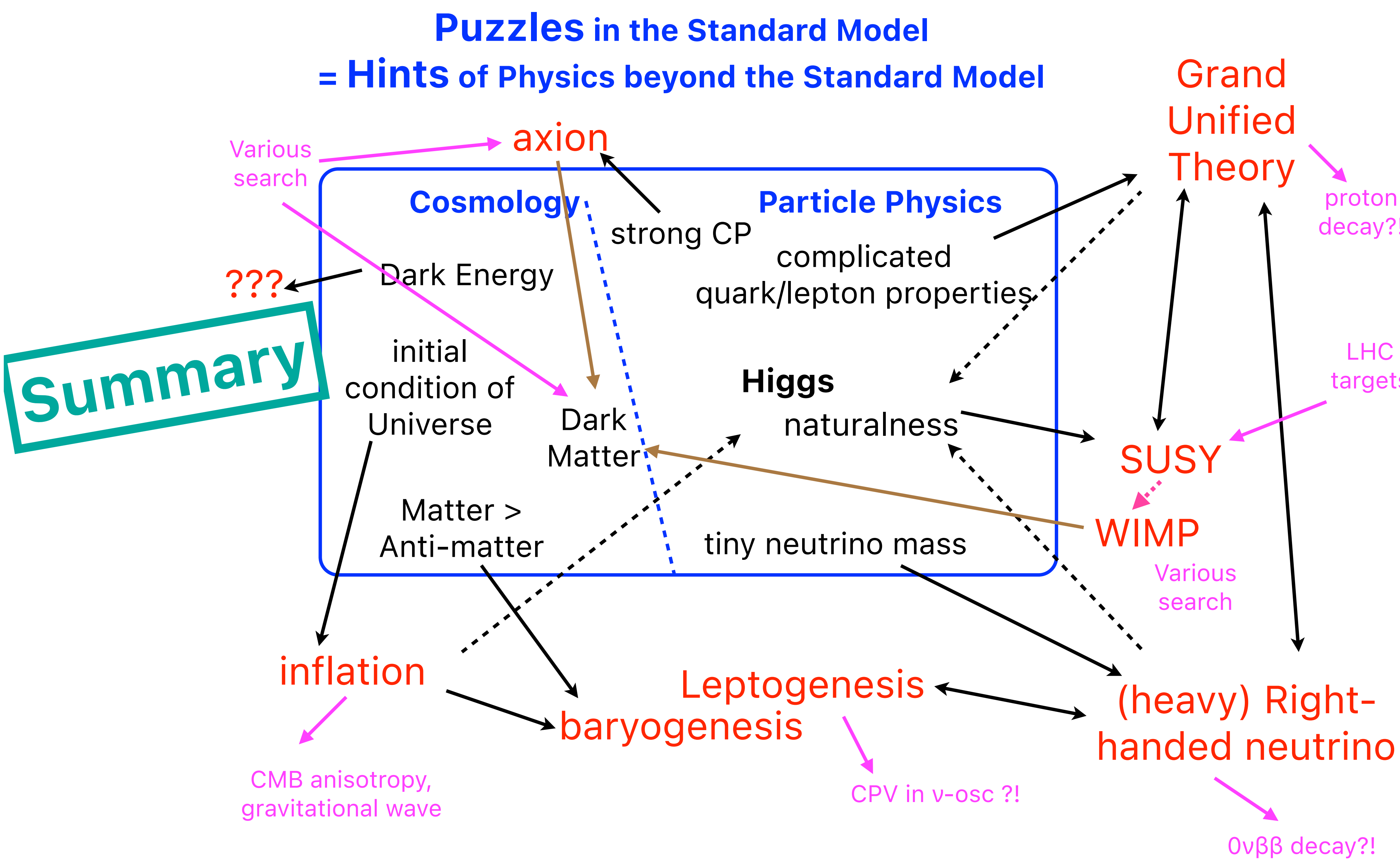
20225-03-01 SUSY informal lecture, Koichi Hamaguchi

@The 4th International Iwate Collider School (ICS2025)

Iwate, Feb.24-Mar.1, 2025.



That's all… Thank you!