The Standard Model of Particle Physics

2025.02.24

@ ICS 2025 (Appi, Iwate, Japan)

Kaoru.Hagiwara@kek.jp

Elementary Particles (~~before~~ above EW symmetry breakdown)

					# of real fields
Fermions (spin $= 1/2$)	$Q_k = \begin{pmatrix} u_{Lk} \\ d_{Lk} \end{pmatrix}$	3	2	$\frac{1}{6}$	$6 \times 2 \times 2$
	u_{Rk}	3	1	$\frac{2}{3}$	$3 \times 2 \times 2$
$k = 1, 2, 3$ ↓ (generation, family, flavor)	d_{Rk}	3	1	$-\frac{1}{3}$	$3 \times 2 \times 2$
	$L_k = \begin{pmatrix} \nu_{Lk} \\ l_{Lk} \end{pmatrix}$	1	2	$-\frac{1}{2}$	$2 \times 2 \times 2$
	l_{Rk}	1	1	-1	$1 \times 2 \times 2$ ↕ spin
Gauge bosons (spin $= 1$)		A_μ^a ($a = 1 \dots 8$)	W_μ^i ($i = 1, 2, 3$)	B_μ	$(8+3+1) \times 2$
Higgs boson (spin $= 0$)	ϕ	1	2	$-\frac{1}{2}$	$2 \times 2 = 4$

$SU(3)_c \times SU(2)_L \times U(1)_Y$: gauge symmetry

All the above elementary particles are massless above the EW symmetry breaking except the Higgs boson.

Elementary particles (below EW symmetry breakdown scale : $v = 246\,\mathrm{GeV}$)

Fermions (spin = 1/2)	$u,\ c,\ t$	3	2/3	quarks
	$d,\ s,\ b$	3	−1/3	quarks
	$\nu_e,\ \nu_\mu,\ \nu_\tau$	1	0	leptons*
	$e,\ \mu,\ \tau$	1	−1	leptons*
Gauge bosons (spin = 1)	A^a_μ $(a=1\ldots8)$	8	0	gluons
	$W^\pm_\mu$	1	±1	weak bosons
	Z_μ	1	0	weak bosons
	A_μ	1	0	photon
Higgs boson (spin = 0)	H	1	0	Higgs boson
		$SU(3)_c$	$\times\ U(1)_{EM}$	

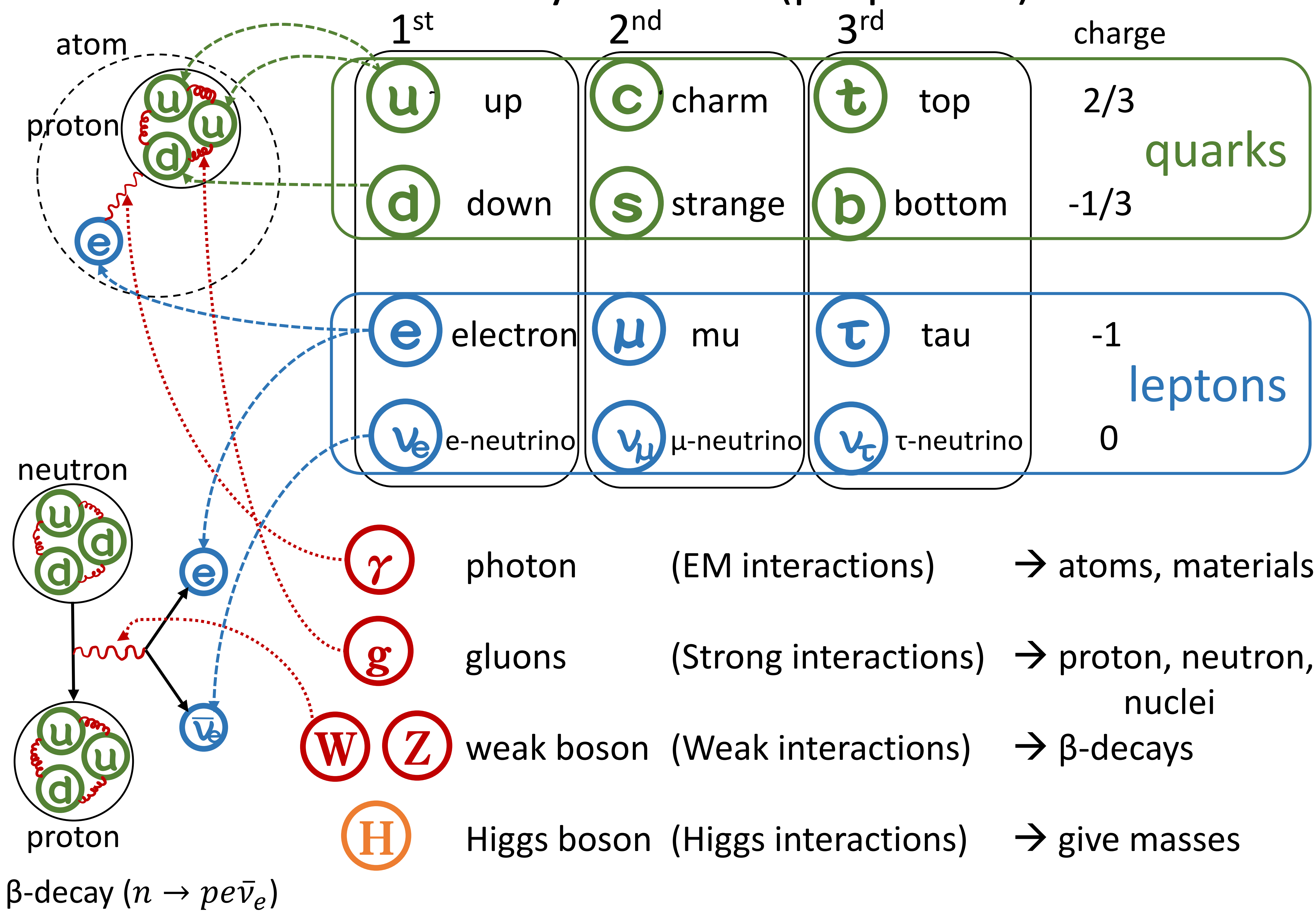
* ν-mixing appears @EFT

Elementary particles (below the QCD confinement scale : $(1\,\mathrm{fm})^{-1} \approx 200\,\mathrm{MeV}$)

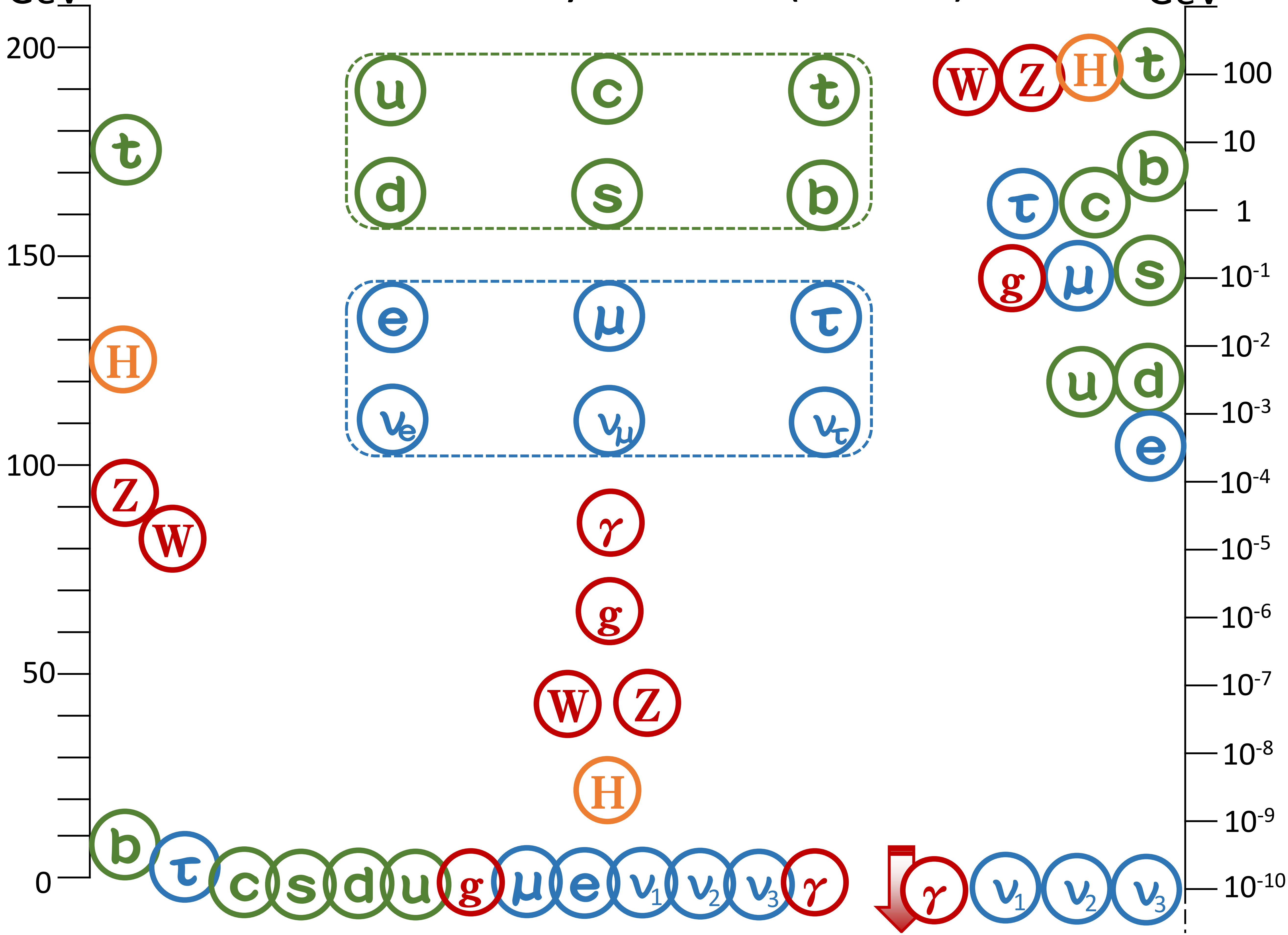
Hadrons	Baryons	spin = 1/2	$p\,(uud)$, $n\,(udd)$, $\Lambda\,(uds)$, $\Lambda_c\,(udc)$, $\Lambda_b\,(udb)$, …
(color singlets)	(qqq)	spin = 3/2	$\Delta^{++}(uuu)$, $\Delta^{+}(uud)$, $\Delta^{0}(udd)$, $\Delta^{-}(ddd)$, …
	Mesons	spin = 0	$\pi^{+}(u\bar{d})$, $\pi^{-}(\bar{u}d)$, $\pi^{0}(u\bar{u}+d\bar{d})$, $K^{+}(u\bar{s})$, …, $\eta_c(c\bar{c})$, $\eta_b(b\bar{b})$, …
	$(q\bar{q})$	spin = 1	$\rho^{+}(u\bar{d})$, $\rho^{-}(\bar{u}d)$, $\rho^{0}(u\bar{u}+d\bar{d})$, $K^{*+}(u\bar{s})$, $\phi(s\bar{s})$, $J/\psi(c\bar{c})$, $\Upsilon(b\bar{b})$, …
Leptons	$Q=-1$	spin = 1/2	e, μ, τ
	$Q=0$	spin = 1/2	ν_e, ν_μ, ν_τ
Gauge bosons (spin 1)		$Q=\pm 1$	$W^{\pm}$
		$Q=0$	Z, γ
Higgs boson (spin 0)		$Q=0$	H

These are the particles which are ~~produced and~~ detected by collider experiments.

Elementary Particles (properties)



Elementary Particles (masses)



Quantum Field Theory (QFT) is the language with which the particles are described.

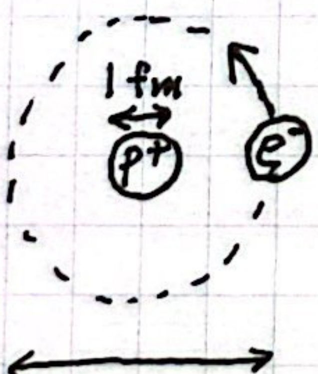
$\approx$ Quantum Mechanics (QM) + Special Relativity

19 th Century : Classical Physics (Newton + Maxwell)

$\Downarrow$ $\Downarrow$ $\leftarrow$ Rutherford probed atom by α beam

20 th Century : Quantum Mechanics

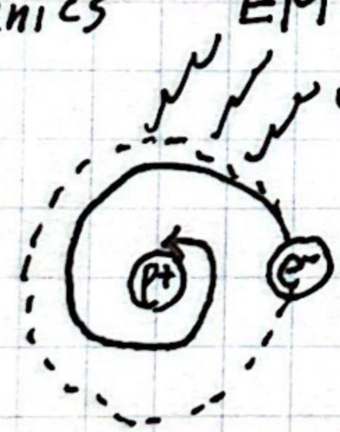
atom



$1\,\mathrm{fm} = 10^{-15}\,\mathrm{m}$

$1\,\mathring{A} = 10^{-10}\,\mathrm{m}$

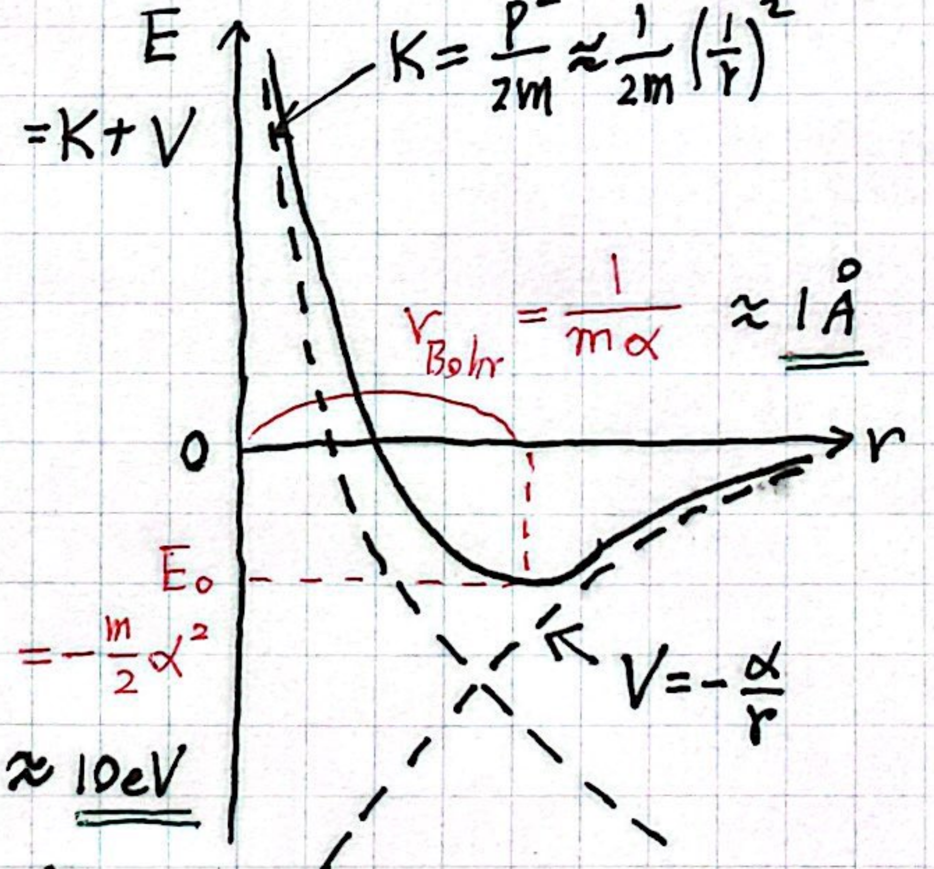
Classical Physics $\Longrightarrow$



All atoms are instantly dead!

But we are alive!

$[c = \hbar = 1 \text{ unit}]$



QM : e^- momentum ($\vec{P}$) and location ($\vec{x}$) cannot be determined simultaneously, because $[x, p_x] = i\hbar$ ($\hbar$ is the quantum of $\int dx\, p$)

$$\underset{\substack{\uparrow \\ \text{Energy}}}{H}\,\psi(x) = \Big[\underset{\substack{\uparrow \\ \text{Kinetic}}}{\frac{\vec{p}^{\,2}}{2m}} + \underset{\substack{\uparrow \\ \text{Potential}}}{V(r)}\Big]\psi(x) = \Big[\underbrace{\frac{(i\vec{\nabla})^2}{2m}}_{\text{blows up at } r \approx 0} + V(r)\Big]\psi(x) = E\,\psi(x)$$

$|\psi(x)|^2$ determines the probability of finding e^- at $\vec{x}$.

experiments!

QFT 2 : Special Relativity

Illusion : time and space are independent and absolute (idea, phillosophy, ... based on ...)

Reality : mearure of time and space depends on the relative velocity of the (Lorentz) frames.

Fact = Reality check : The velocity of light, c, is exactly the same in all frames ⇐ Michelson-Morley

Partulate (Einstein) : Physics should be the same in all frames whose relative velocity is a constant.

(~~no~~ acceleration. ⇒ General Relativity)

Poincare : Physics should be the same under

$$\begin{cases} \text{Translation} : & x^\mu \to x'^\mu = x^\mu + a^\mu \\ \text{Lorentz transformation} : & x^\mu \to x'^\mu = L^\mu{}_\nu x^\nu \end{cases}$$

4 generators : P^μ ($\mu = 0, 1, 2, 3$) ⇒ Energy-Momentum conservation

6 generators : $J_x, J_y, J_z, K_x, K_y, K_z$

⇒ Angular Momentum conservation + ...

$$\Rightarrow \quad E^2 - |\vec{p}|^2 = m^2 \quad \text{is invariant !}$$

$$E = \sqrt{m^2 + |\vec{p}|^2} \approx mc^2$$

⇒ particles are labeled by their invariant mass and spin because they have the same values in all Lorentz frames.

QFT 3: What happens when QM and Special Relativity are combined?

particles are created and annihilated when $E \gg mc^2$!

particle numbers are not conserved. $\Longleftrightarrow$ QM studies one or two particle system when $v \ll c$.

$$\gamma\gamma \longleftrightarrow e^+e^- \quad \text{when } E_{tot} \gtrsim 2m_e c^2$$

QM: $\psi(t,\vec{x})$ is the wave function of an electron $\Rightarrow$ $|\psi(t,\vec{x})|^2$ measures the probability of finding e at...

QFT:
$$\psi_i(t,\vec{x}) = \sum_h \int \frac{d^3\vec{k}}{(2\pi)^3 2E}\left[\underbrace{a(\vec{k},h)}_{\text{annihilate } e^-} u_i(\vec{k},h)\, e^{-i(Et-\vec{k}\cdot\vec{x})} + \underbrace{b^\dagger(\vec{k},h)}_{\text{creat } e^+} v_i(\vec{k},h)\, e^{i(Et-\vec{k}\cdot\vec{x})} \right]_{E=\sqrt{m^2+|\vec{k}|^2}}$$

$(i=1,2,3,4)$

$\Rightarrow$ annihilation & creation are quantized: $[a(\vec{k},h), a^\dagger(\vec{k}',h')] = [b(\vec{k},h), b^\dagger(\vec{k}',h')] = (2\pi)^3 2E\, \delta^3(\vec{k}-\vec{k}')$

$\Rightarrow$ one e^- state $\quad \langle 0|\,\psi(t,\vec{x})\, a^\dagger(\vec{k},h)\,|0\rangle = u(\vec{k},h)\, e^{-i(Et-\vec{k}\cdot\vec{x})}$

one e^+ state $\quad \langle 0|\,\psi^\dagger(t,\vec{x})\, b^\dagger(\vec{k},h)\,|0\rangle = v(\vec{k},h)\, e^{-i(Et-\vec{k}\cdot\vec{x})}$

positive energy!

$\Rightarrow$ spin!

Dirac found that e^- wave functions should have 4 components, which should mix under Lorentz transformation.

Many physicists contribute to make QFT to solve the negative energy problem! $\Rightarrow$ e^+ is predicted!

QFT 4 : Perturbative calculation of scattering amplitudes

$$\mathcal{L}(\phi_K(x), \partial^\mu \phi_K(x)) = \underbrace{\mathcal{L}^{(0)}(\phi_K(x), \partial^\mu \phi_K(x))} + \mathcal{L}_{int}(x)$$

solve E.O.M. $\partial^\mu \dfrac{\delta \mathcal{L}}{\delta \partial^\mu \phi_K} + \dfrac{\delta \mathcal{L}}{\delta \phi_K} = 0$

plane wave solution : $\phi_K(x) = \displaystyle\int \frac{d^3\vec{p}}{(2\pi)^3 2E} \left[a_K(\vec{p}) \epsilon_K(\vec{p}) e^{-ipx} + b_K^+(\vec{p}) \epsilon_K^*(\vec{p}) e^{ipx} \right]_{p^0 = E = \sqrt{m^2 + |\vec{p}|^2}}$

if $\phi_K(x)$ has spin, then under Lorentz transformation : $\begin{cases} \phi_K(x) \to \phi'_{K'}(x) = L_{K'K} \phi_K(L^{-1}x) \\ \epsilon_K(\vec{p}) \to \epsilon'_{K'}(\vec{p}) = L_{K'K} \epsilon_K(L^{-1}p) \end{cases}$

⇒ obtain the Green's function of the E.O.M. operator for free (plain wave) solution.

propagator

Time ordering

$$\langle f | S | i \rangle = \langle f | (1 + iT) | i \rangle = \langle f | T e^{i\int d^4x \mathcal{L}_{INT}(x)} | i \rangle = \langle f | \sum_{n=0}^{\infty} T \left(i \int d^4x \mathcal{L}_{INT}(x) \right)^n | i \rangle$$

$|i\rangle = a^+(\vec{p}_1) b^+(\vec{p}_2) |0\rangle$

$$= i (2\pi)^4 \delta^4 \left(\sum_{in} p_K^\mu - \sum_{out} p_K^\mu \right) M_{fi}$$

$|f\rangle = a^+(\vec{p}_3) b^+(\vec{p}_4) \cdots |0\rangle$

$\Rightarrow |M_{fi}|^2$ gives $\underbrace{\text{cross section}}_{2\to n}$ & $\underbrace{\text{decay width}}_{1\to n}$

(Fermi's Golden Rule for plane-wave normalization.)


synchrotron accelators
cycrotron
pN
p̄p
pp colliders
Sp̄pS, TeV
LHC
β decay, α rays
e+e− colliders
SPEAR, SLC, LEP
Future colliders
e+e− (ILC, CEPC, FCC-ee)
pp (FCC-hh)
μ+μ−, μ+μ+
atomic physics
cosmic rays
KEKB
γ rays
Z
W H t
c
τ
b
μ
Λ
n
p
K
π
GUT (no new physics up to ~10^16 GeV)
ideas!
a few nm device
e
atom
nuclei
New physics at TeV scale!
m_e α^2
0.5 Å ≈ (m_e α)^−1
1 fm ≈ (√2 m_π)^−1
eV
keV
MeV
GeV
TeV
PeV
EeV
10^−9 GeV
10^−6 GeV
10^−3 GeV
1 GeV
10^3 GeV
10^6 GeV
10^9 GeV
μm
nm
Å
pm
fm
10^−6 m
10^−9 m
10^−12 m
10^−15 m
10^−18 m
10^−21 m
10^−24 m