UCLouvain

Institut de recherche en mathématique et physique

Centre de Cosmologie, Physique des Particules et Phénoménologie

MCnet

1 Go to **wooclap.com**

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Event code

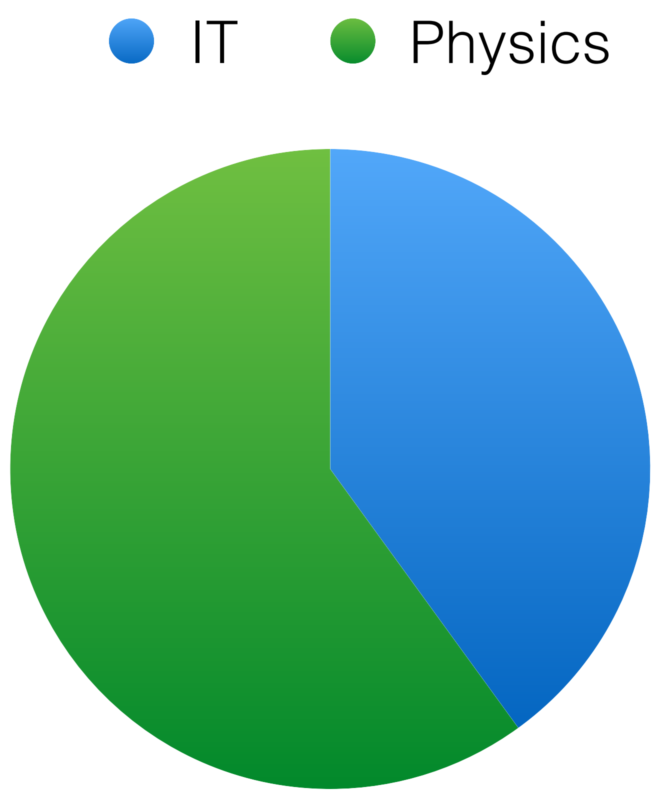
MADGRAPH

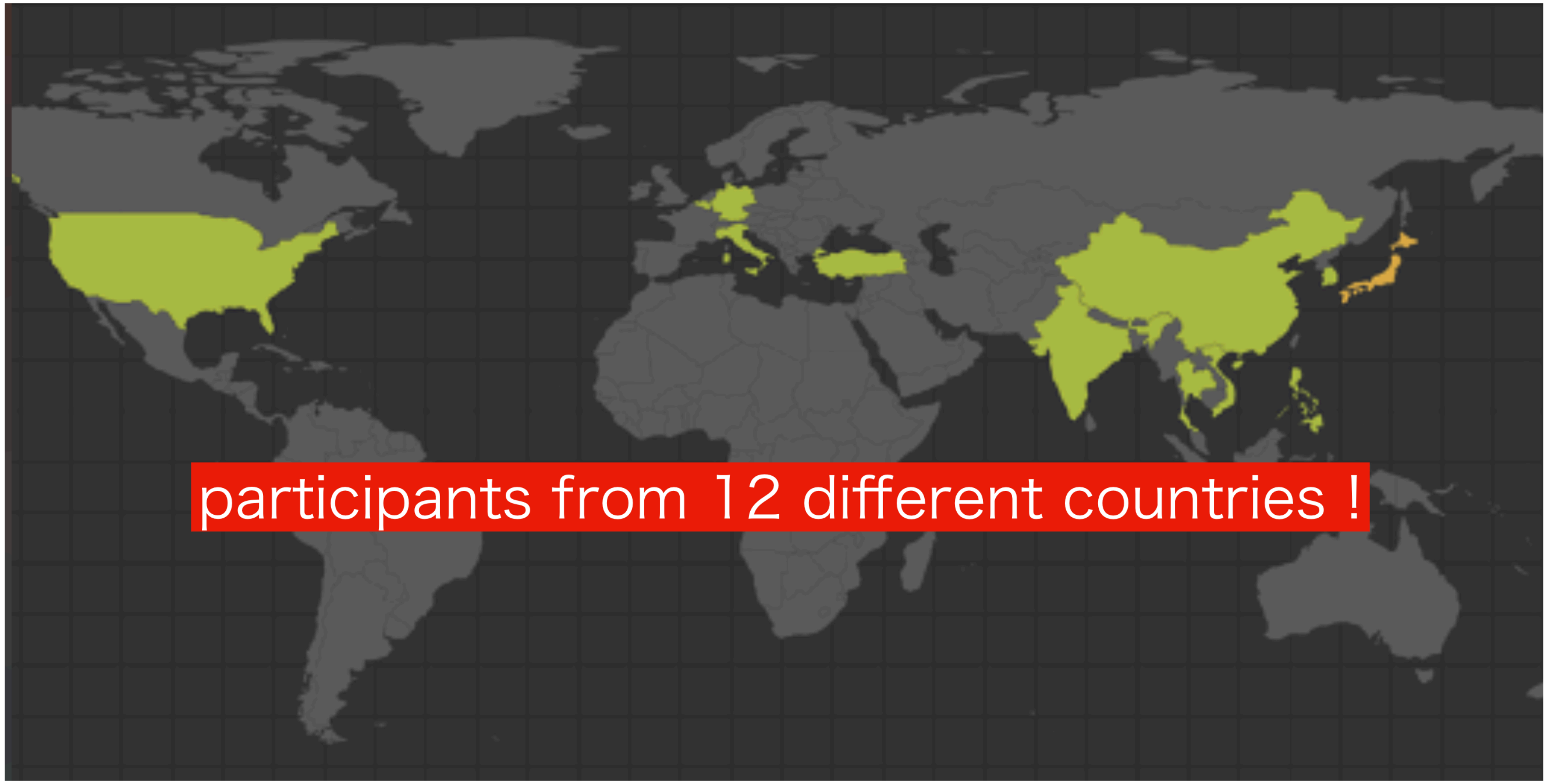
MonteCarlo Simulation

Olivier Mattelaer

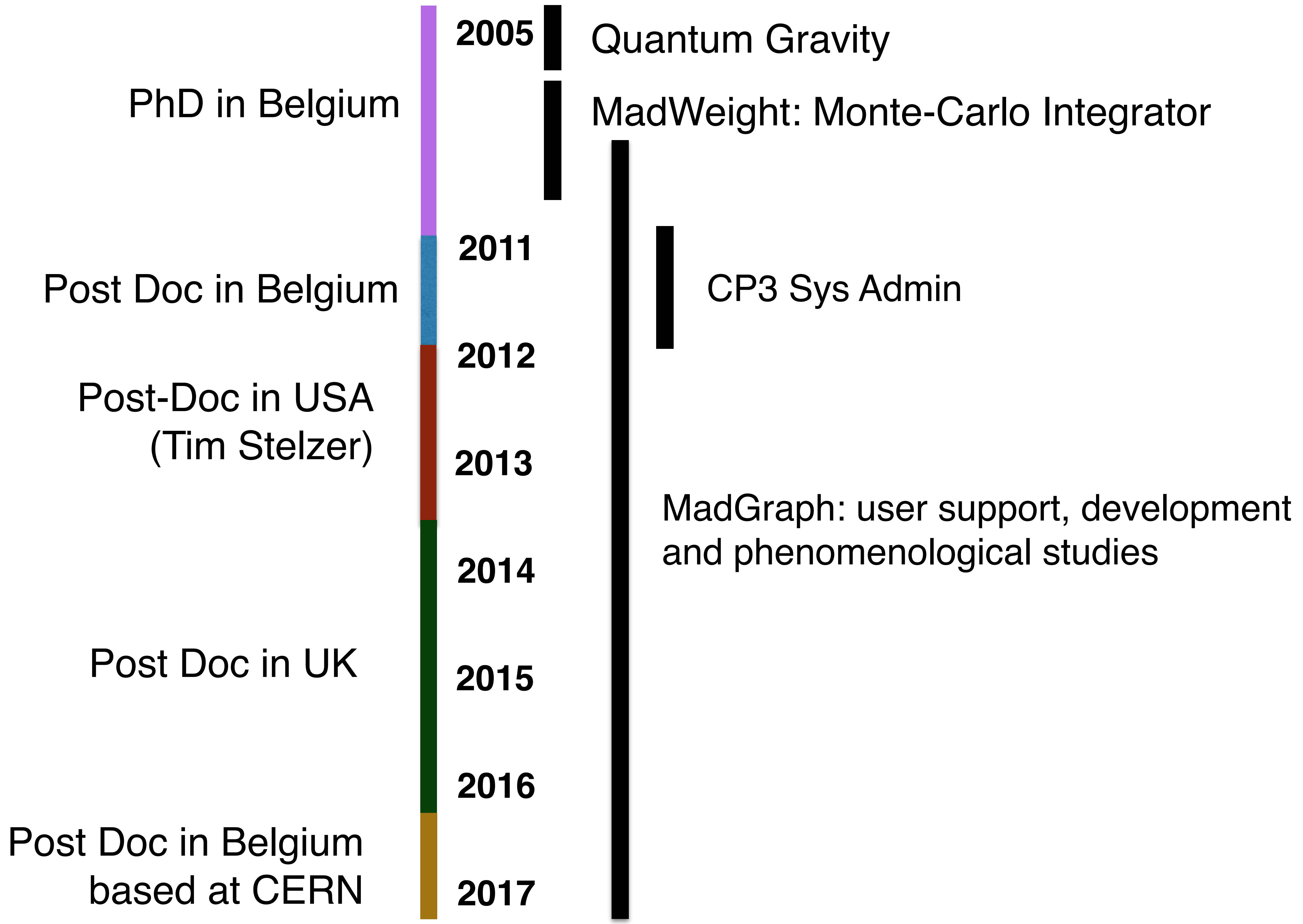
Who am I

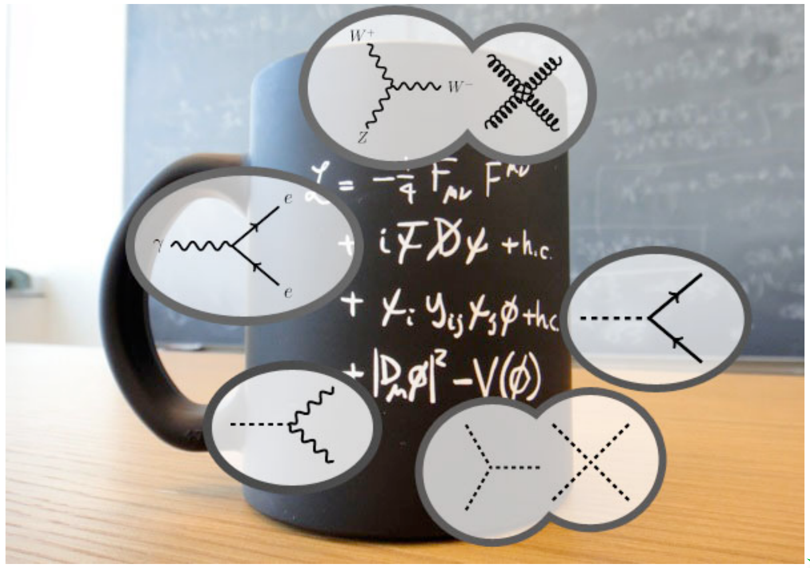
Research scientist at University of Louvain in Belgium
Since 2017
- partially for the Physics department (60%)
- also managing the HPC cluster (40%)



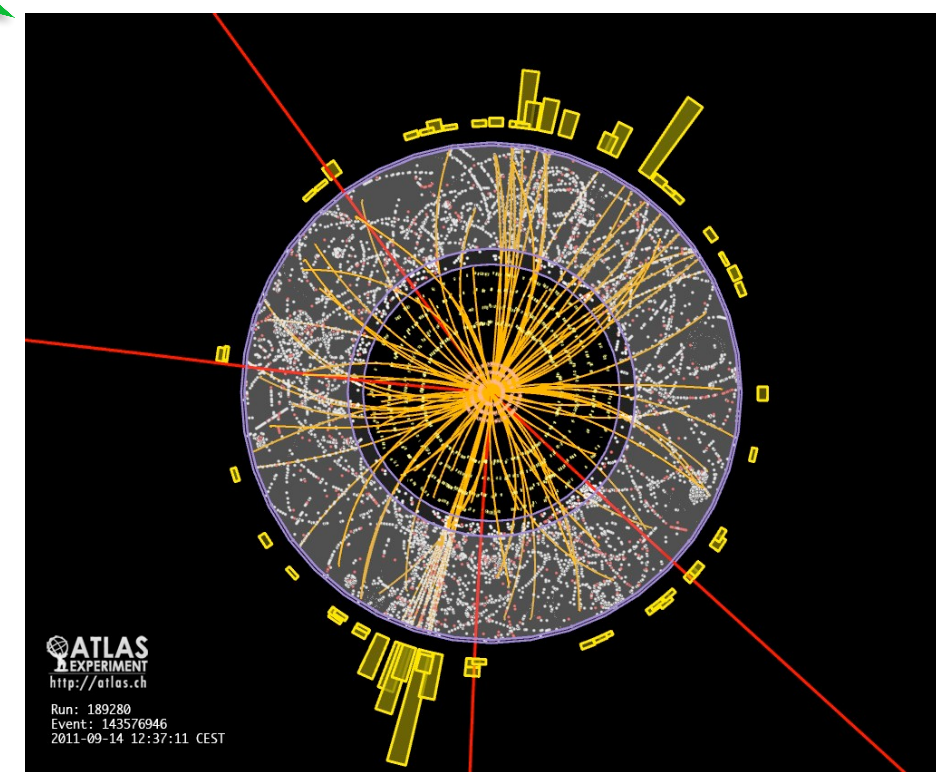


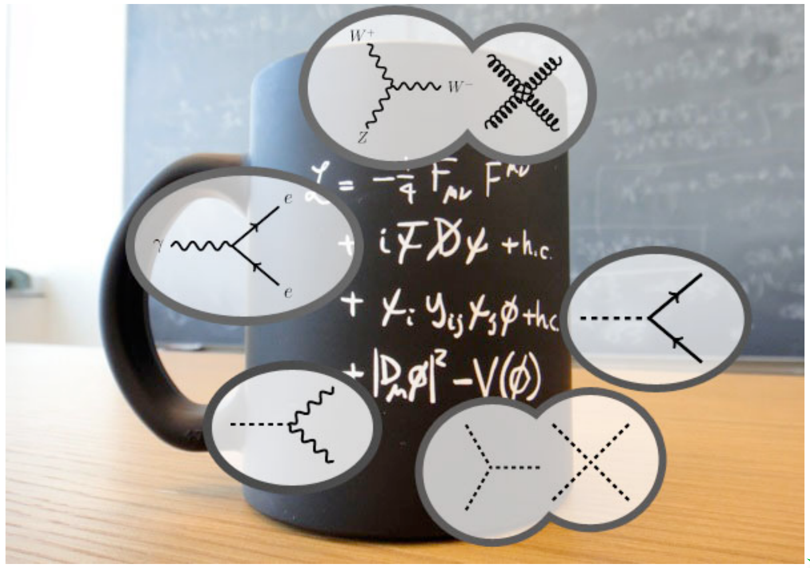
Before my current job



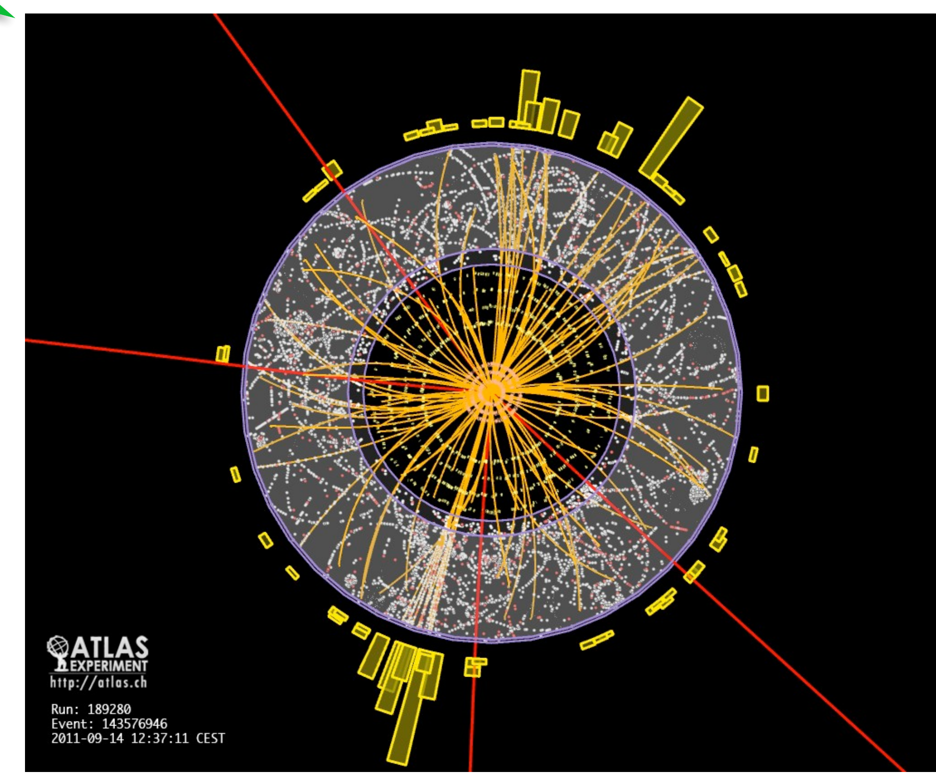


Monte-Carlo Physics





Monte-Carlo Physics

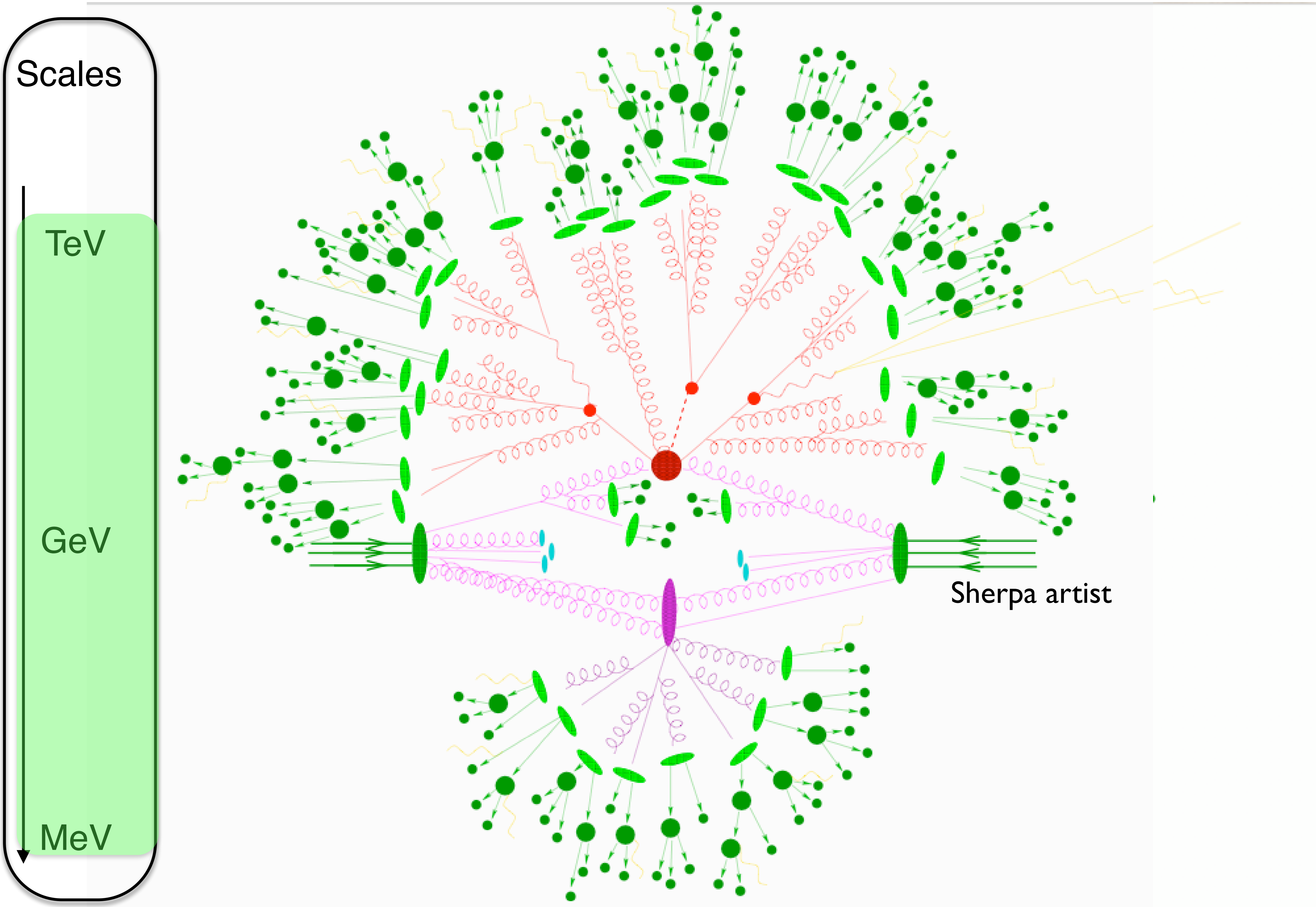


Our goal

- Cross-section
- Differential cross-section
- Un-weighted events

Simulation of collider events

What are the MC for?



What are the MC for?

Scales

TeV

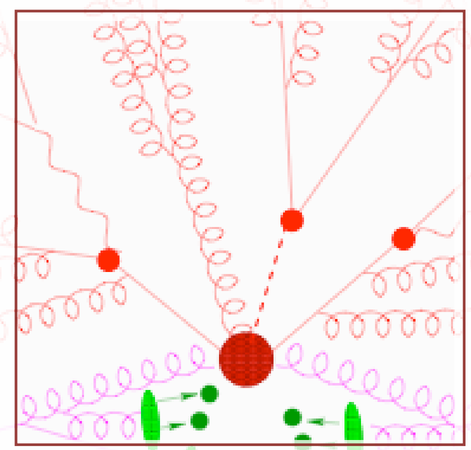
GeV

MeV

1. High-Q^2 Scattering

2. Parton Shower

☞ where BSM physics lies



☞ process dependent

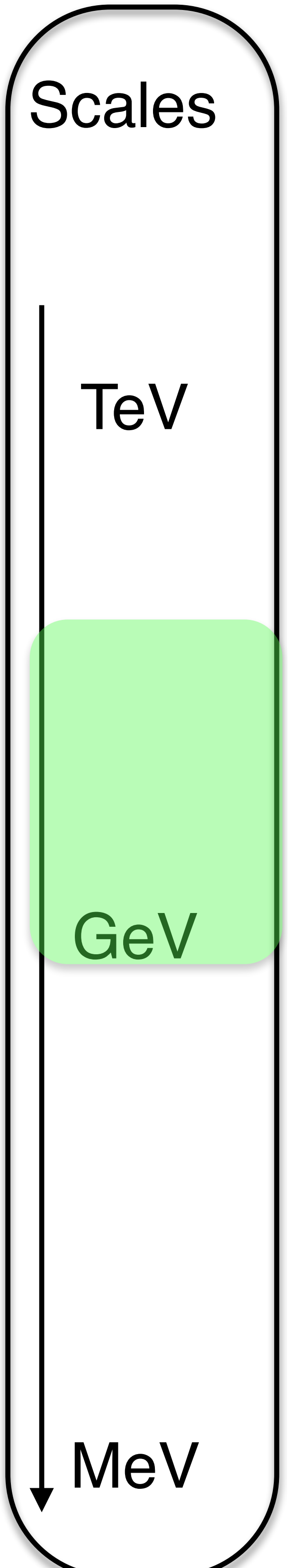
☞ first principles description

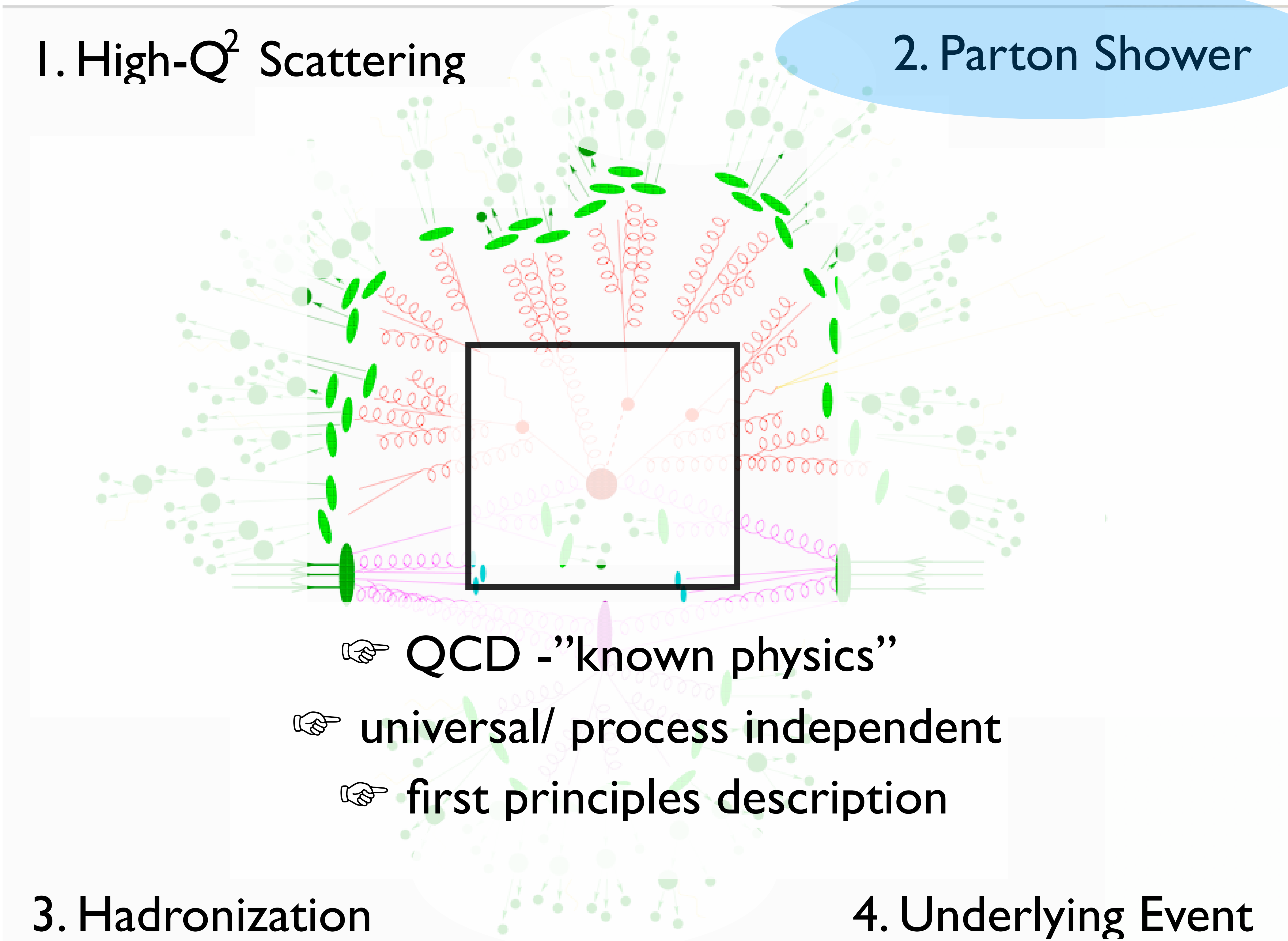
☞ it can be systematically improved

3. Hadronization

4. Underlying Event

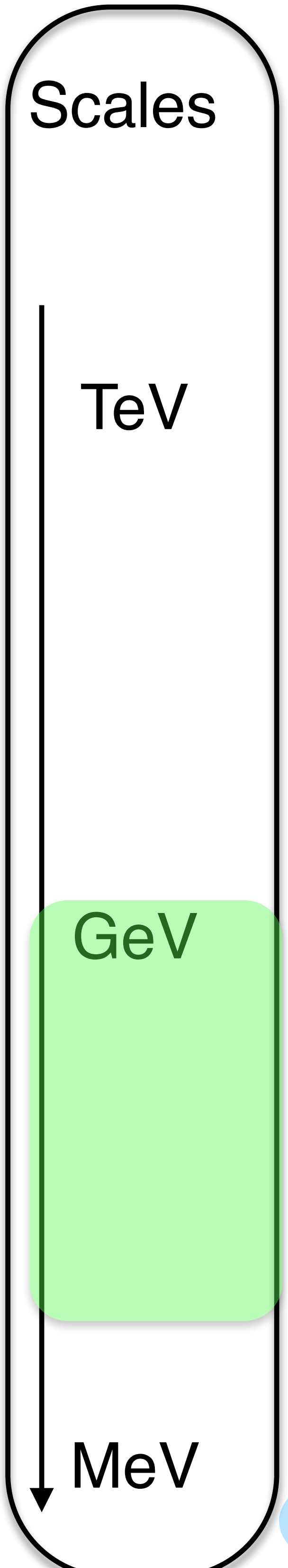
What are the MC for?

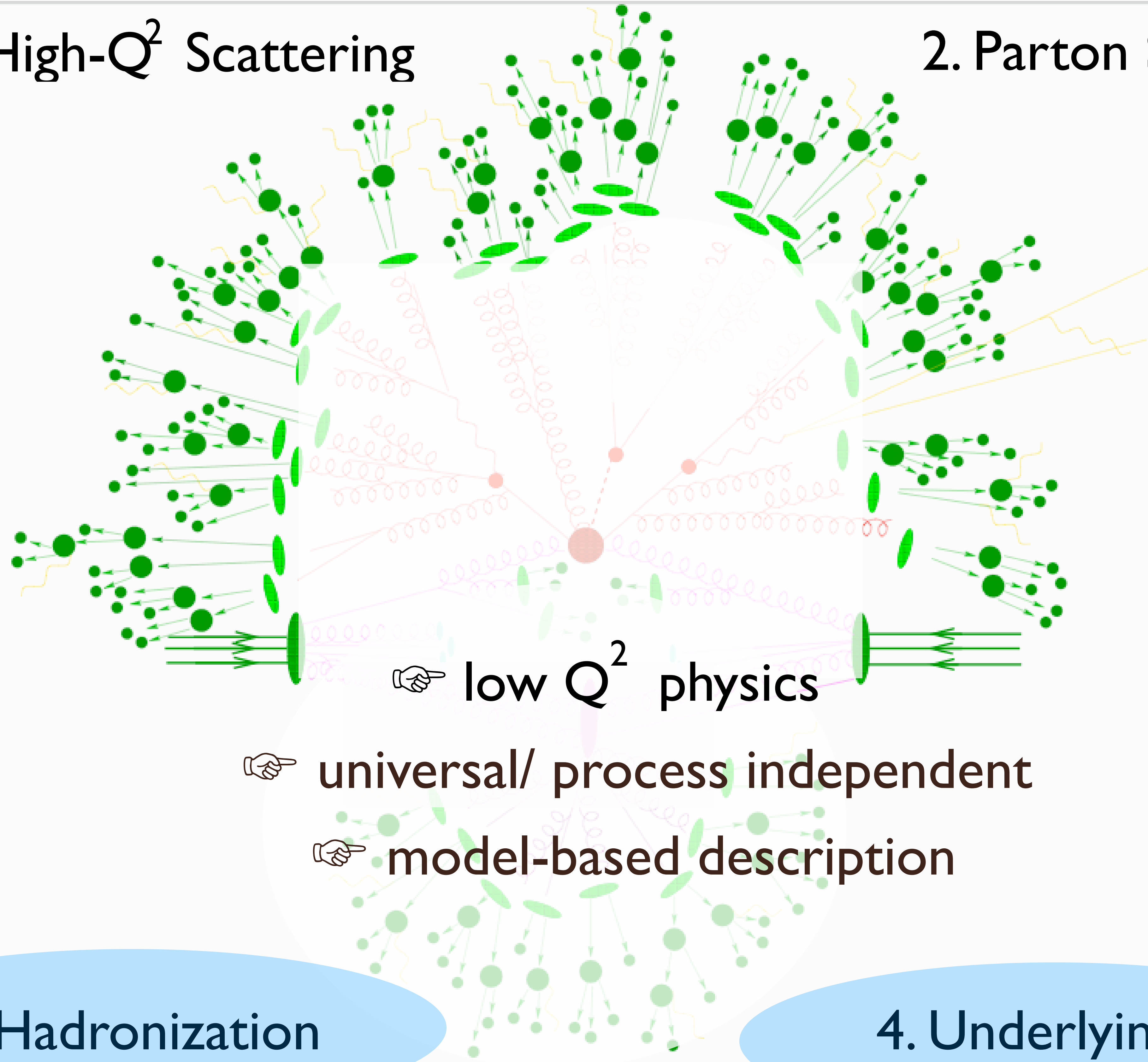




- ☞ QCD -”known physics”
- ☞ universal/ process independent
- ☞ first principles description

What are the MC for?



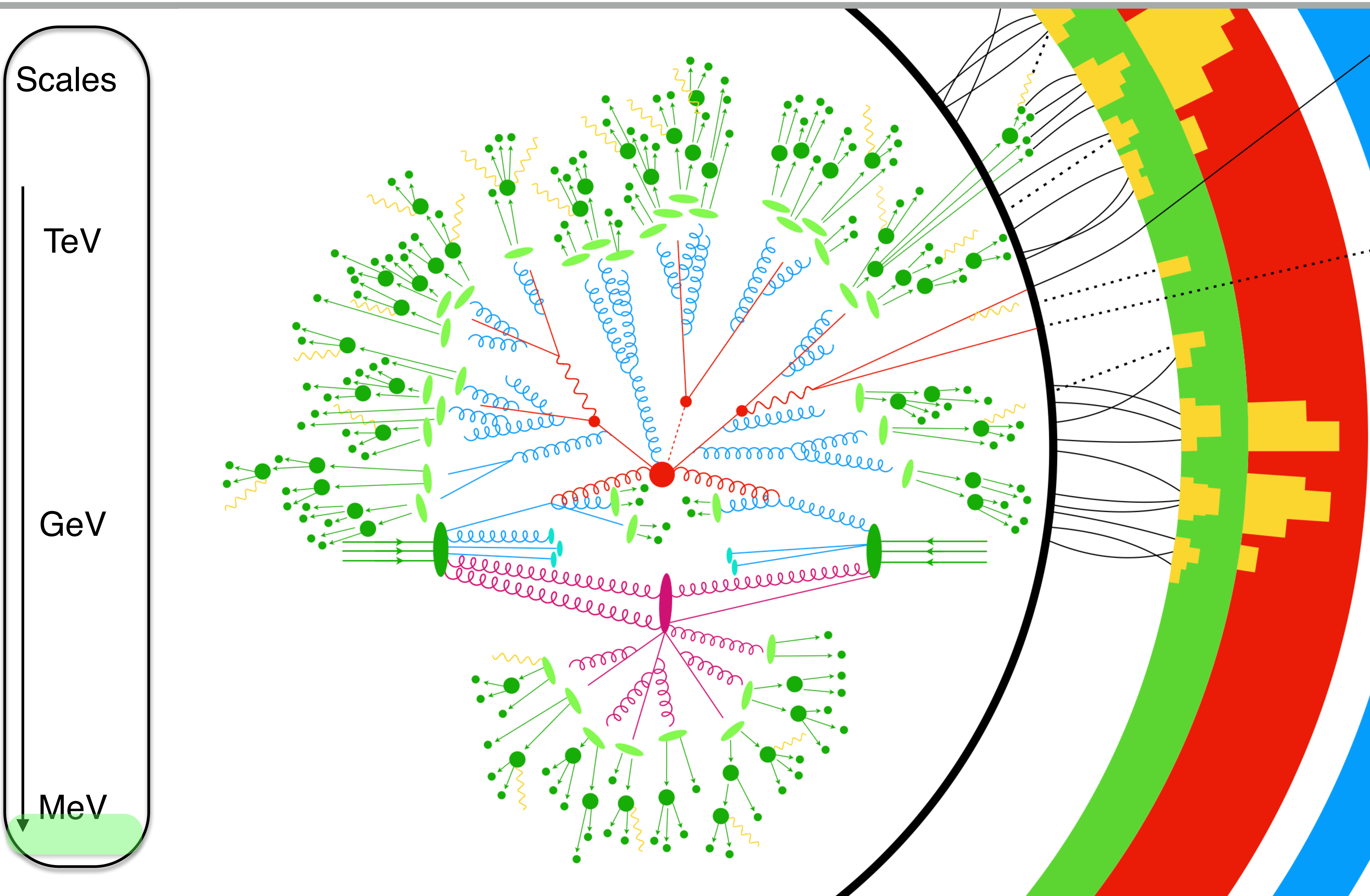


- ☞ low Q^2 physics
- ☞ universal/ process independent
- ☞ model-based description

3. Hadronization

4. Underlying Event

What are the MC for?



Question time

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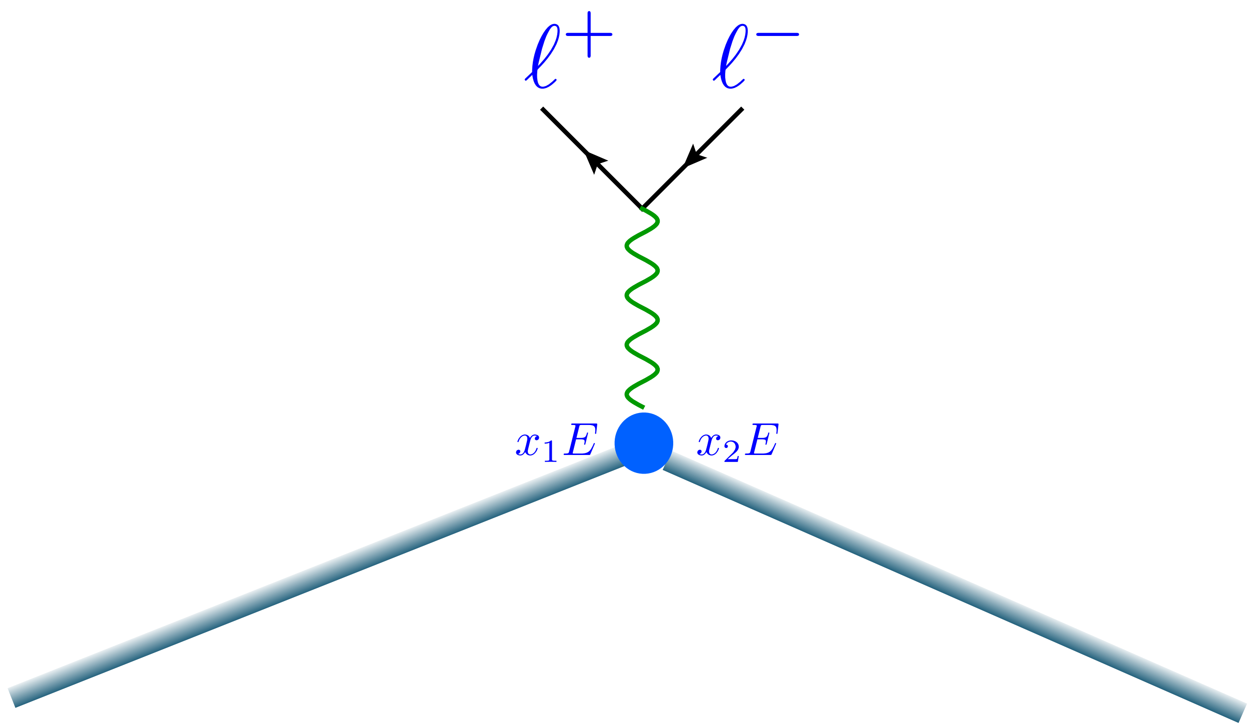
MADGRAPH

To Remember

- Multi-scale problem
  - ➡ New physics visible only at High scale
  - ➡ Problem split in different scale
    - Factorisation theorem

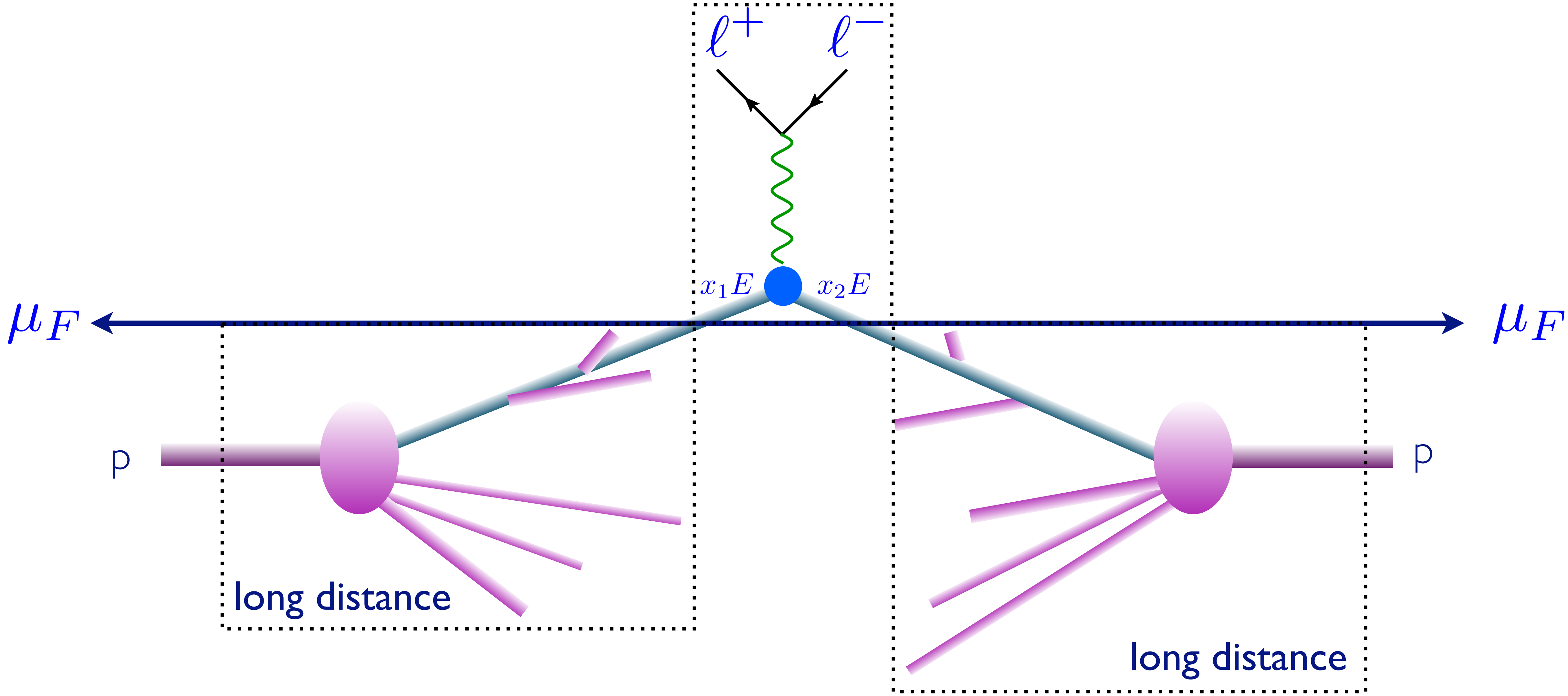
Master formula for the LHC



$$\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

Parton-level cross section

Master formula for the LHC

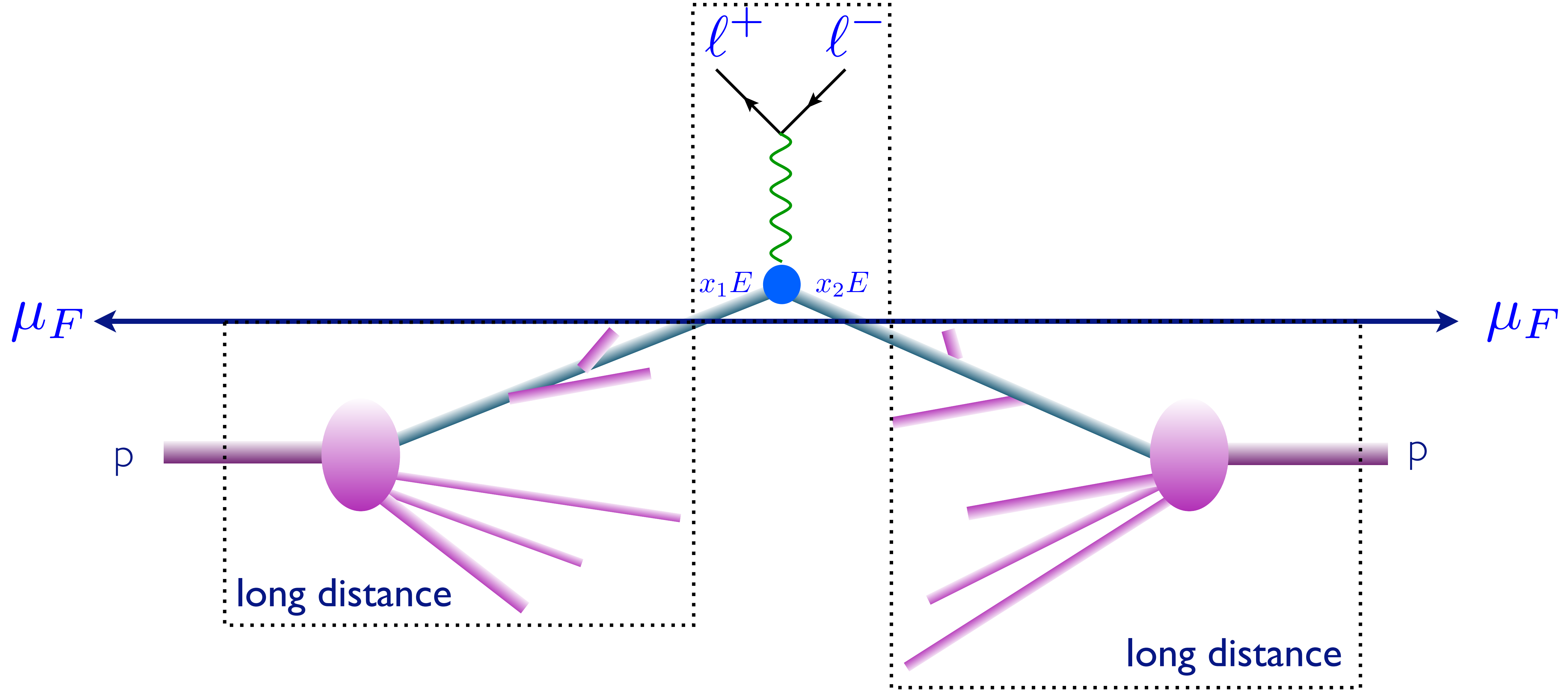


$$f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, \hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

Parton density functions

Parton-level cross section

Master formula for the LHC



$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}}\, f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

Phase-space integral

Parton density functions

Parton-level cross section

Perturbative expansion

$d\hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$ **Parton-level cross section**

- The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter, schematically:

$$\hat{\sigma} = \sigma^{\text{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \dots\right)$$

Perturbative expansion

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LO predictions

Perturbative expansion

$d\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$ **Parton-level cross section**

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LO predictions

NLO corrections

NNLO corrections

N3LO or NNNLO corrections

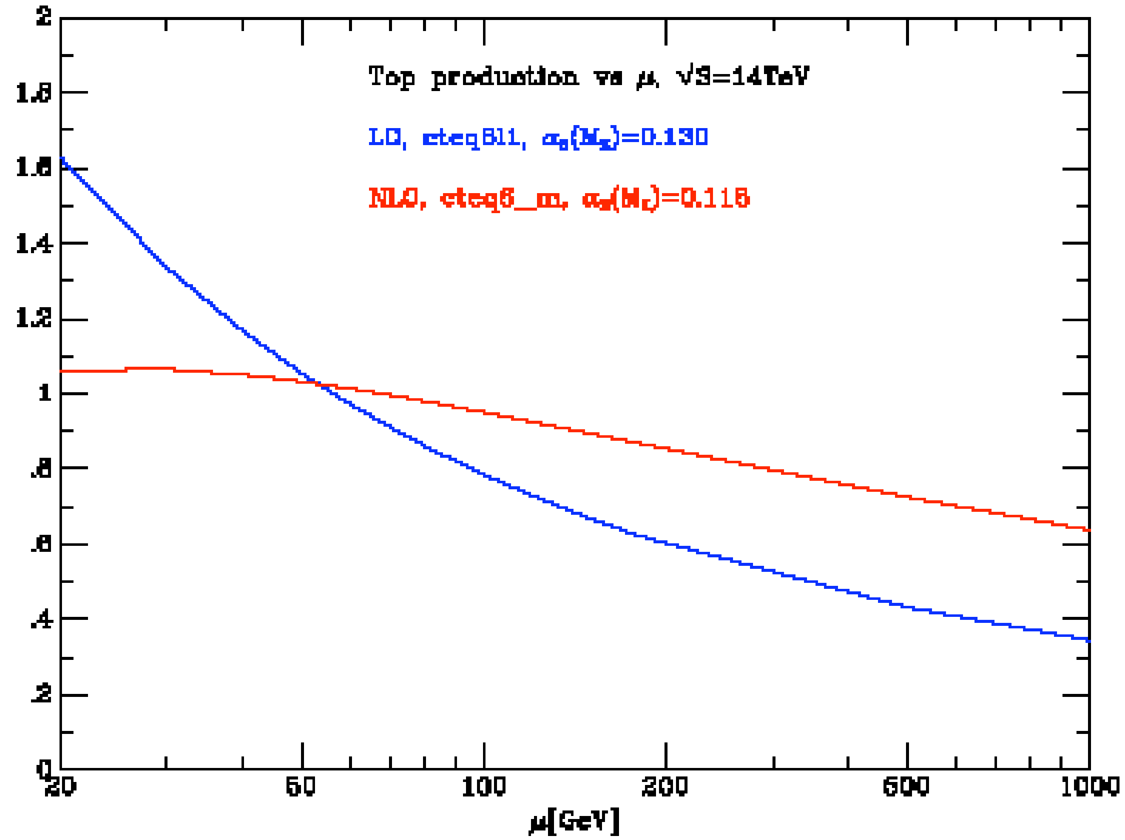
- Including higher corrections improves predictions and reduces theoretical uncertainties

Improved predictions

$$d\sigma = \sum_{a,b} \int dx_1 dx_2 \;\; f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, d\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

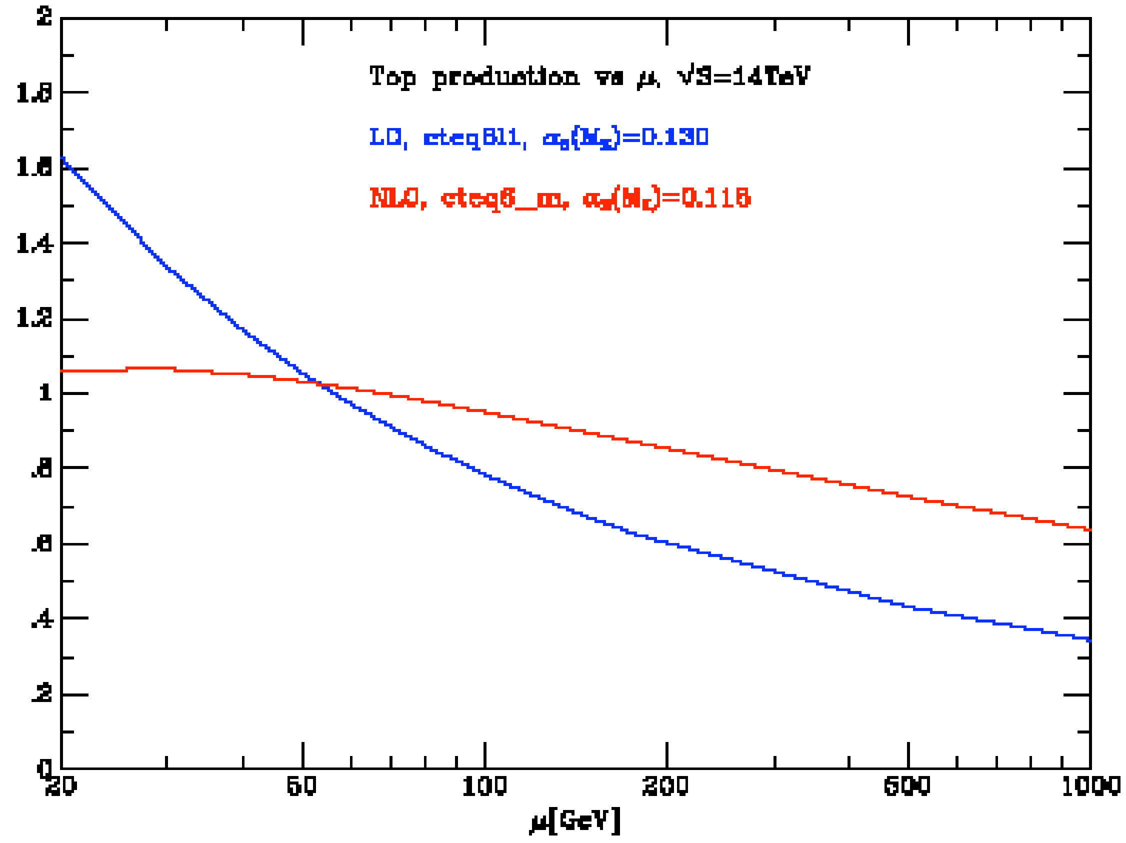
$$\hat{\sigma} = \sigma^{\mathrm{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \ldots\right)$$

- Leading Order predictions can depend strongly on the renormalization and factorization scales
- Including higher order corrections reduces the dependence on these scales



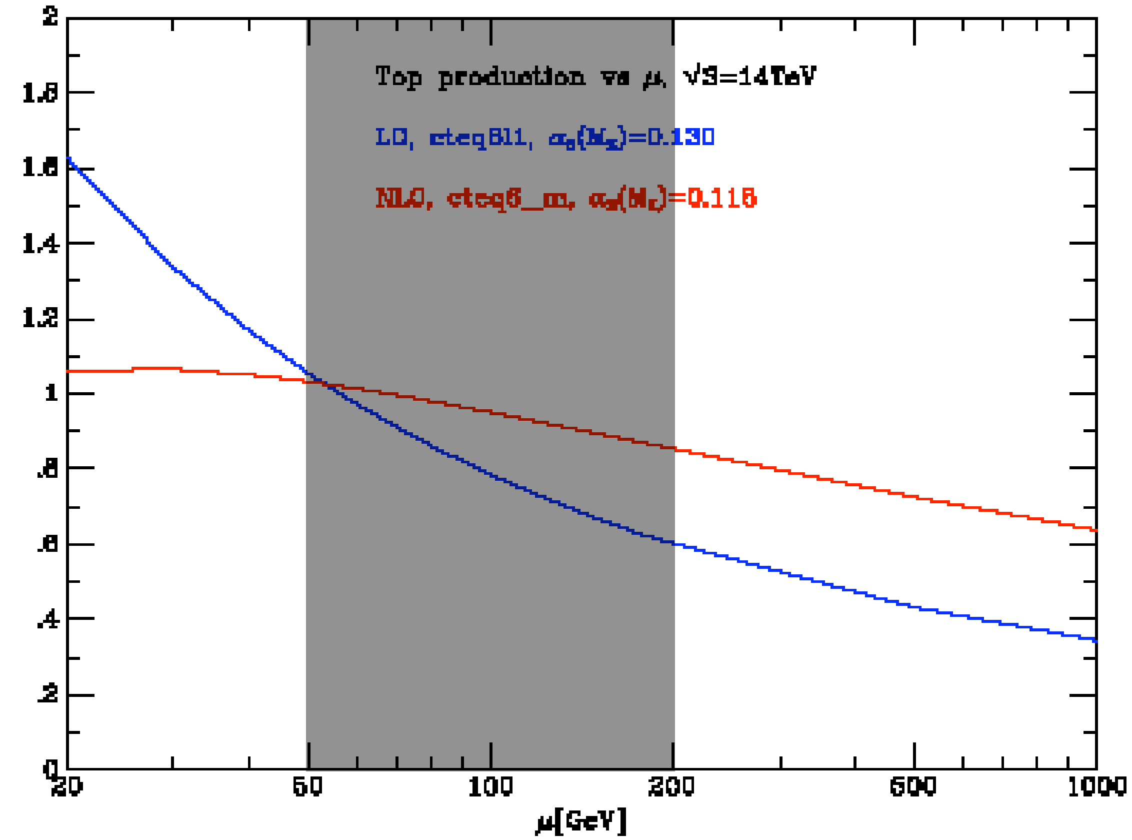
Improved predictions

$$\hat{\sigma} = \sigma^{\text{Born}} \left(1 + \frac{\alpha_s}{2\pi} \sigma^{(1)} + \left(\frac{\alpha_s}{2\pi} \right)^2 \sigma^{(2)} + \left(\frac{\alpha_s}{2\pi} \right)^3 \sigma^{(3)} + \ldots \right)$$



Improved predictions

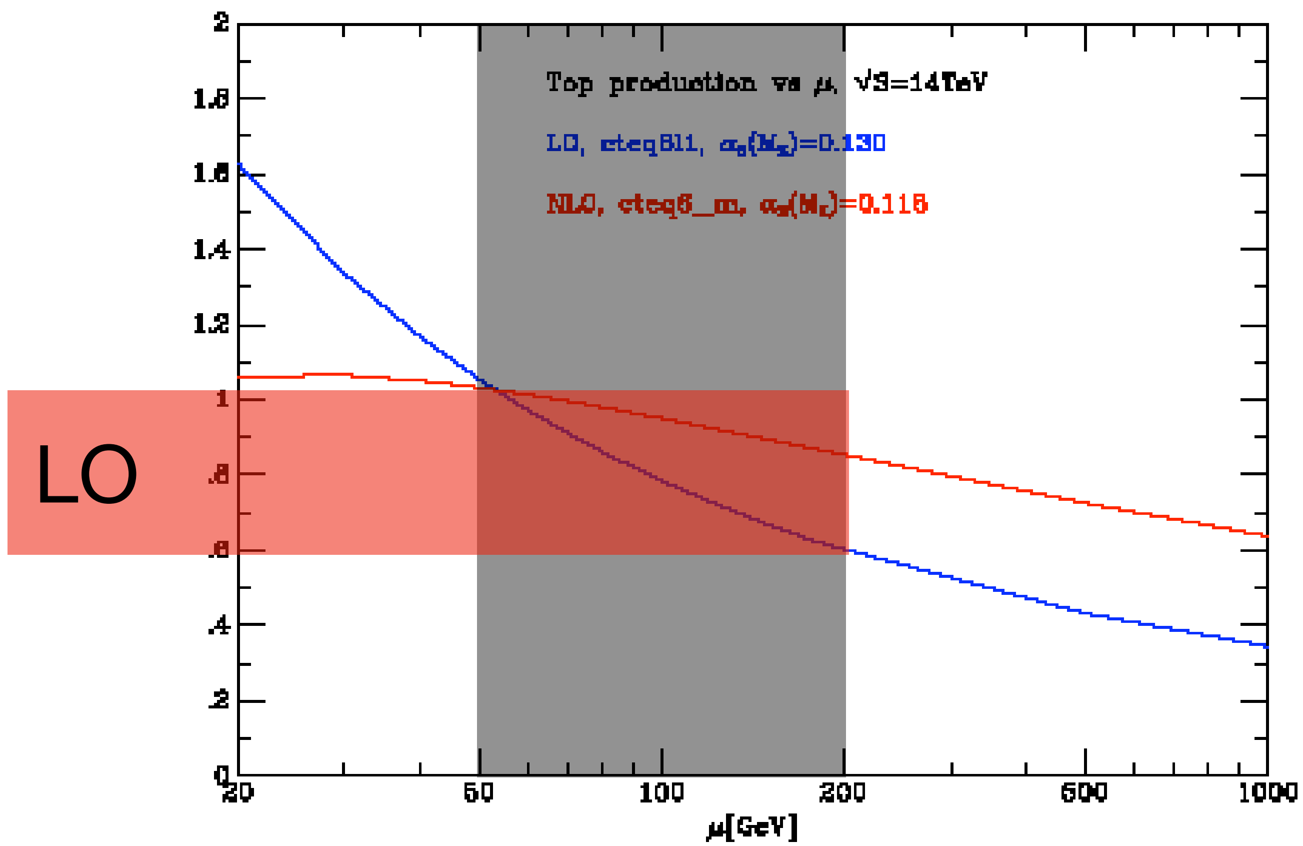
$$\hat{\sigma} = \sigma^{\text{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \dots\right)$$



Improved predictions

LO

$$\hat{\sigma} = \sigma^{\text{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \dots\right)$$

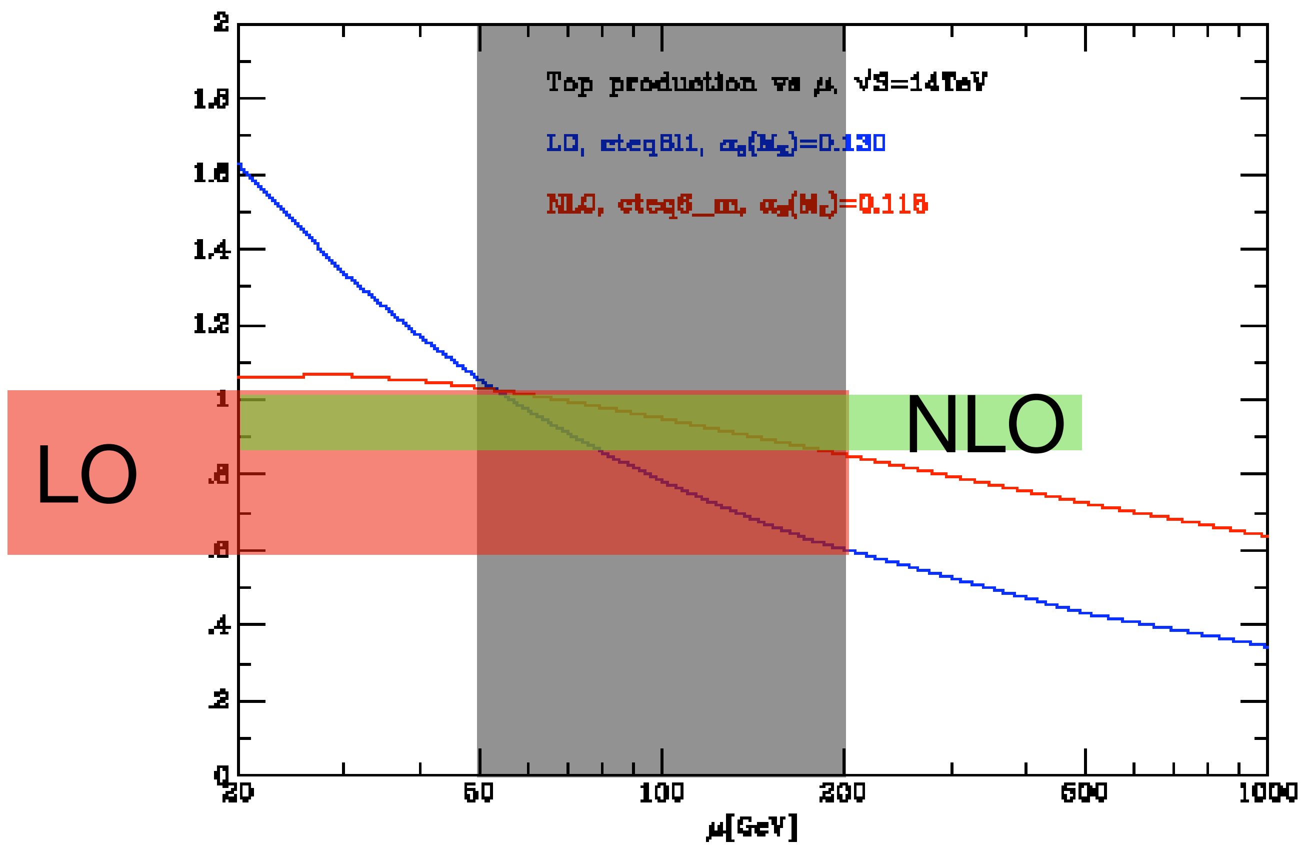


- Leading Order predictions depend strongly on arbitrary scales
  - Poor accuracy (error at ~40%)

Improved predictions

NLO

$$\hat{\sigma} = \sigma^{\text{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \dots\right)$$



- Leading Order predictions depend strongly on arbitrary scales
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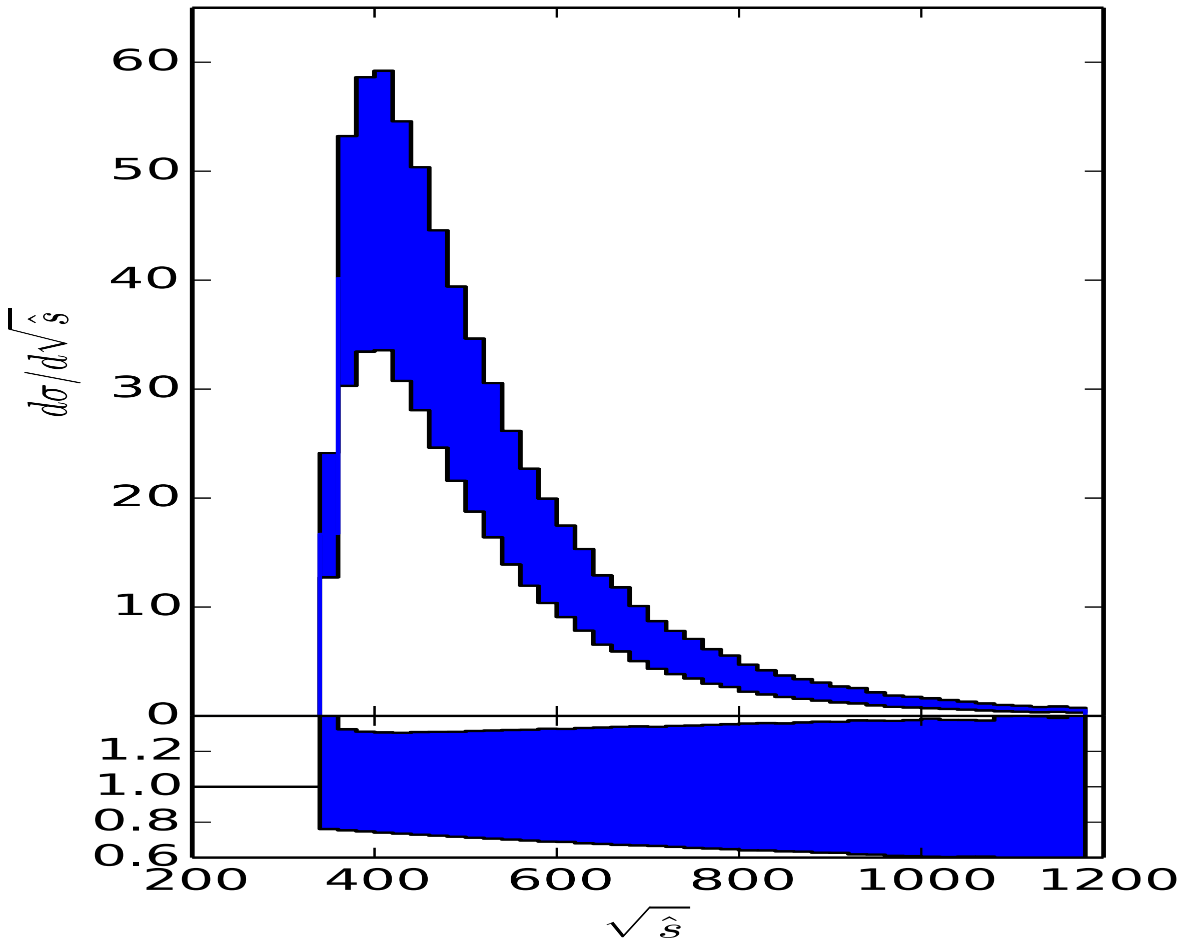
LO

Error on LO prediction (top quark pair)

Error on Normalisation

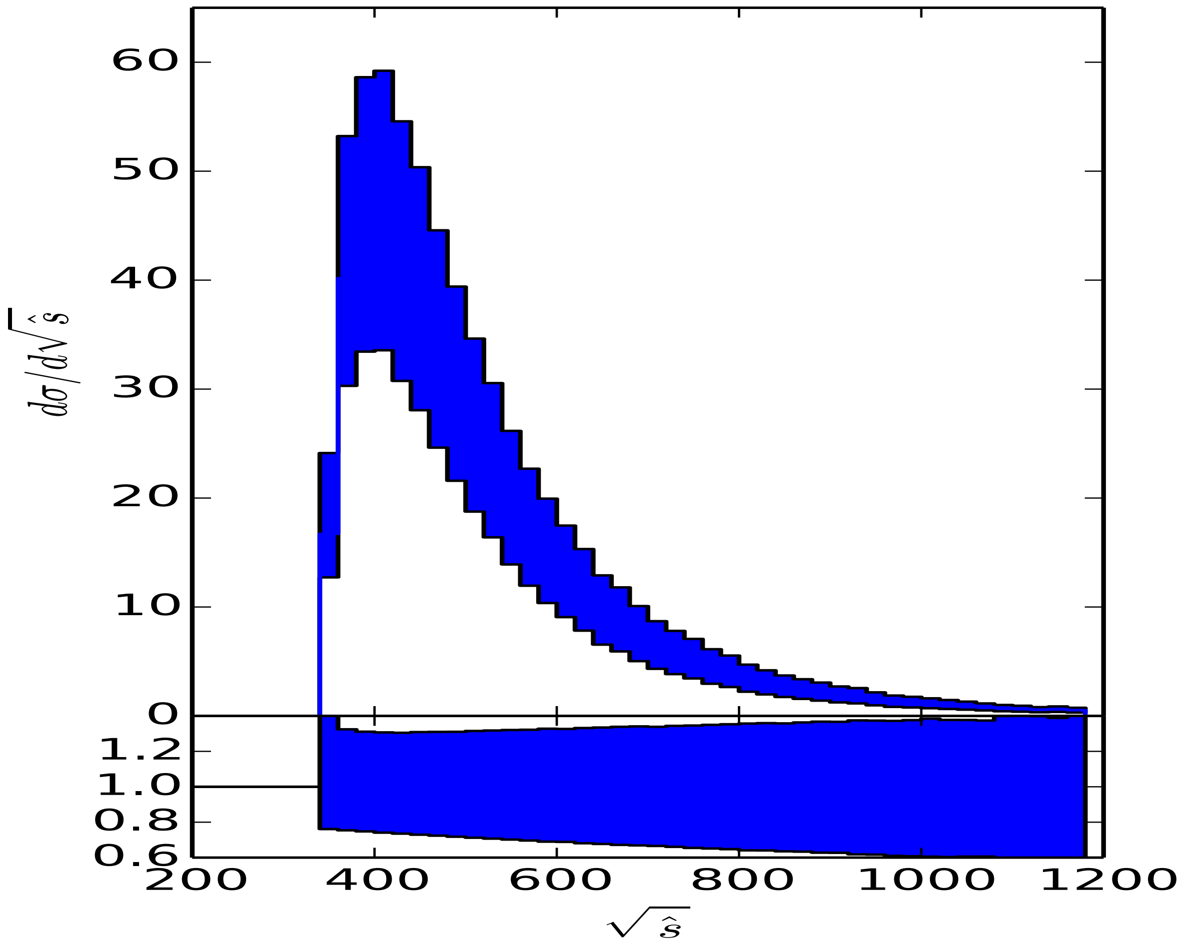


- Large scale uncertainty
- LO is good for shape

LO

Error on LO prediction (top quark pair)

Error on Normalisation



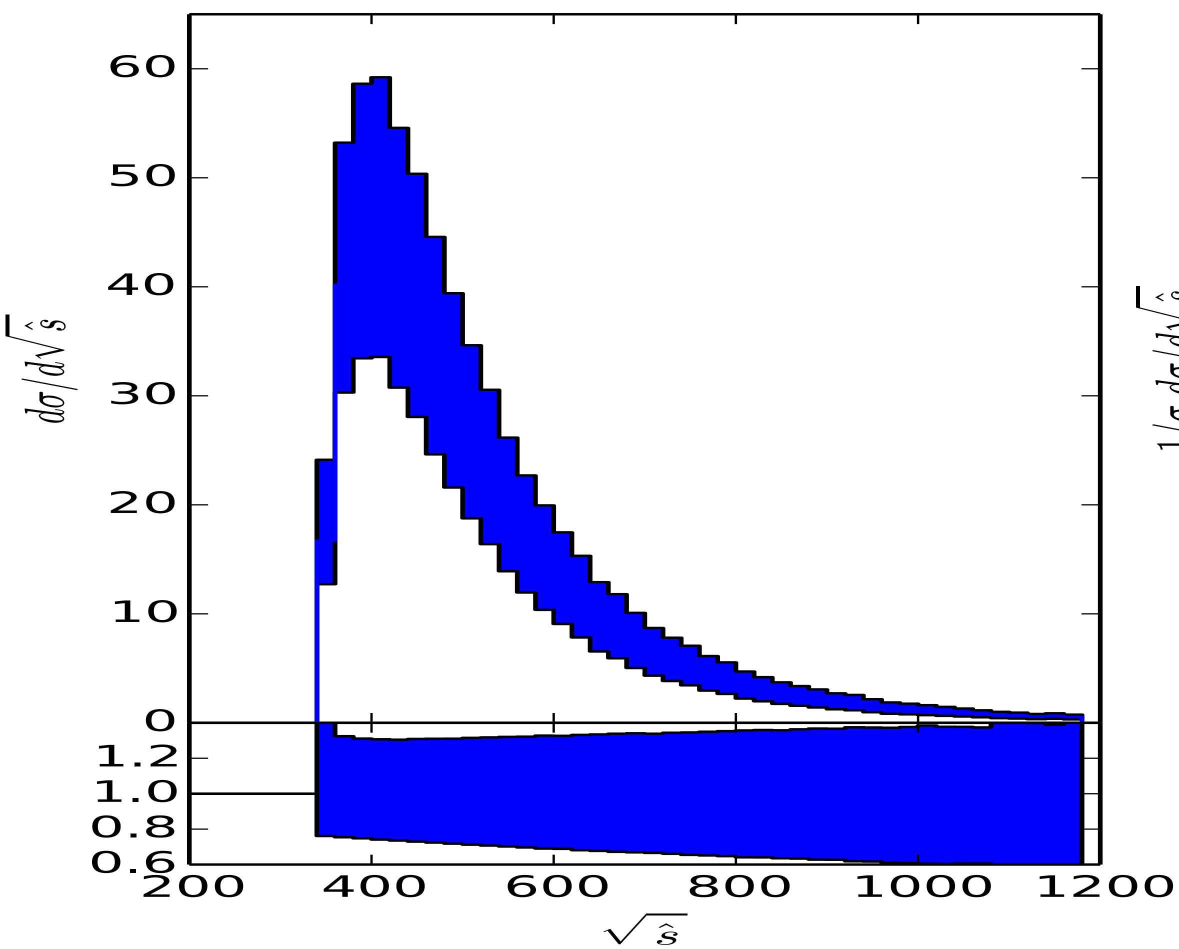
Large Error (30-50%)

- Large scale uncertainty
- LO is good for shape

LO

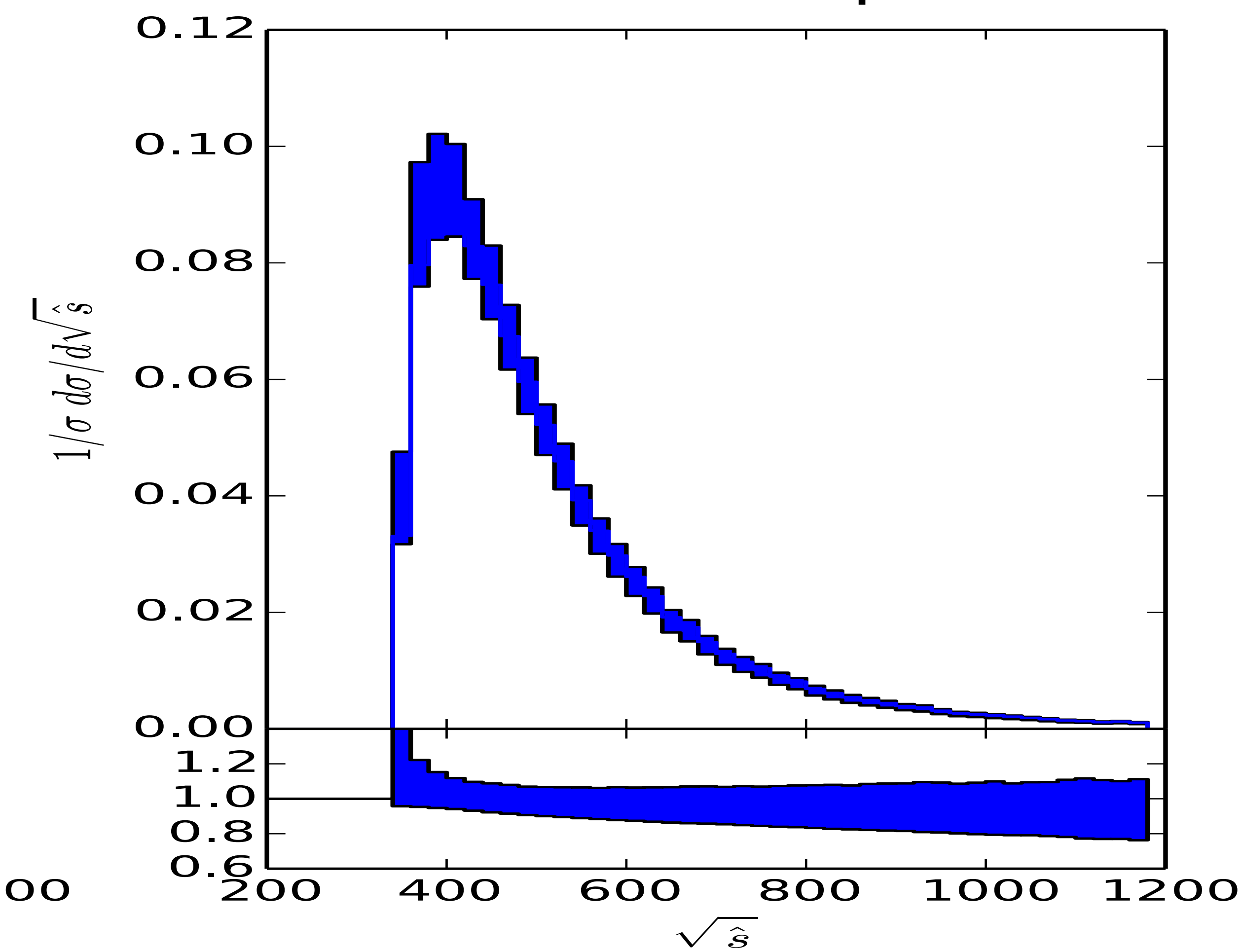
Error on LO prediction (top quark pair)

Error on Normalisation



Large Error (30-50%)

Error on Shape



More reasonable (~10%)

- Large scale uncertainty
- LO is good for shape

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MADGRAPH

To Remember

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat\sigma_{ab\to X}(\hat s, \mu_F, \mu_R)$$

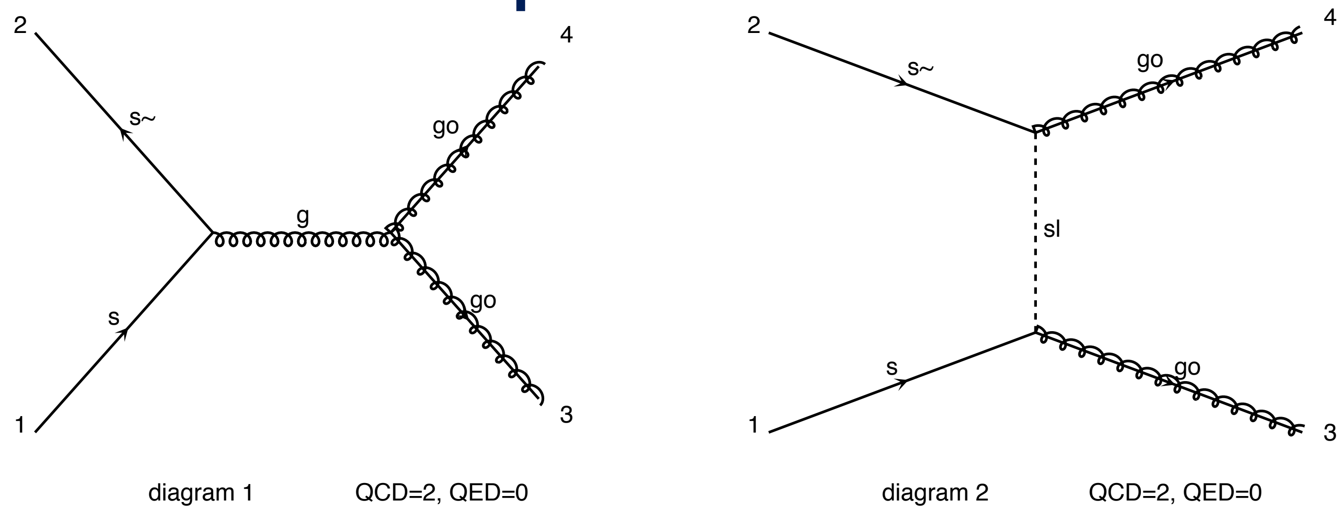
Phase-space integral — Parton density functions — Parton-level cross section

- PDF: content of the proton
  - ➡ Define the physics/processes that will dominate on your accelerator
- LO: good for shape
- NLO/NNLO: Reduce scale uncertainty
- Computation are inclusive (+ any jet) due to renormalization/factorization scale

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



- Evaluate the matrix-element

$$|\mathcal{M}|^2$$ ➡Need Feynman Rules!

- Phase-Space Integration

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

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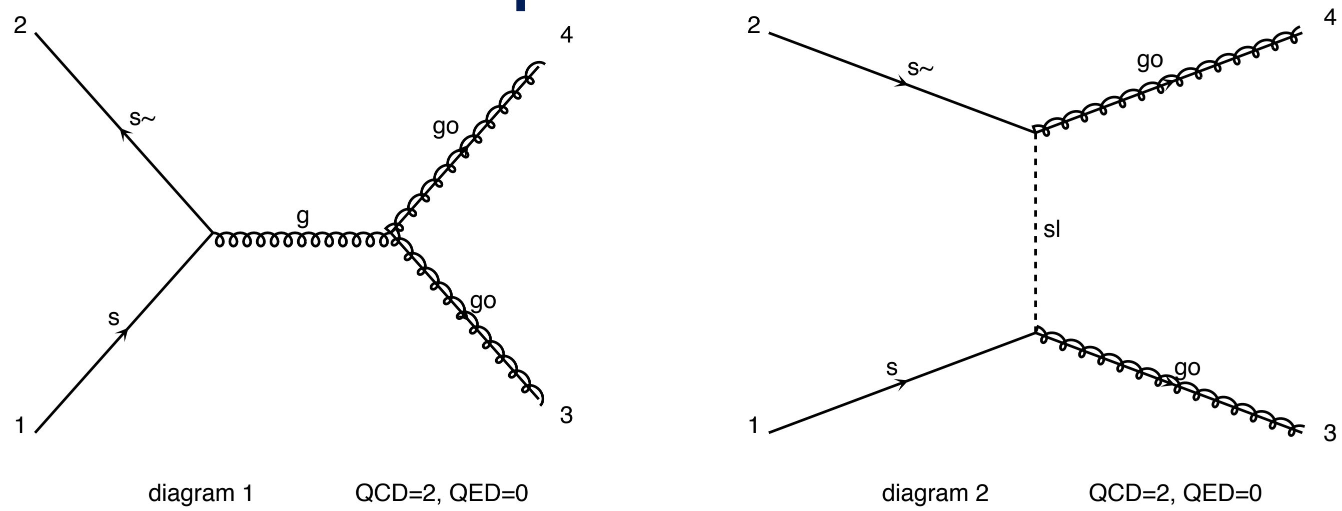

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$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

Easy enough

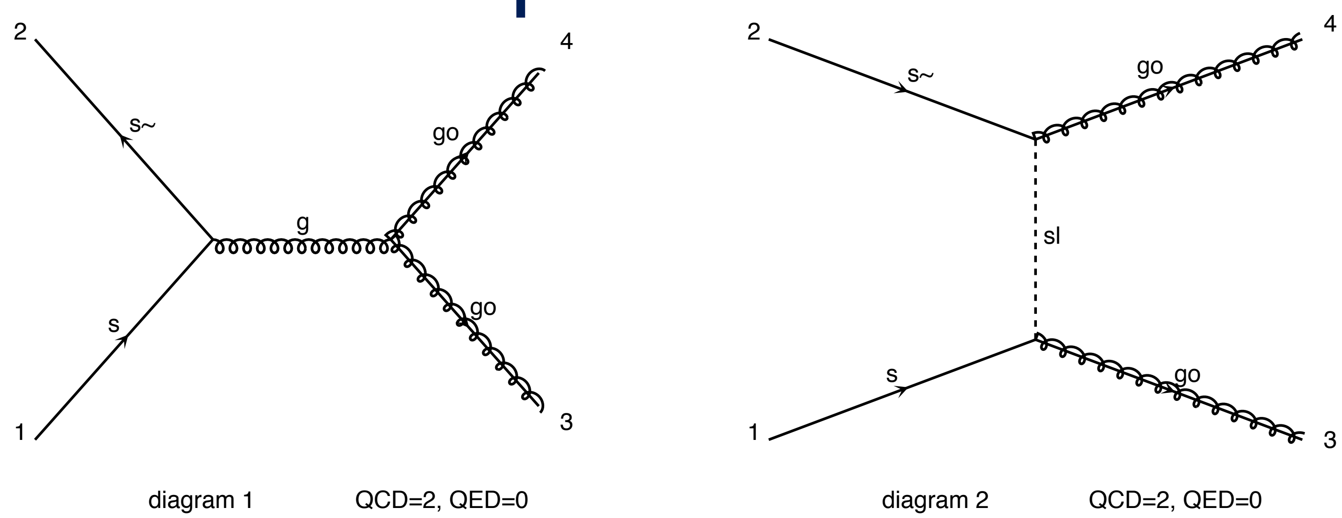
Hard

Very Hard (in general)

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



Easy enough

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Hard

- Phase-Space Integration

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

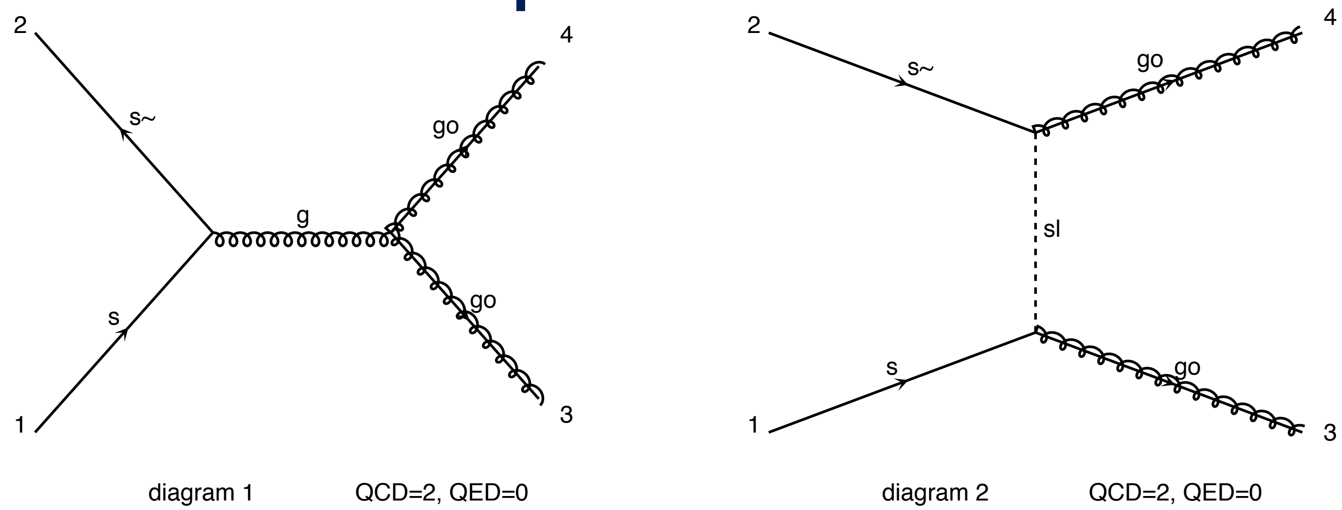
Very Hard (in general)

Now

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



Easy enough

- Evaluate the matrix-element

$$|\mathcal{M}|^2$$

➡Need Feynman Rules!

Hard

Next

- Phase-Space Integration

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

Very Hard (in general)

Now

Monte Carlo Integration

Monte Carlo Integration

Calculations of cross section or decay widths involve integrations over high-dimension phase space of very peaked functions:

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

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General and flexible method is needed

Monte Carlo Integration

Calculations of cross section or decay widths involve integrations over high-dimension phase space of very peaked functions:

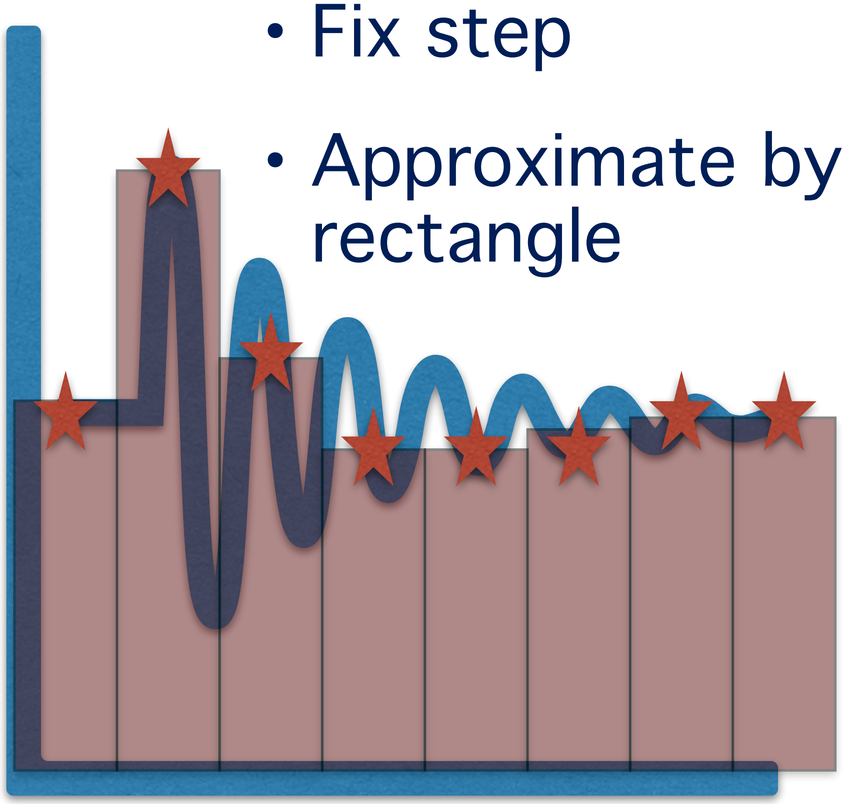
$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

$$Dim[\Phi(n)] \sim 3n$$

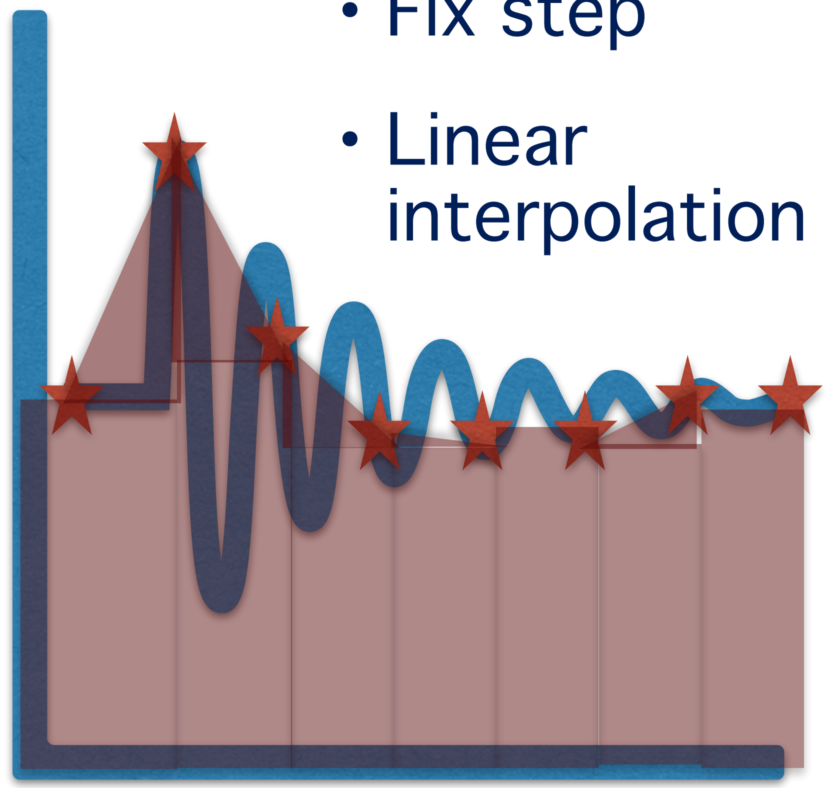
General and flexible method is needed

Not only integrating but also **generates events**

How to integrate

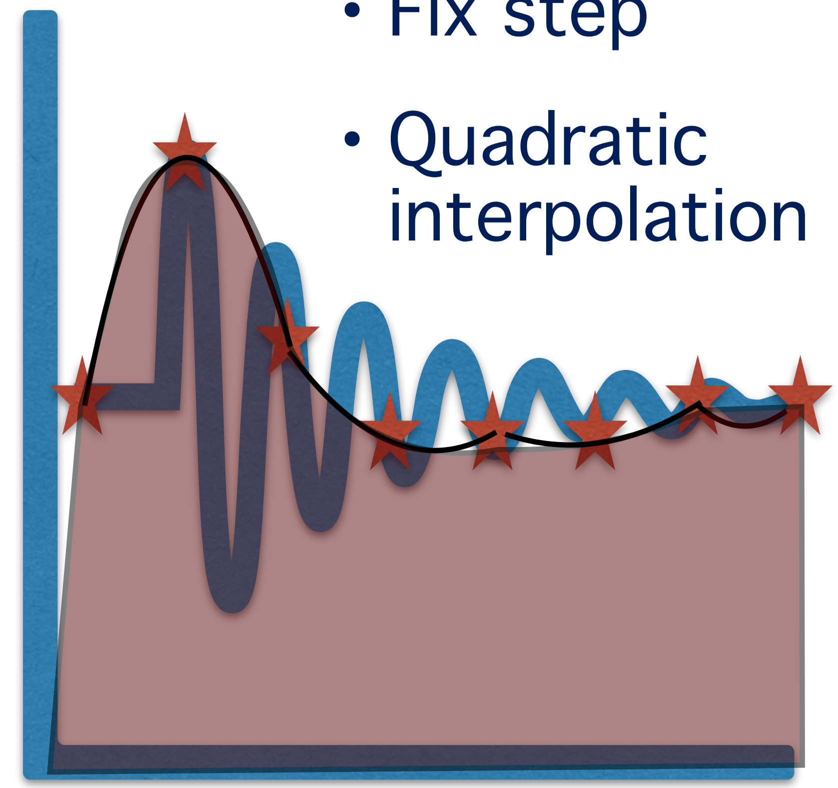


- Fix step
- Approximate by rectangle



- Fix step
- Linear interpolation

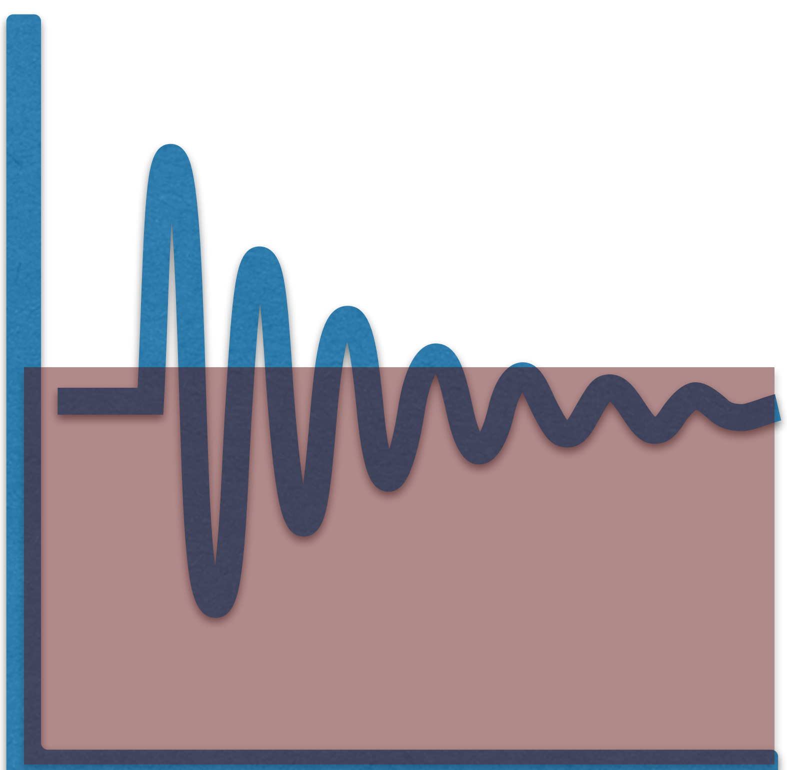
- Trapezium



- Fix step
- Quadratic interpolation

- Simpson

Average



Compute the average instead
-> Use same method as pool estimate
-> Use random number point in the x direction

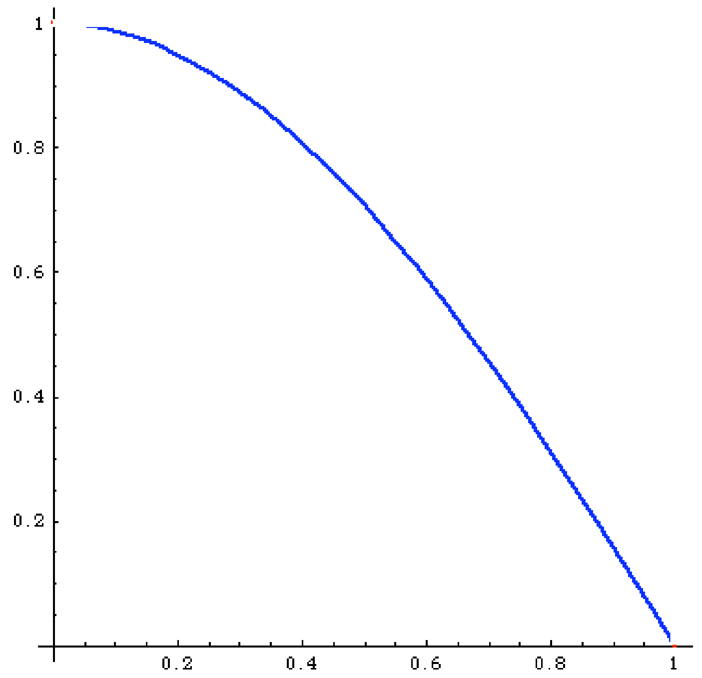
- Monte-Carlo

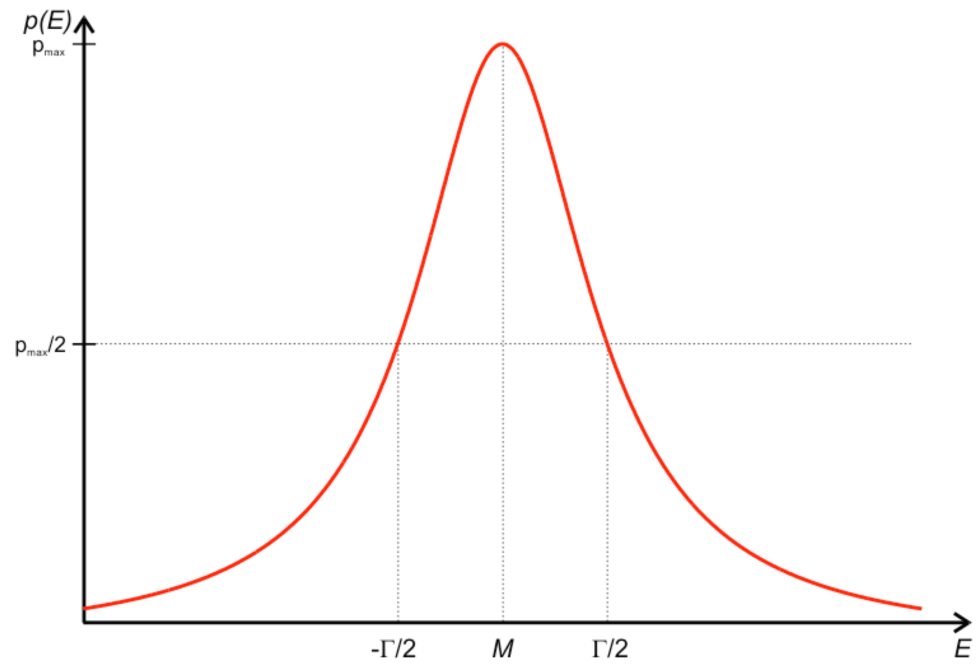
Integration

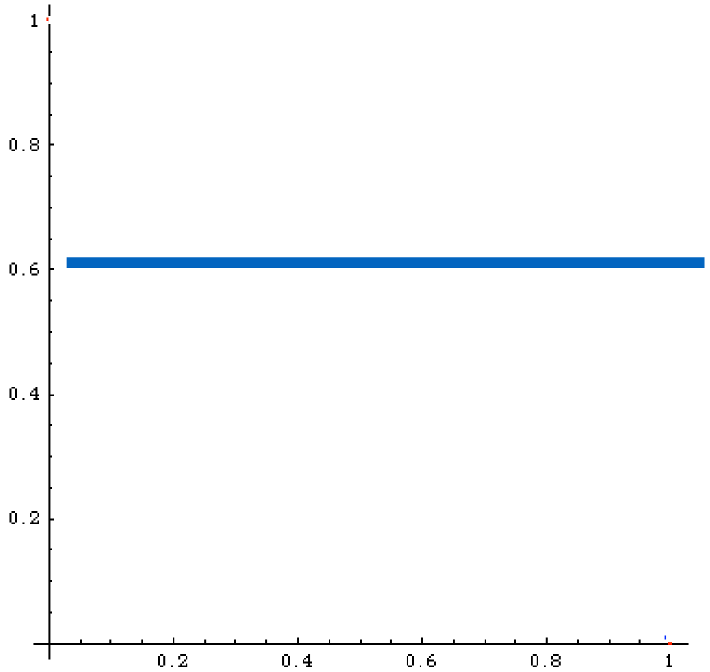
$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\int dx\, C$$

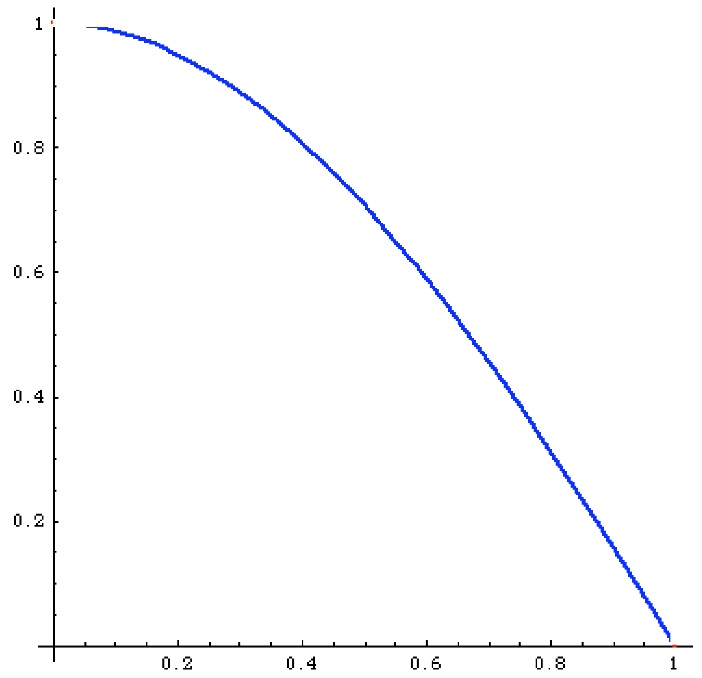




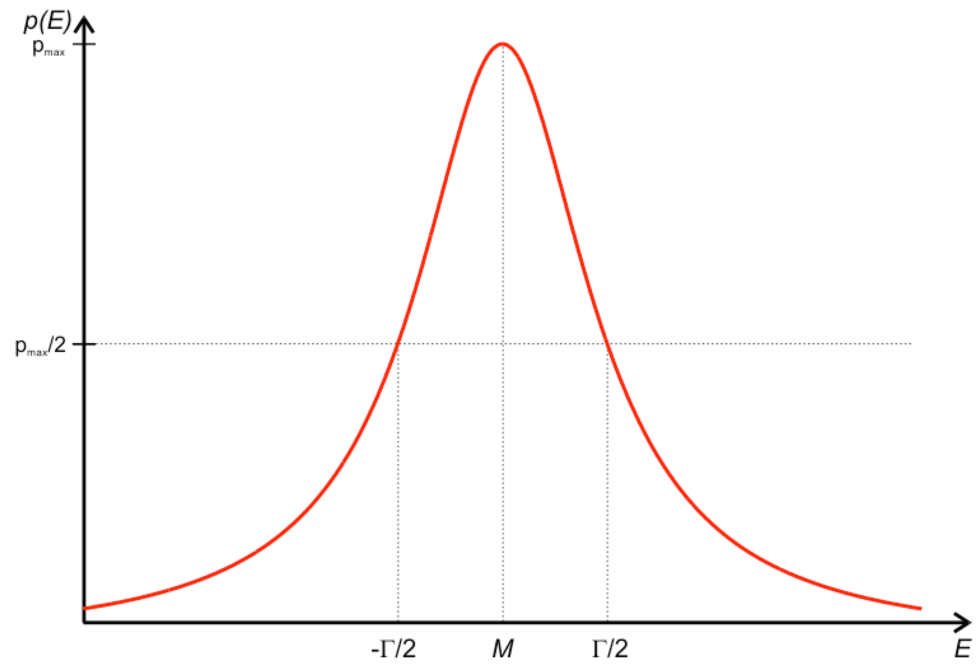


Integration

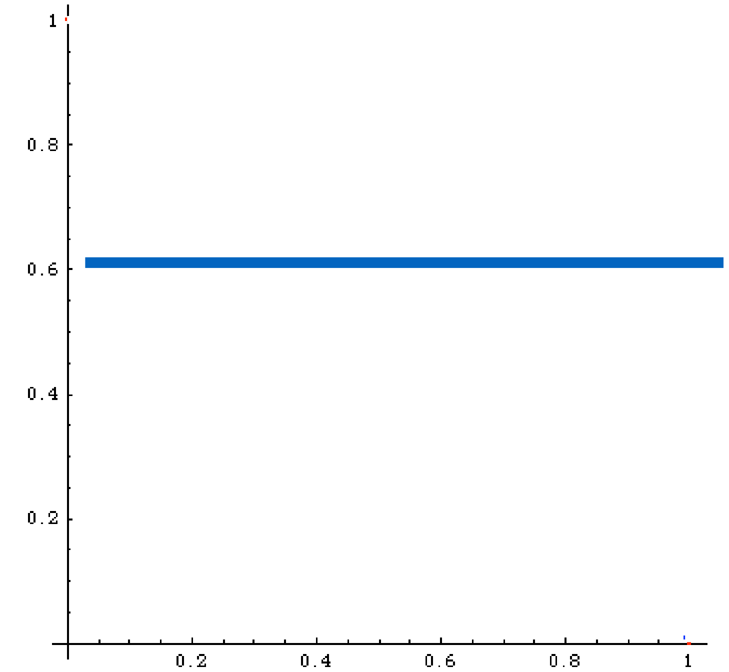
$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$



$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$



$$\int dx\, C$$



Method of evaluation

- MonteCarlo $1/\sqrt{N}$
- Trapezium $1/N^2$
- Simpson $1/N^4$

Integration

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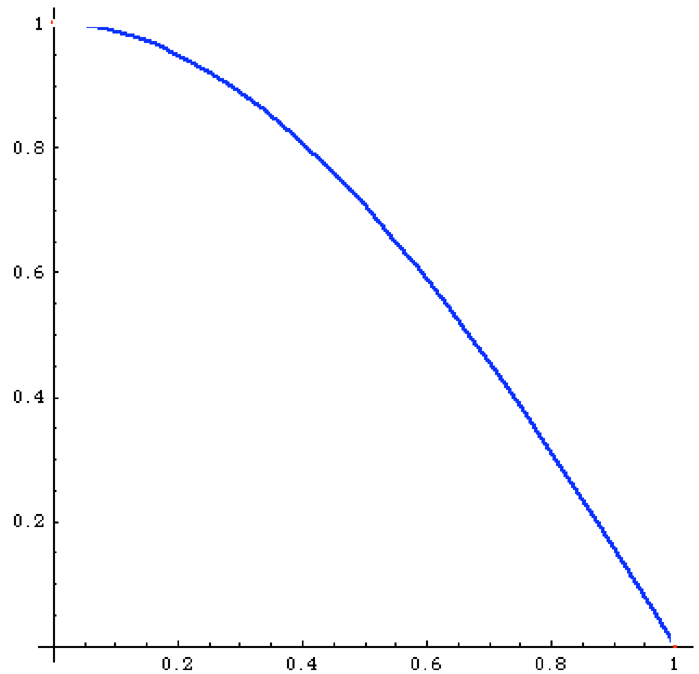

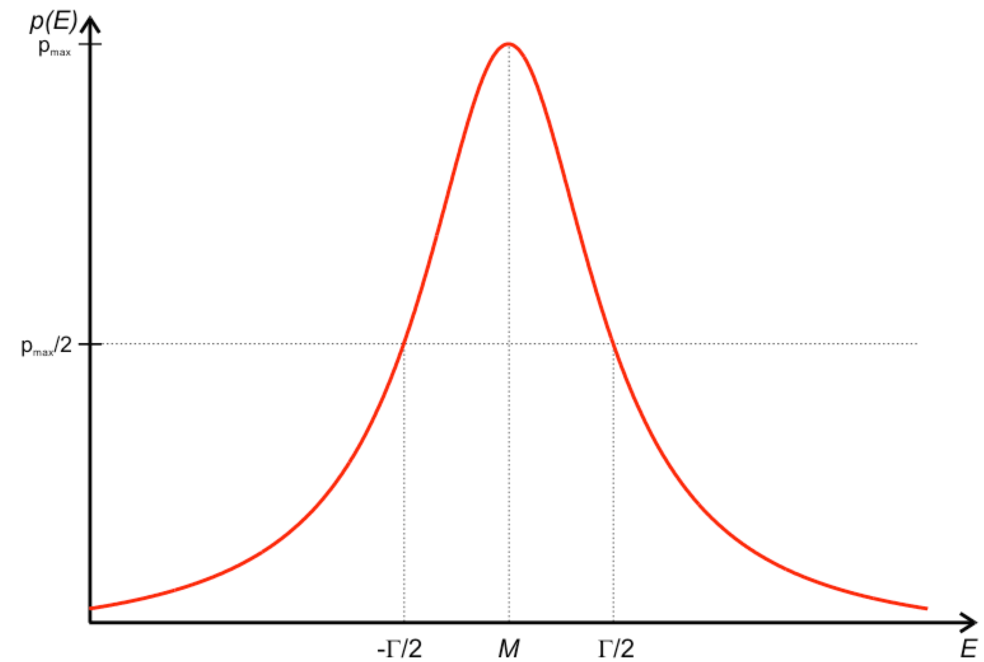

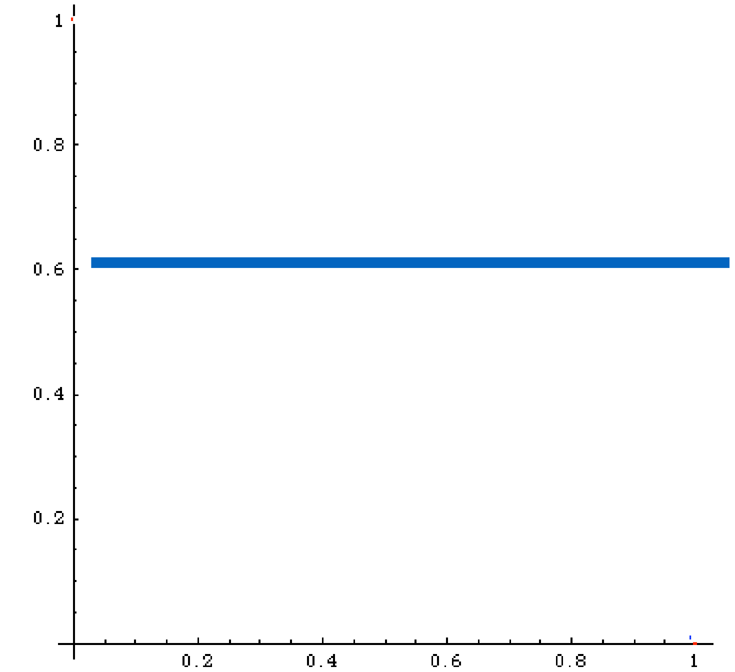

	simpson	MC
3	0,638	0,3
5	0,6367	0,8
20	0,63662	0,6
100	0,636619	0,65
1000	0,636619	0,636

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Integration

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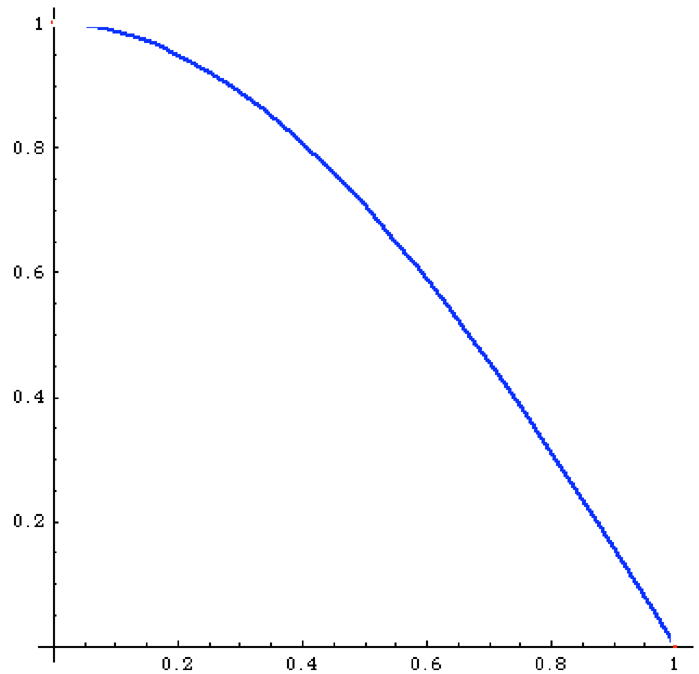
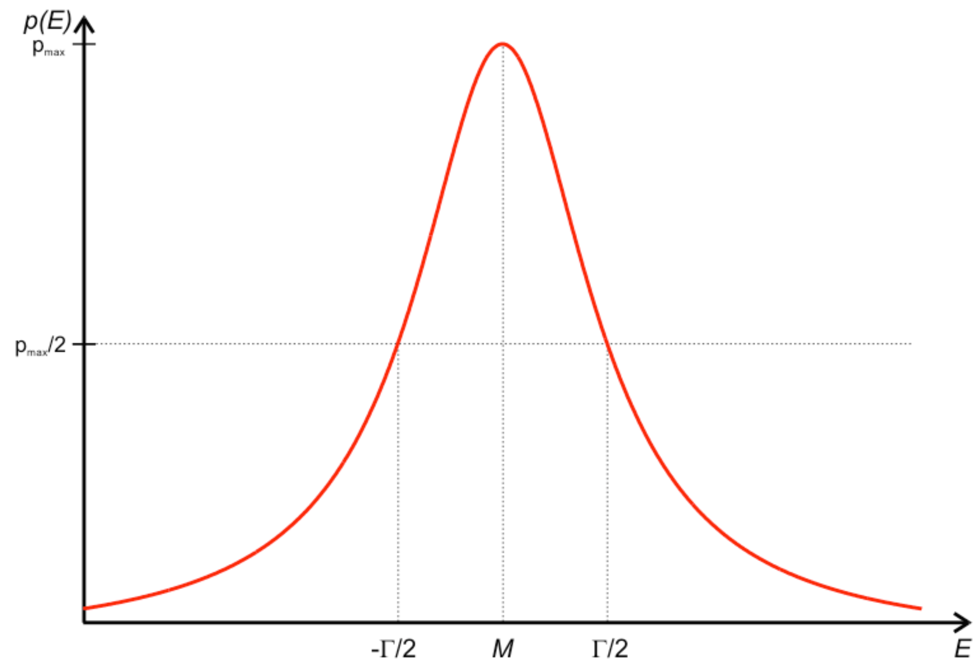
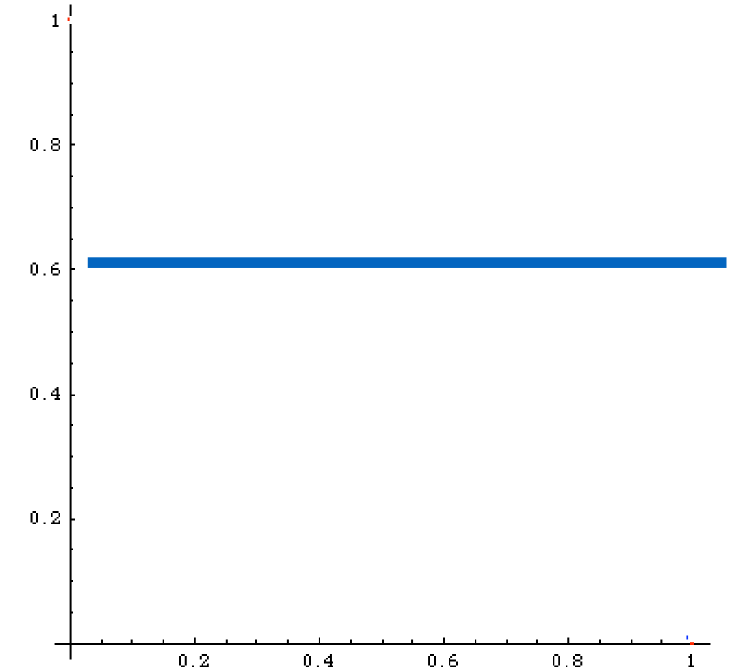
$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\int dx\, C$$







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- MonteCarlo $1/\sqrt{N}$
- Trapezium $1/N^2$
- Simpson $1/N^4$

More Dimension →

$1/\sqrt{N}$

$1/N^{2/d}$

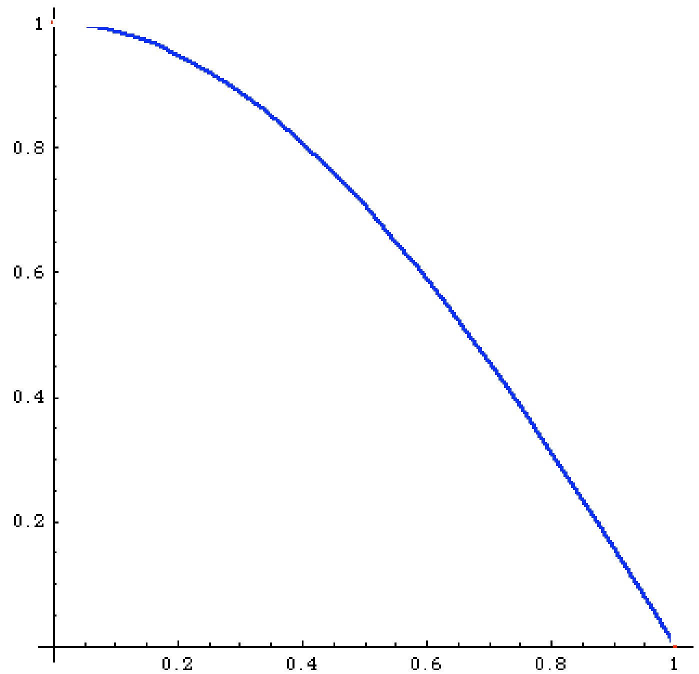
$1/N^{4/d}$

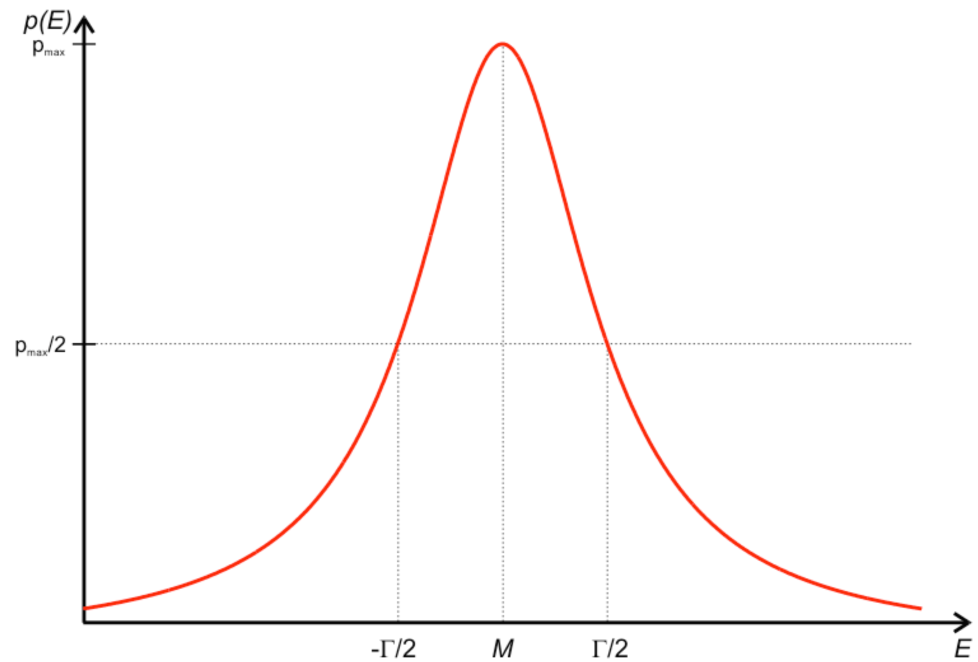
Integration

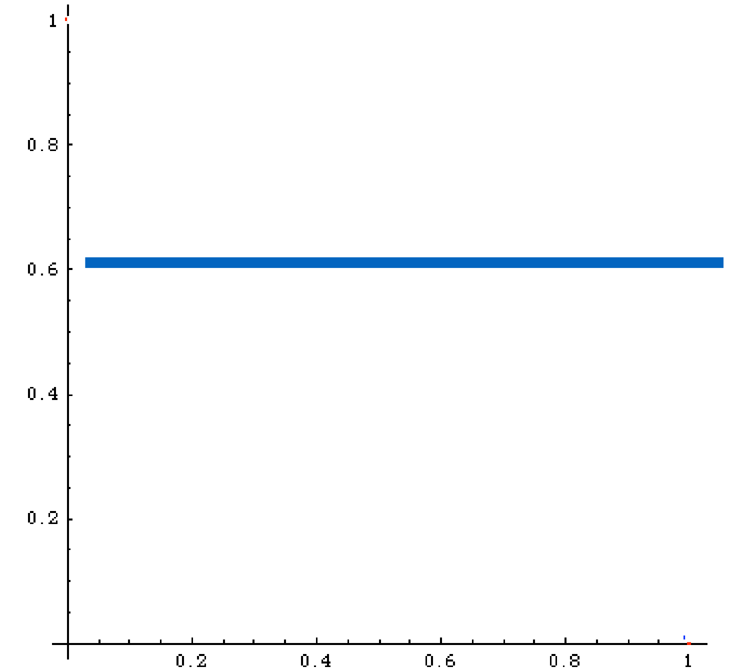
$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

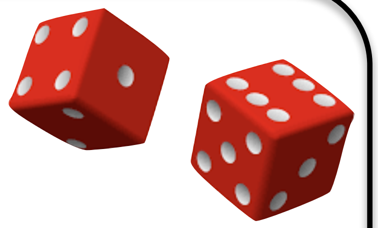
$$\int dx\, C$$







$$I = \int_{x_1}^{x_2} f(x)dx \quad \Rightarrow \quad I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x)$$



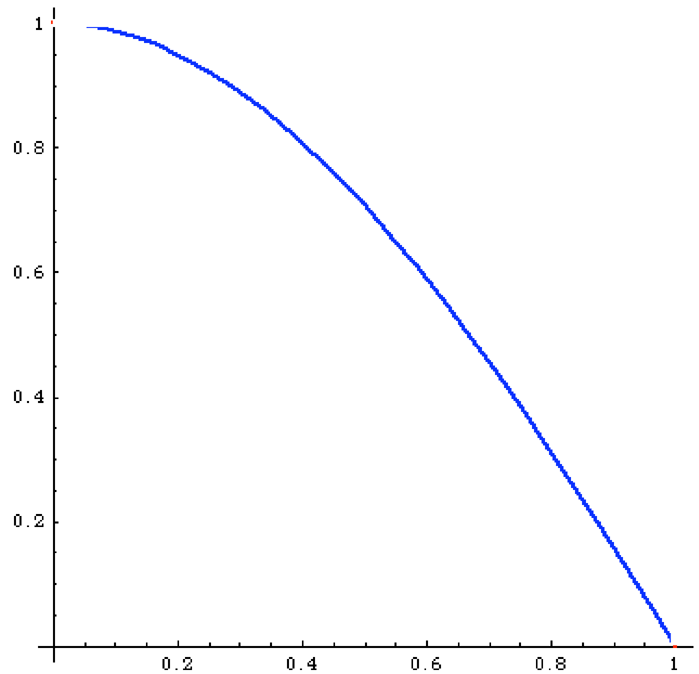
$$V = (x_2 - x_1)\int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \quad \Rightarrow \quad V_N = (x_2 - x_1)^2 \frac{1}{N}\sum_{i=1}^{N}[f(x)]^2 - I_N^2$$

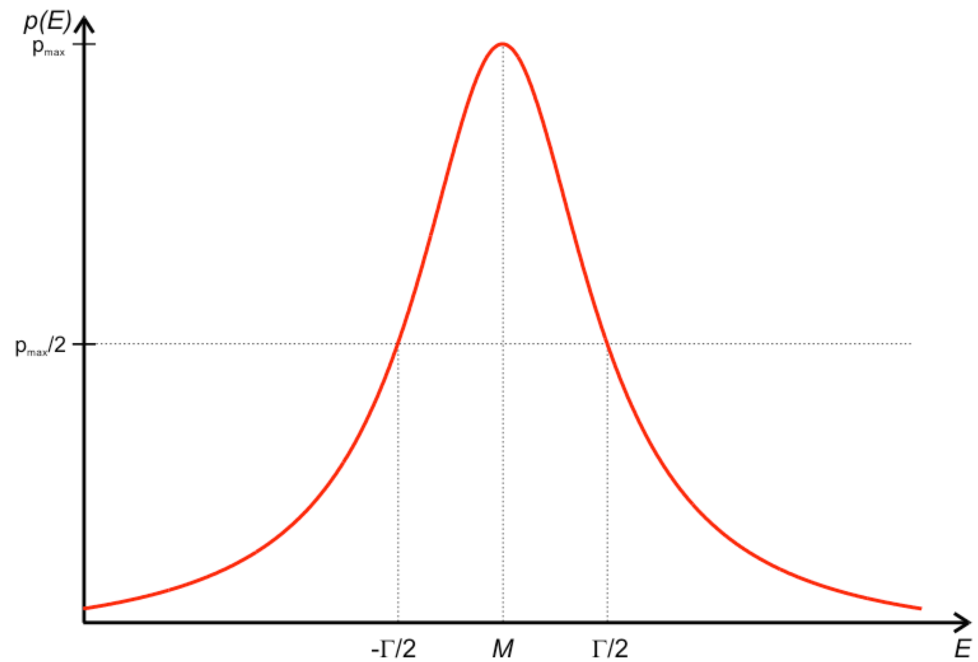
Integration

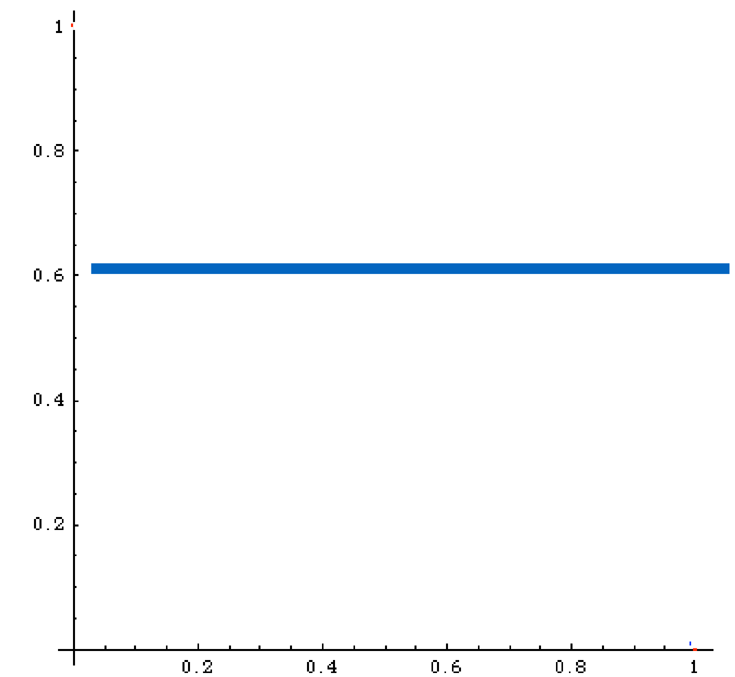
$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

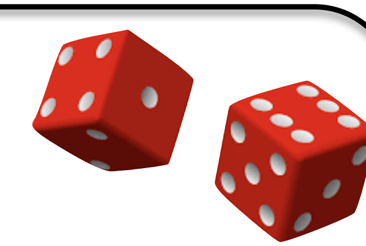
$$\int dx\, C$$







$$I = \int_{x_1}^{x_2} f(x) dx \quad \Rightarrow \quad I_N = (x_2 - x_1) \frac{1}{N} \sum_{i=1}^{N} f(x)$$



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$$I = I_N \pm \sqrt{V_N / N}$$

Integration

$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

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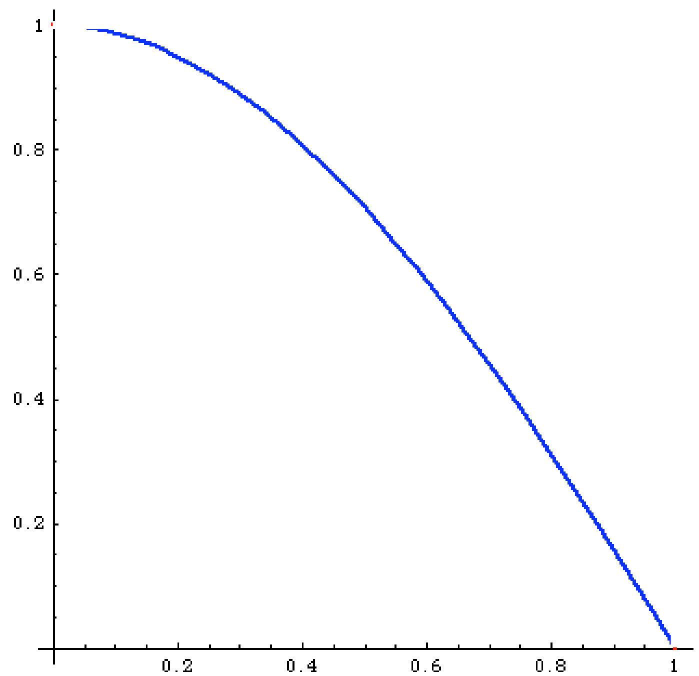

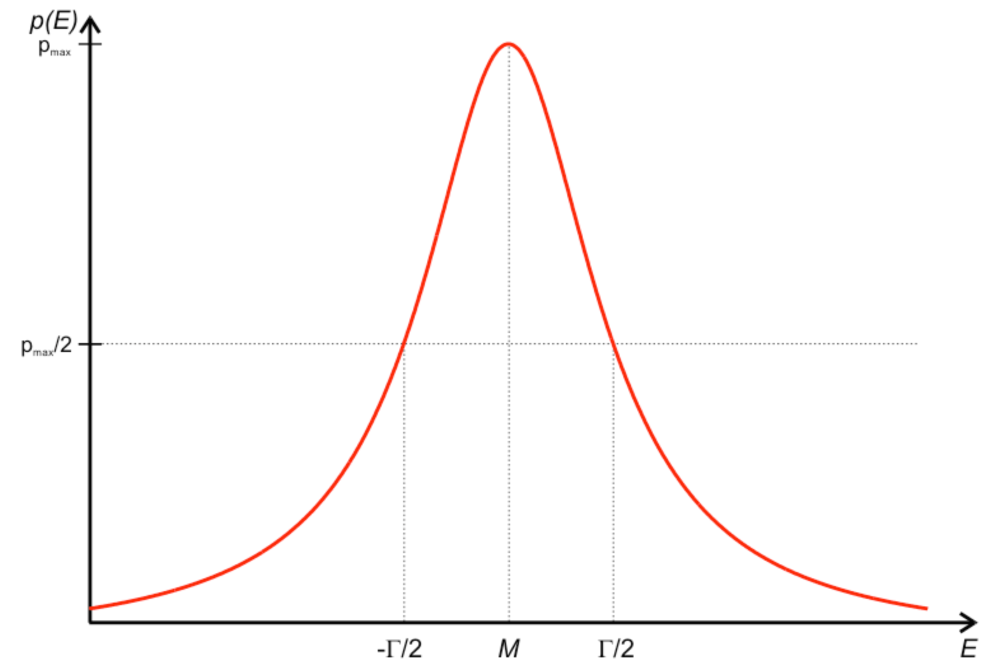

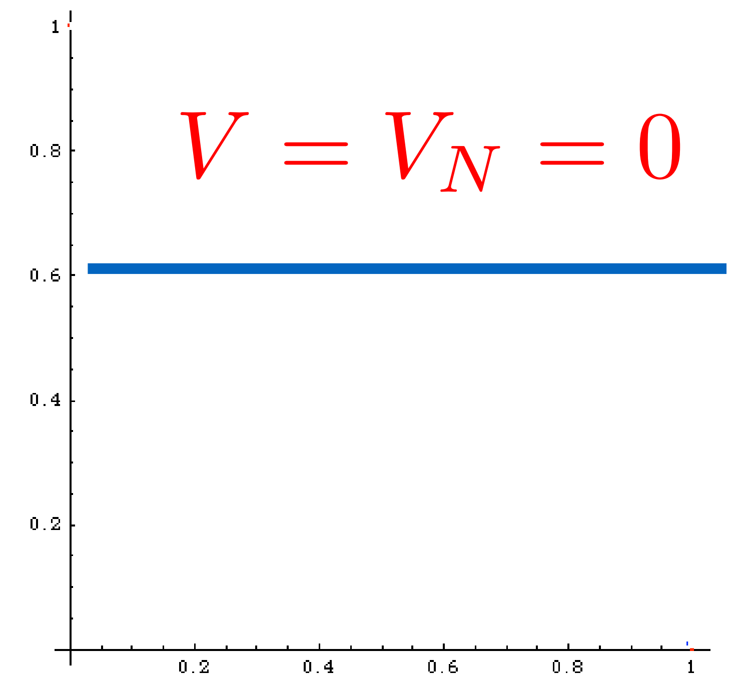

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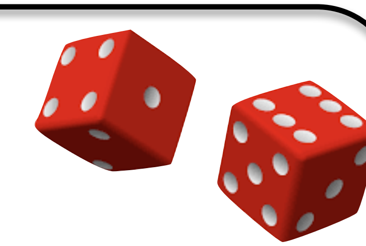

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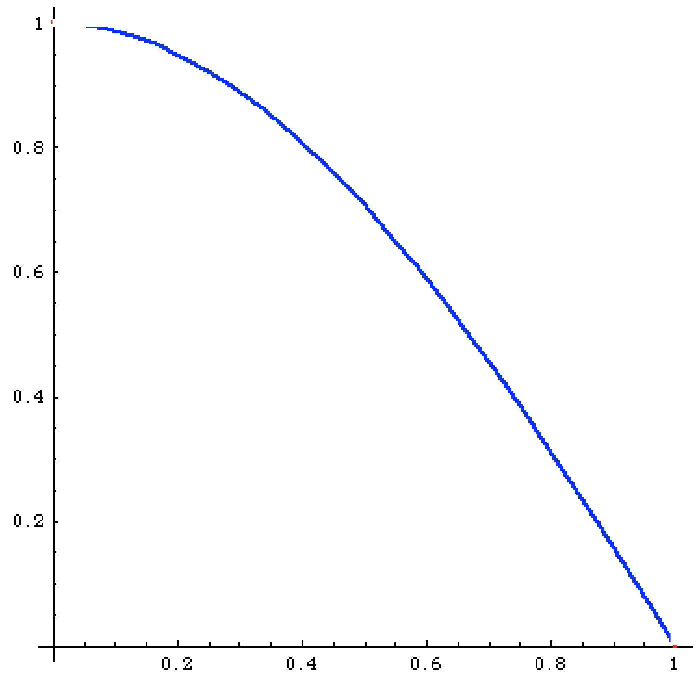

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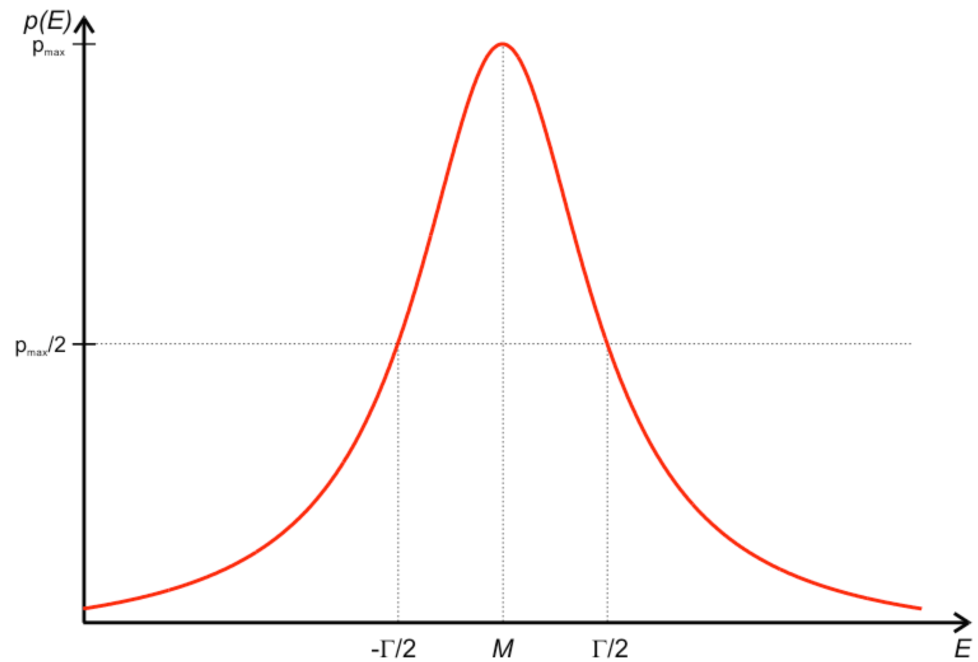

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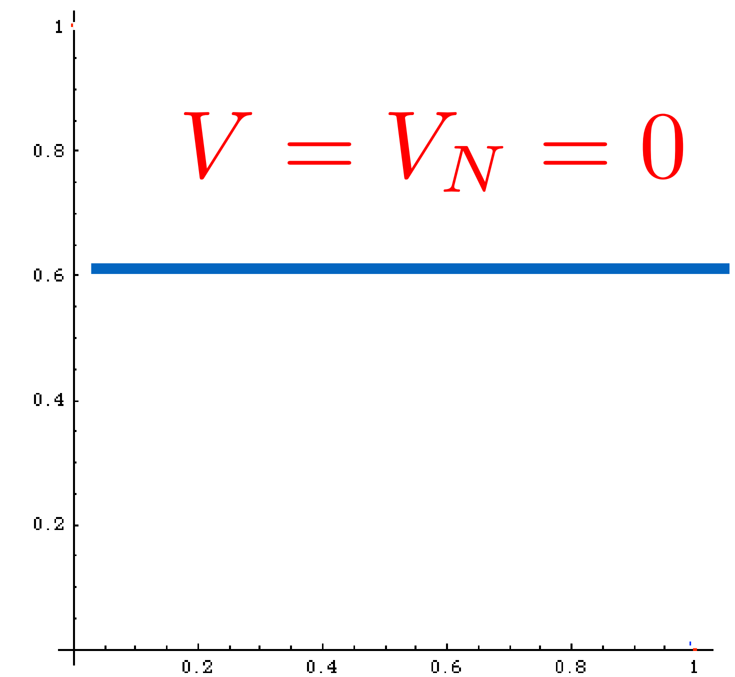

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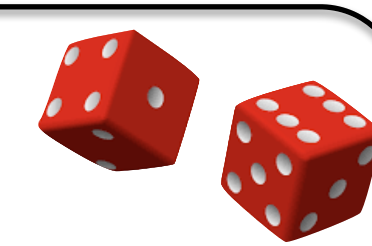

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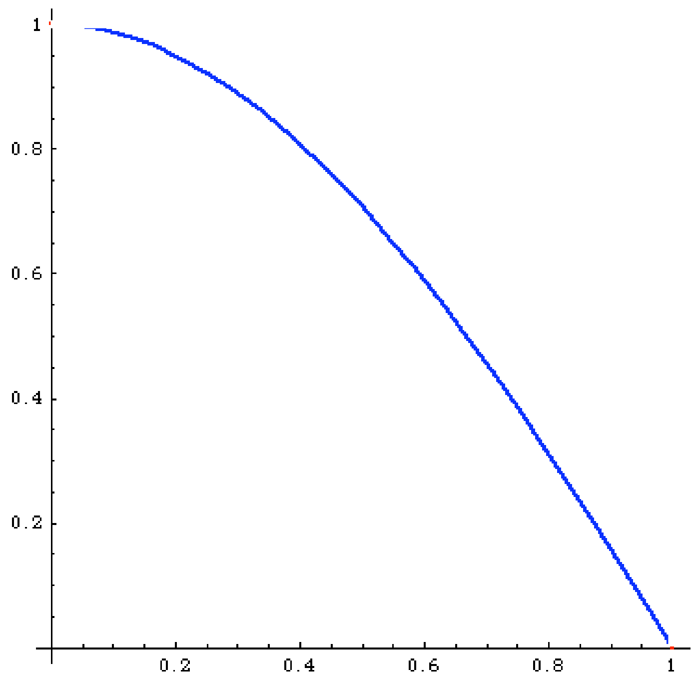
$$I = I_N \pm \sqrt{V_N/N}$$

Can be minimized!

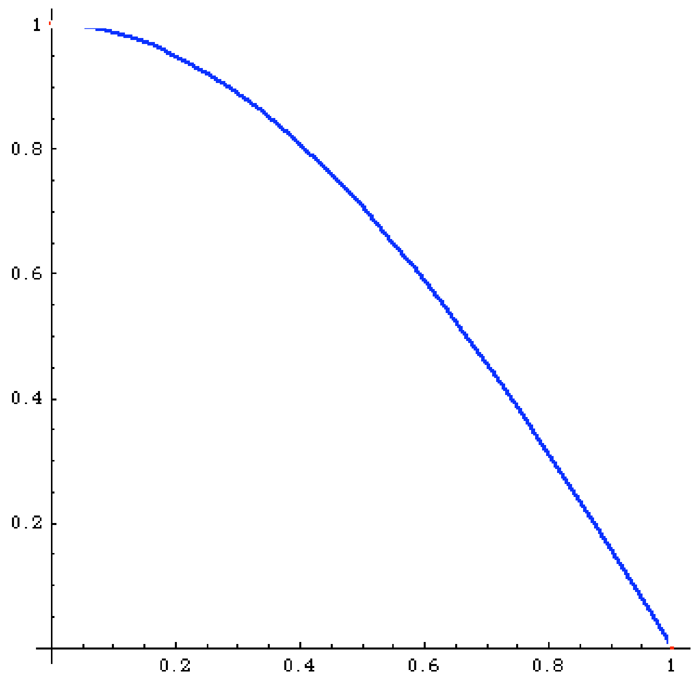
Importance Sampling



$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$I_N = 0.637 \pm 0.307/\sqrt{N}$$

Importance Sampling



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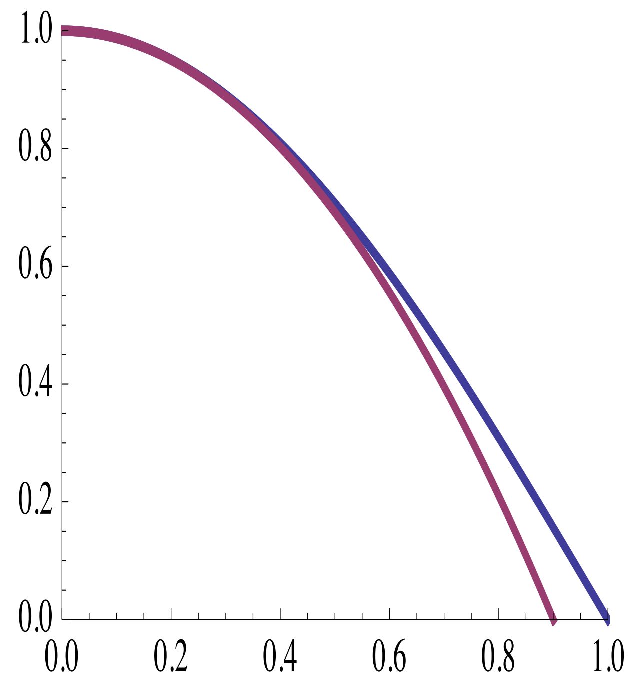
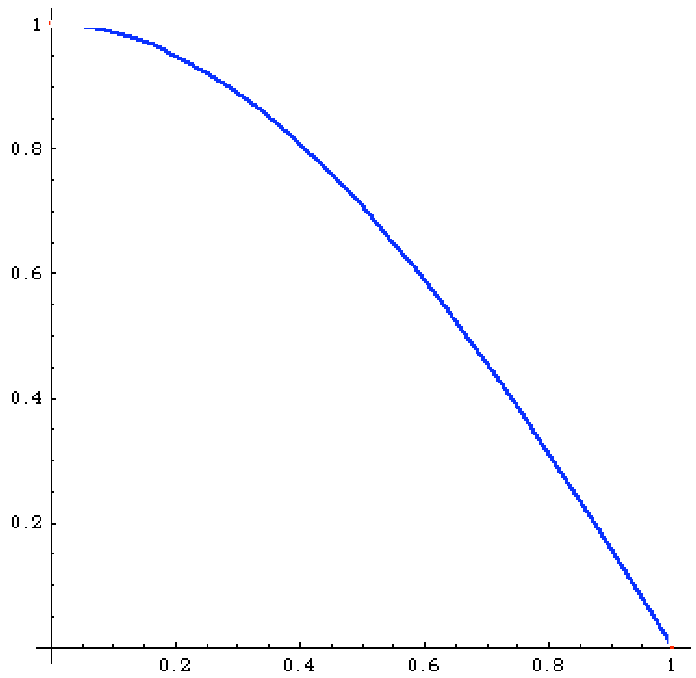


$$I = \int_0^1 dx (1 - cx^2) \frac{\cos\left(\frac{\pi}{2} x\right)}{(1 - cx^2)}$$

Importance Sampling



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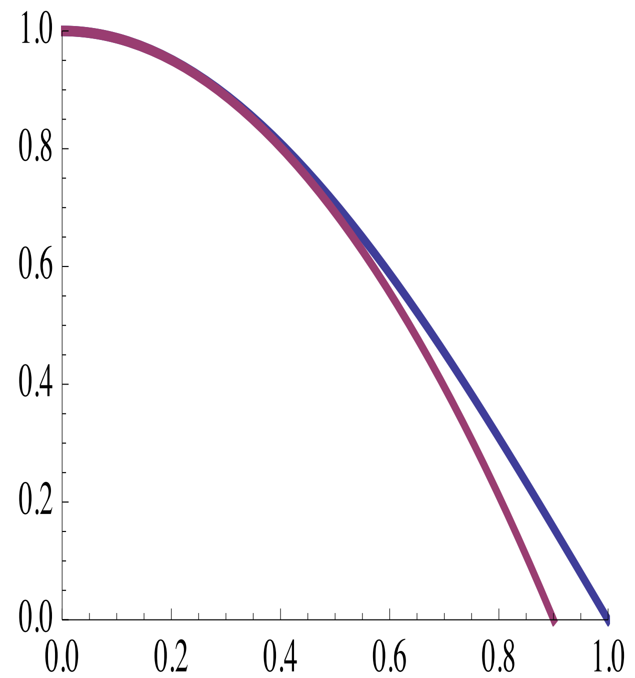

$$I = \int_0^1 dx (1 - cx^2) \frac{\cos\left(\frac{\pi}{2} x\right)}{(1 - cx^2)} = \int_{\xi_1}^{\xi_2} d\xi \frac{\cos \frac{\pi}{2} x[\xi]}{1 - x[\xi]^2 c}$$

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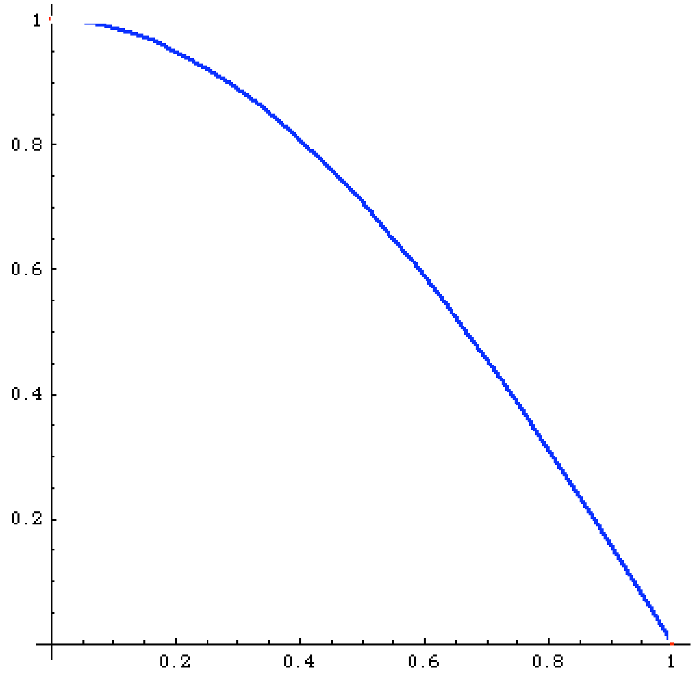

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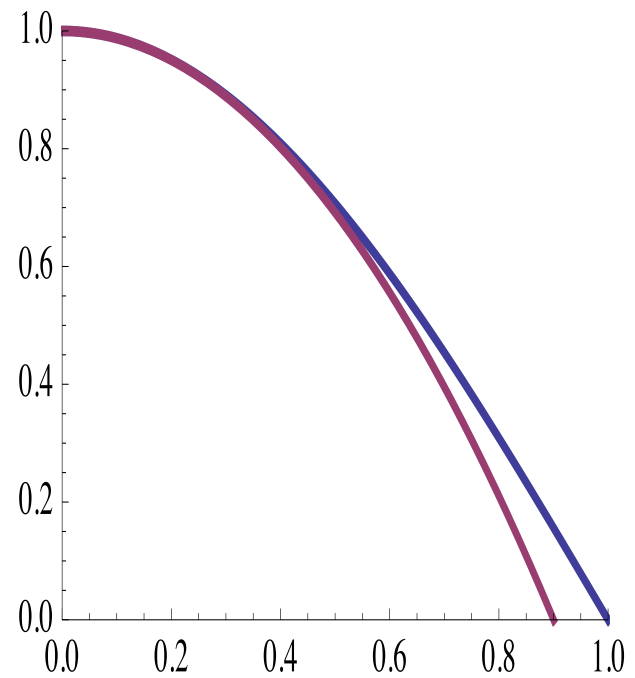

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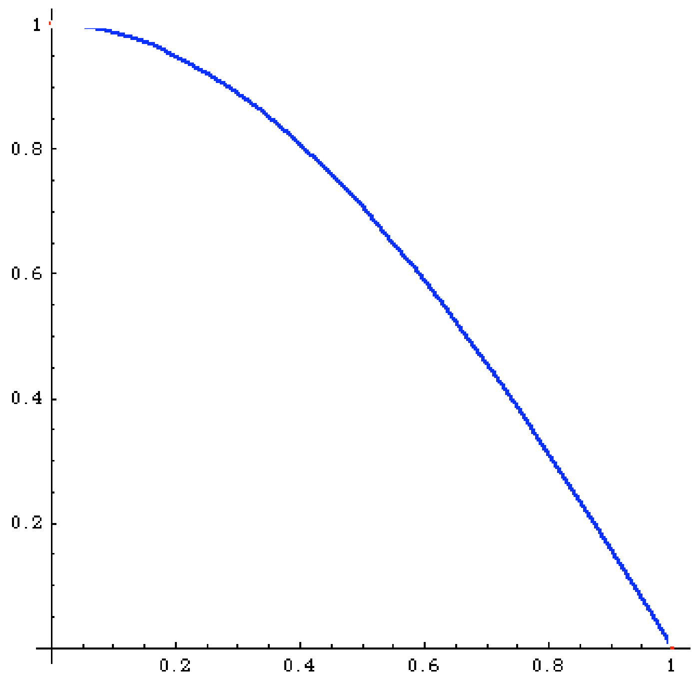
$\Rightarrow \simeq 1$

Importance Sampling



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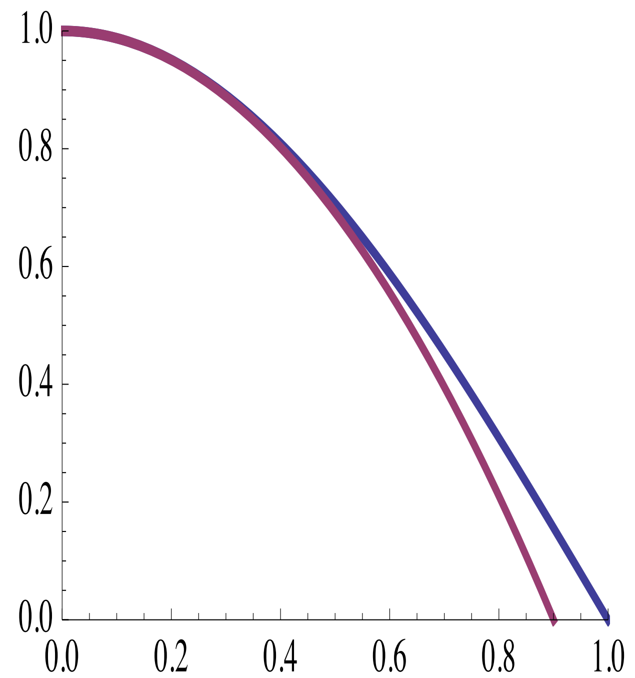

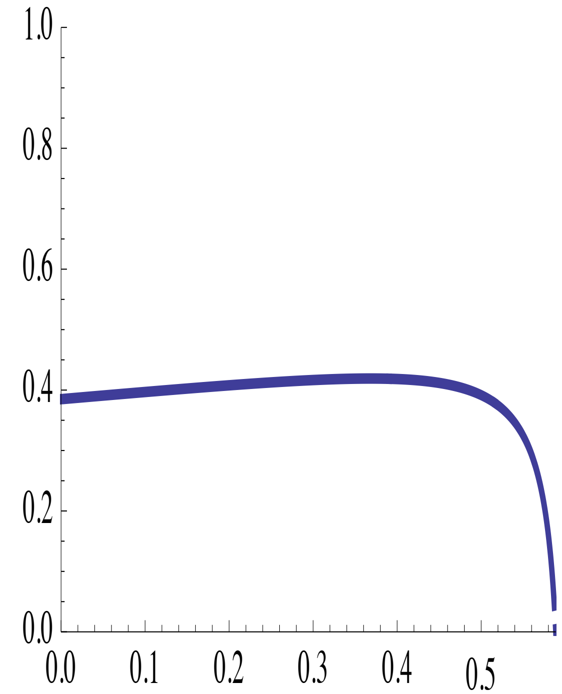

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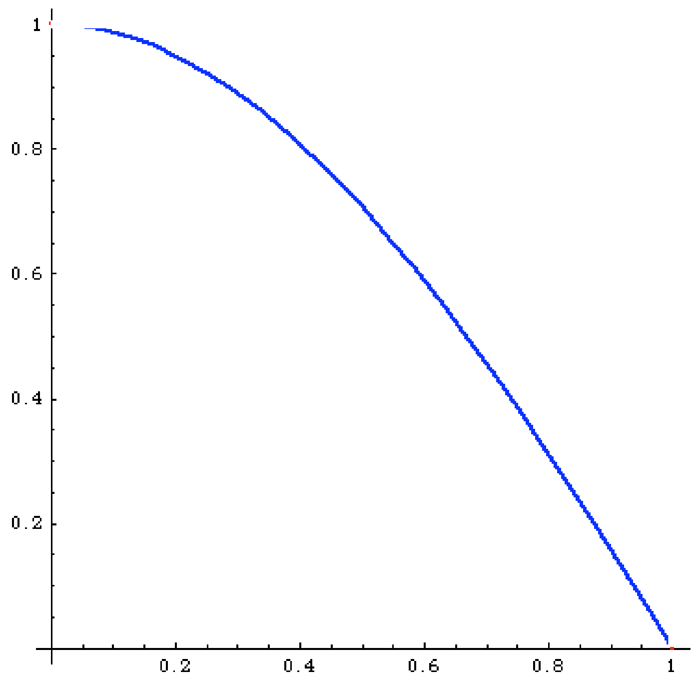

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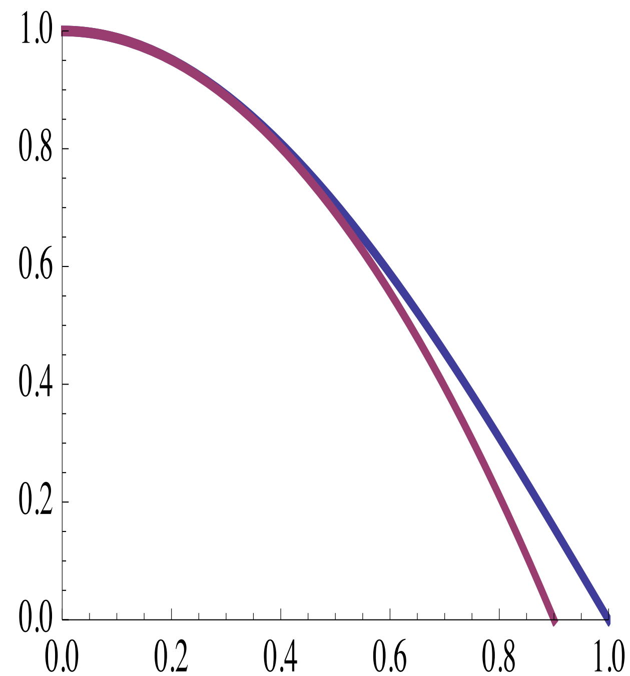

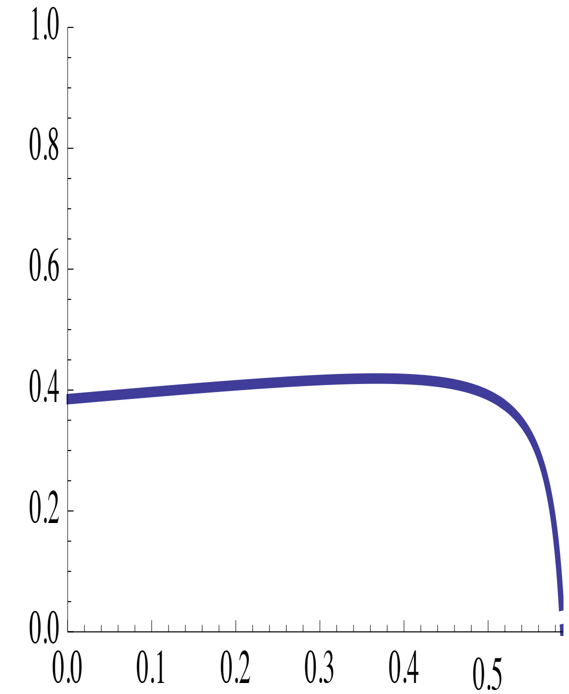

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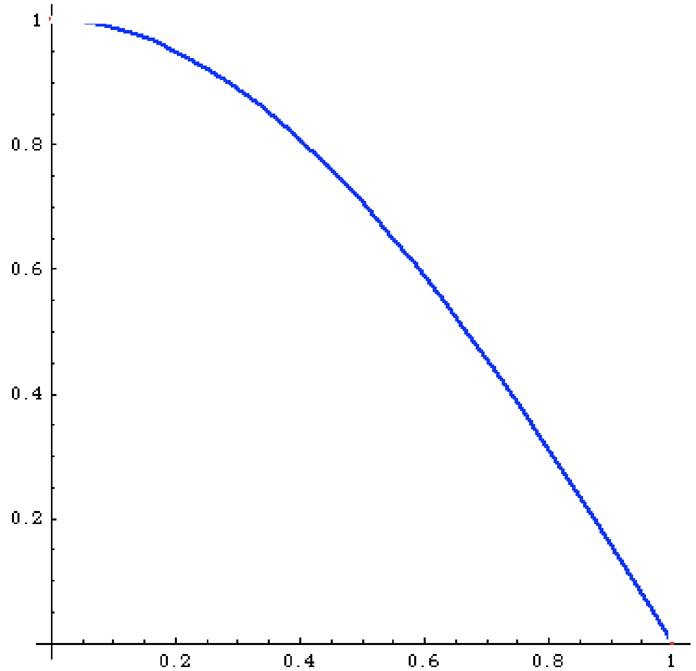
Importance Sampling



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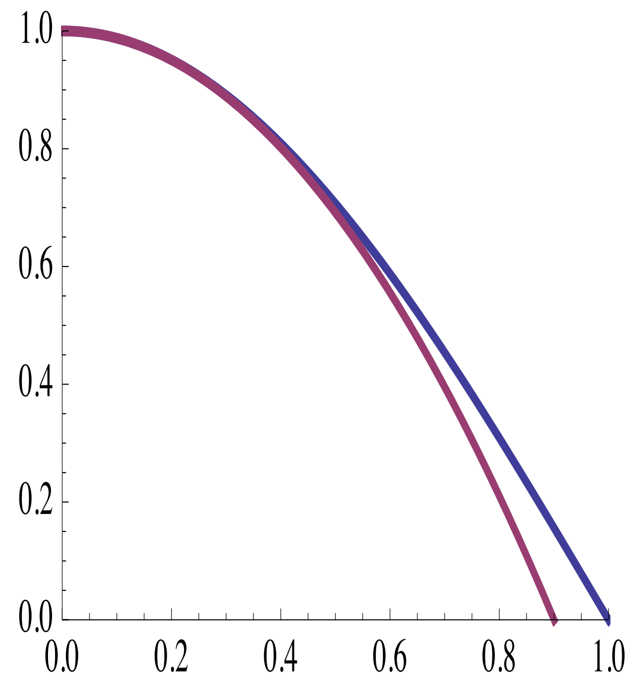
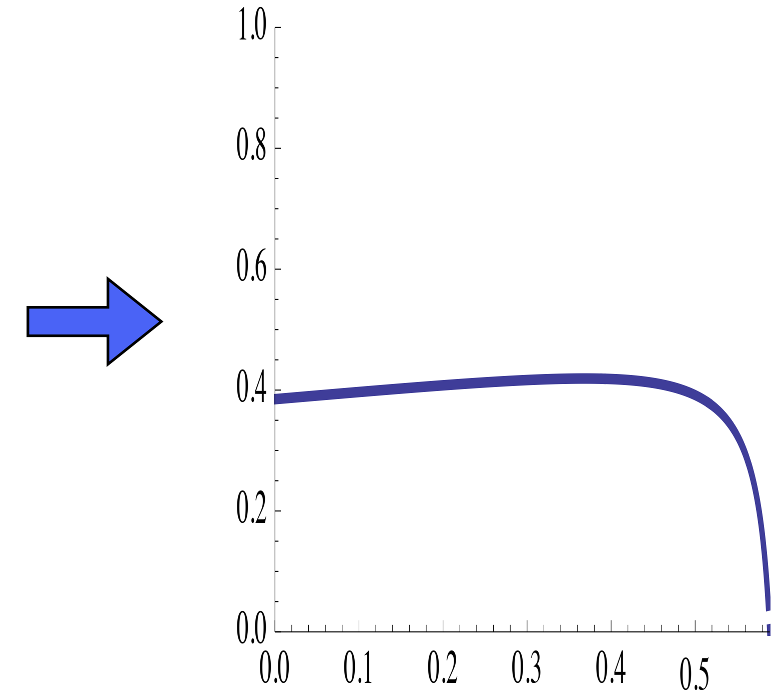




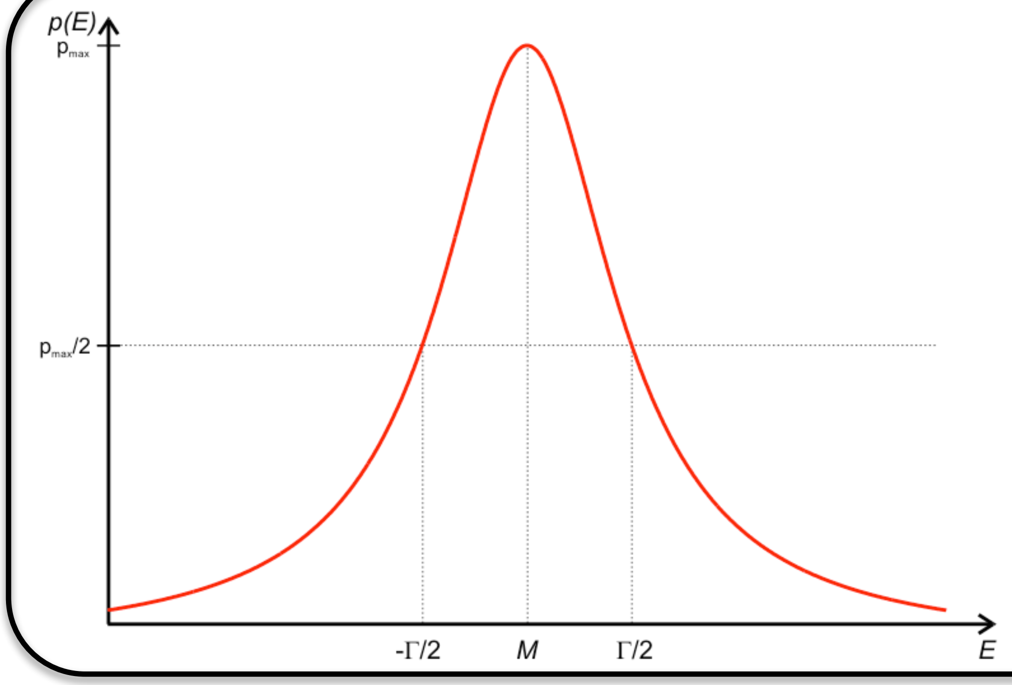
$$I = \int_0^1 dx(1-cx^2)\frac{\cos\left(\frac{\pi}{2}x\right)}{(1-cx^2)} = \int_{\xi_1}^{\xi_2} d\xi \frac{\cos\frac{\pi}{2}x[\xi]}{1-x[\xi]^2 c}$$

$\simeq 1$

$$I_N = 0.637 \pm 0.031/\sqrt{N}$$

The Phase-Space parametrization is important to have an efficient computation!

Importance Sampling



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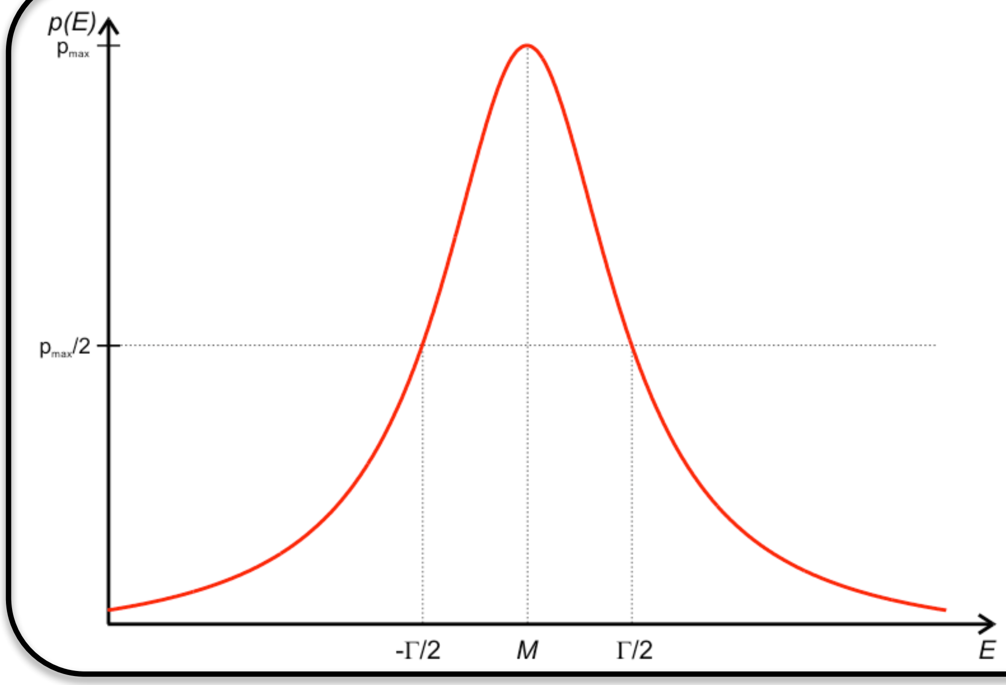
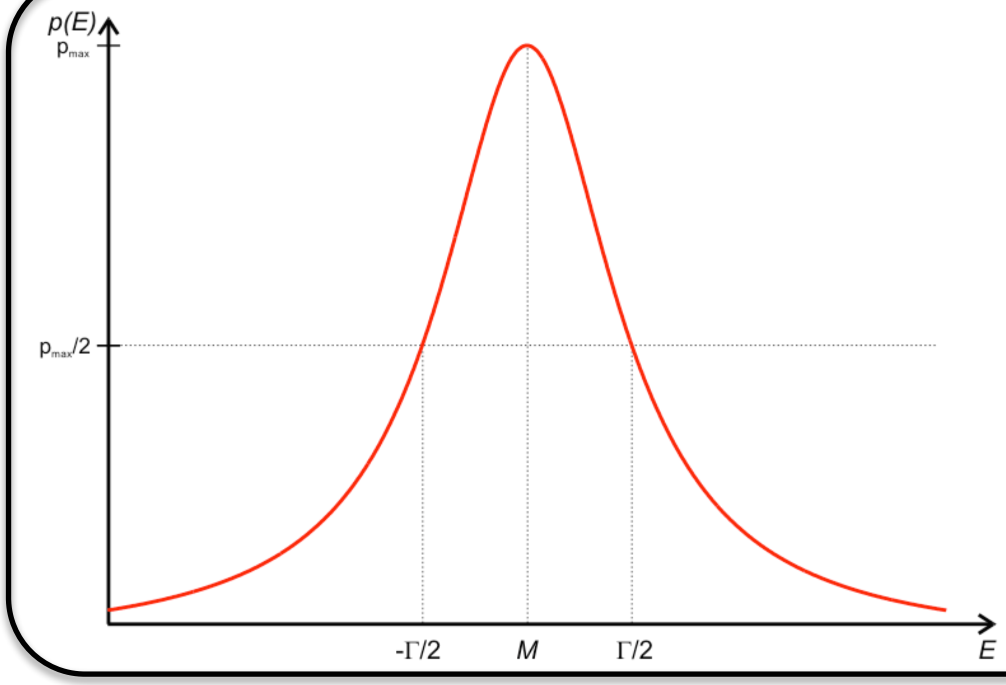


$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\xi = arctan\left(\frac{q^2 - M^2}{\Gamma M}\right)$$

Importance Sampling



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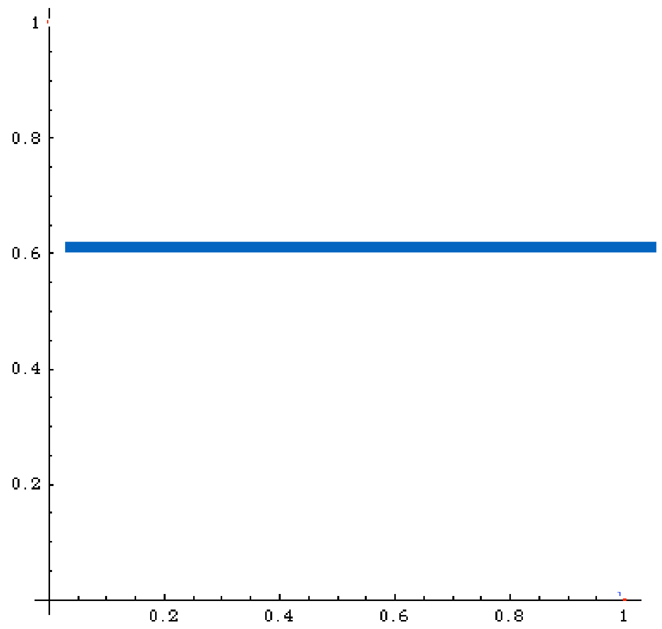
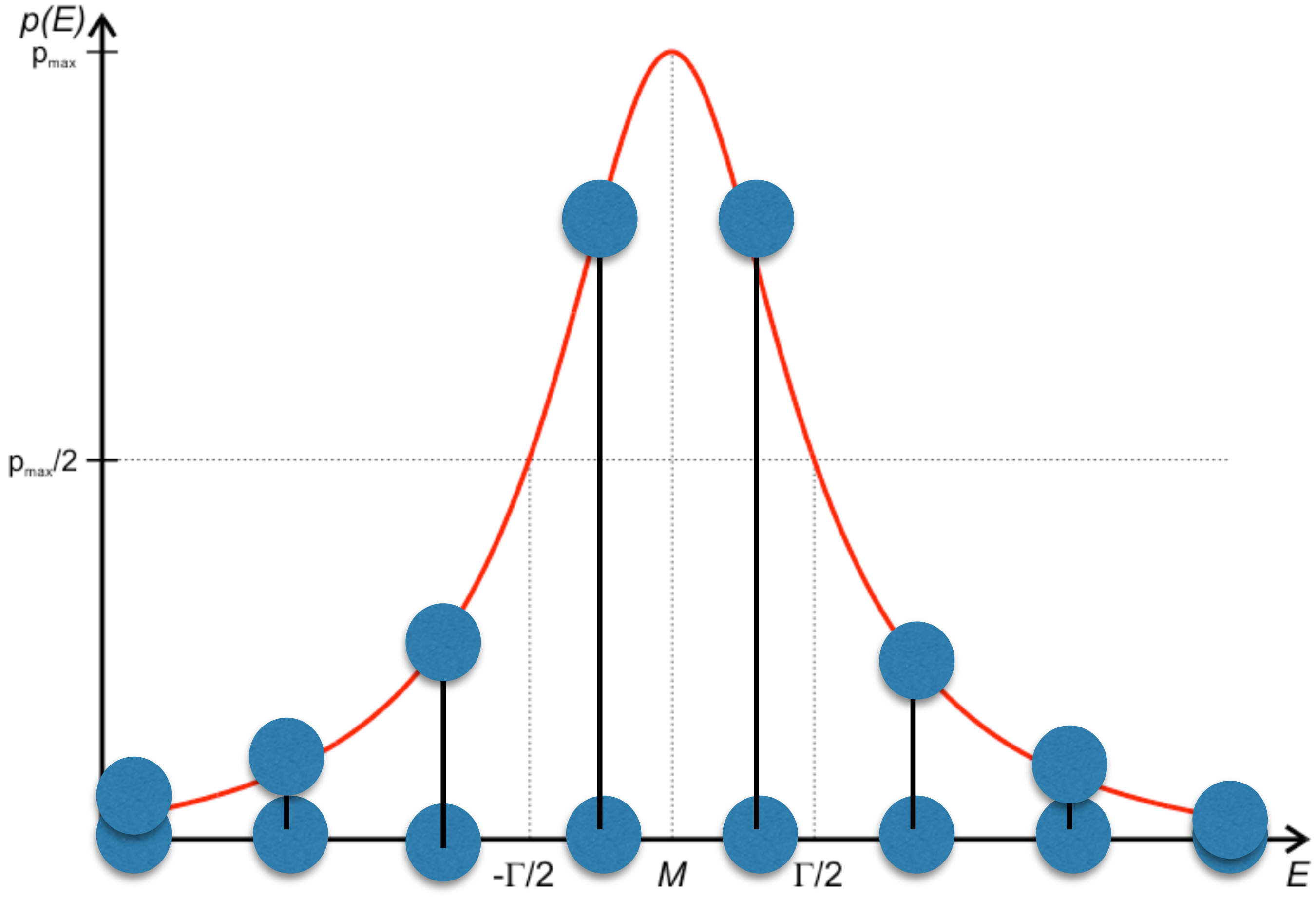


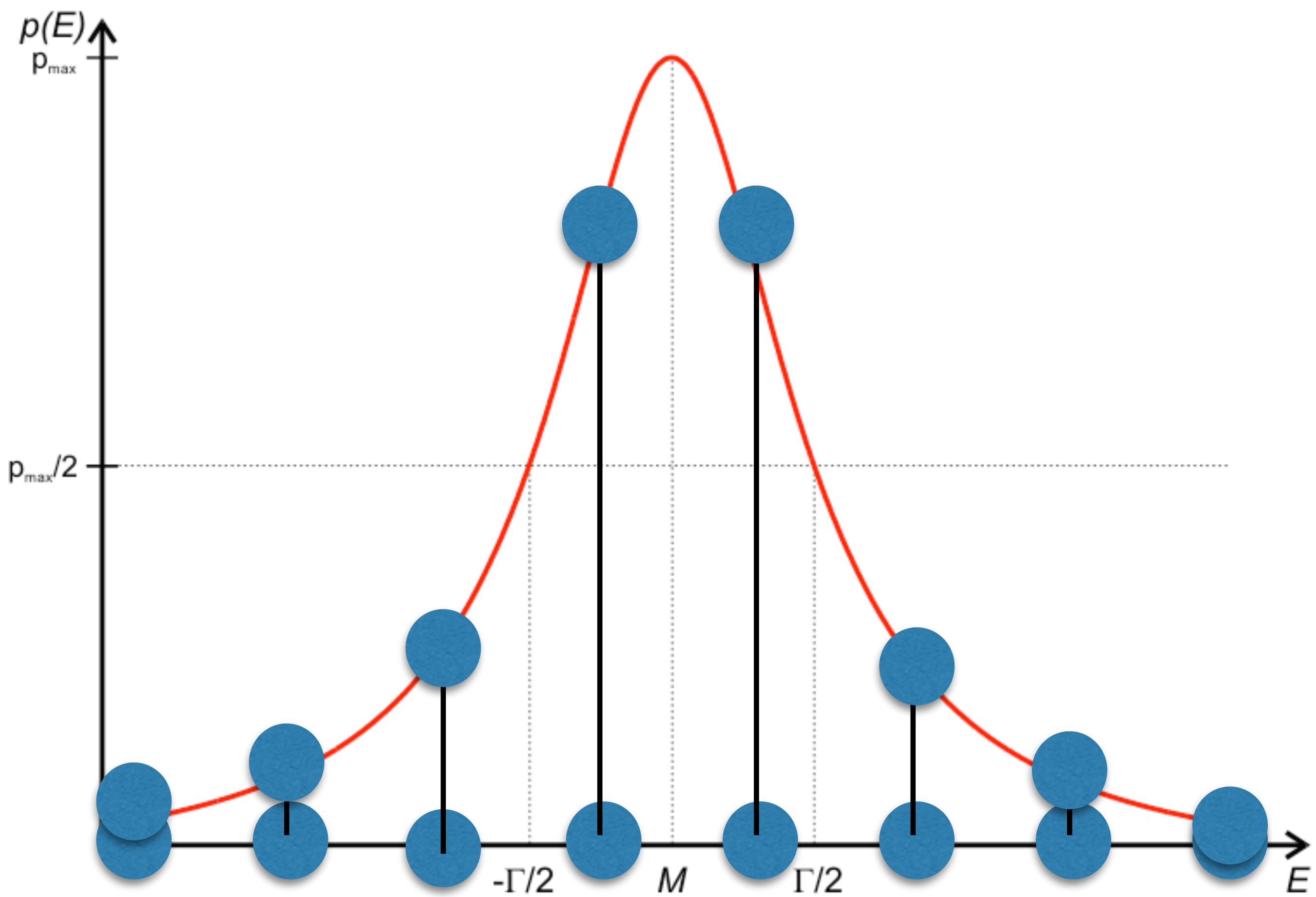
Why importance?

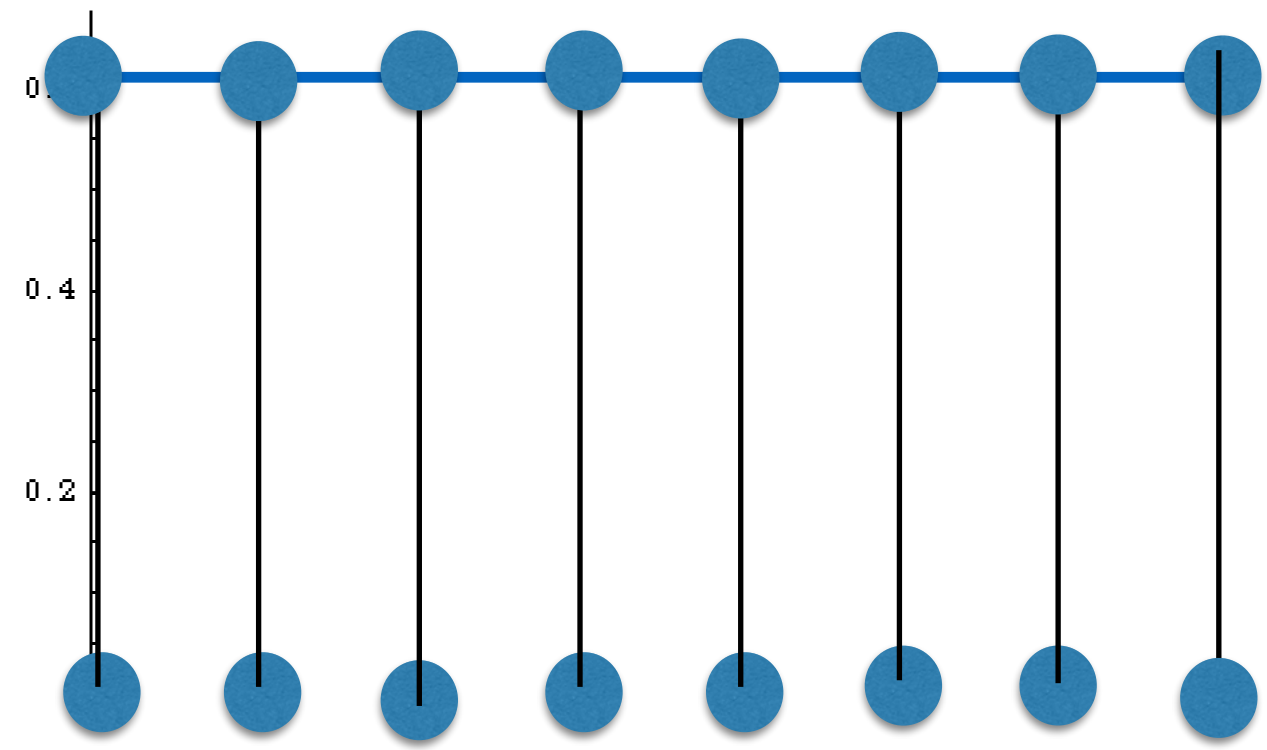


Why Importance Sampling?

We probe more often the region where the function is high!

Why importance?





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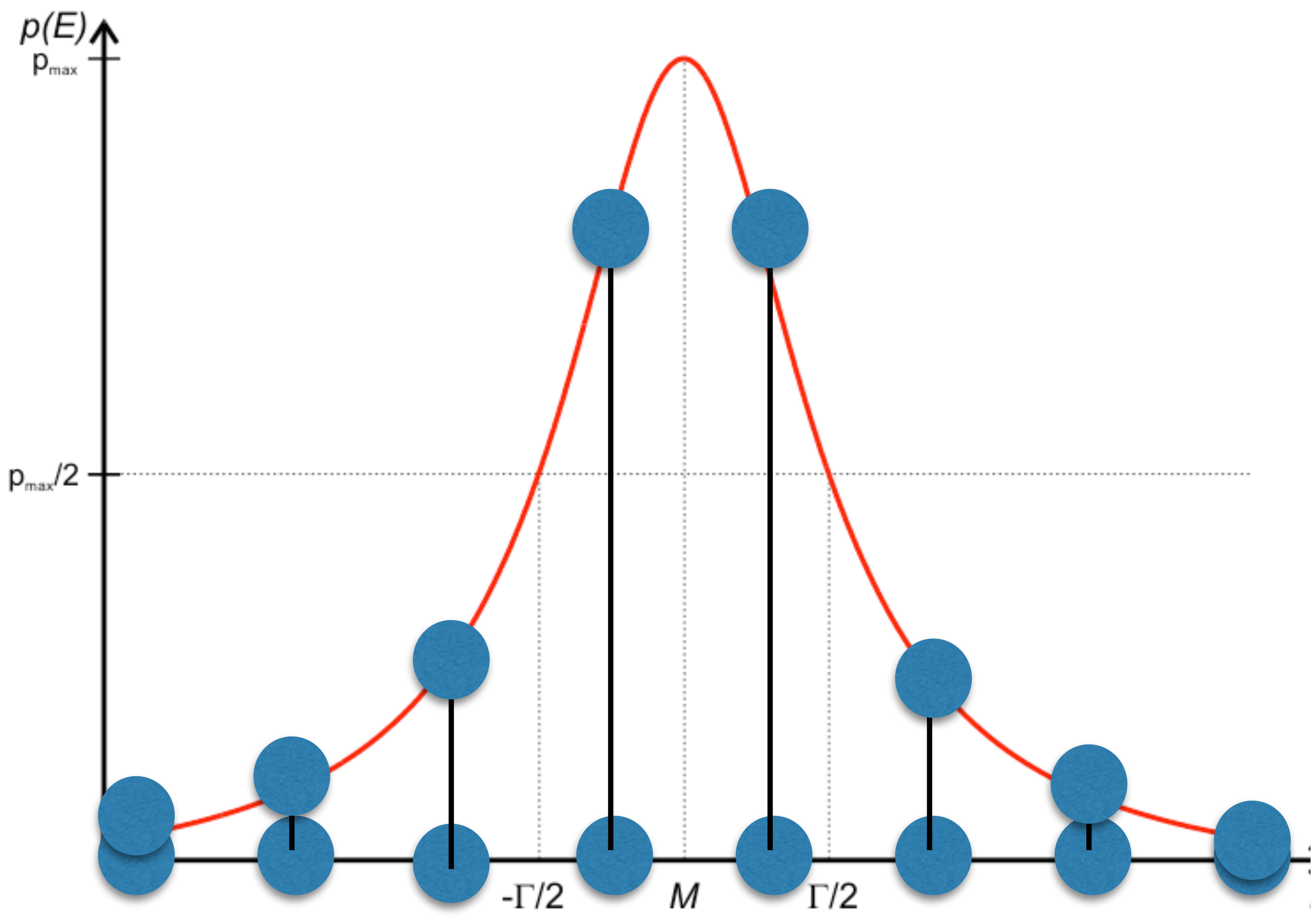

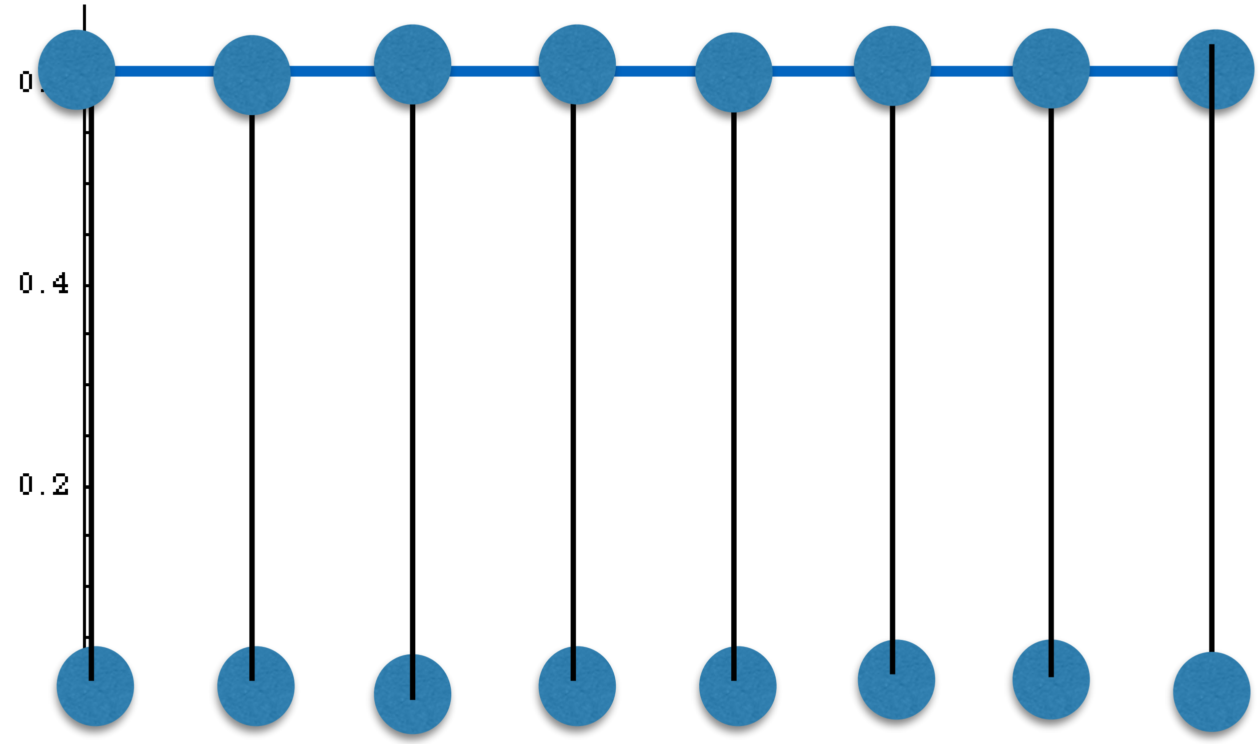

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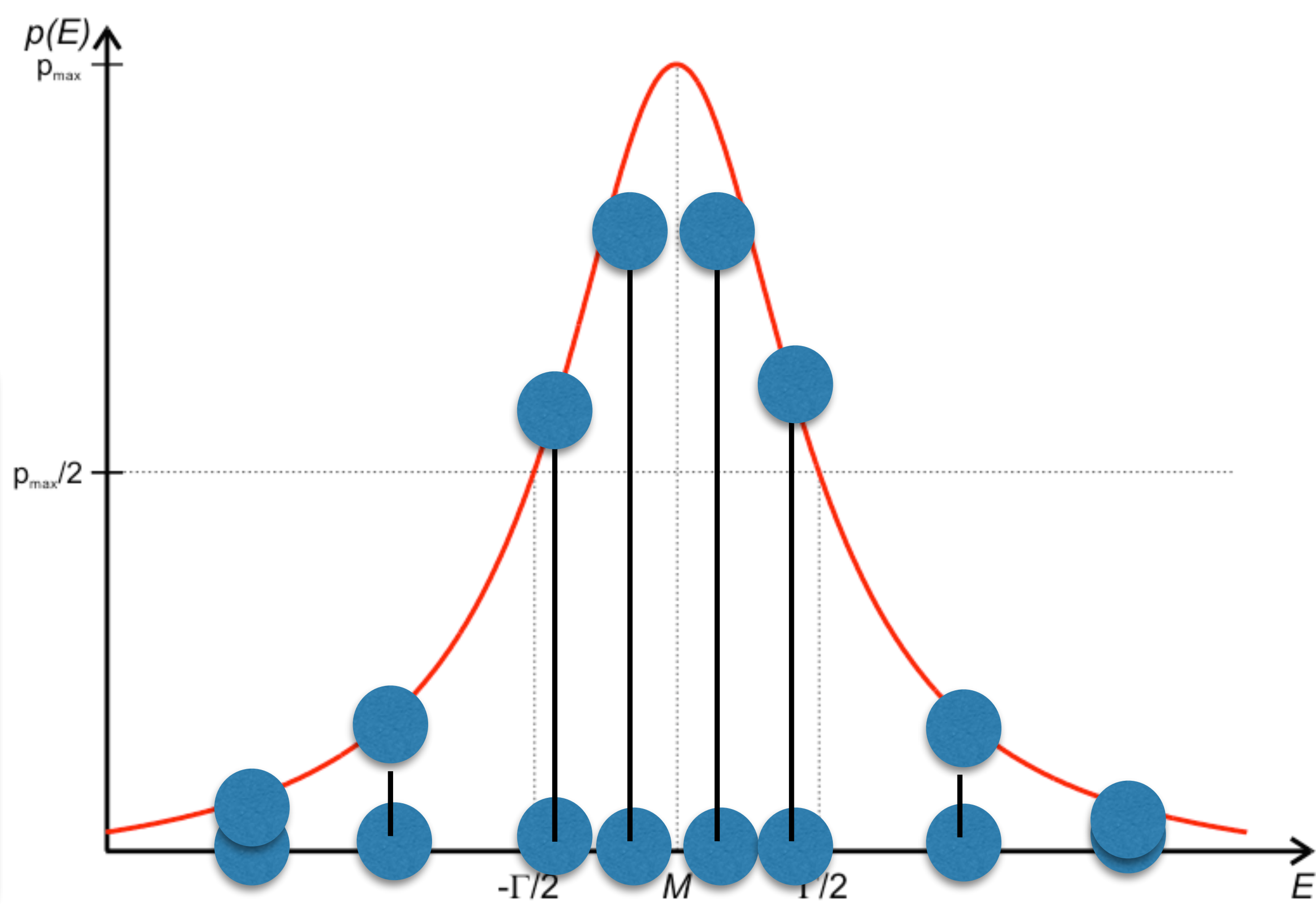


Question time

1 Go to **wooclap.com**

2 Enter the event code in the top banner

Event code

MADGRAPH

Importance Sampling

Key Point

- Generate the random point in a distribution which is close to the function to integrate.
- This is a change of variable, such that the function is flatter in this new variable.
- Needs to know an approximate function.

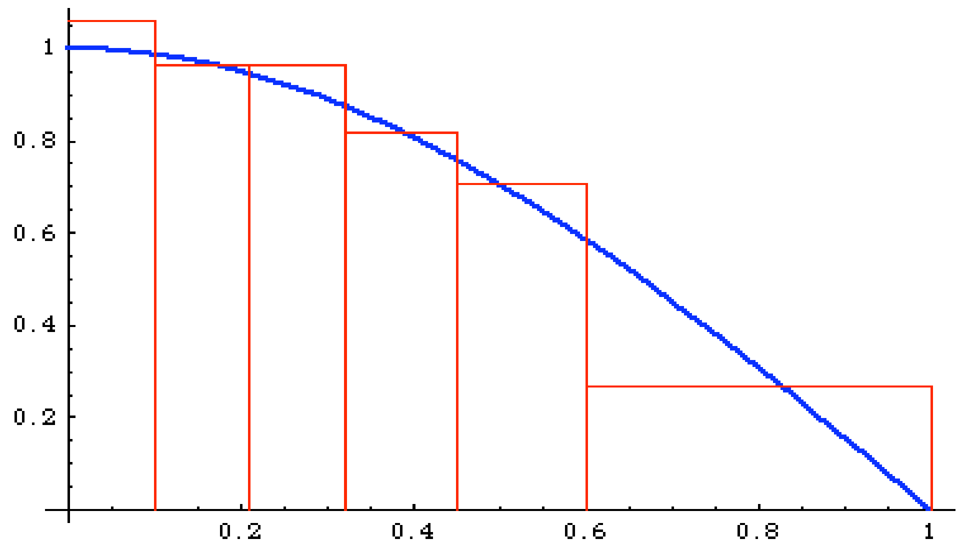
Adaptative Monte-Carlo

- Create an approximation of the function on the flight!

VEGAS

Adaptative Monte-Carlo

- Create an approximation of the function on the flight!



Algorithm

1. Creates bin such that each of them have the same contribution.

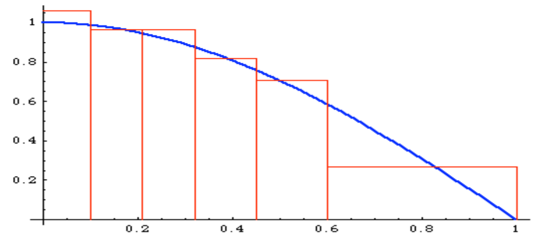
   ➡Many bins where the function is large

2. Use the approximate for the importance sampling method.

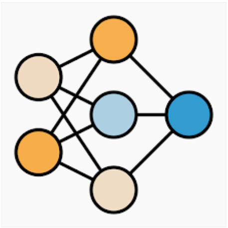
Can we do Better?

- Importance sampling/VEGAS is learning a function
  - ➡ HOT TOPIC: Machine Learning
  - ➡ Lot of work in progress

VEGAS

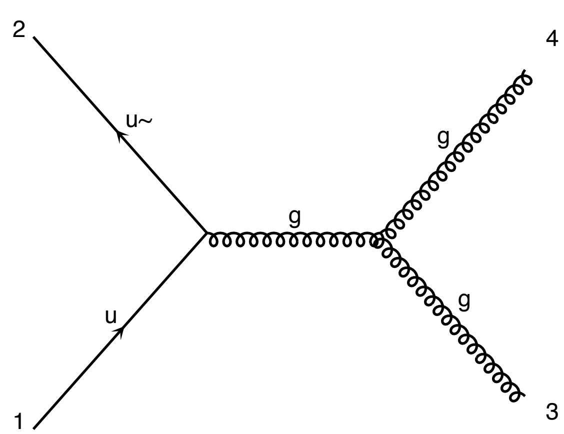


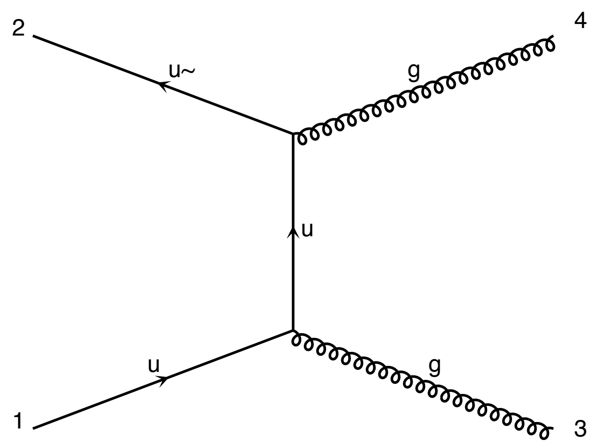
MadNIS

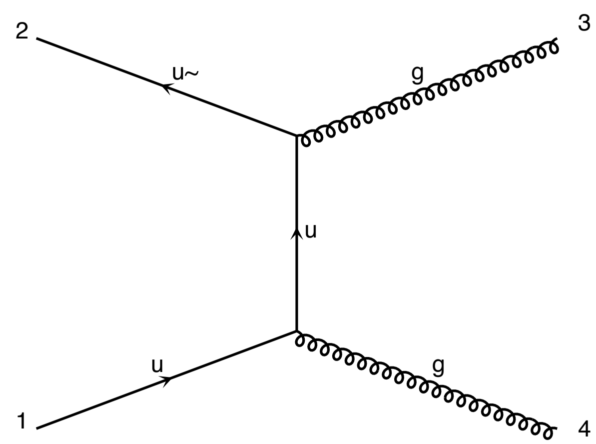


~20 times more efficient

Example: QCD 2 → 2







$$\propto \frac{1}{\hat{s}} = \frac{1}{(p_1 + p_2)^2} \qquad \propto \frac{1}{\hat{t}} = \frac{1}{(p_1 - p_3)^2} \qquad \propto \frac{1}{\hat{u}} = \frac{1}{(p_1 - p_4)^2}$$

Three very different pole structures contributing to the same matrix element.

*Method used in MadGraph

Trick in MadEvent: Split the complexity

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

*Method used in MadGraph

Trick in MadEvent: Split the complexity

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$$\frac{|M_{tot}|^2}{\sum_j |M_j|^2} \approx 1$$

- Any single diagram is "easy" to integrate (pole structures/suitable integration variables known from the propagators)
- Divide integration into pieces, based on diagrams
- All other peaks taken care of by denominator sum

*Method used in MadGraph

Trick in MadEvent: Split the complexity

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≈ 1

- Any single diagram is “easy” to integrate (pole structures/suitable integration variables known from the propagators)
- Divide integration into pieces, based on diagrams
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N Integral

- Errors add in quadrature so no extra cost
- “Weight” functions already calculated during $|M|^2$ calculation
- Parallel in nature

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

P1_qq_wpwm

s= 725.73 ± 2.07 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G2.2	377.6	1.67	142.285	7941.0	21
G3	239	1.16	220.04	10856.0	45.5
G1	109.1	0.378	70.88	3793.0	34.8

P1_gg_wpwm

s= 20.714 ± 0.332 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G1.2	20.71	0.332	7.01	373.0	18

term of the above sum.

each term might not be gauge invariant

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Event code

MADGRAPH

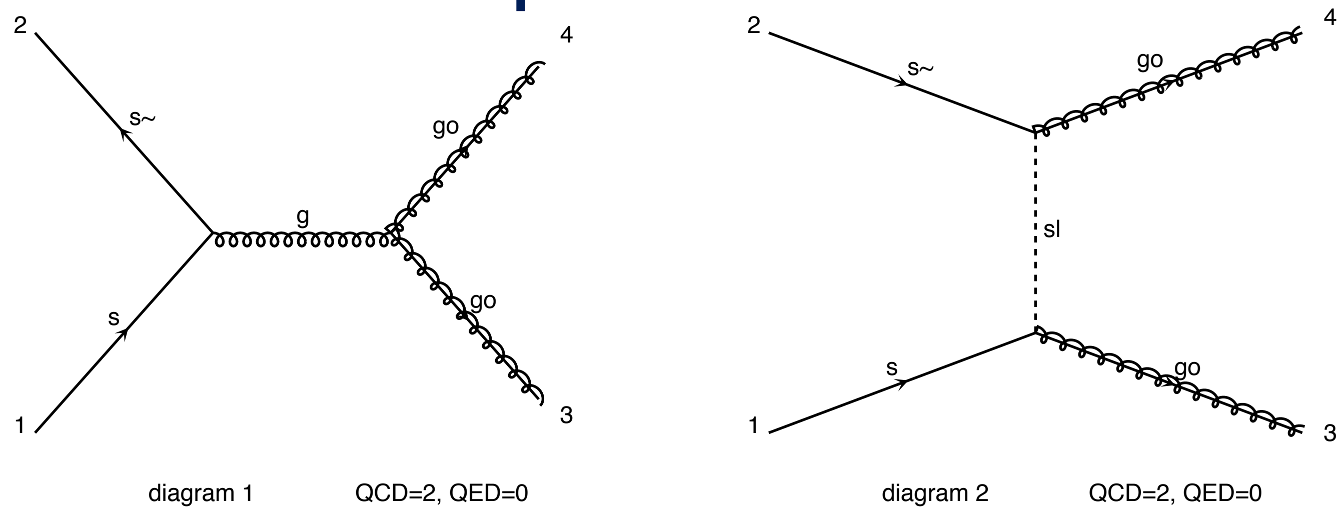
To Remember

- Phase-Space integration is difficult
- We need to know the function
  - ➡ Be careful with cuts
- MadGraph split the integral in different contribution linked to the Feynman Diagram
  - ➡Those are not the contribution of a given diagram

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



- Evaluate the matrix-element

$$|\mathcal{M}|^2$$ ➡Need Feynman Rules!

- Phase-Space Integration

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

Matrix-Element

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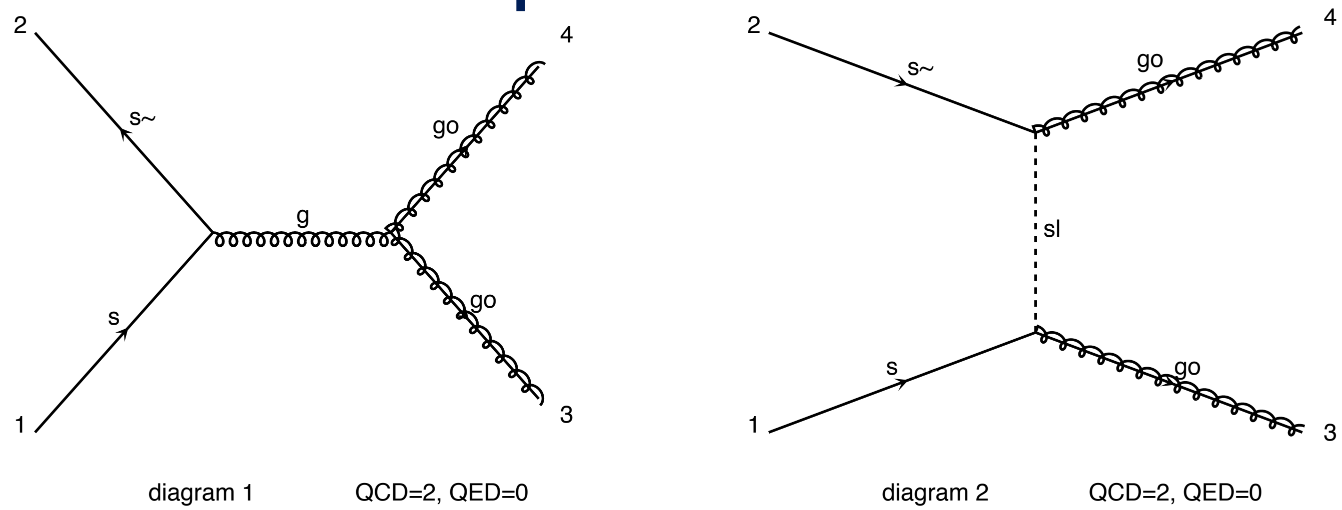

Easy enough

- Evaluate the matrix-element

$|\mathcal{M}|^2$ ➡Need Feynman Rules!

Hard

- Phase-Space Integration

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

Very Hard (in general)

Matrix-Element

Calculate a given process (e.g. gluino pair)

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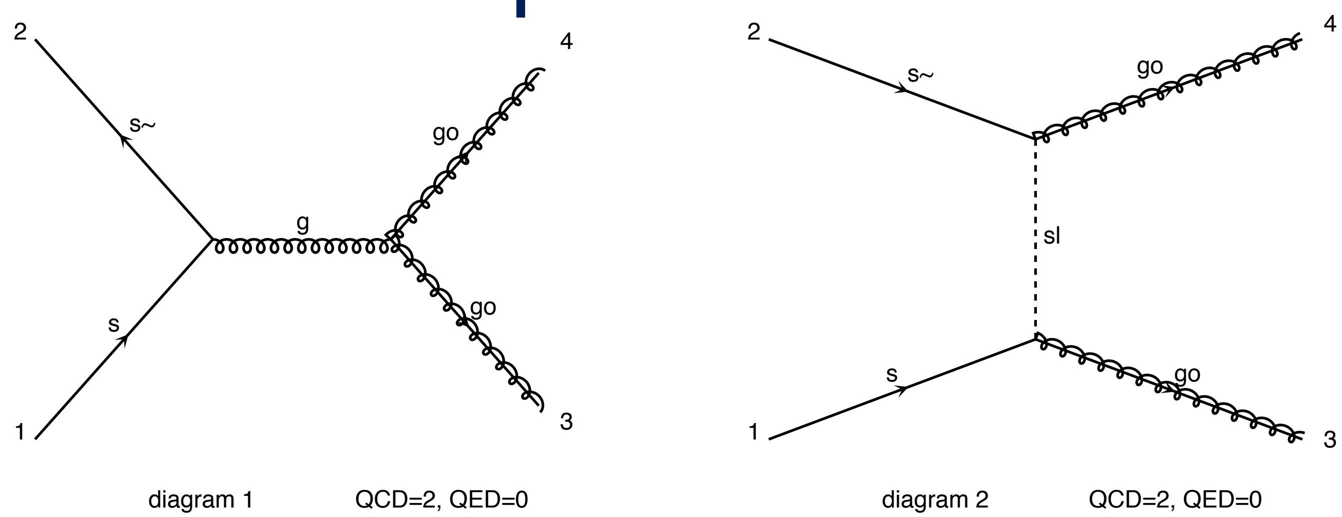

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$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

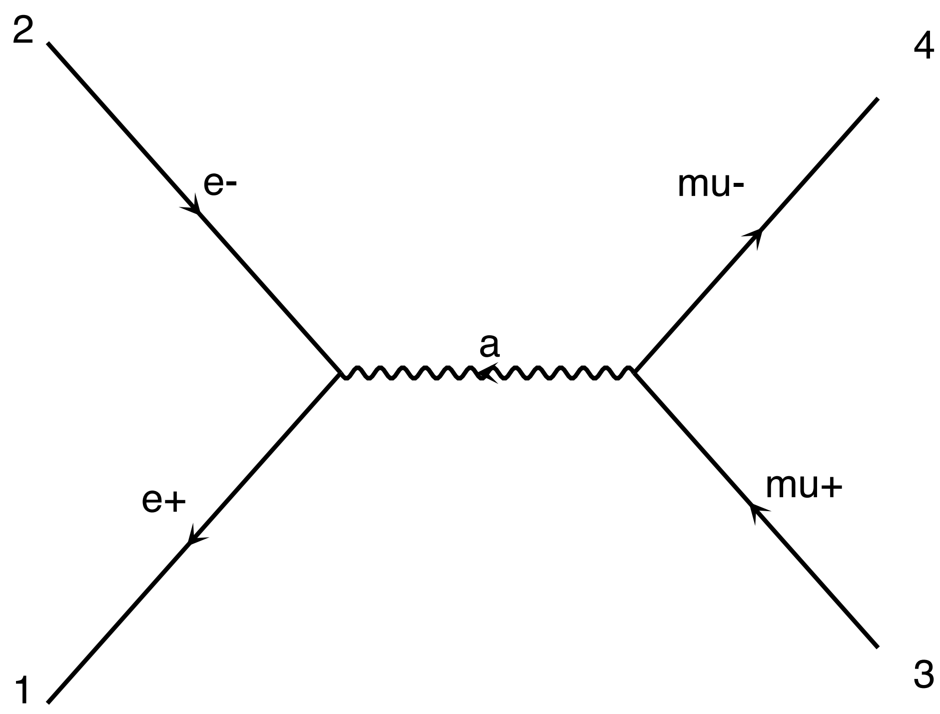
Easy enough

Hard

Now

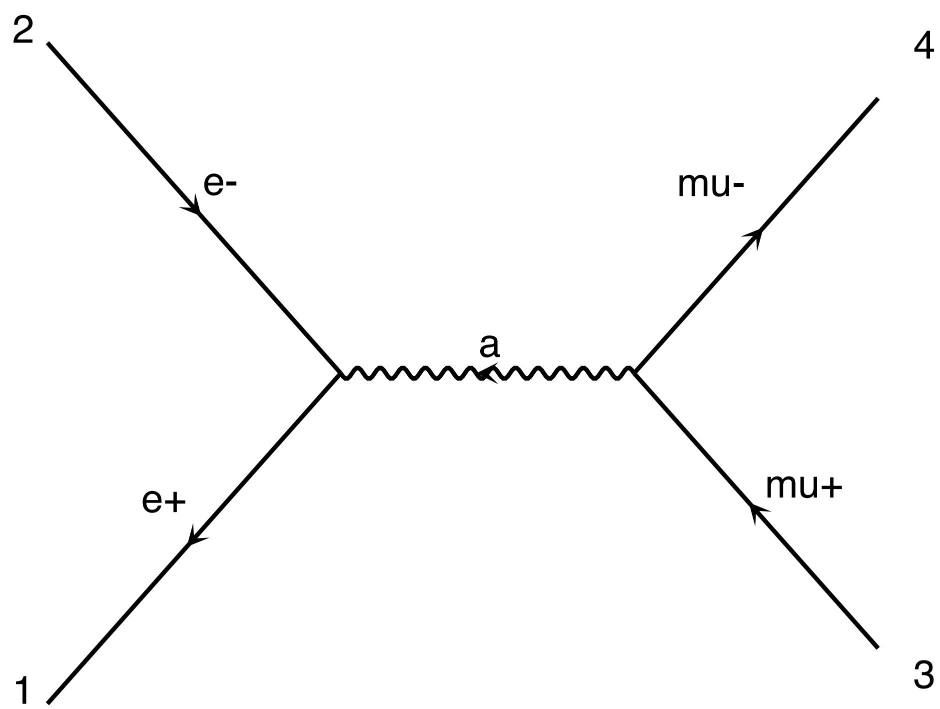
Very Hard
(in general)

Matrix Element



$$\mathcal{M} = e^2 (\bar{u}\gamma^\mu v)\frac{g_{\mu\nu}}{q^2}(\bar{v}\gamma^\nu u)$$

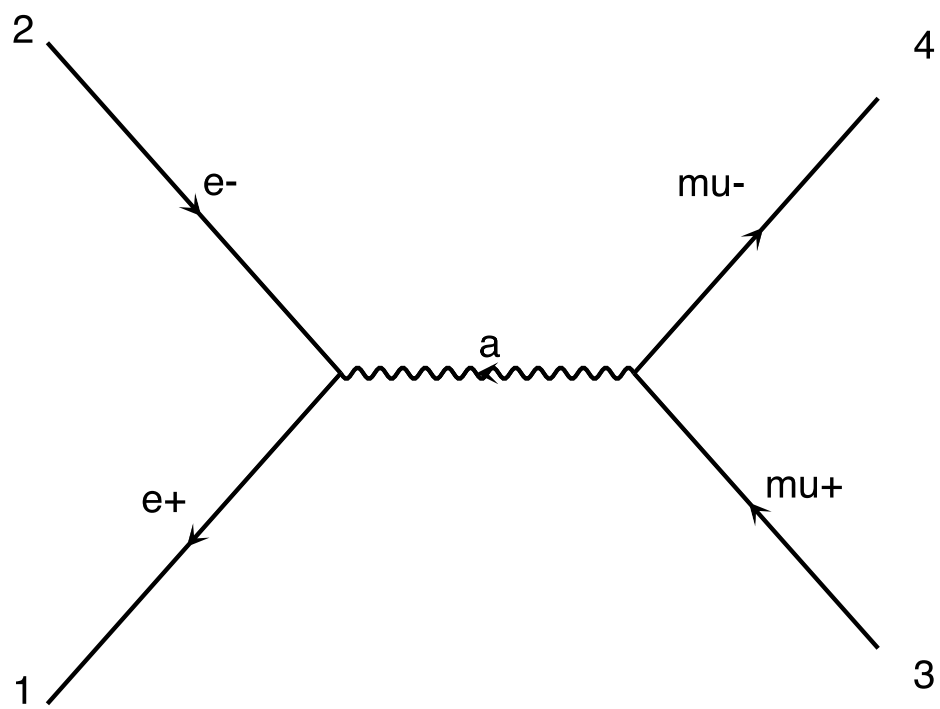
Matrix Element



$$\mathcal{M} = e^2 (\bar{u}\gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\gamma^\nu u)$$

$$\frac{1}{4} \sum_{pol} |\mathcal{M}|^2 = \frac{1}{4} \sum_{pol} \mathcal{M}^* \mathcal{M}$$

Matrix Element



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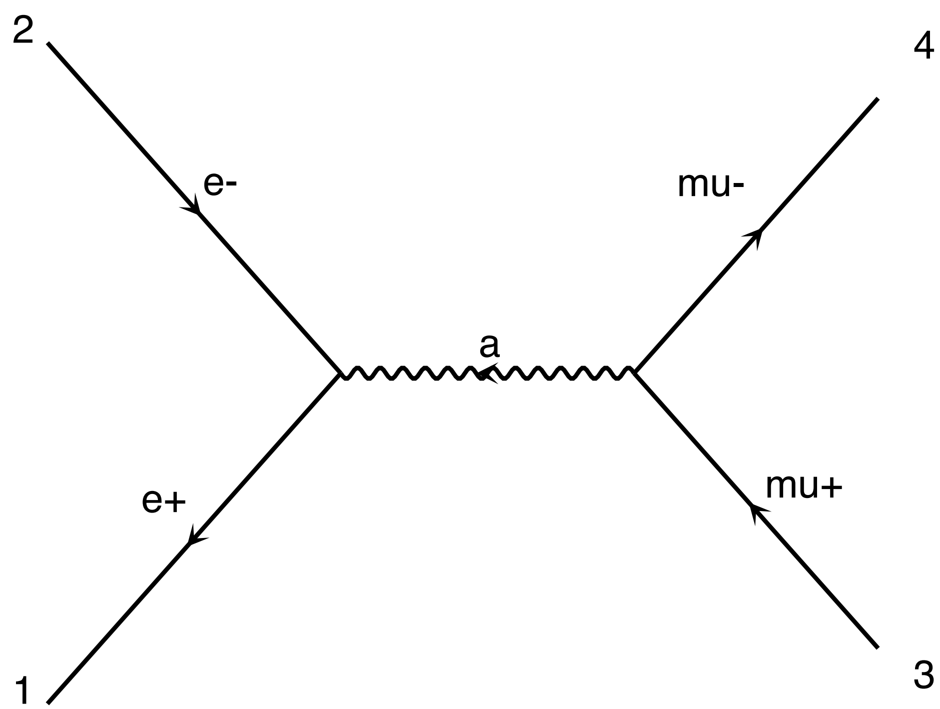
$$\sum_{pol} u\bar{u} = \not{p} + m$$

$$\rightarrow \quad \frac{e^4}{4q^4} Tr[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu] Tr[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu]$$

Matrix Element



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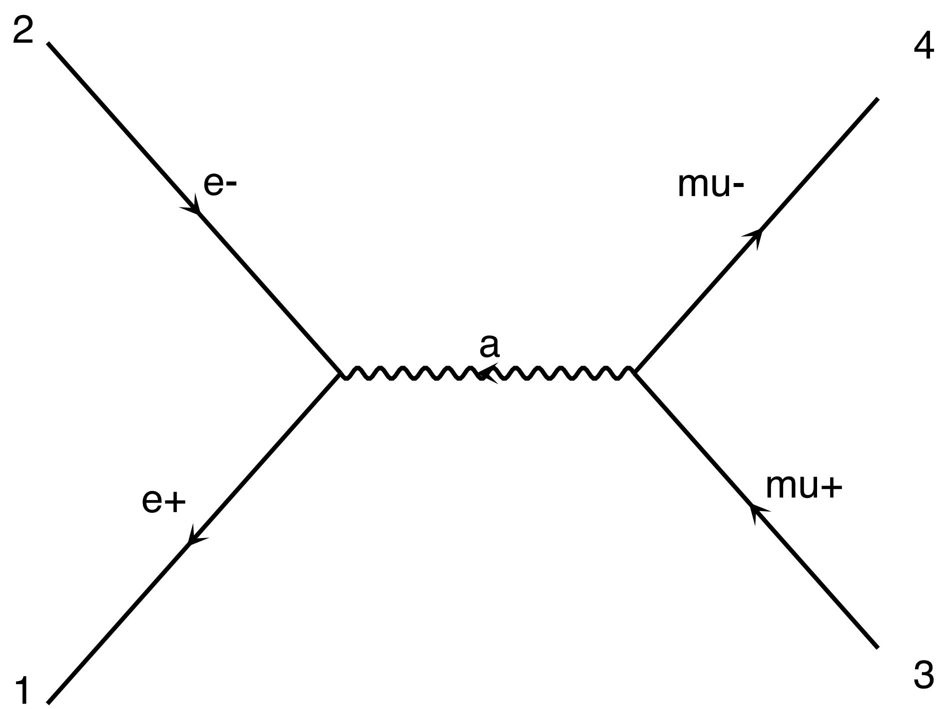
$$\rightarrow \frac{8e^4}{q^4} [(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3)]$$

Matrix Element



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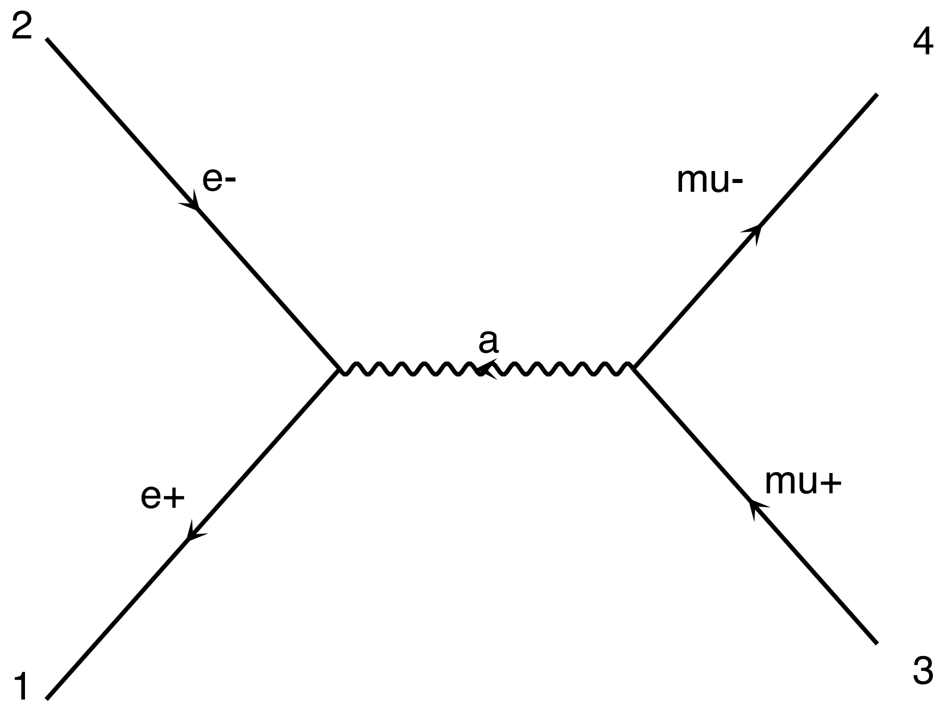
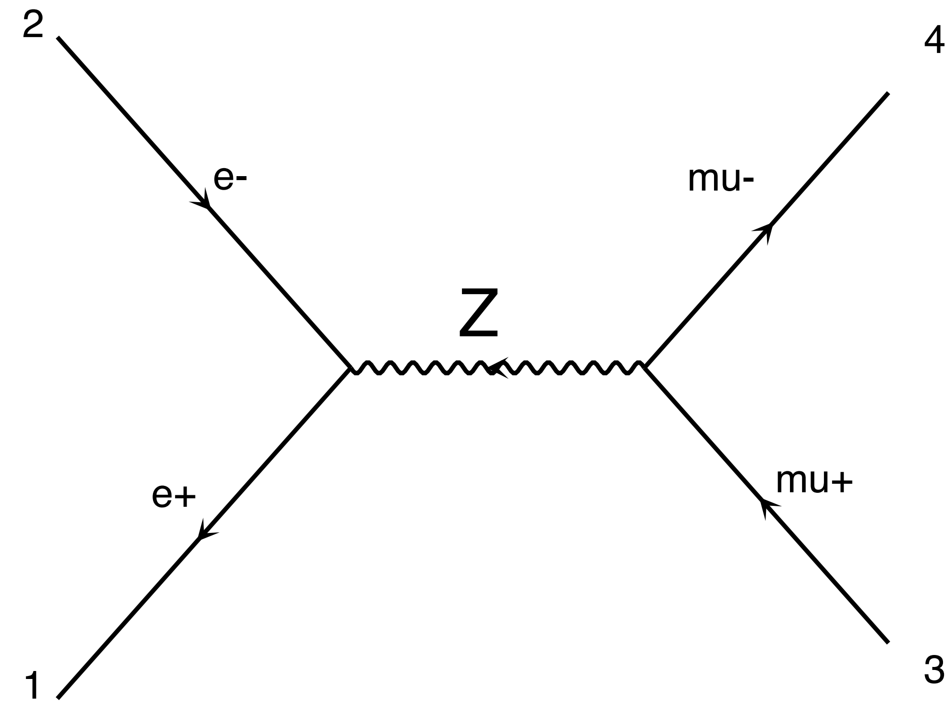
$$\rightarrow \frac{8e^4}{q^4}[(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3)]$$

Very Efficient !!!



2
e-
e+
1
a
mu-
4
mu+
3



2
e-
e+
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Z
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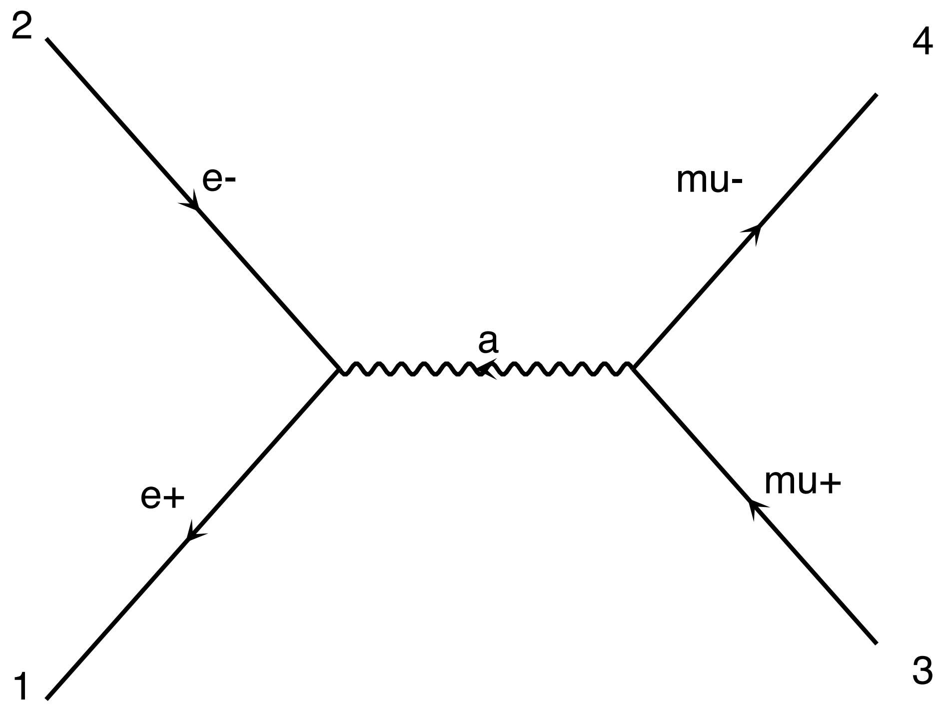
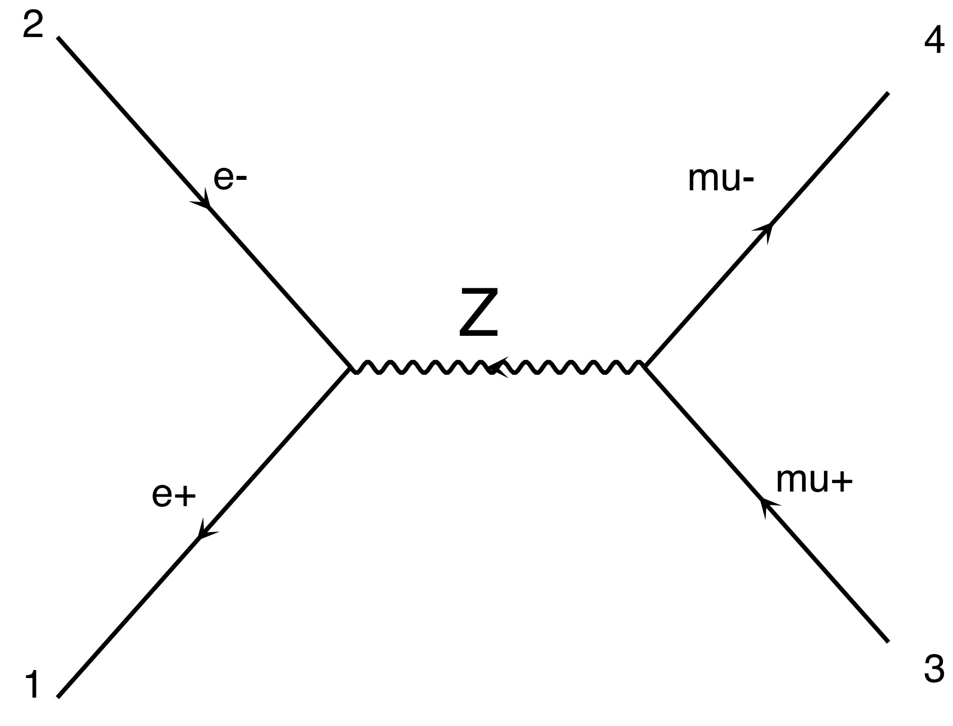
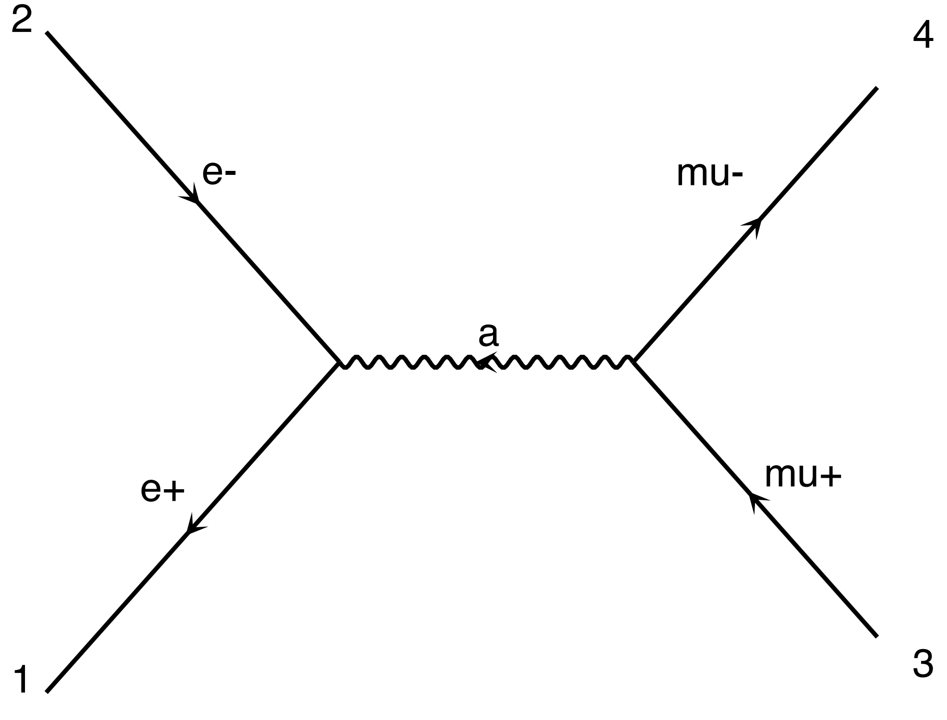
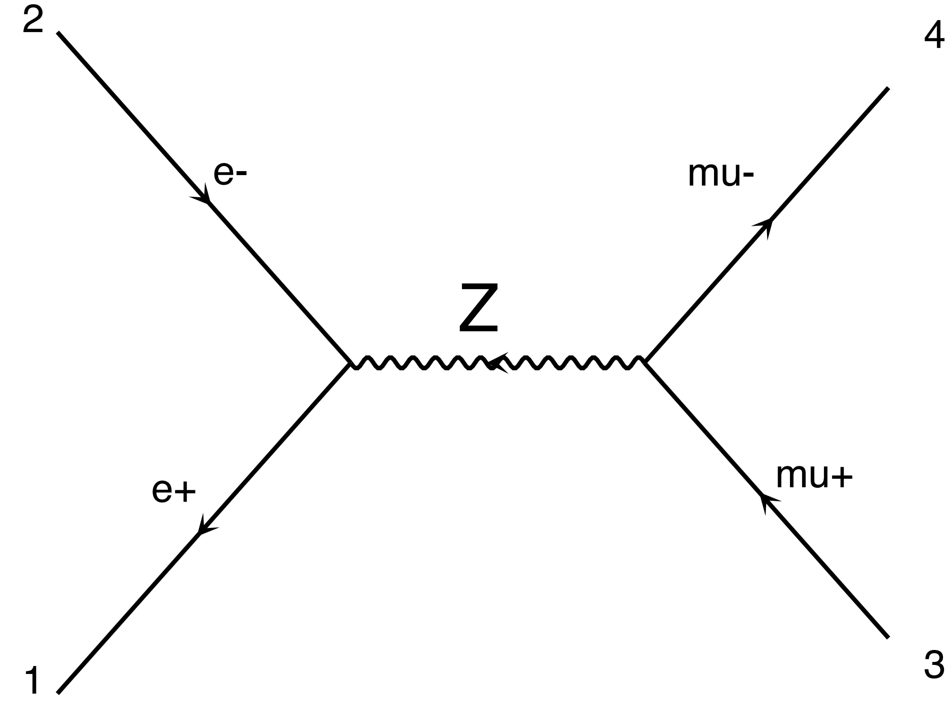
3





Need to compute $|M_a|^2$ $|M_z|^2$ $2Re(M_a^* M_z)$





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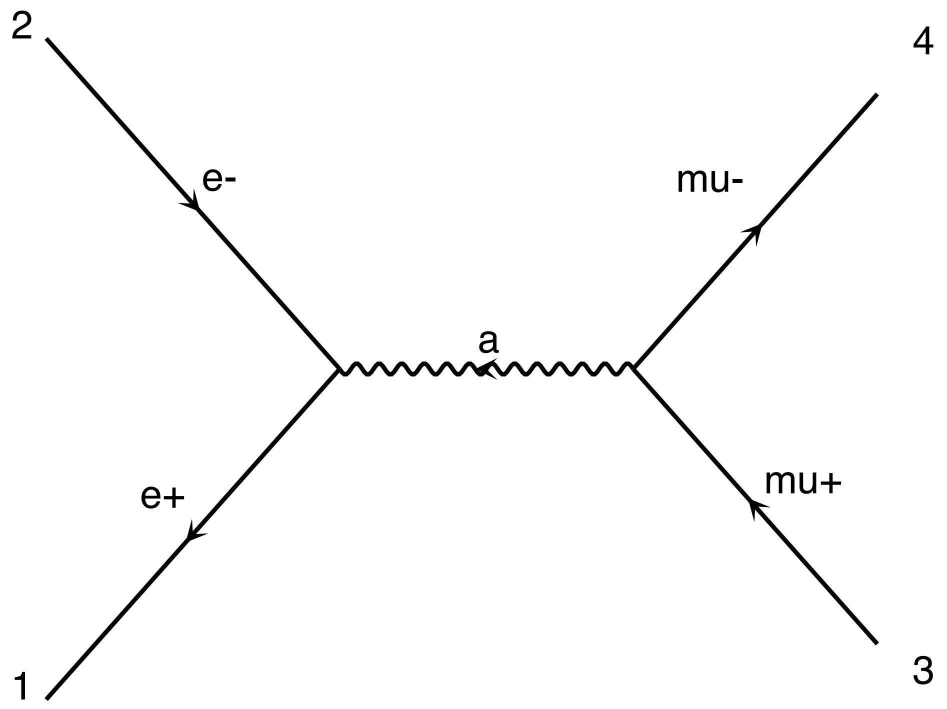
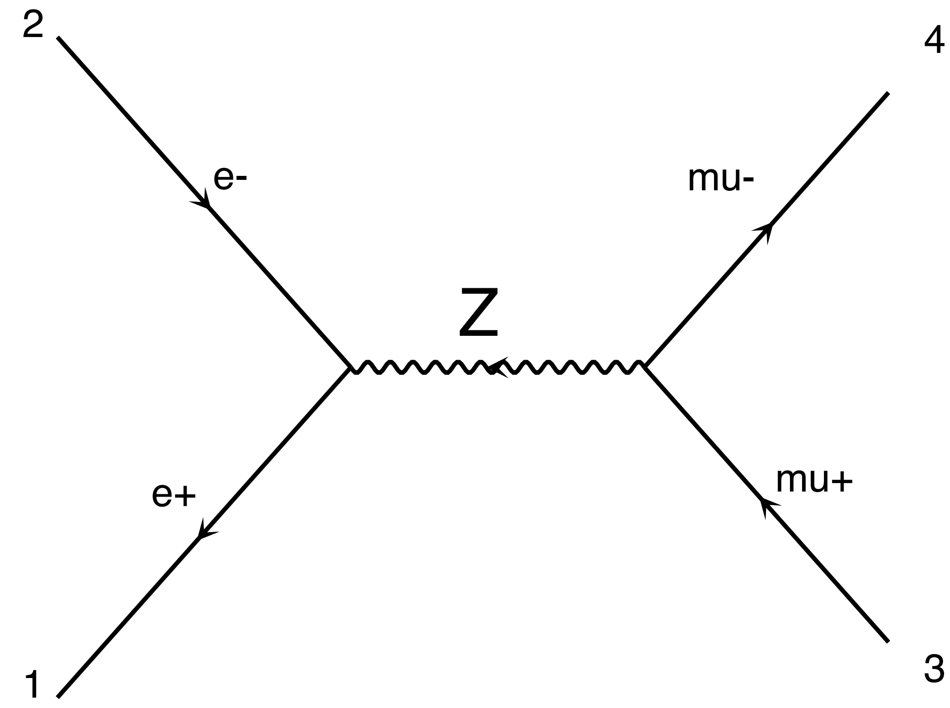
So for M Feynman diagram we need to compute M^2 different term





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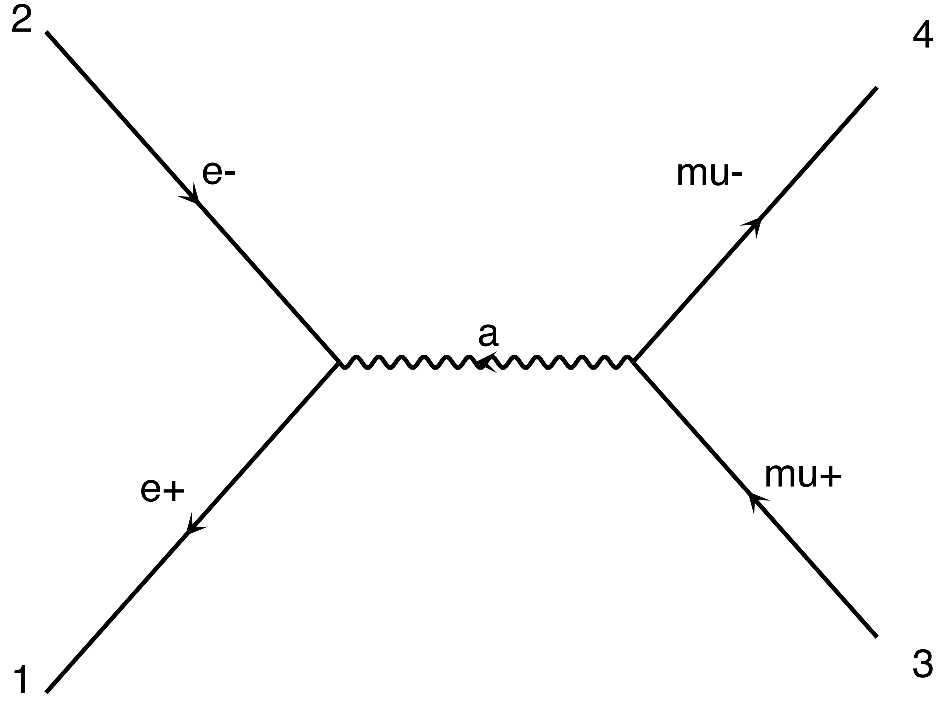
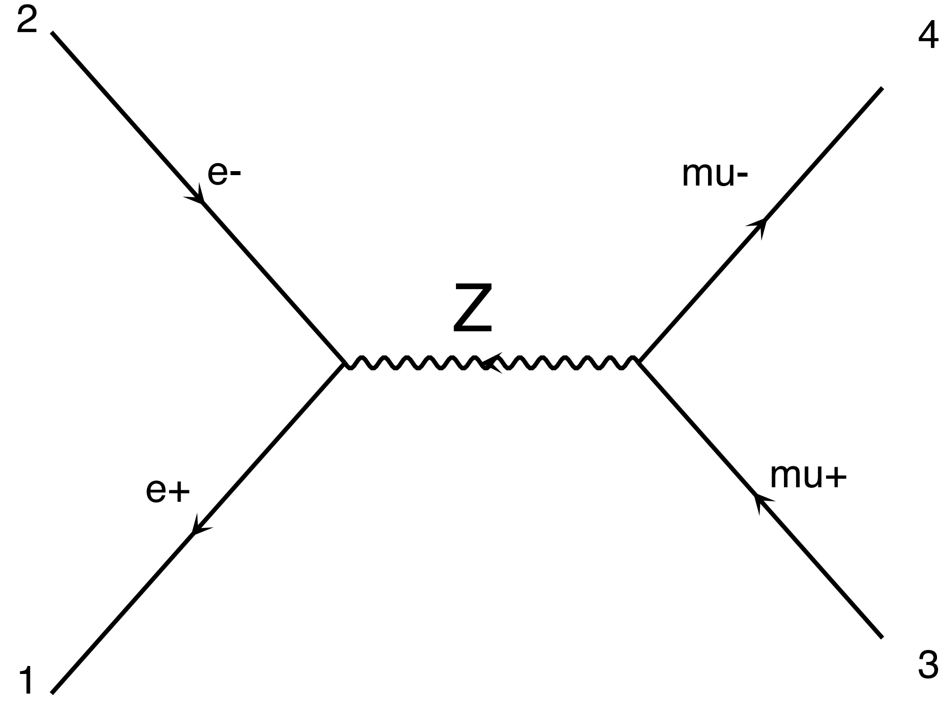
The number of diagram scales factorially with the number of particle





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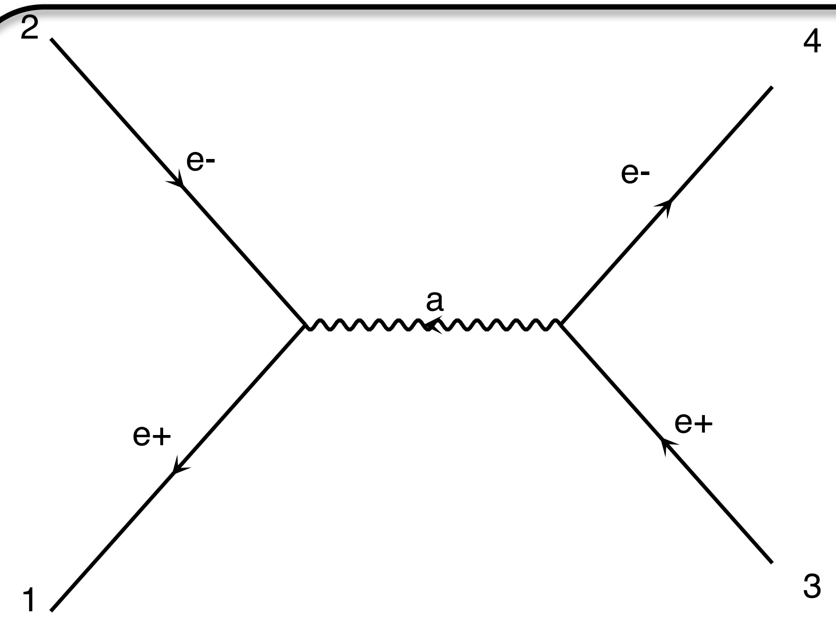
The number of diagram scales factorially with the number of particle

In practise possible up to 2>4

Helicity Amplitude

Idea
- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡Loop on Helicity and average the results



$$\mathcal{M} = ((\bar{u} e \gamma^\mu v) \frac{g_{\mu\nu}}{q^2})(\bar{v} e \gamma^\nu u))$$

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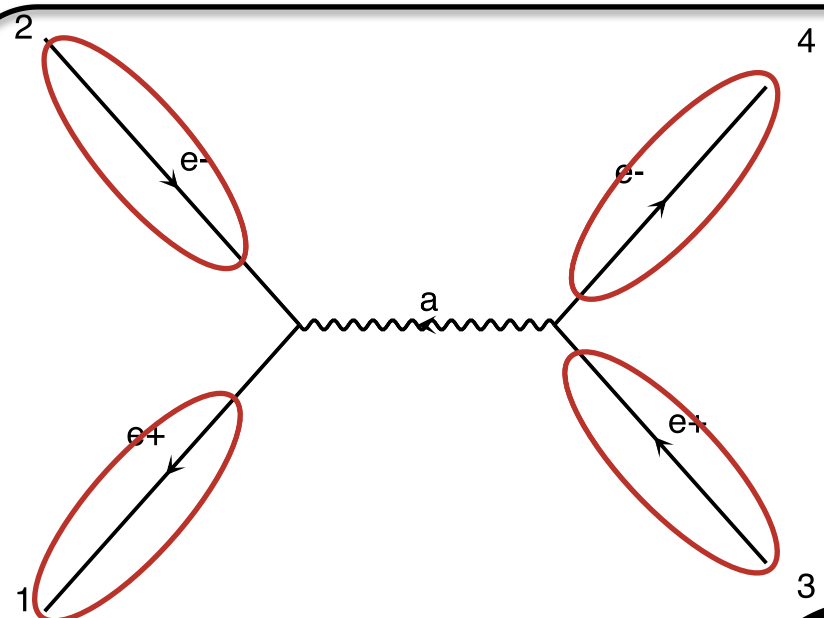

$$\mathcal{M} = ((\bar{u} e \gamma^\mu v)\frac{g_{\mu\nu}}{q^2})(\bar{v} e \gamma^\nu u))$$

Numbers for given helicity and momenta

Lines present in the code.

$$\begin{cases} \bar{v}_1 = fct(\vec{p}_1, m_1) \\ u_2 = fct(\vec{p}_2, m_2) \\ v_3 = fct(\vec{p}_3, m_3) \\ \bar{u}_4 = fct(\vec{p}_4, m_4) \end{cases}$$

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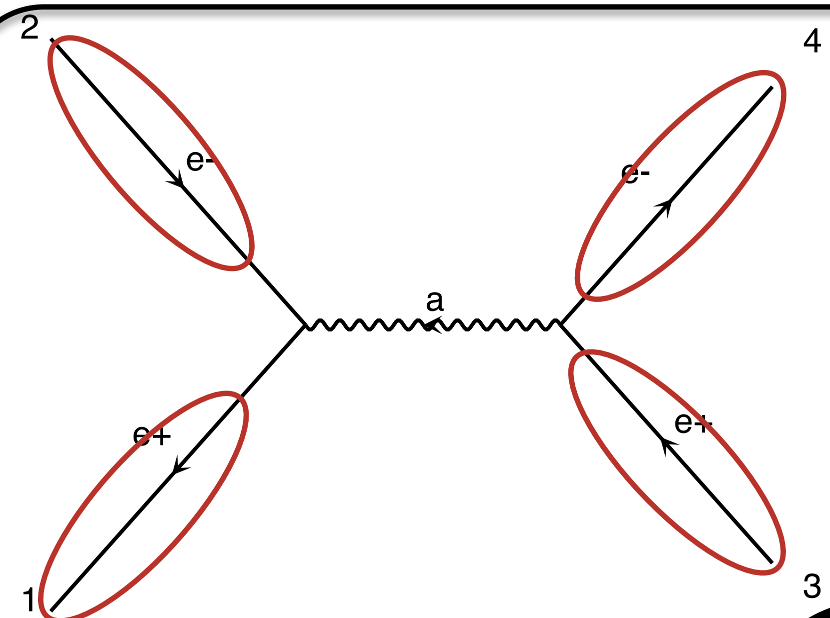

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$$u(p) = \begin{pmatrix} \omega_{-\lambda}(p)\chi_{\lambda}(\vec{p}) \\ \omega_{\lambda}(p)\chi_{\lambda}(\vec{p}) \end{pmatrix}$$

$$\omega_{\pm}(p) \equiv \sqrt{E \pm |\vec{p}|}.$$

$$\chi_+(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}} \begin{pmatrix} |\vec{p}|+p_z \\ p_x + i p_y \end{pmatrix},$$

$$\chi_-(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}} \begin{pmatrix} -p_x + i p_y \\ |\vec{p}|+p_z \end{pmatrix}.$$

Helicity Amplitude

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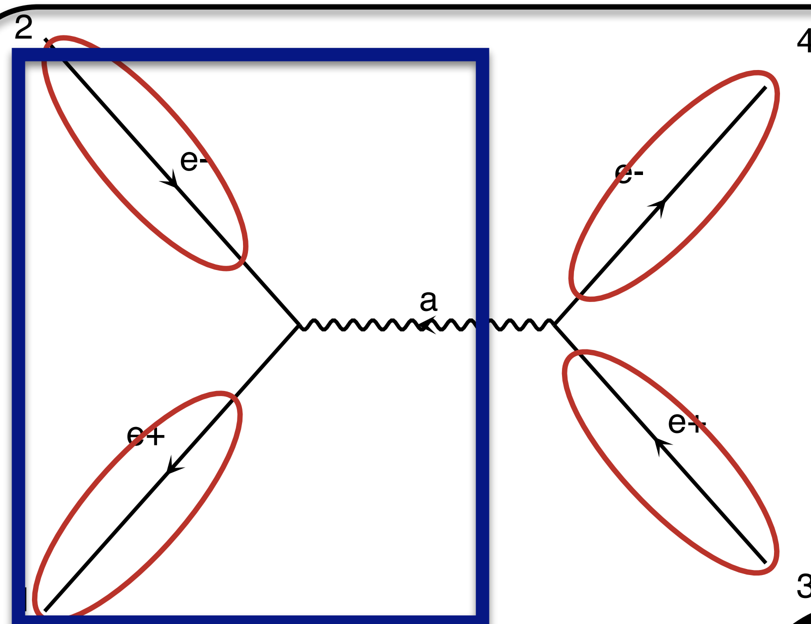

$$\mathcal{M} = ((\bar{u} e \gamma^\mu v)\frac{g_{\mu\nu}}{q^2})(\bar{v} e \gamma^\nu u))$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

Lines present in the code.

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Helicity Amplitude

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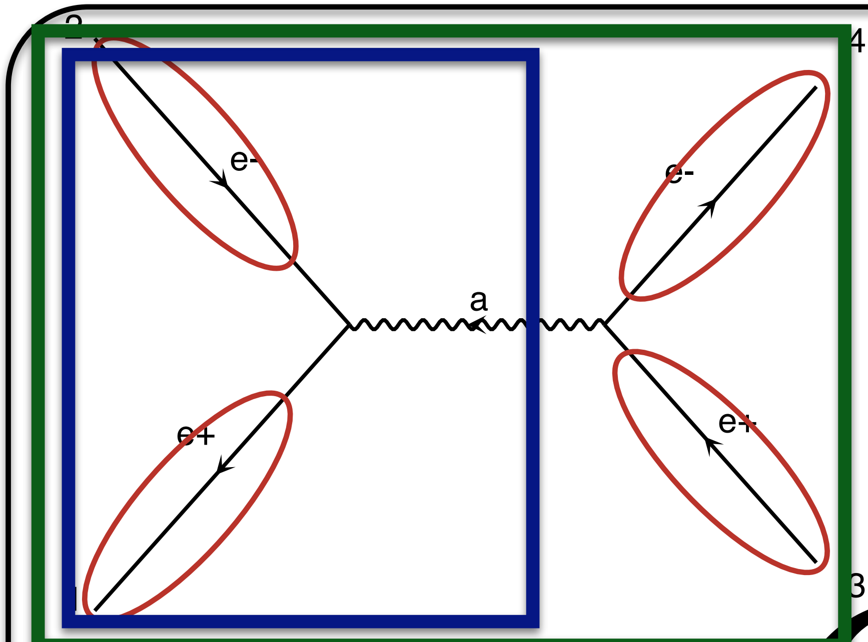

$$\mathcal{M} = ((\bar{u} e \gamma^\mu v)\frac{g_{\mu\nu}}{q^2})(\bar{v} e \gamma^\nu u))$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

Finally evaluate amplitude (c-number)

Lines present in the code.

$$\bar{v}_1 = fct(\vec{p}_1, m_1)$$
$$u_2 = fct(\vec{p}_2, m_2)$$
$$v_3 = fct(\vec{p}_3, m_3)$$
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$$W_a = fct(\bar{v}_1, u_2, e, m_a, \Gamma_a) = e \bar{v}_1 \gamma^\mu u_2 \frac{g_{\mu\nu}}{q^2 - m_a^2 + i m_a \Gamma_a}$$
$$\mathcal{M} = fct(\bar{v}_3, u_4, W^a_\nu, e) = e \bar{v}_3 \gamma_\nu u_4 W^a_\nu$$

Question time

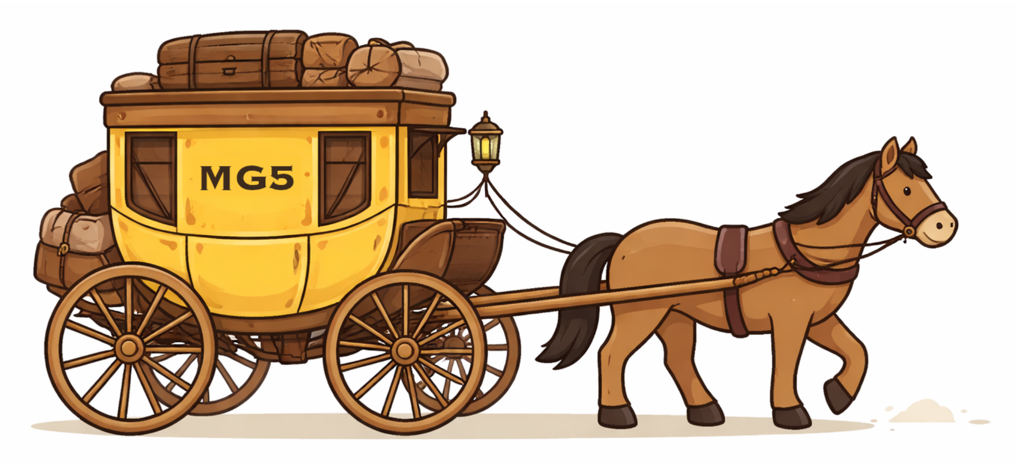
1 Allez sur wooclap.com

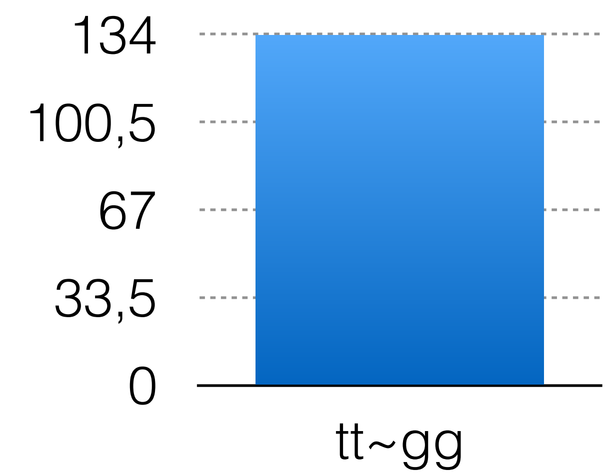
2 Entrez le code d'événement dans le bandeau supérieur

Code d'événement
MADGRAPH

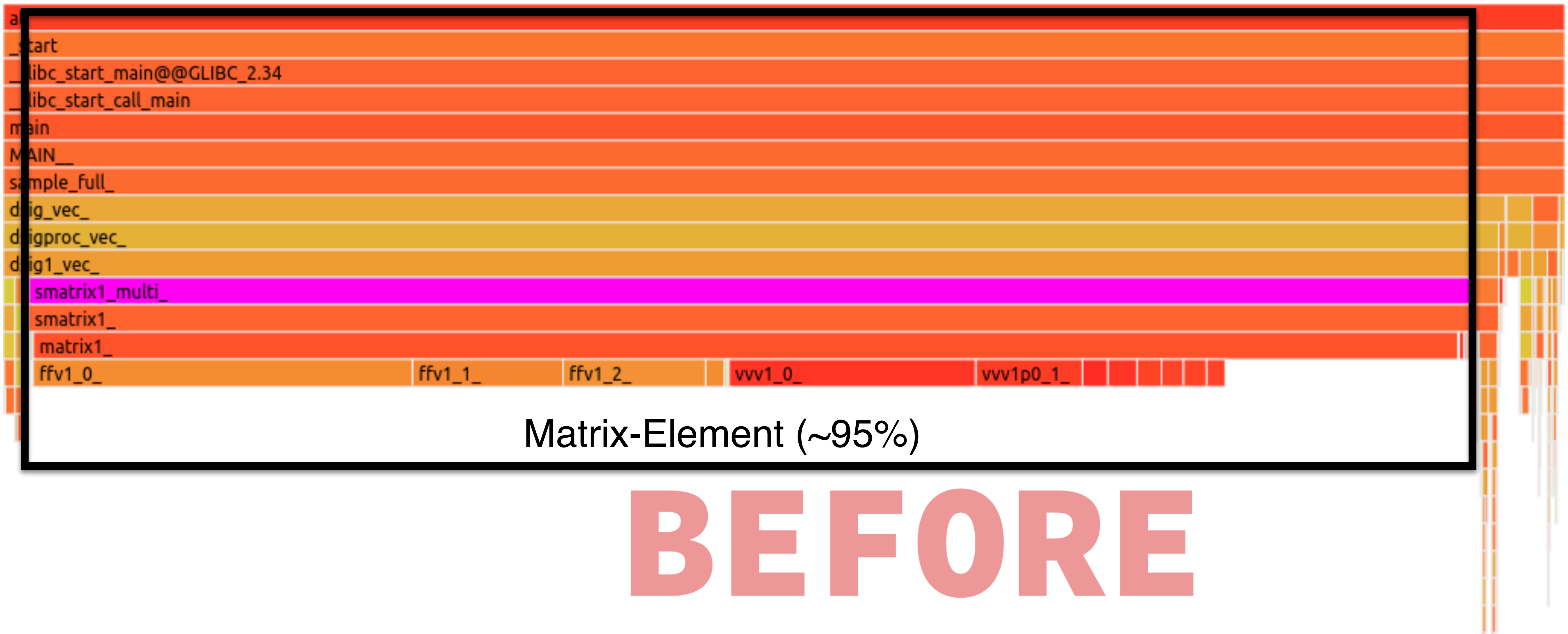
Activer les réponses par SMS

Let's modernise

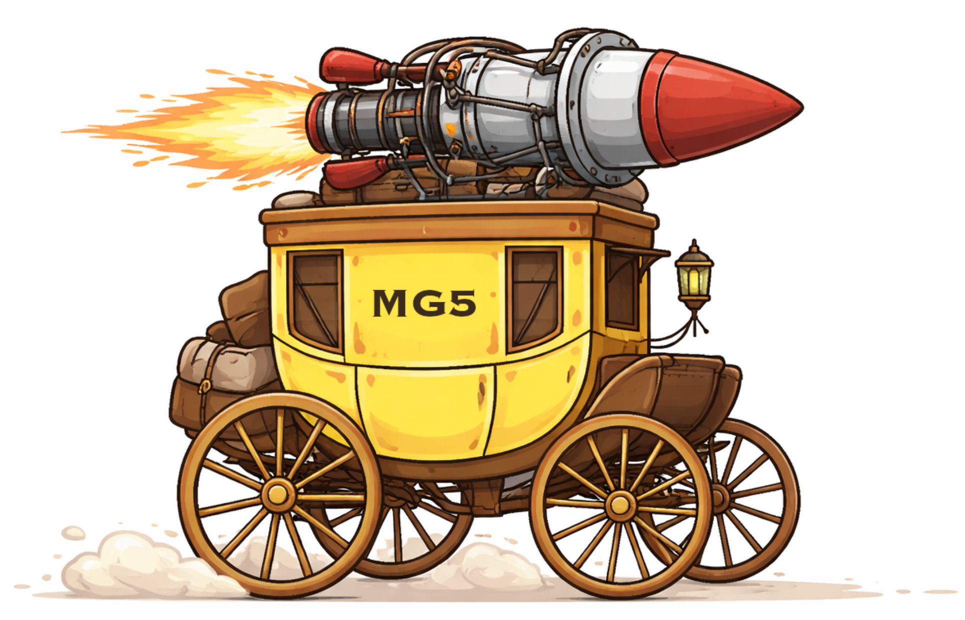




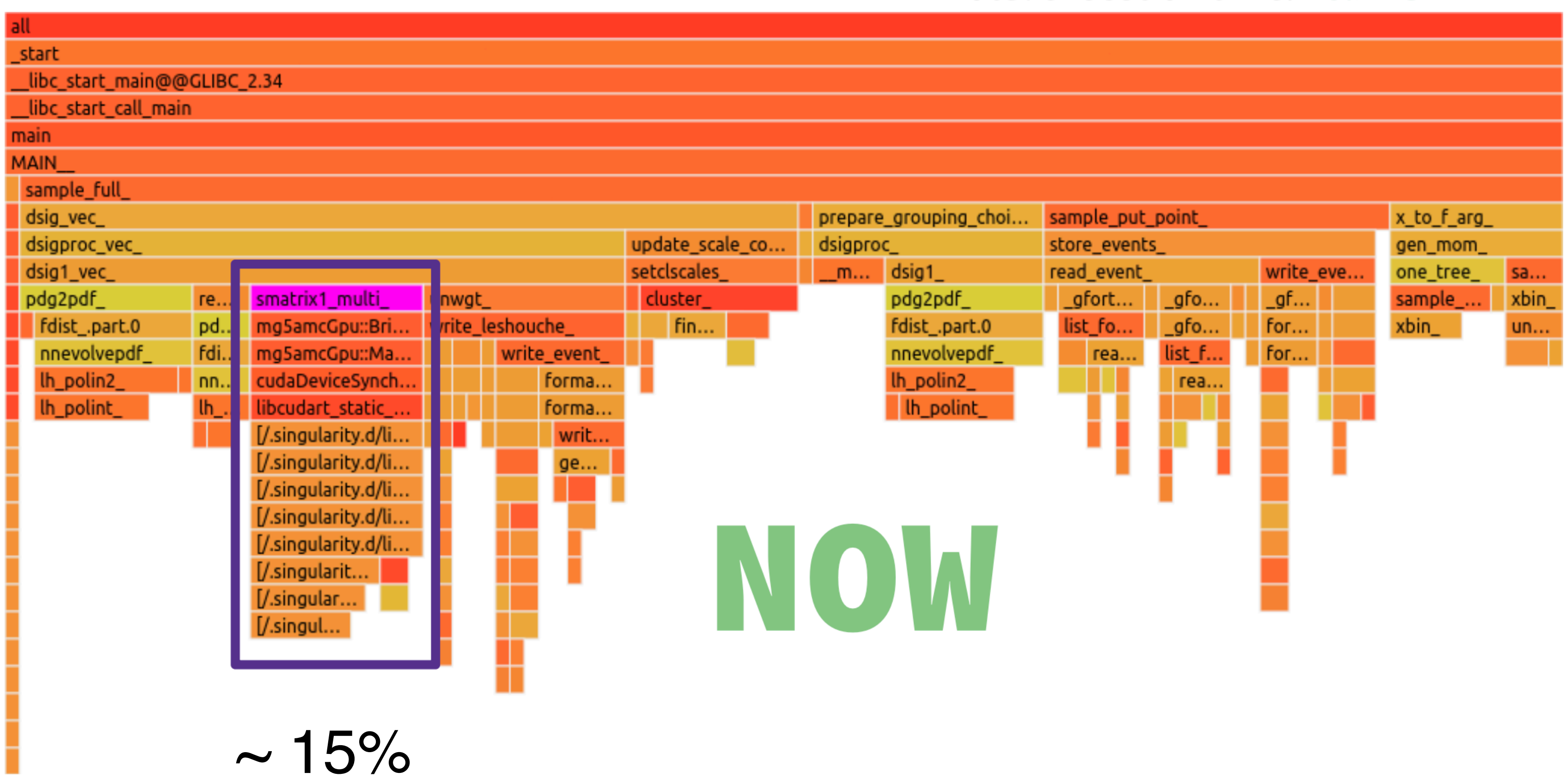
Total execution time: **134.83 s**



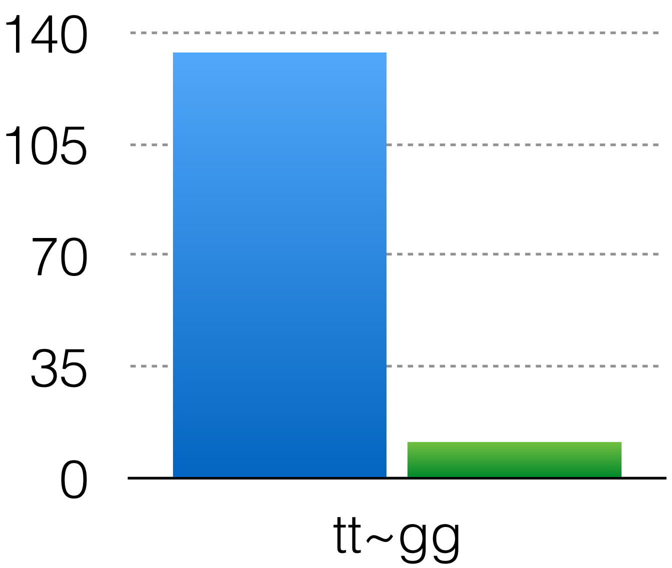
Let's modernise



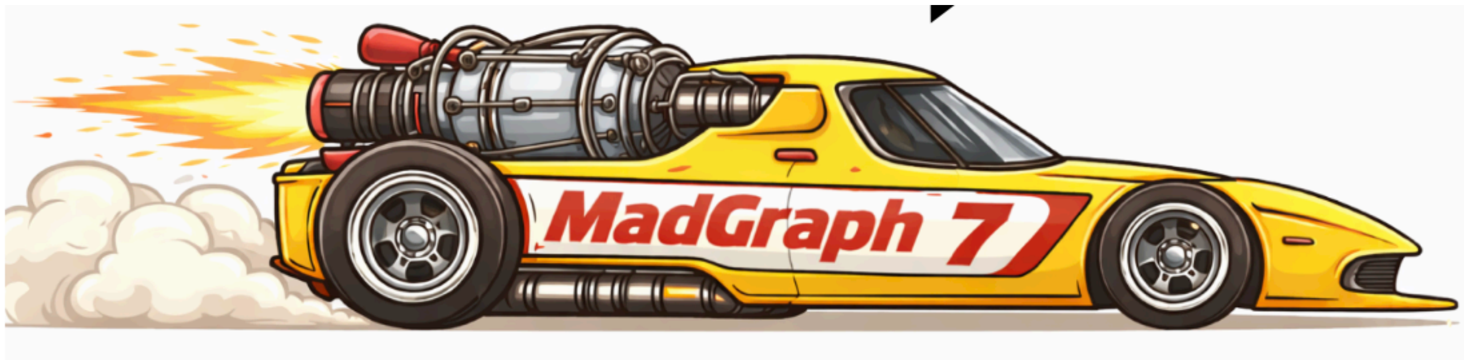
Total execution time: **10.94 s**

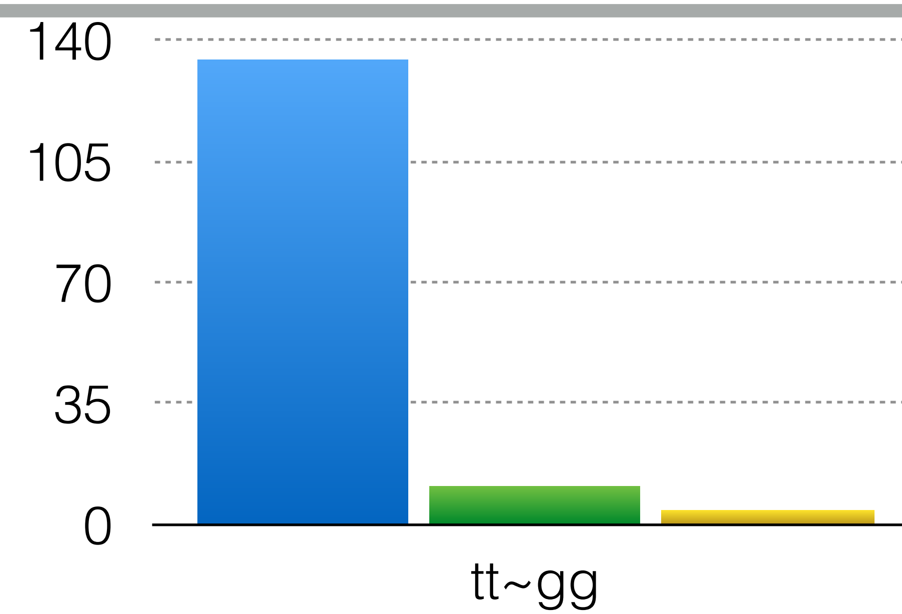


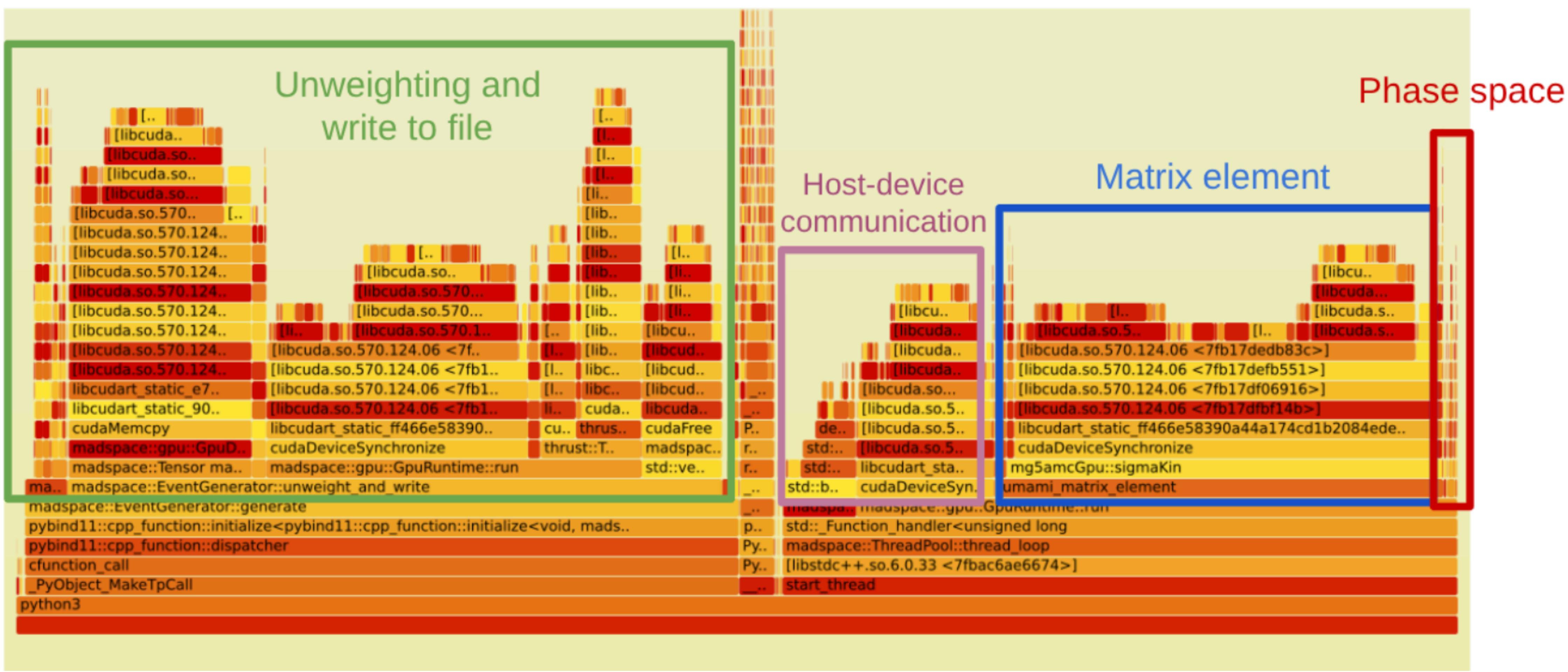
~ 15%



Soon


MadGraph 7


140
105
70
35
0
tt~gg

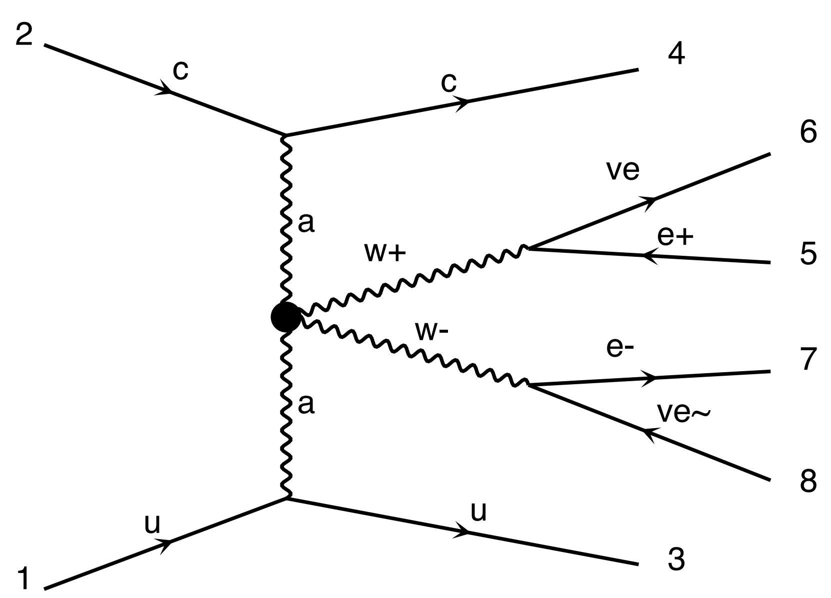

Unweighting and write to file
Host-device communication
Matrix element
Phase space
madspace::EventGenerator::unweight_and_write
madspace::EventGenerator::generate
pybind11::cpp_function::initialize<pybind11::cpp_function::initialize<void, mads..
pybind11::cpp_function::dispatcher
cfunction_call
_PyObject_MakeTpCall
python3
cudaMemcpy
cudaDeviceSynchronize
madspace::gpu::GpuRuntime::run
libcudart_static_ff466e58390a44a174cd1b2084ede..
mg5amcGpu::sigmaKin
umami_matrix_element
std::_Function_handler<unsigned long
madspace::ThreadPool::thread_loop
[libstdc++.so.6.0.33 <7fbac6ae6674>]
start_thread

To Remember

- Numerical computation faster than analytical computation
- We are able to compute matrix-elment
  - ➡ for large number of final state
  - ➡ for any BSM theory
  - ➡ actually also for loop
- Big Improvement for MadGraph7

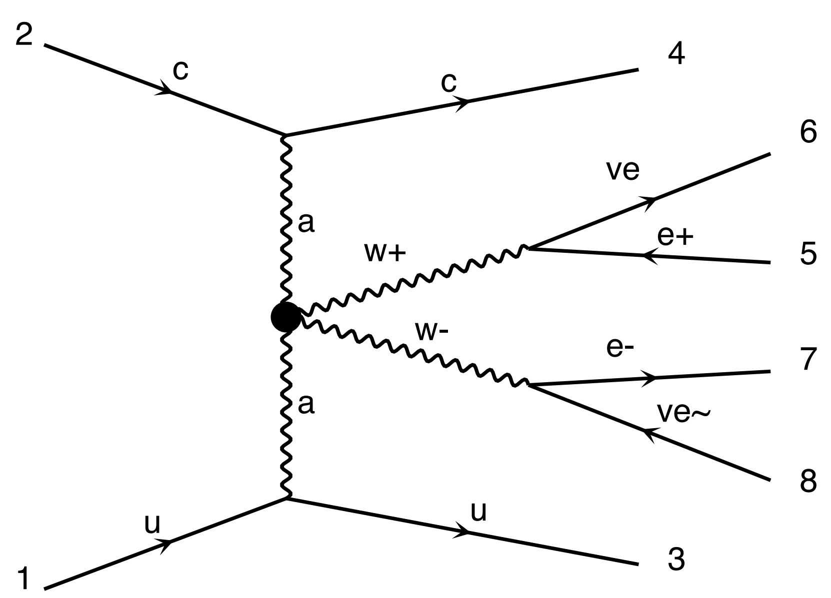
Decay

Resonant Diagram

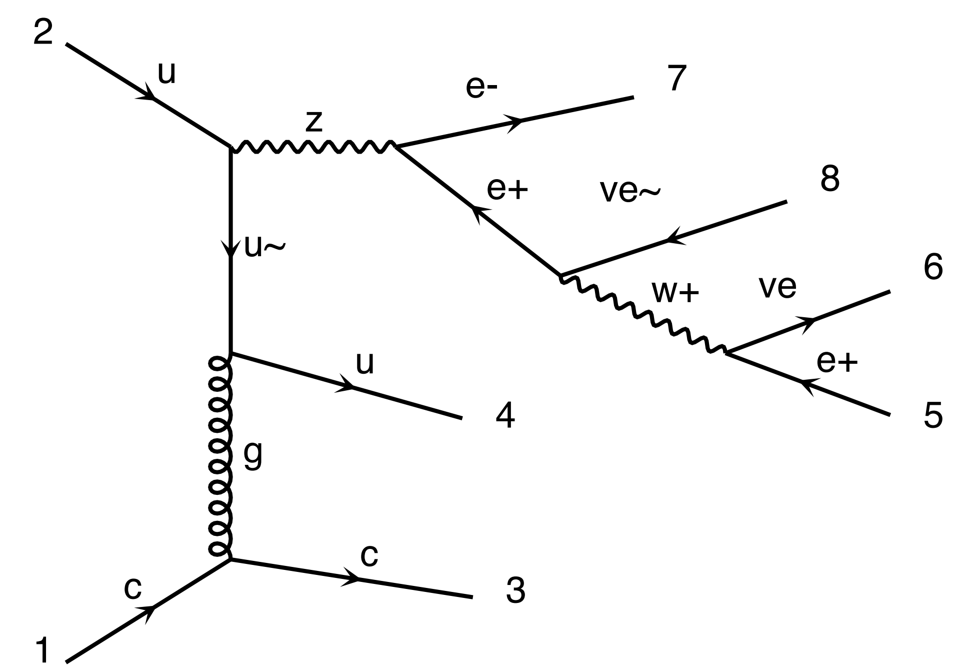


Decay

Resonant Diagram



Non Resonant Diagram



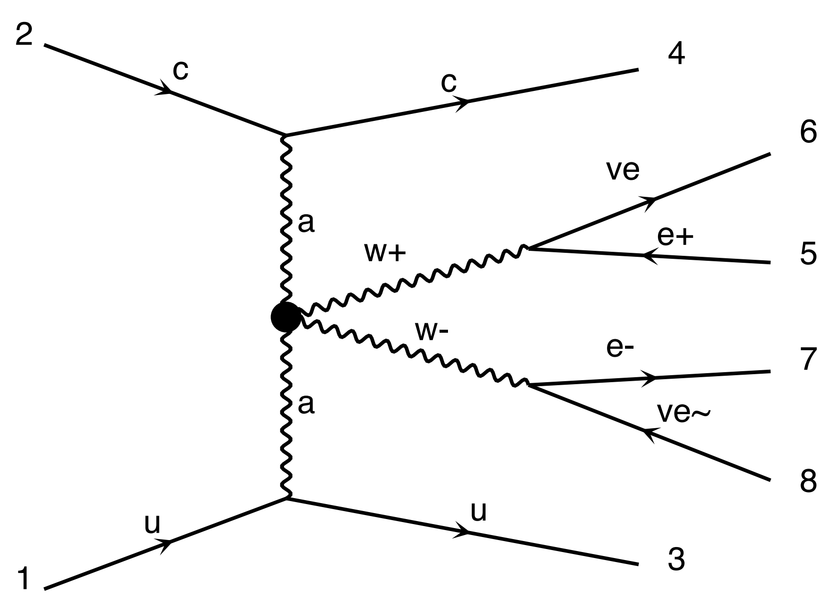
Problem

- Process complicated to have the full process

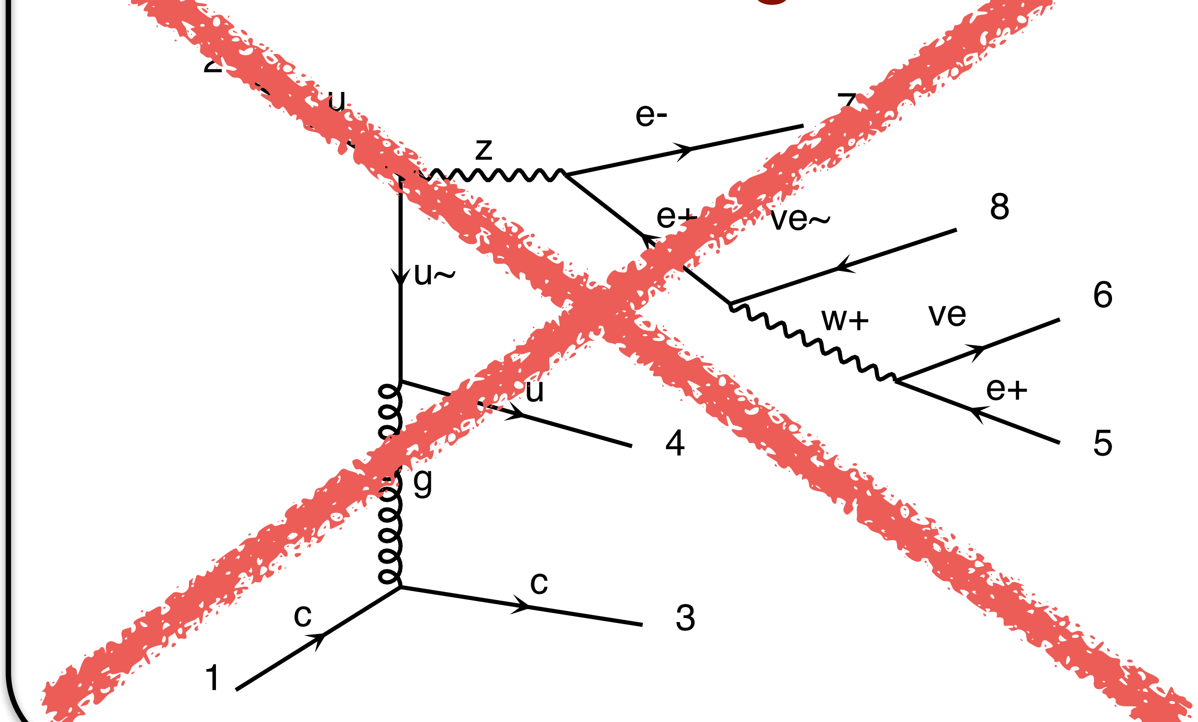
➡Including off-shell contribution

Decay

Resonant Diagram



Non Resonant Diagram



Problem

- Process complicated to have the full process

➡Including off-shell contribution

Solution

- Only keep on-shell contribution

Narrow-Width Approx.

Theory

$$\int dq^2 \left| \frac{1}{q^2 - M^2 + iM\Gamma} \right|^2 \approx \frac{\pi}{M\Gamma}\ \delta(q^2 - M^2)$$

$$\sigma_{full} = \sigma_{prod} * (BR + \mathcal{O}(\frac{\Gamma}{M}))$$

Comment

Narrow-Width Approx.

Theory

$$\int dq^2 \left| \frac{1}{q^2 - M^2 + iM\Gamma} \right|^2 \approx \frac{\pi}{M\Gamma}\, \delta(q^2 - M^2)$$

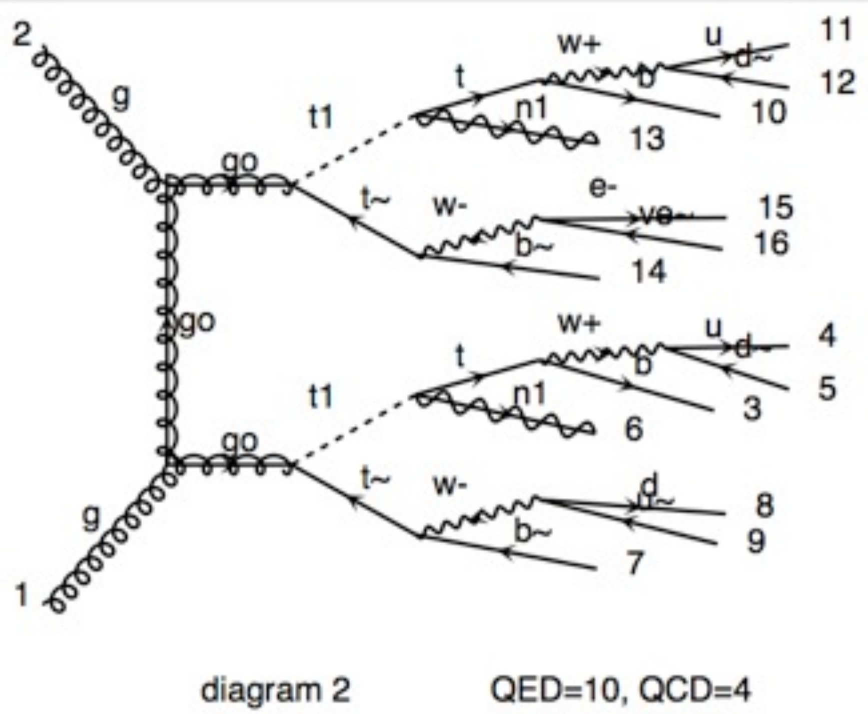
$$\sigma_{full} = \sigma_{prod} * (BR + \mathcal{O}(\frac{\Gamma}{M}))$$

Comment

- This is an Approximation!
- This force the particle to be on-shell!
  - Recover by re-introducing the Breit-wigner up-to a cut-off
- The loop is not a free parameter

Decay chain

- p p > t t~ w+, (t > w+ b, w+ > l+ vl), \
  (t~ > w- b~, w- > j j), \
  w+ > l+ vl



very long
decay chains possible to simulate
directly in MadGraph!

Decay chain

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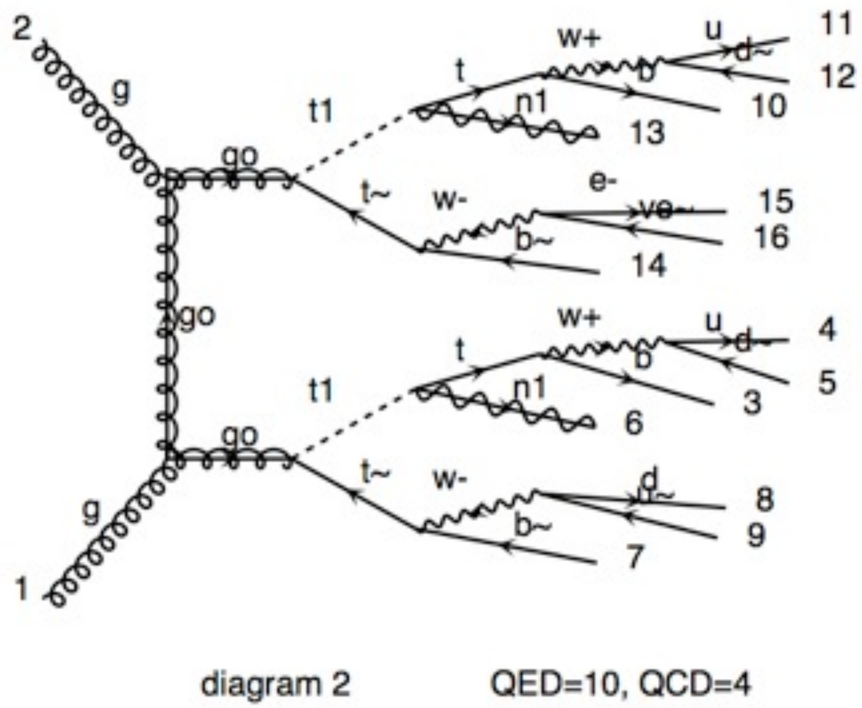

very long
decay chains possible to simulate
directly in MadGraph!

- This syntax has an invariant mass cut associated to t/t~/W
- Other syntax/tools exists for NWA (like MadSpin)

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Code d'événement

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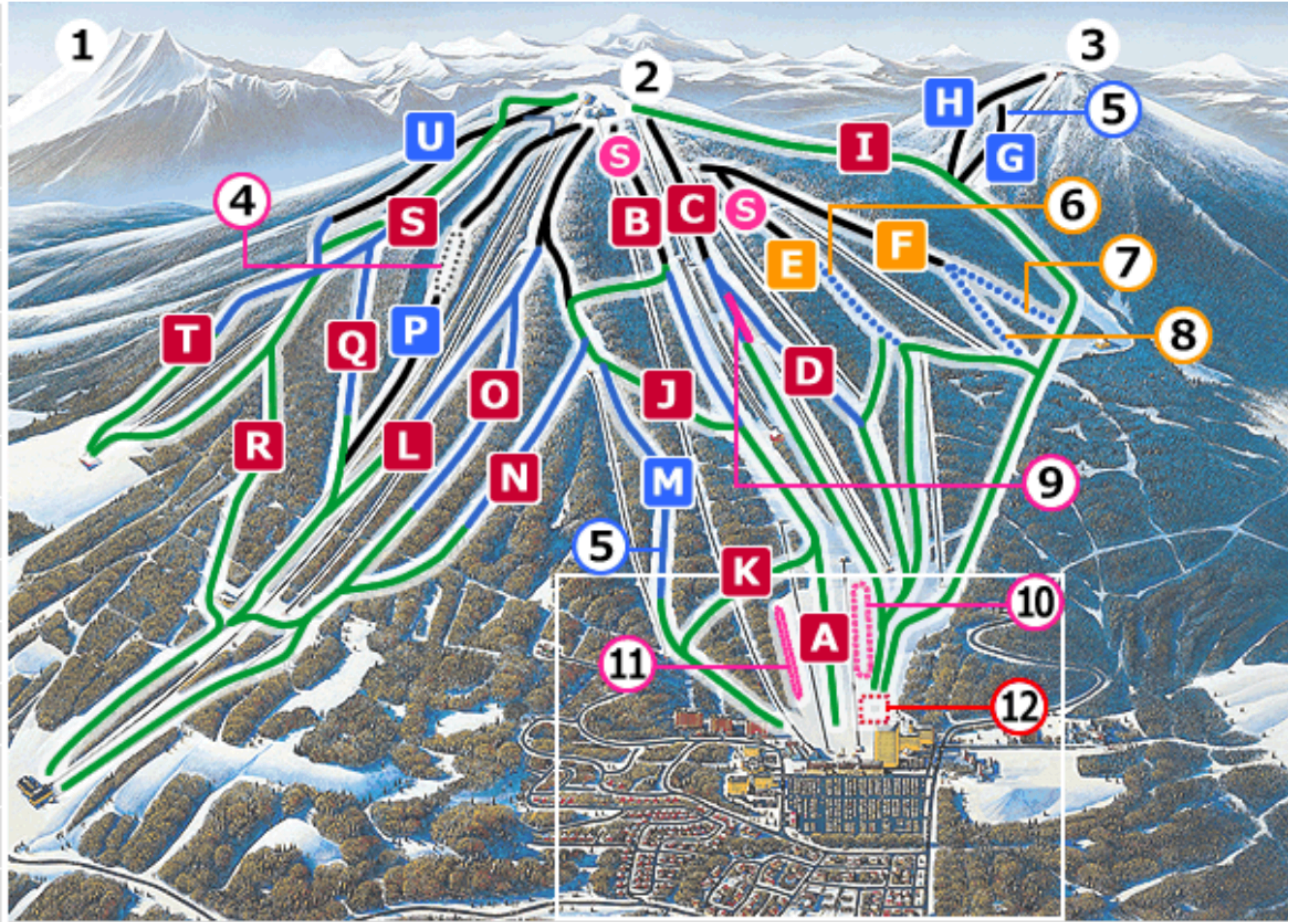
To Remember

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS} \, f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, \hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

Phase-space integral — Parton density functions — Parton-level cross section

- PDF: content of the proton
  - ➡ Define the physics/processes that will dominate on your accelerator
- LO: good for shape
- NLO/NNLO: Reduce scale uncertainty
- Computation are inclusive (+ any jet) due to renormalization/factorization scale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UCLouvain
Institut de recherche en mathématique et physique
Centre de Cosmologie, Physique des Particules et Phénoménologie

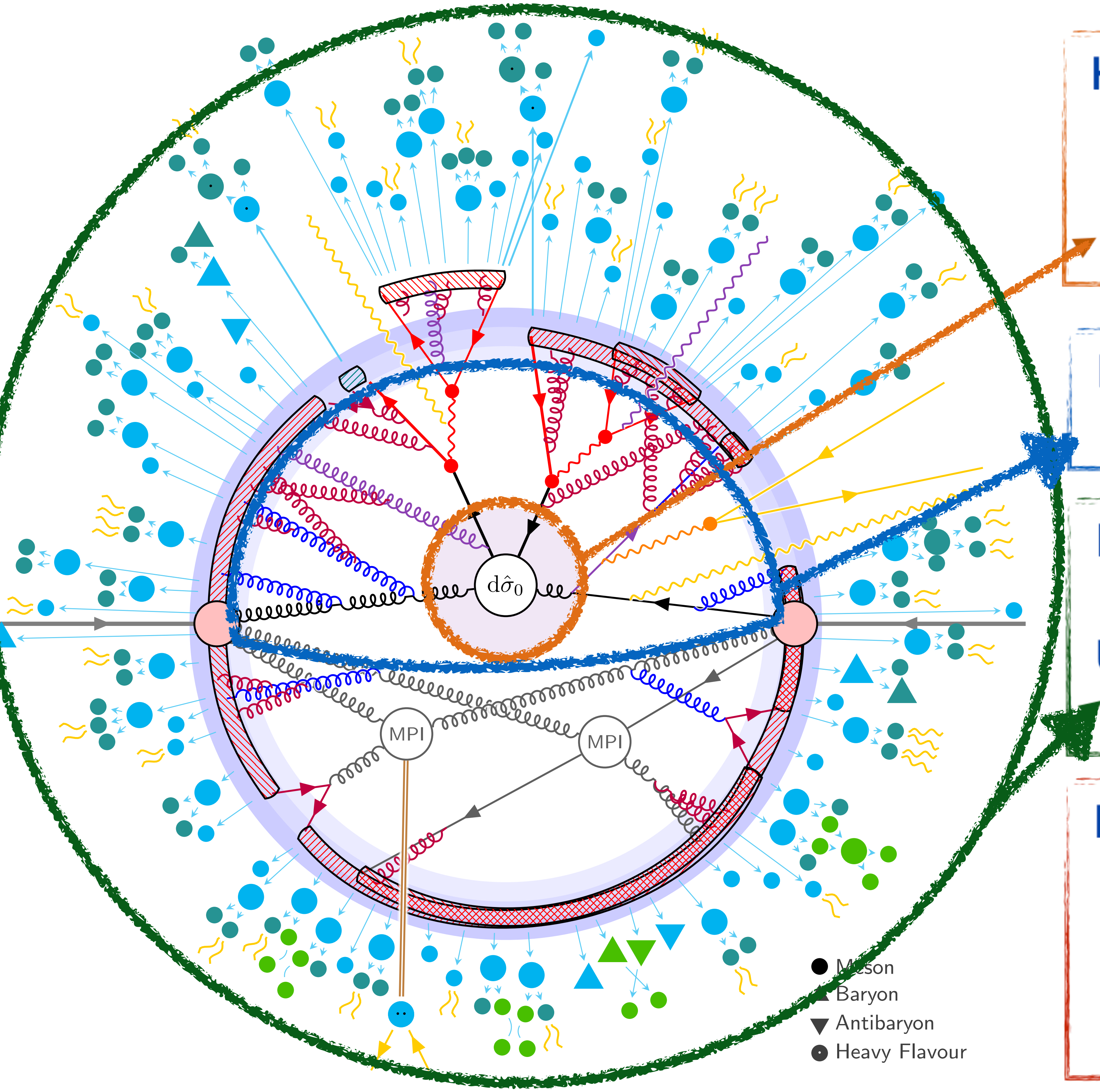
1 Allez sur wooclap.com
2 Entrez le code d'événement dans le bandeau supérieur

Code d'événement
MADGRAPH

MadGraph5_aMC@NLO

Olivier Mattelaer

Collider Physics



From pythia8 manual

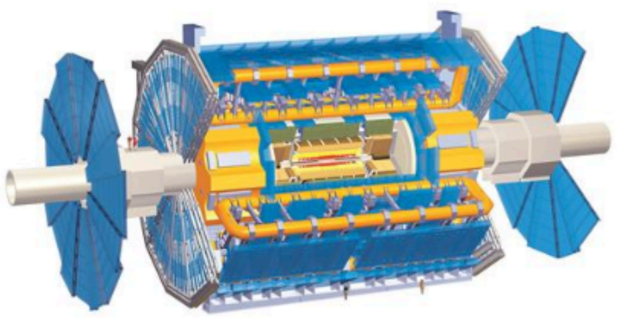
Hard process
- Depends on the model (SM/BSM)
- Perturbative QCD
- Core #1 of this talk

Parton showering
- Universal (QCD)

Hadronisation
- Model-based, universal

Underlying event
- Model-based, non-universal

Detector simulation



To Remember

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}}\, f_a(x_1, \mu_F) f_b(x_2, \mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

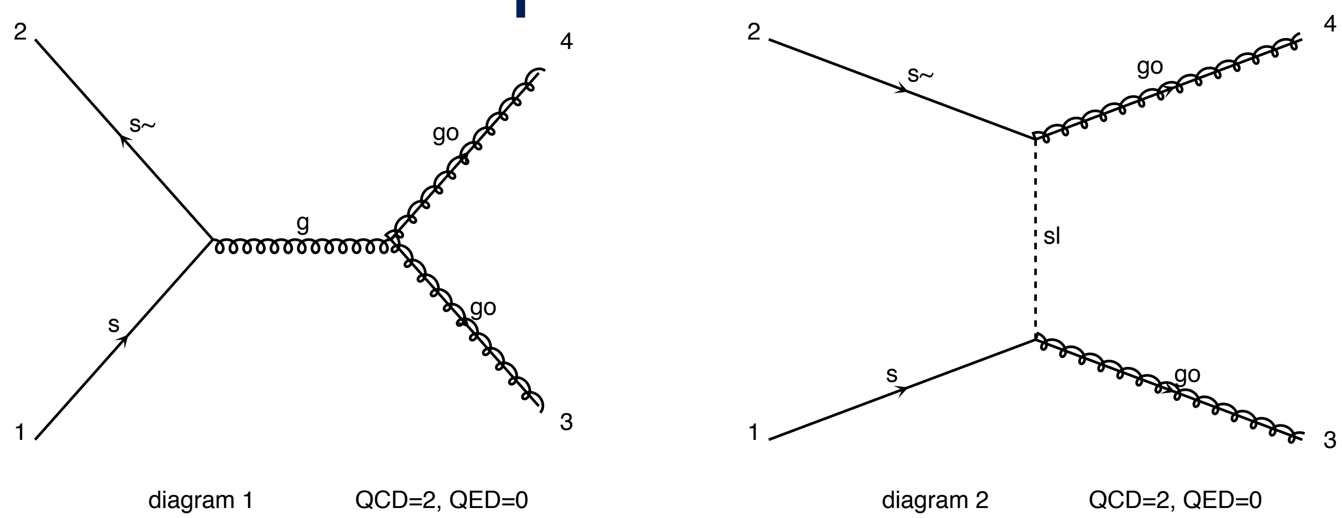
Phase-space integral — Parton density functions — Parton-level cross section

- PDF: content of the proton
  - ➡ Define the physics/processes that will dominate on your accelerator
- LO: good for shape
- NLO/NNLO: Reduce scale uncertainty
- Computation are inclusive (+ any jet) due to renormalization/factorization scale

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



- Evaluate the matrix-element

$$|\mathcal{M}|^2$$ ➡Need Feynman Rules!

- Phase-Space Integration

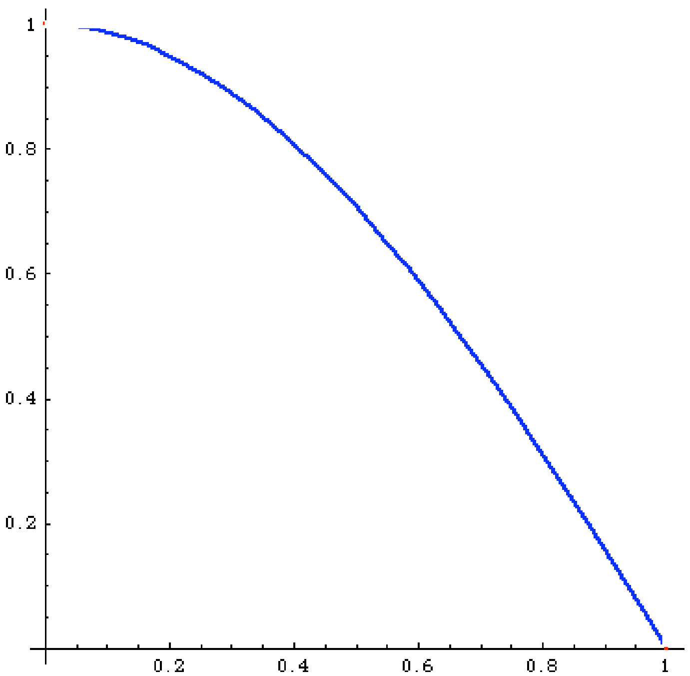
$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

Easy enough

Hard

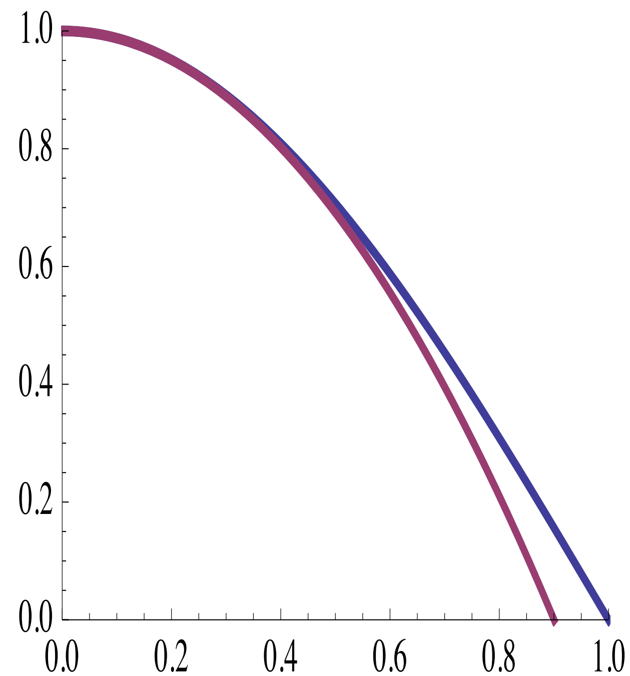
Very Hard (in general)

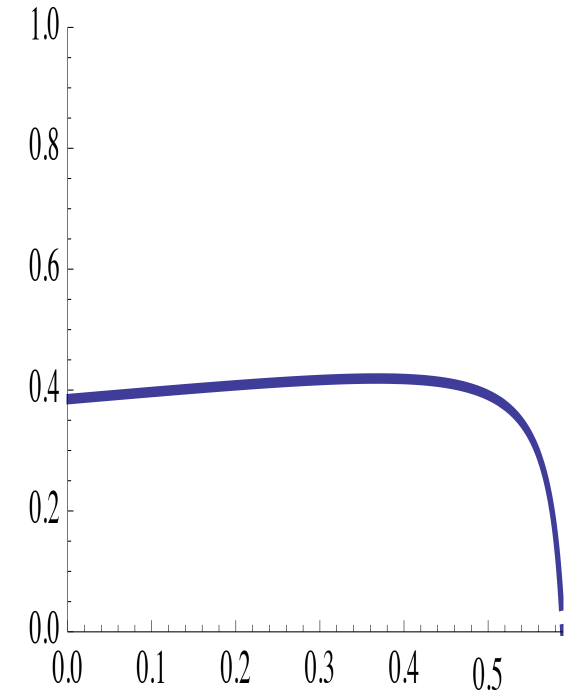
Importance Sampling



$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$I_N = 0.637 \pm 0.307/\sqrt{N}$$





$$I = \int_0^1 dx (1 - cx^2) \frac{\cos\left(\frac{\pi}{2} x\right)}{(1 - cx^2)} = \int_{\xi_1}^{\xi_2} d\xi \frac{\cos \frac{\pi}{2} x[\xi]}{1 - x[\xi]^2 c}$$

$\simeq 1$

$$I_N = 0.637 \pm 0.031/\sqrt{N}$$

The Phase-Space parametrization is important to have an efficient computation!

To Remember

- Phase-Space integration is difficult
- We need to know the function
  - ➡ Be careful with cuts
- MadGraph split the integral in different contribution linked to the Feynman Diagram
  - ➡Those are not the contribution of a given diagram

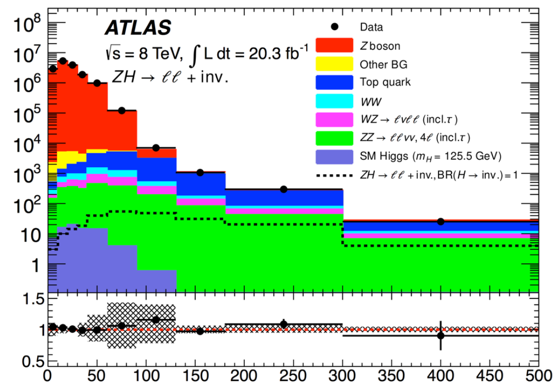
Goal of today

- Event Generation
- Learn how we evaluate (tree-level) matrix-element
- Learn Narrow-width Approximation

Event Generation

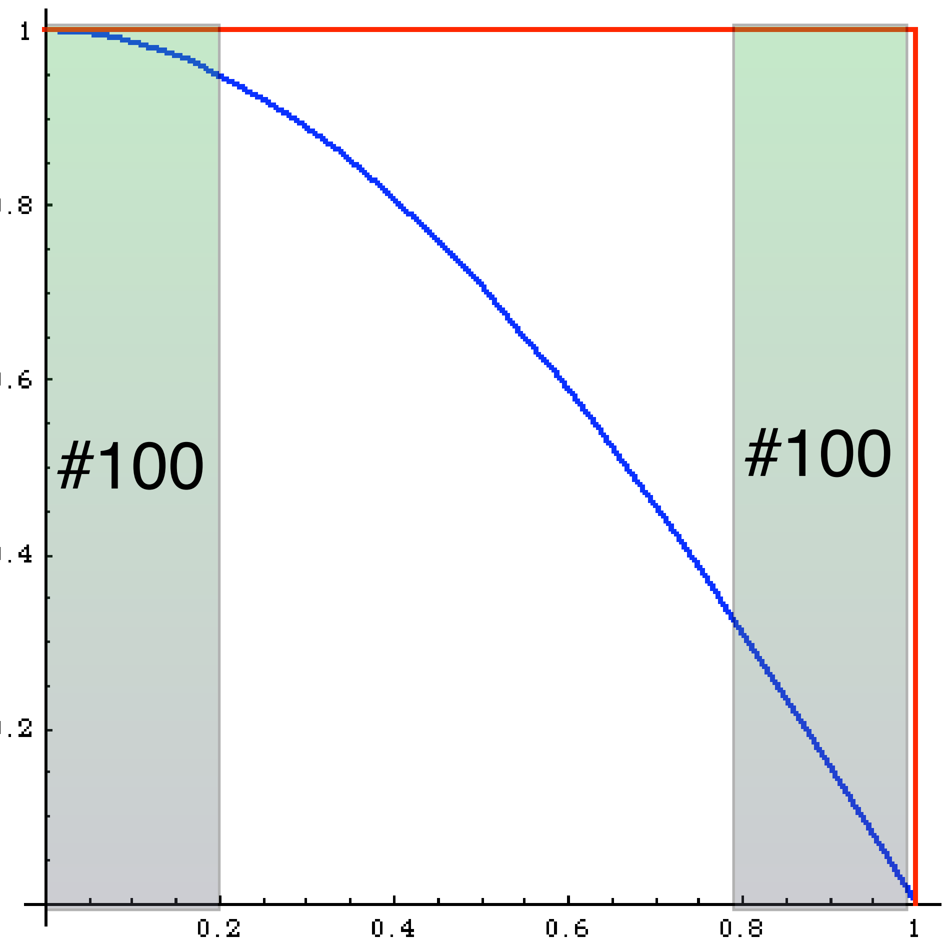
What is the goal?

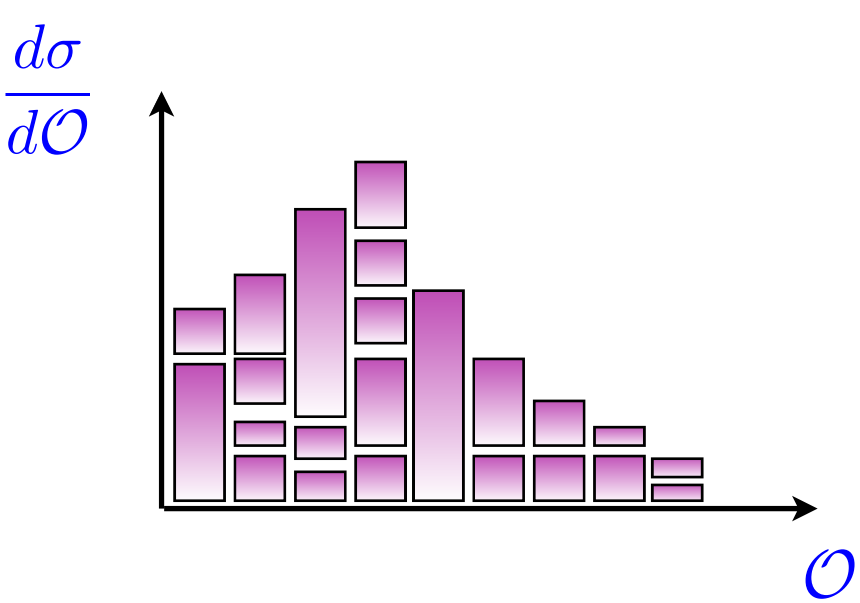
- Cross-section
  - But large theoretical uncertainty



- Differential Cross-Section
  - Provided as sample of events
  - Sample size is problematic
    - Those events will need to have full detector simulation

How to get sample?





- Monte-Carlo integration use random points
  - We can keep those
    - (Uncorrelated) sample

- Points not distributed as the real function
- Need to keep track of the importance of each point (weight)
- Typically a lot of event have low information

Typical choice

Weighted sample

- Output of the MC
- Easy
- Large file
- Histogram with weight

Biased sample

- Modify the weight to enhance (in number of events) a part of the phase-space
- For tails !

Unweighted sample

- Event distributed like in nature
- All event have the same weight.
- Weight is physical (proportional to cross-section of the sample)
- Larger statistical error on tails
- Smaller file

Question time

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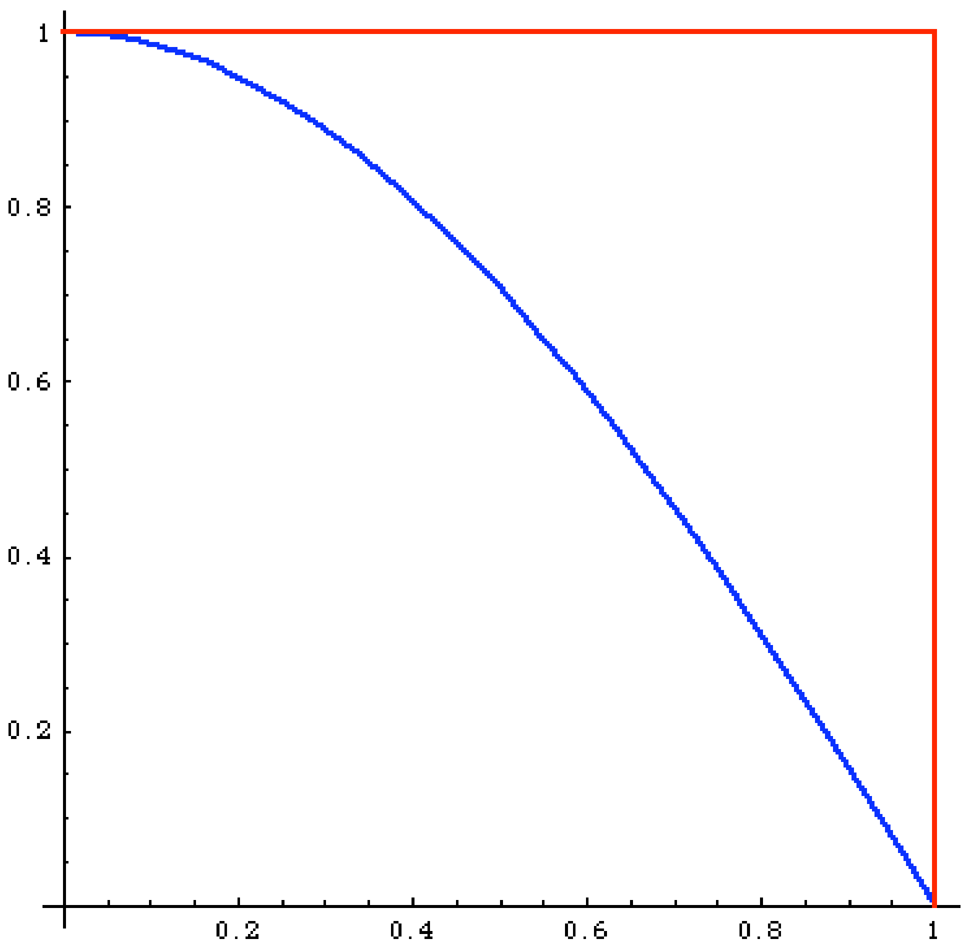
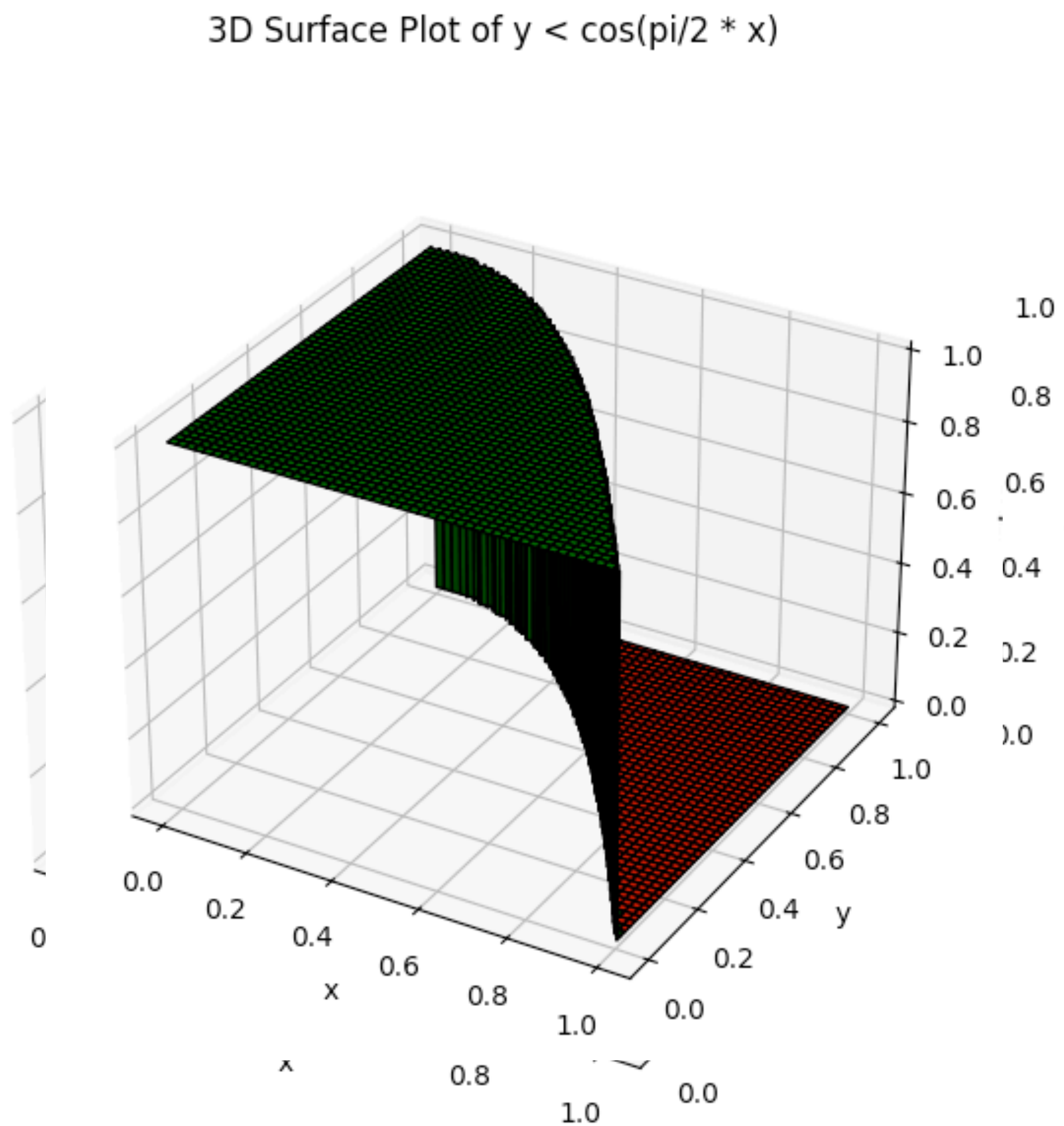
Code d'événement

MADGRAPH

Activer les réponses par SMS

Add one dimension!





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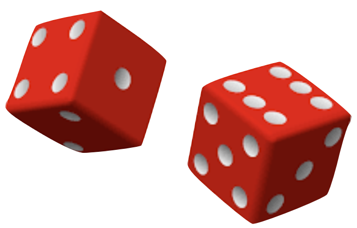
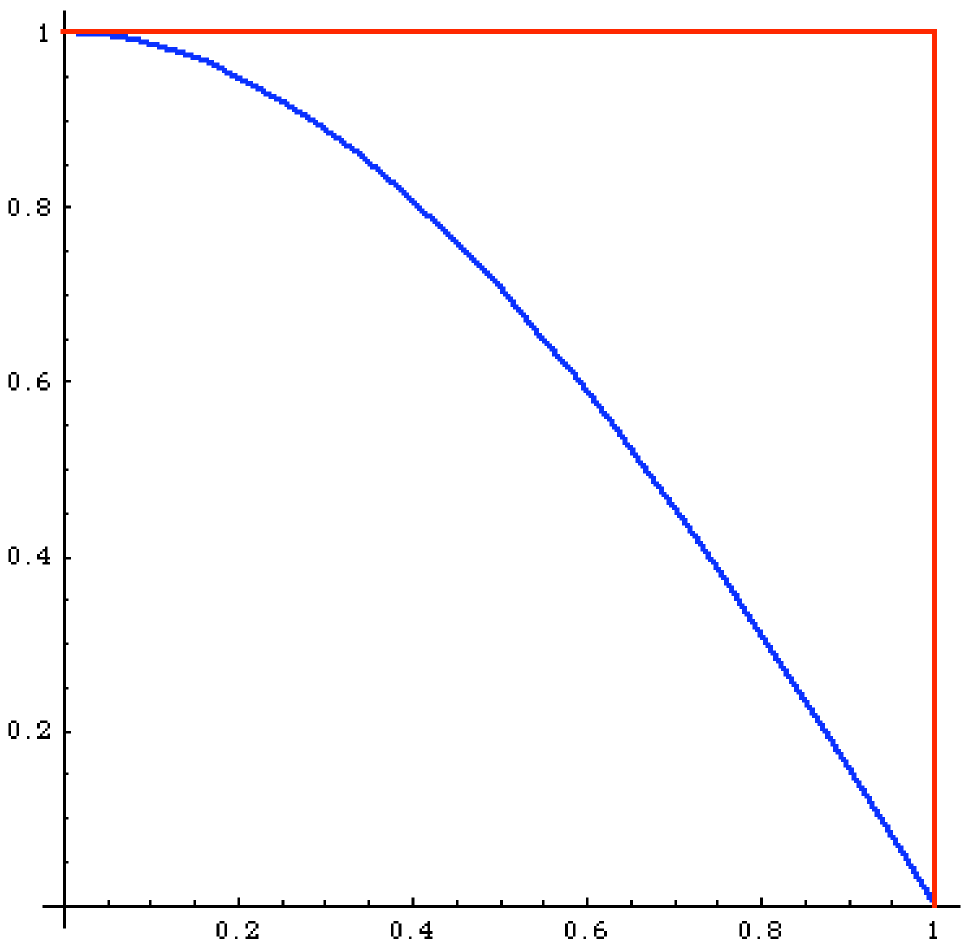
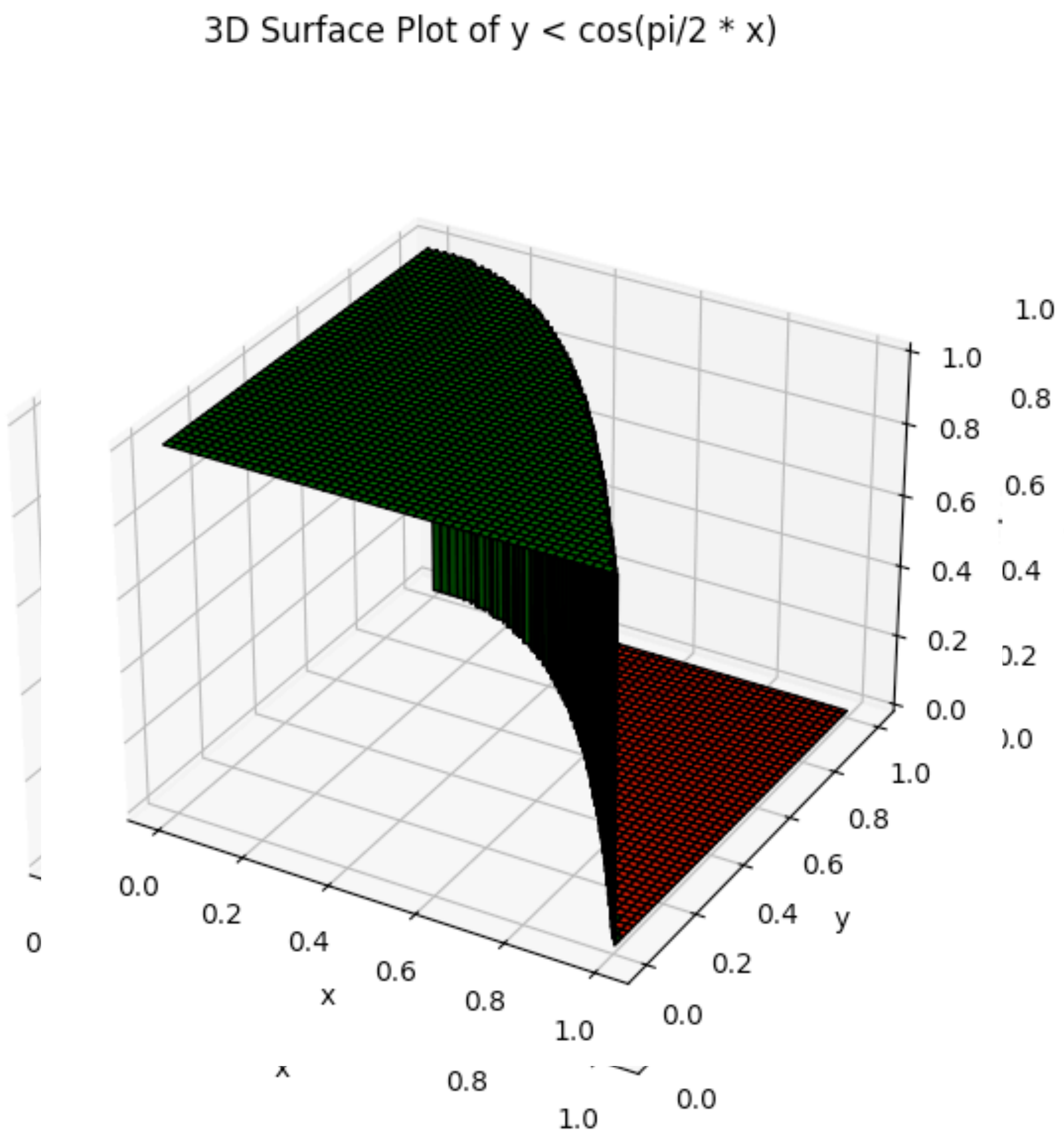
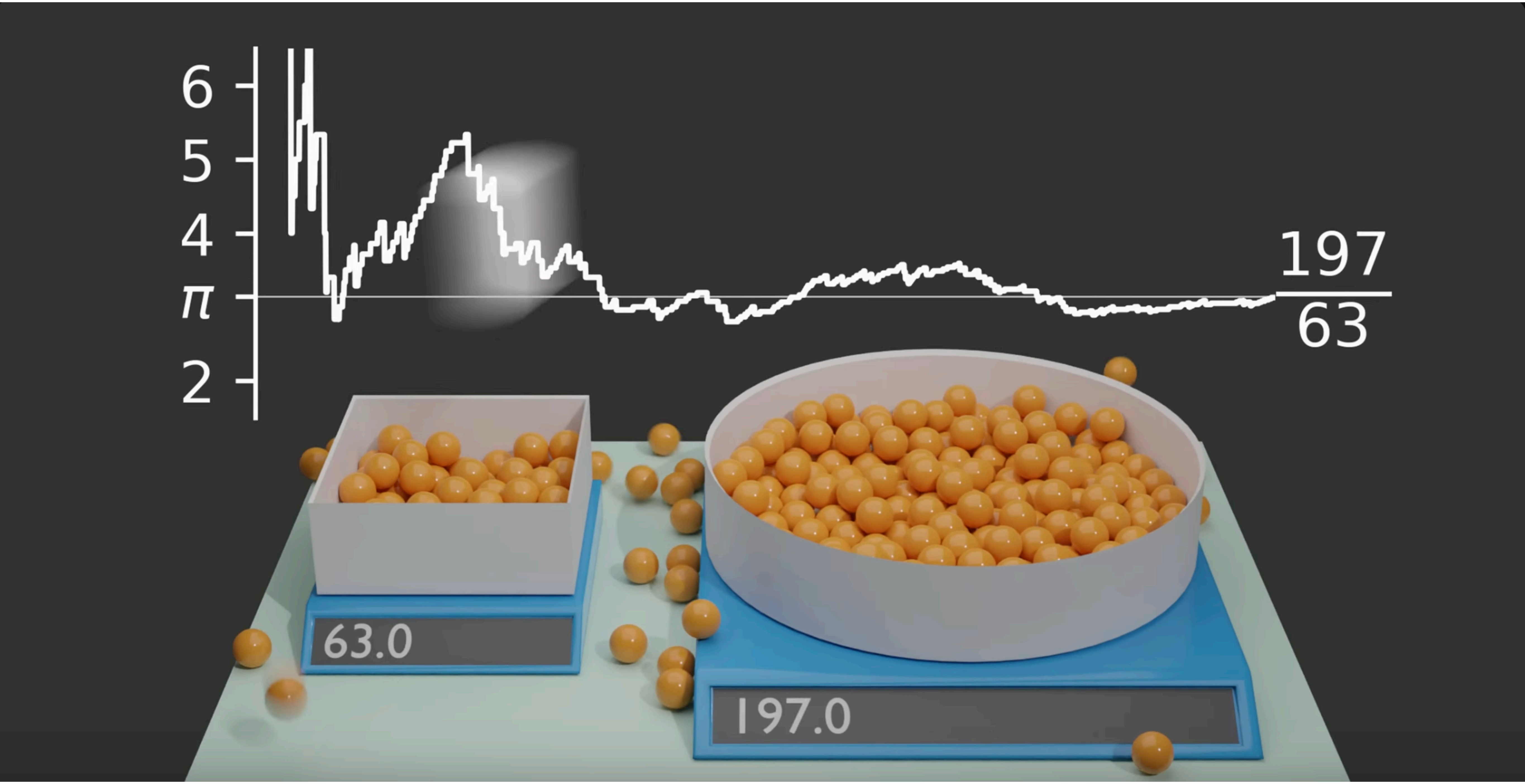


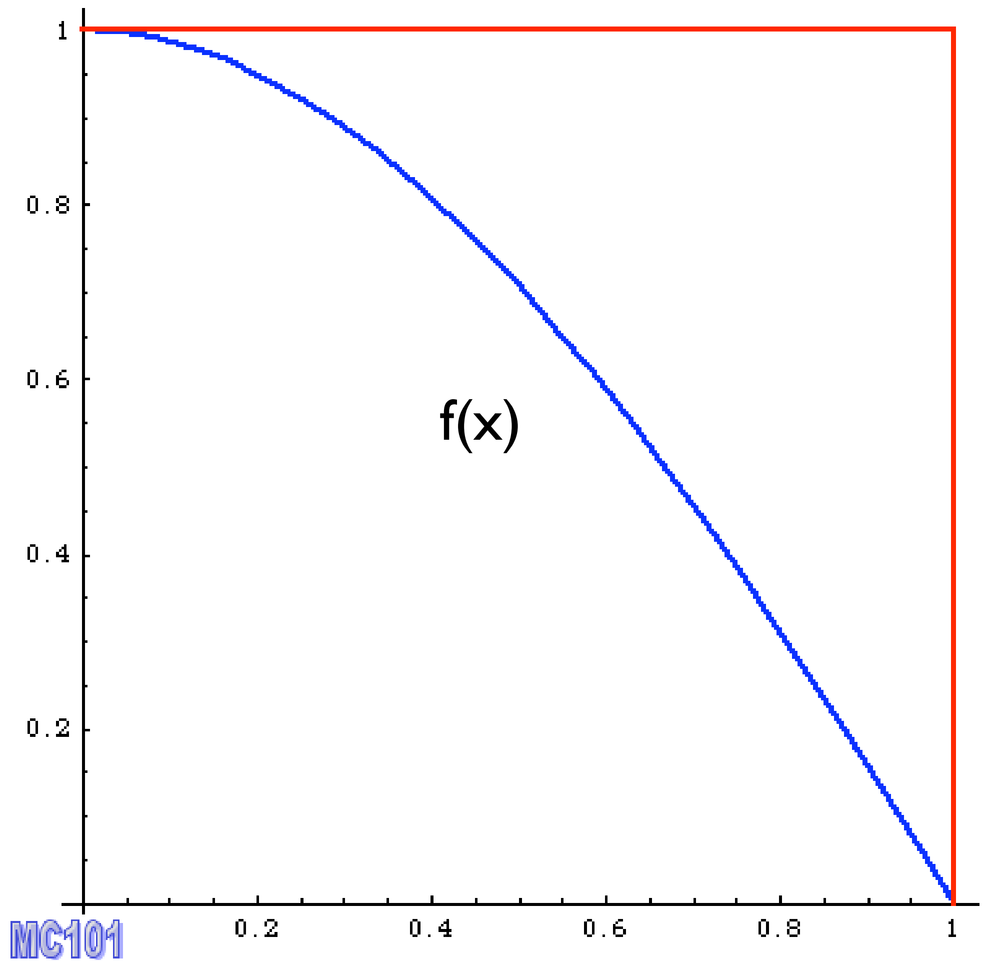


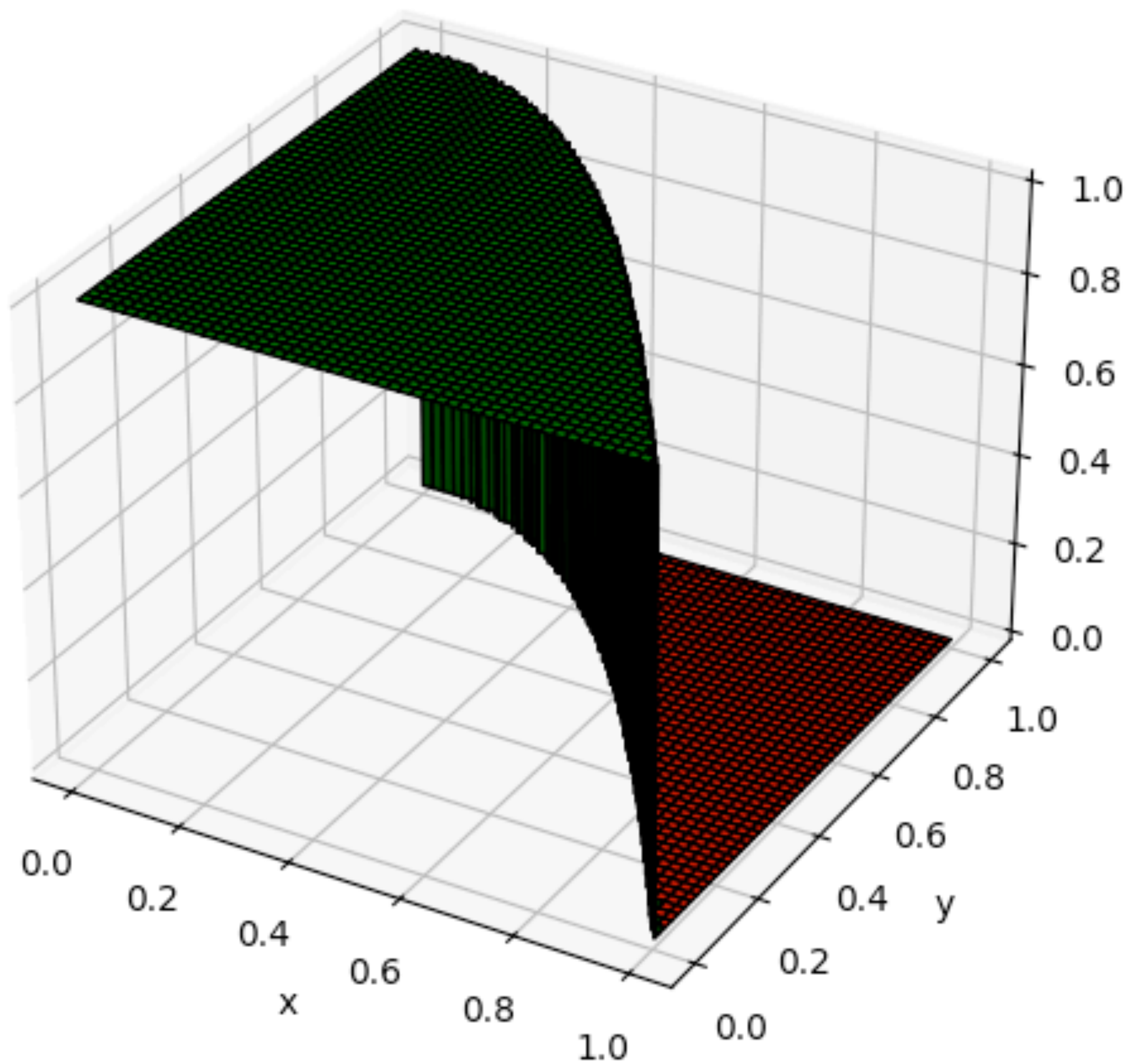


Monte-Carlo



Event generation





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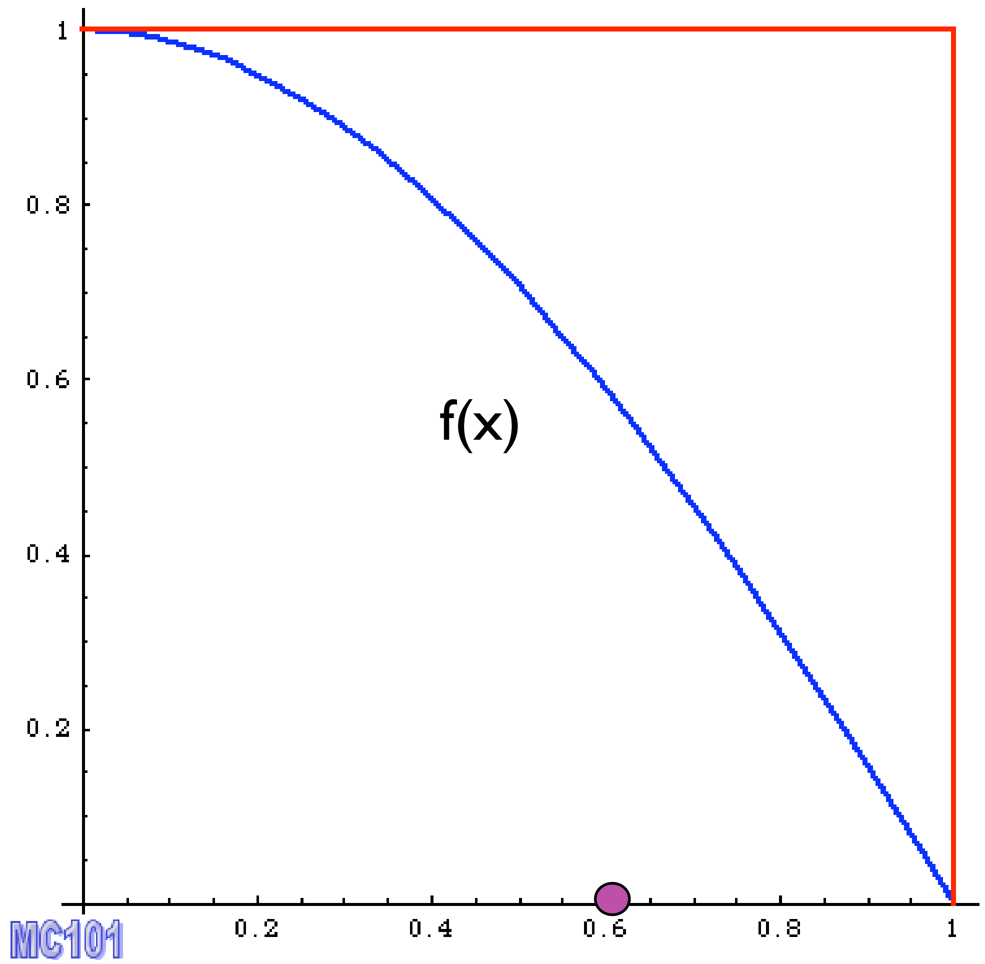
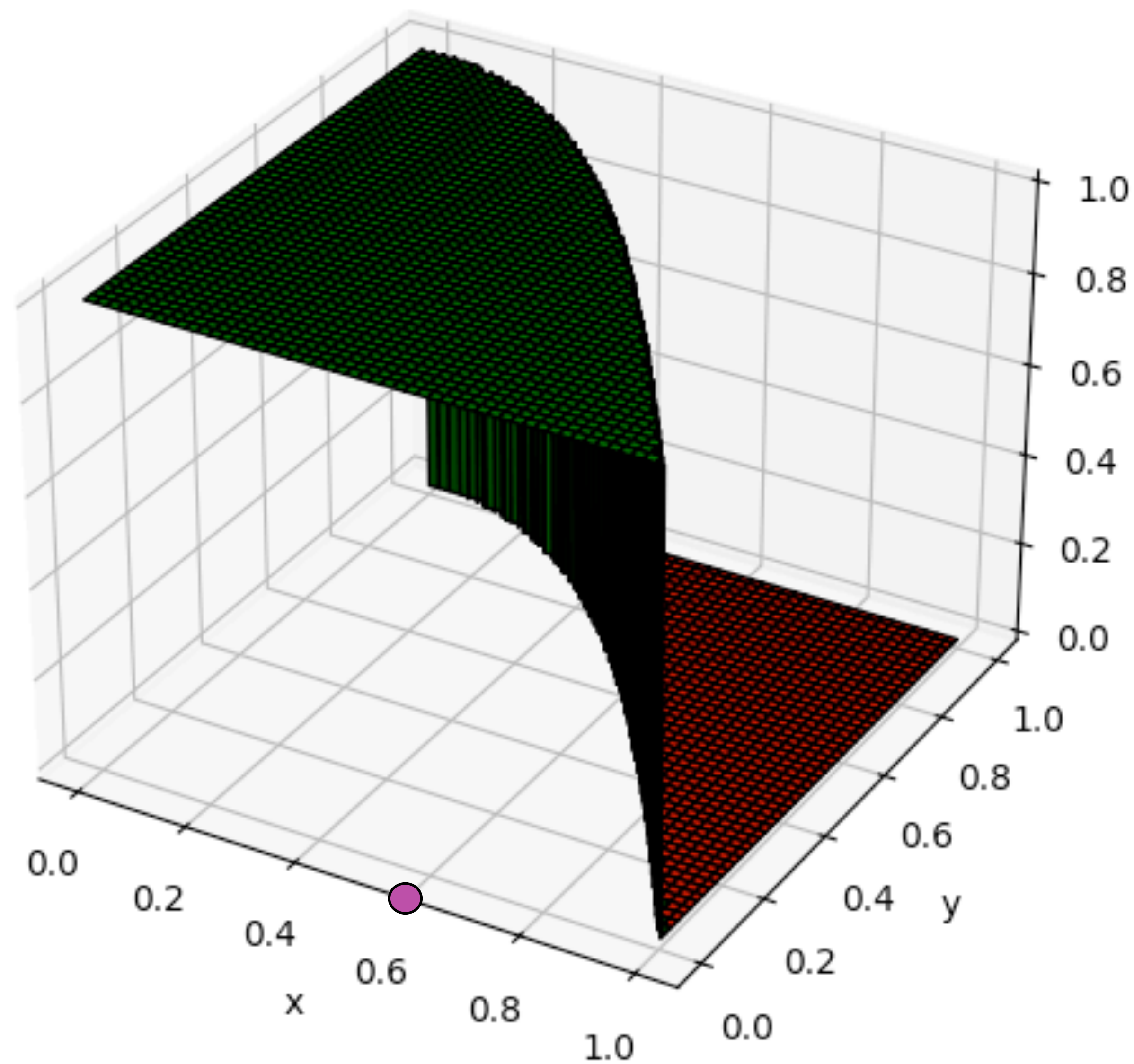


1. pick x_i



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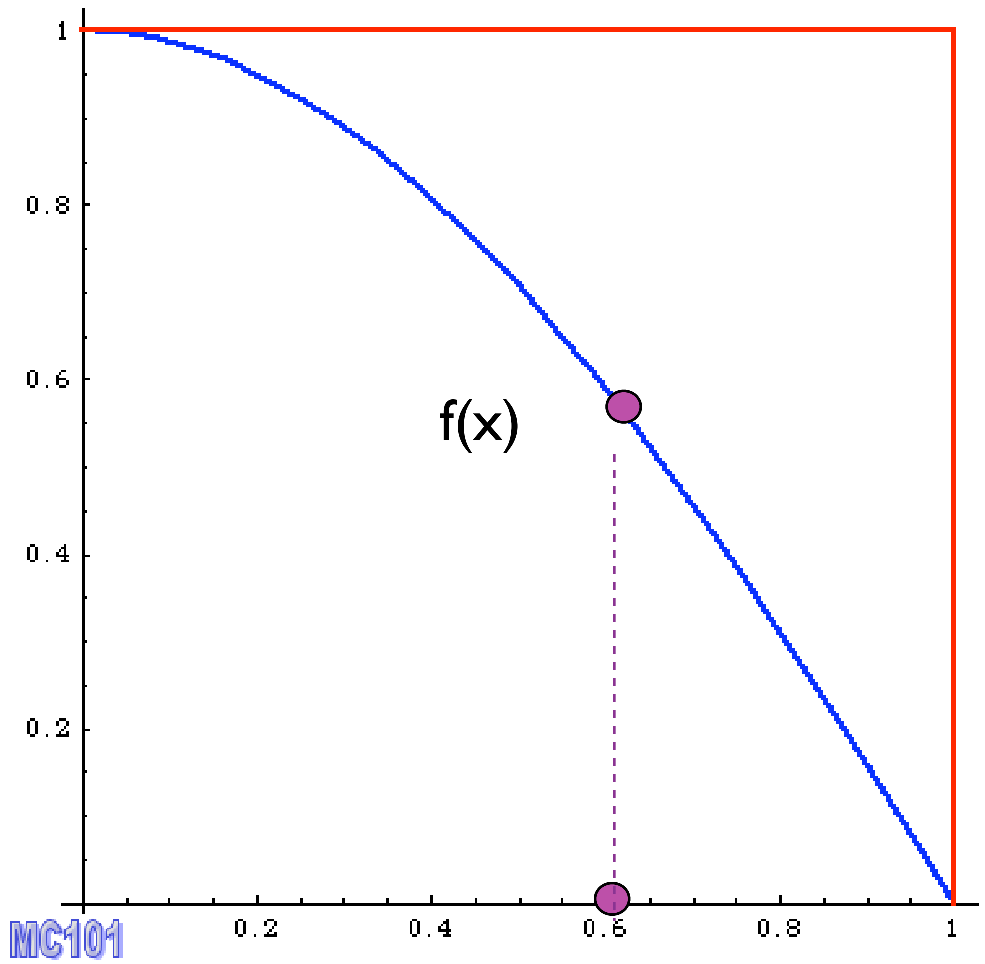
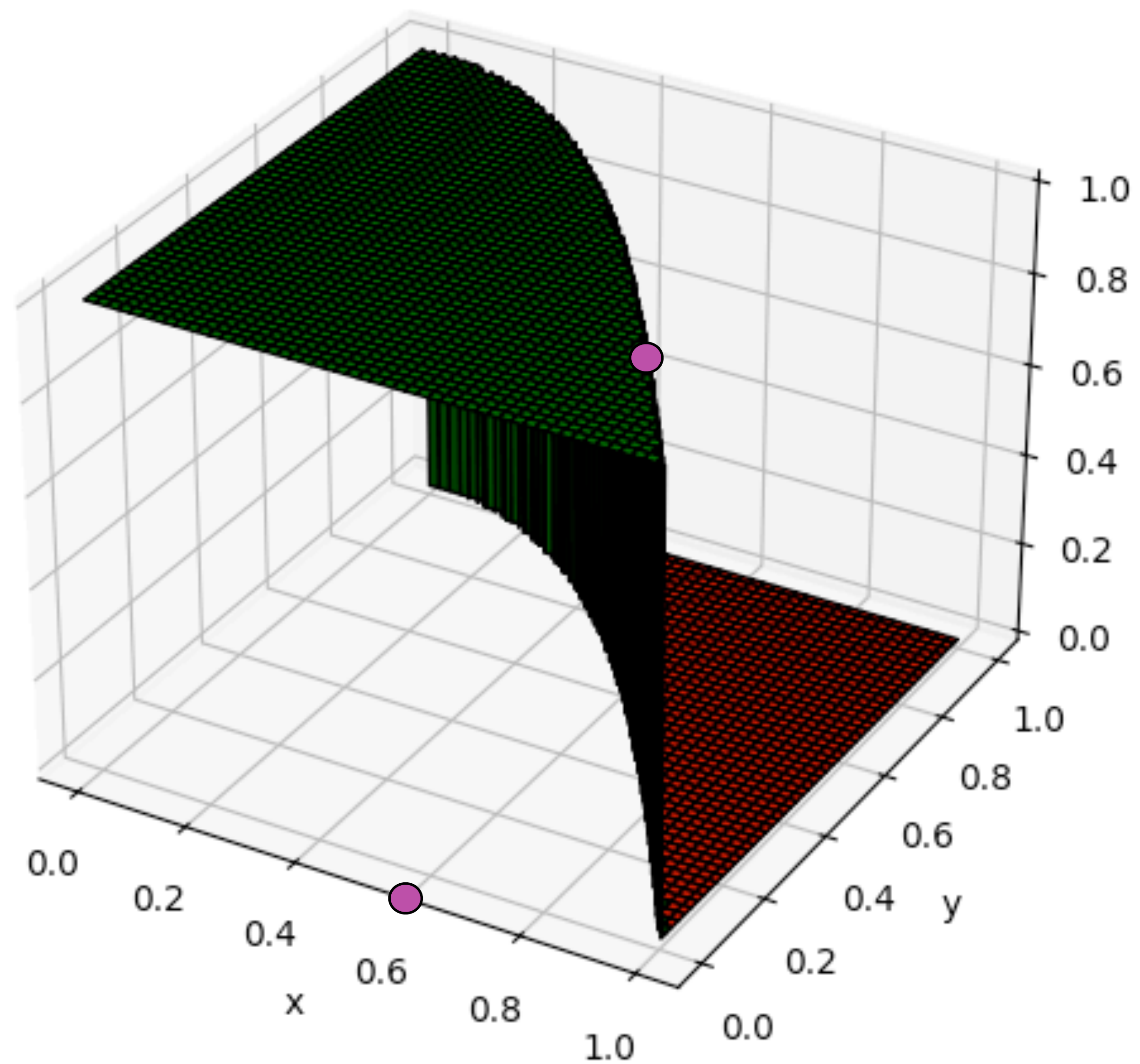
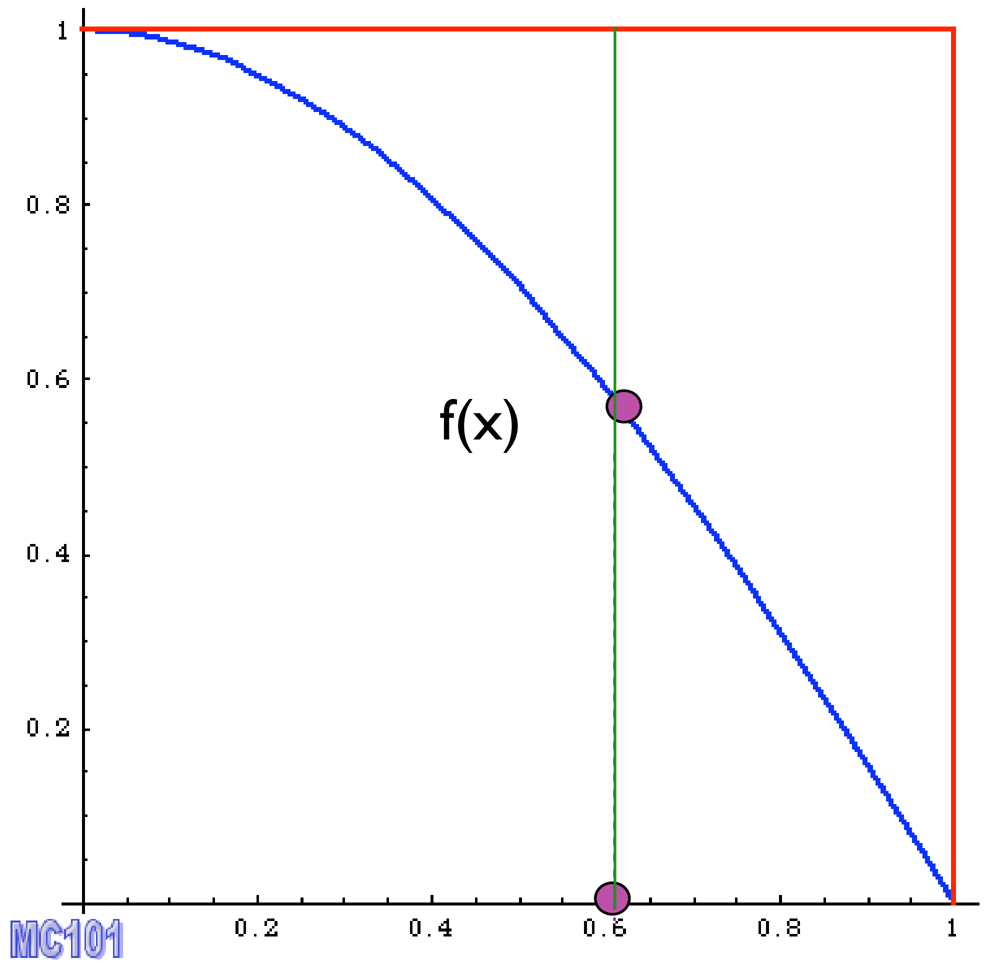


1. pick x_i

2. calculate $f(x_i)$



Event generation

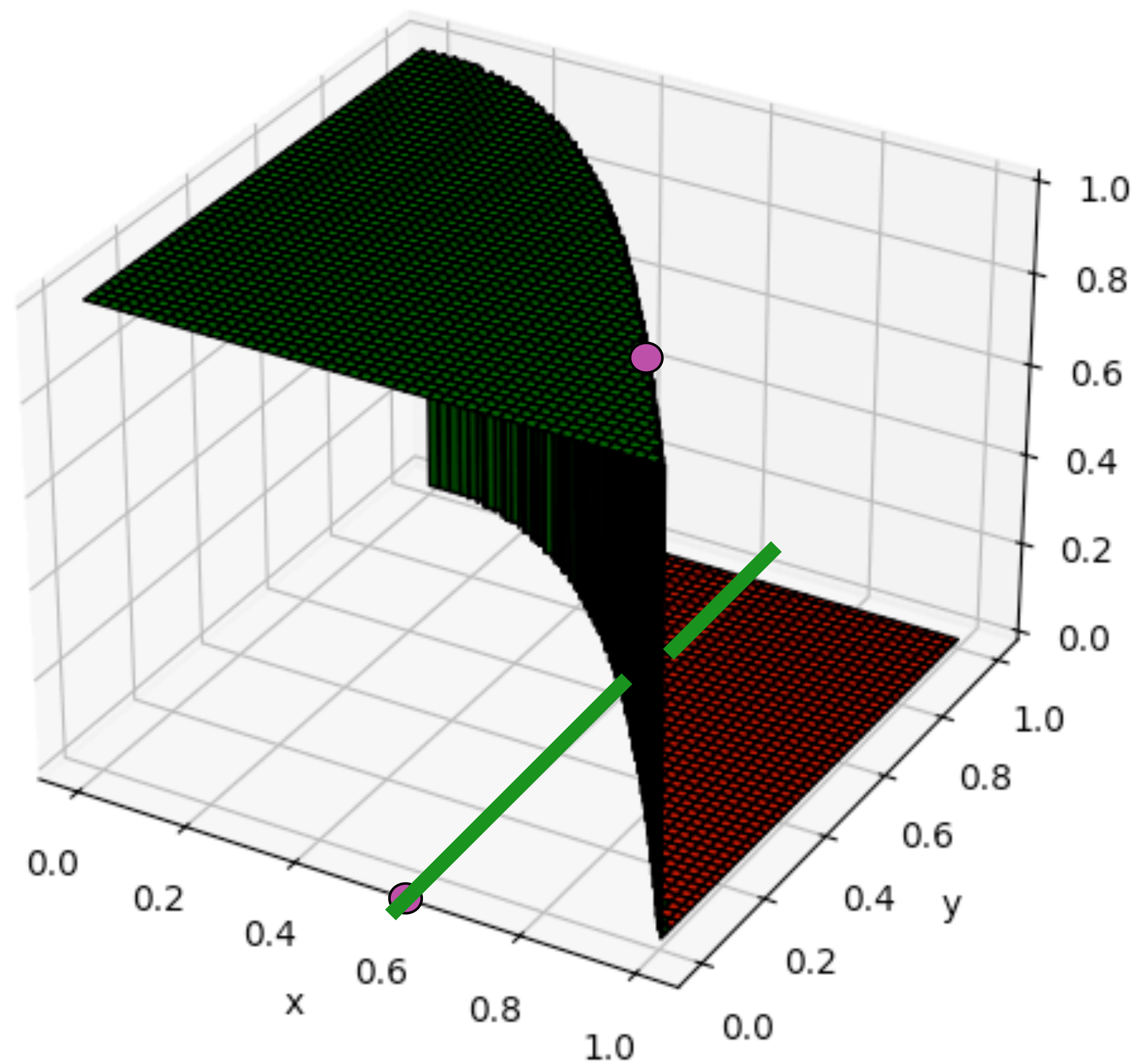


1. pick x_i

2. calculate $f(x_i)$

3. pick $y \in [0, max(f)]$

$$y < f(x_i)$$



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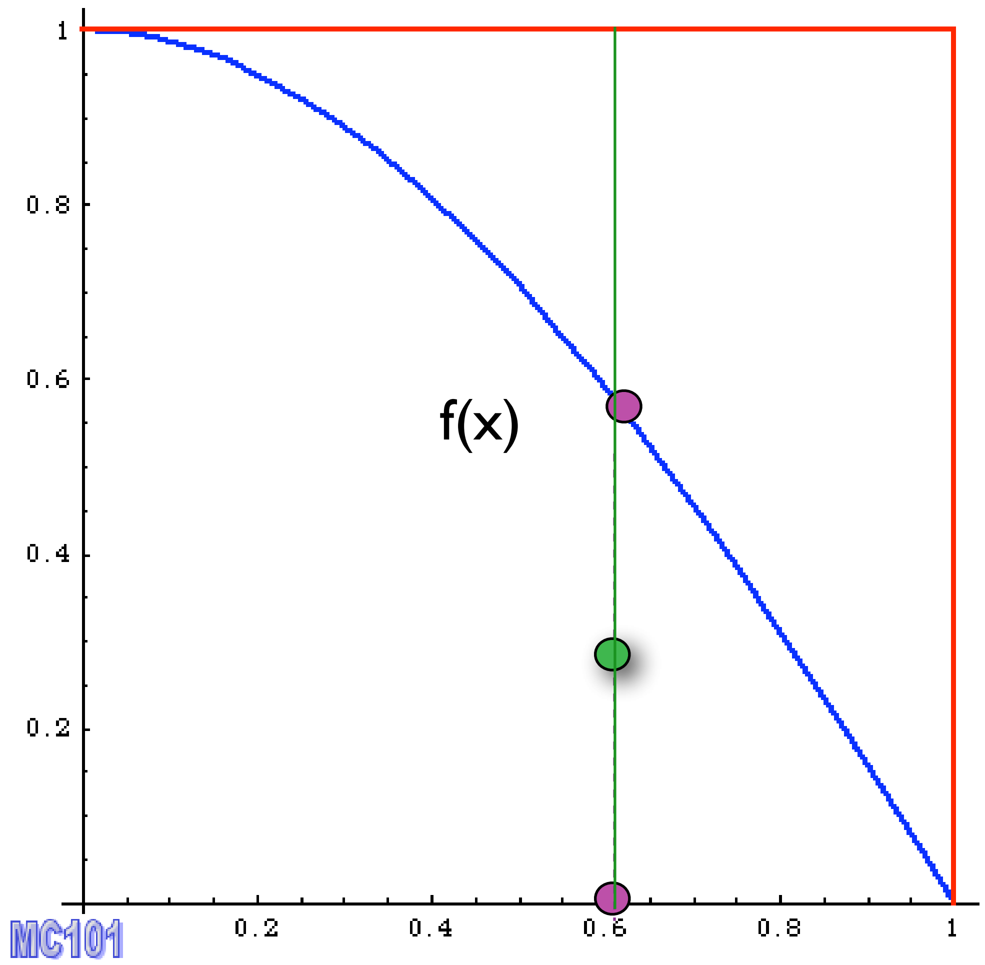

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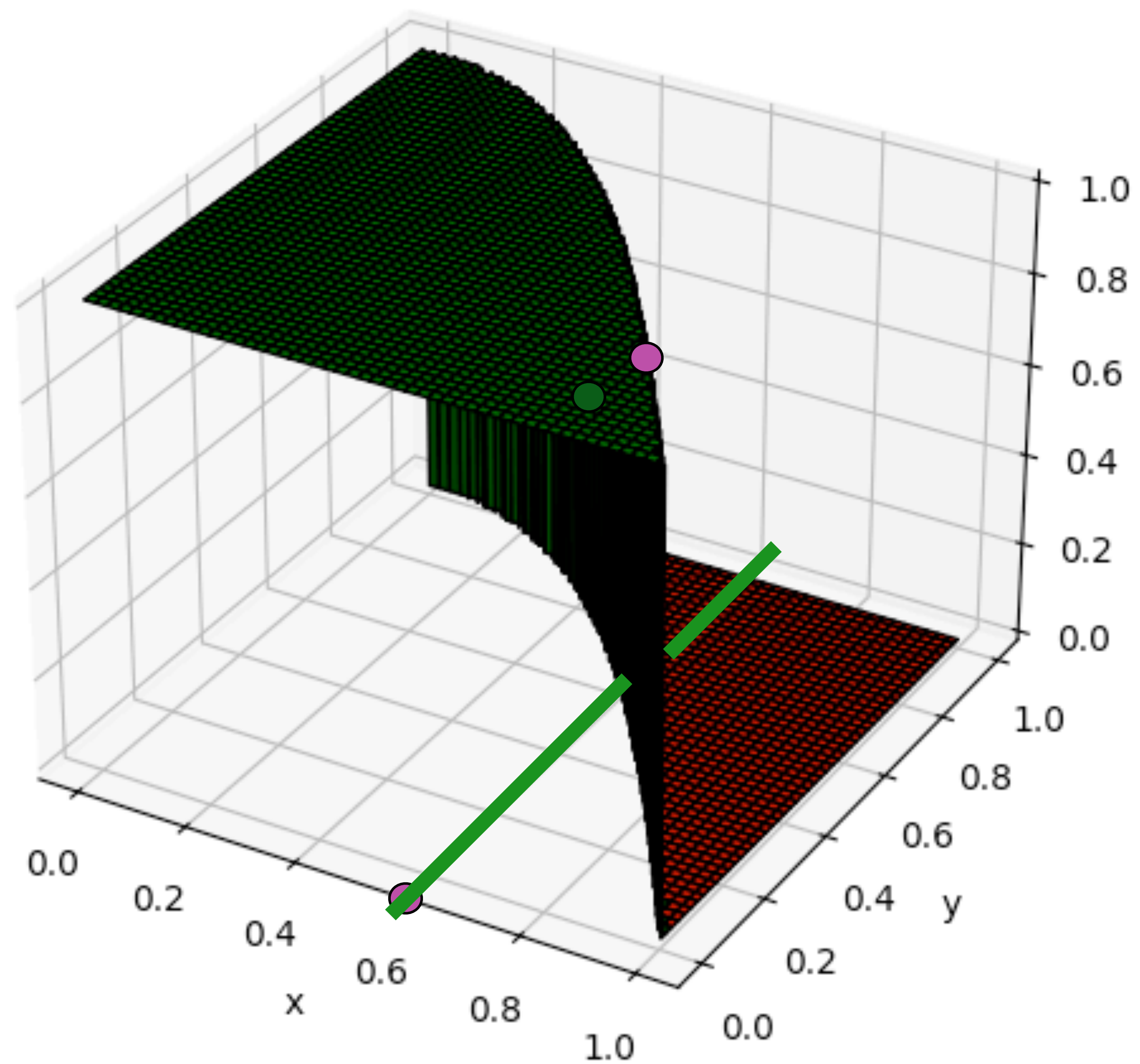
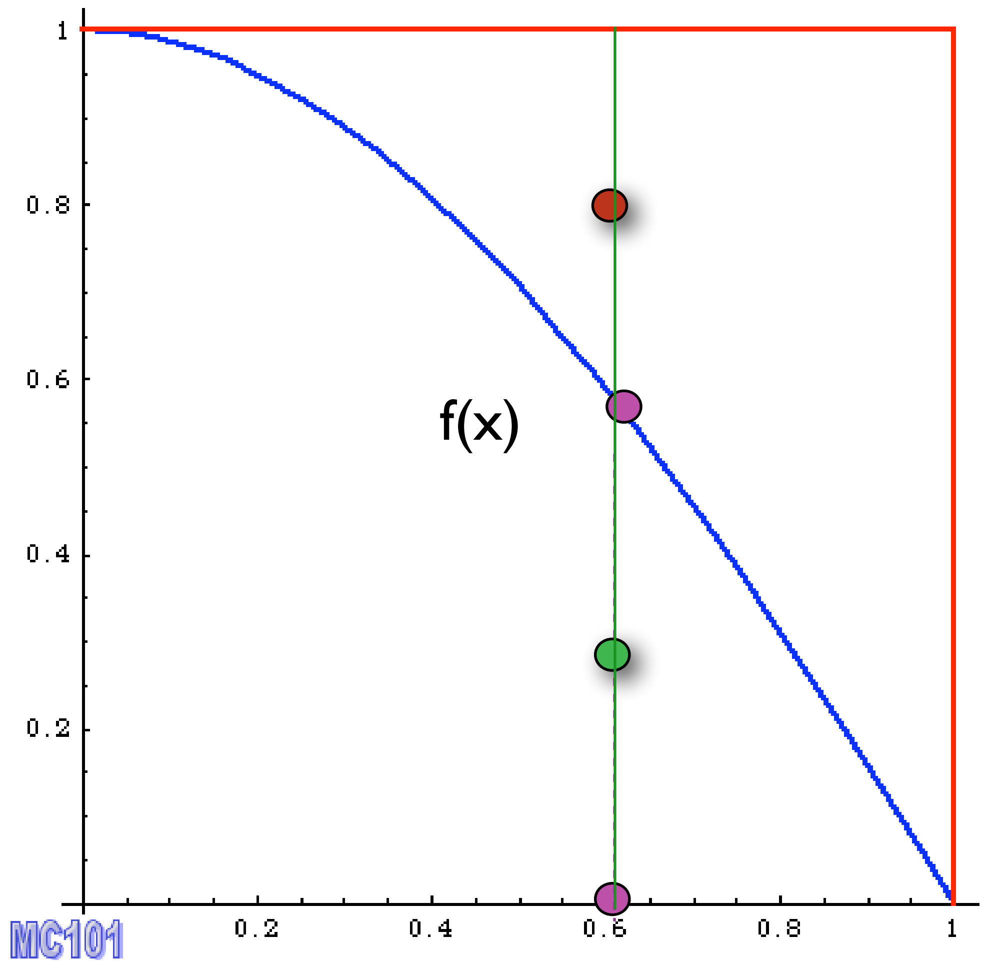


4. Compare:
   if $y < f(x_i)$ accept event,

Event generation



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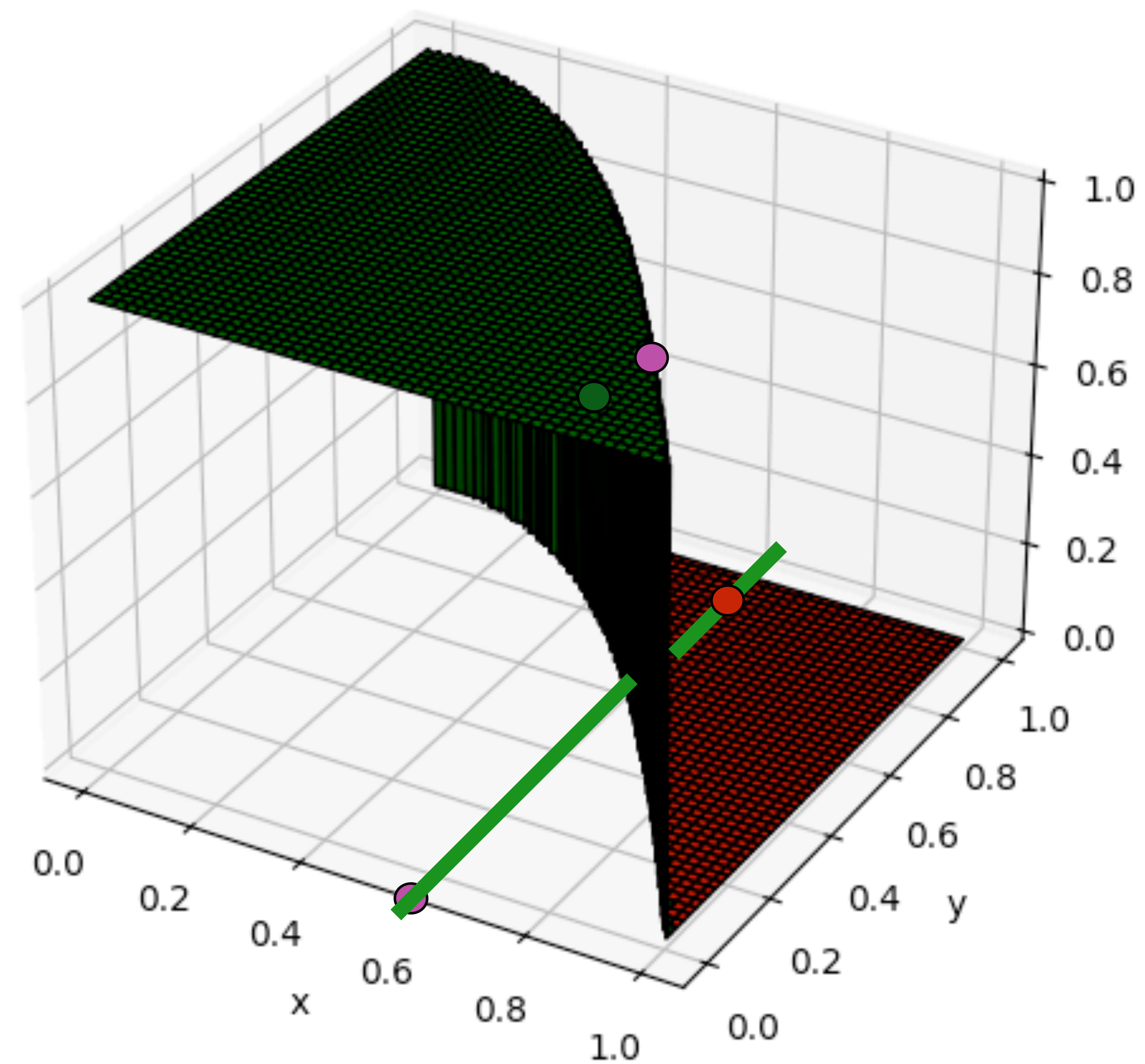
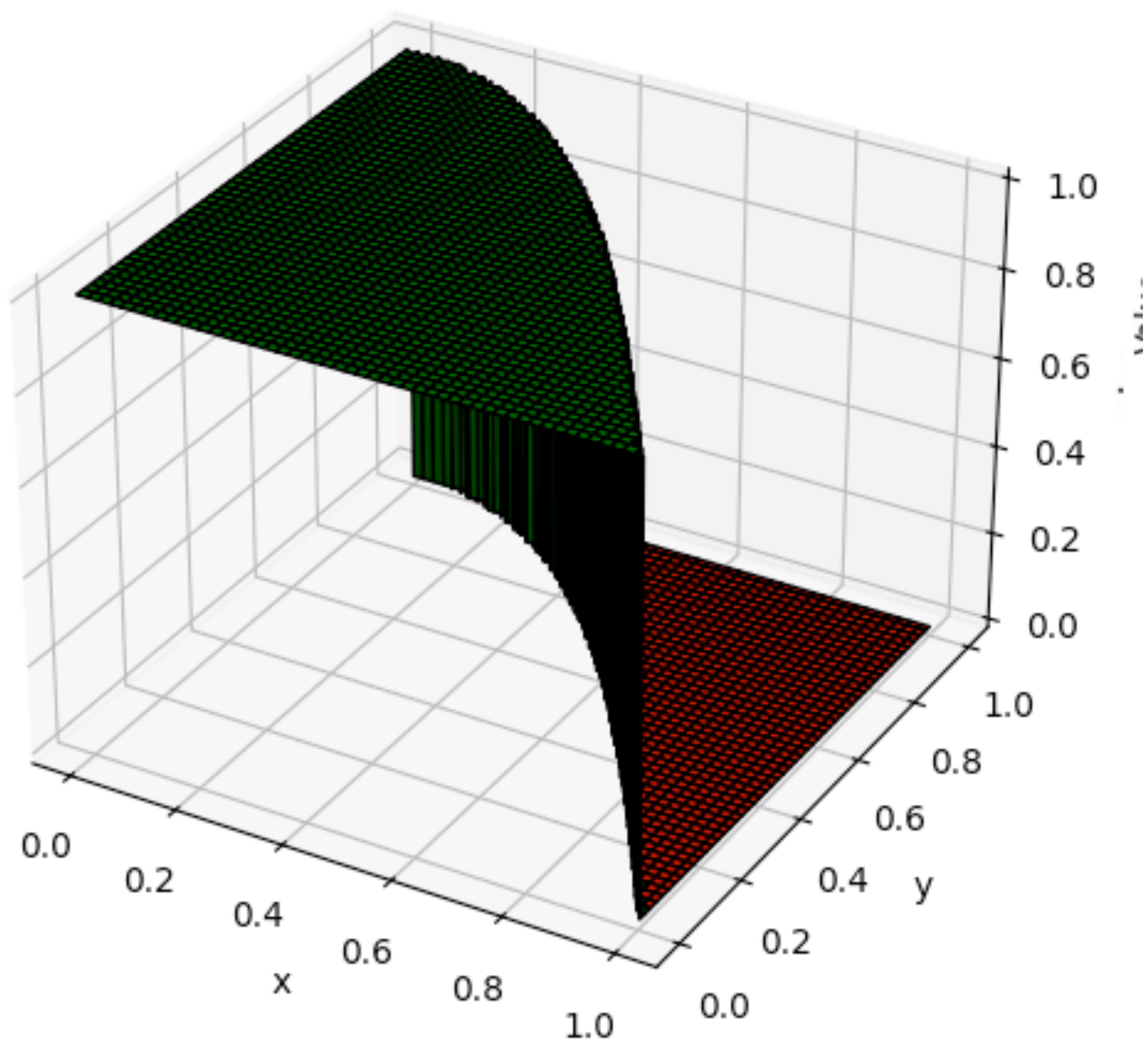
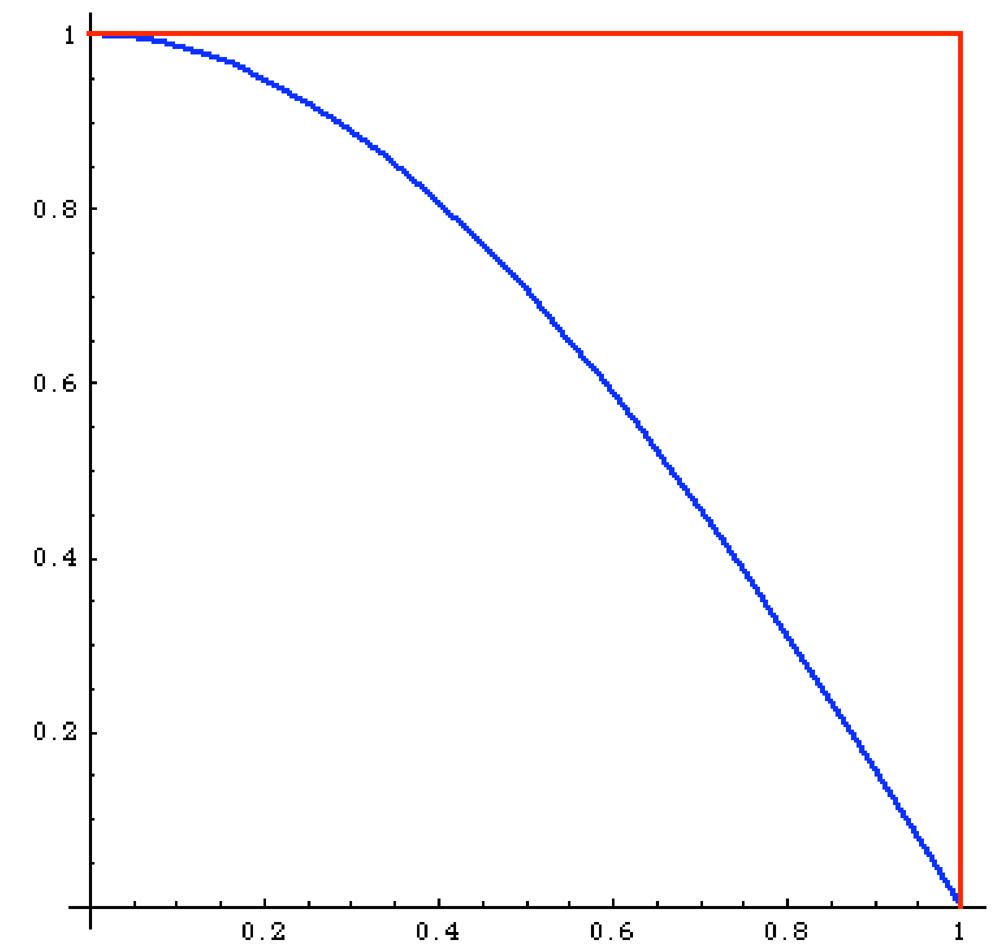


4. Compare:
   if $y < f(x_i)$ accept event,

   else reject it.

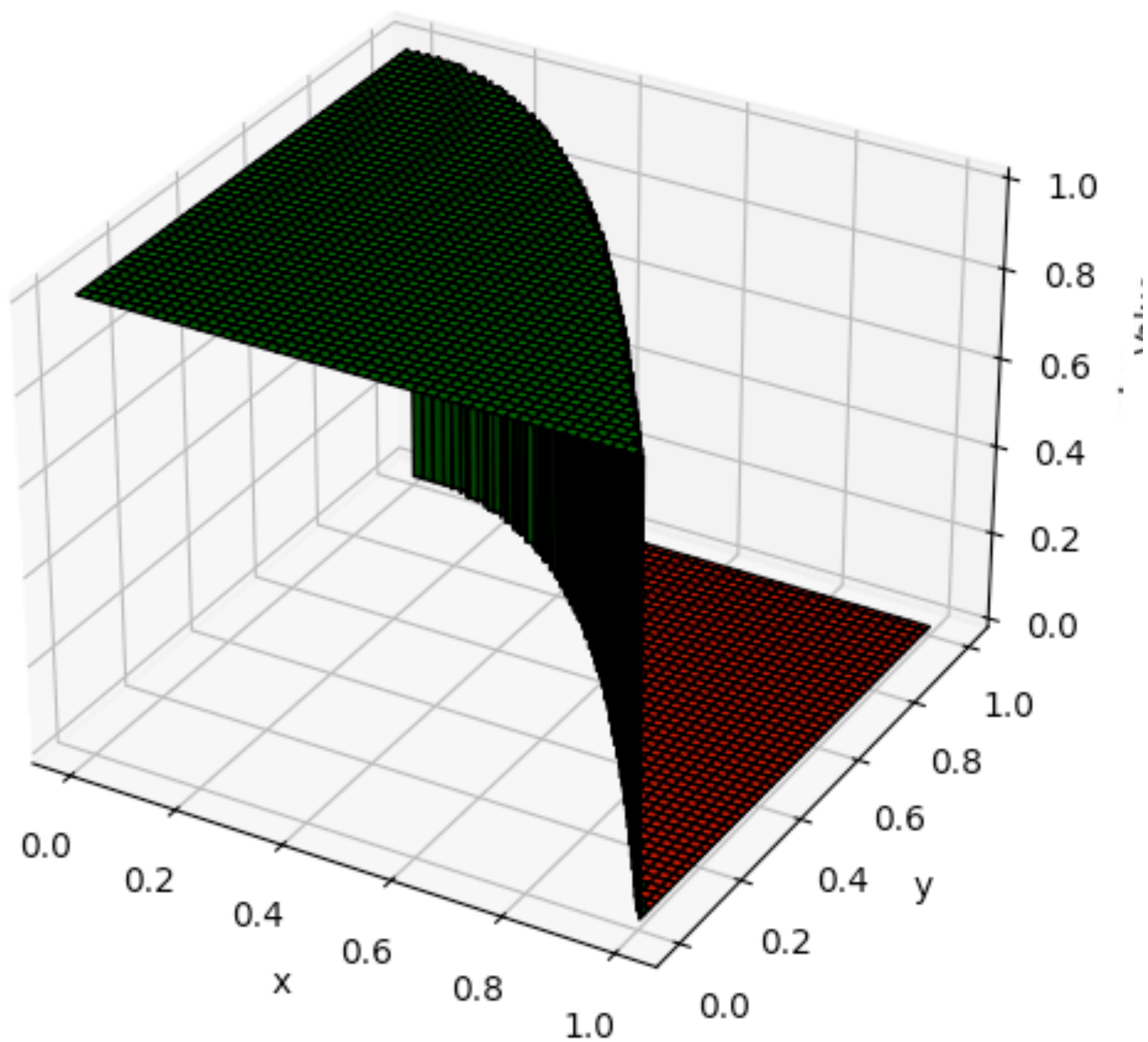
Do we need to keep small weight?

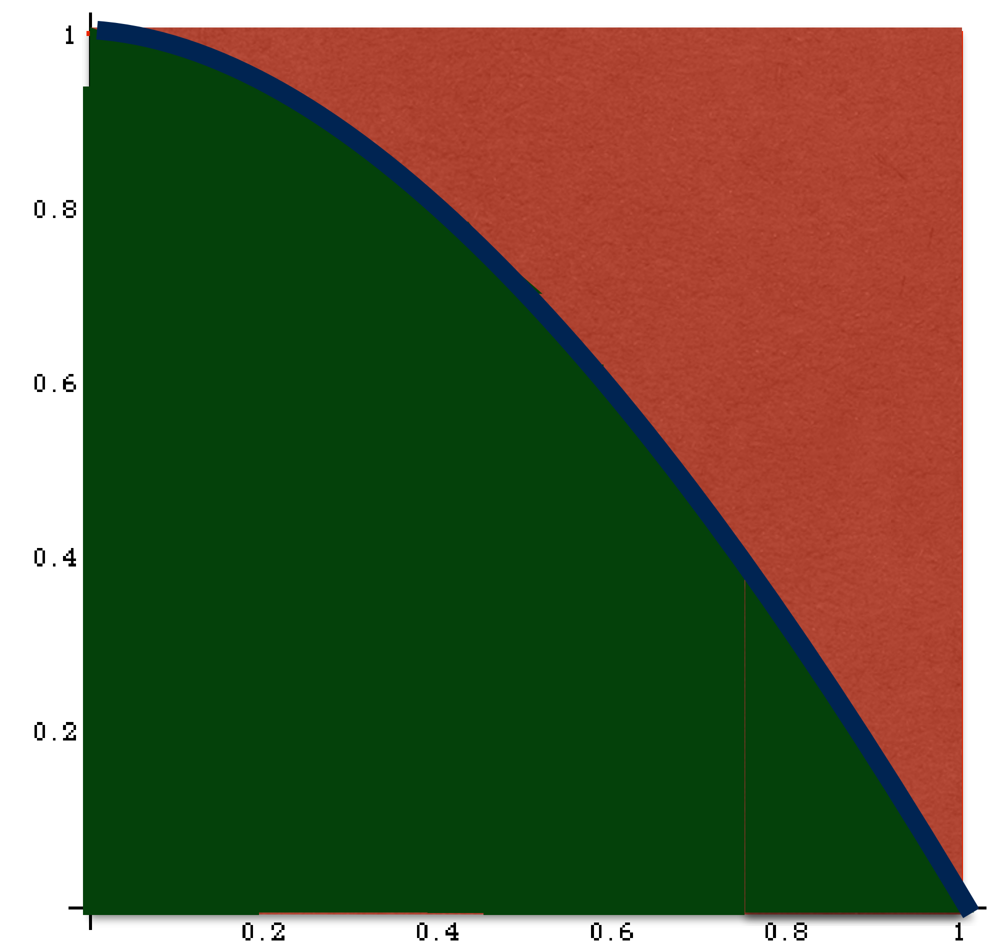




View from above

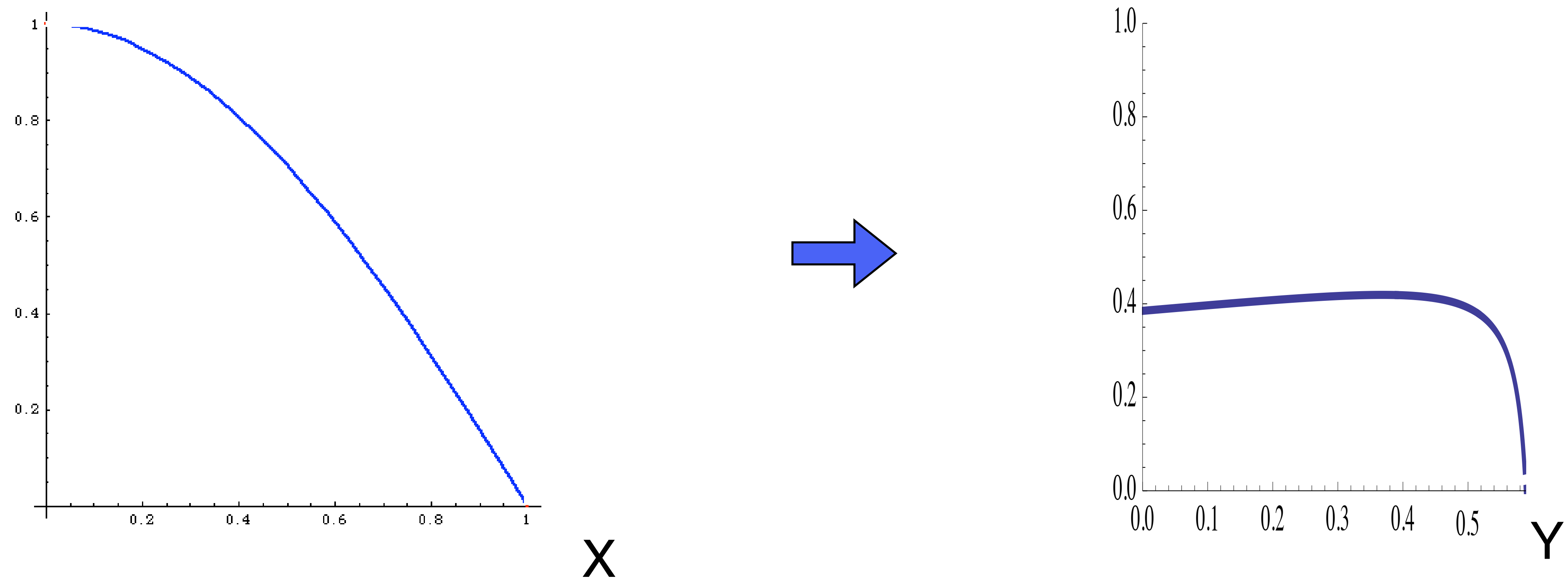
Do we need to keep small weight?





View from above

Importance Sampling



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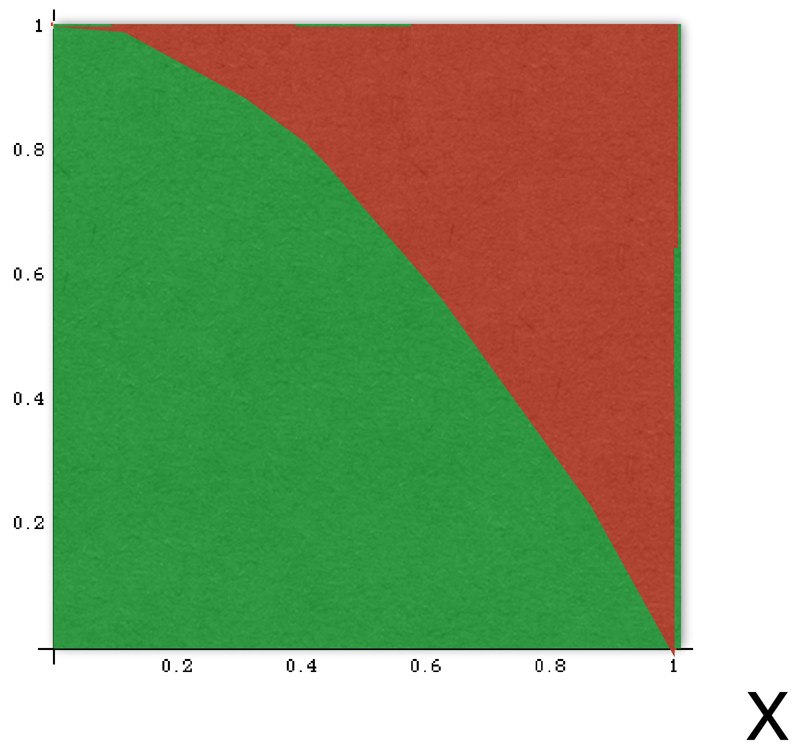

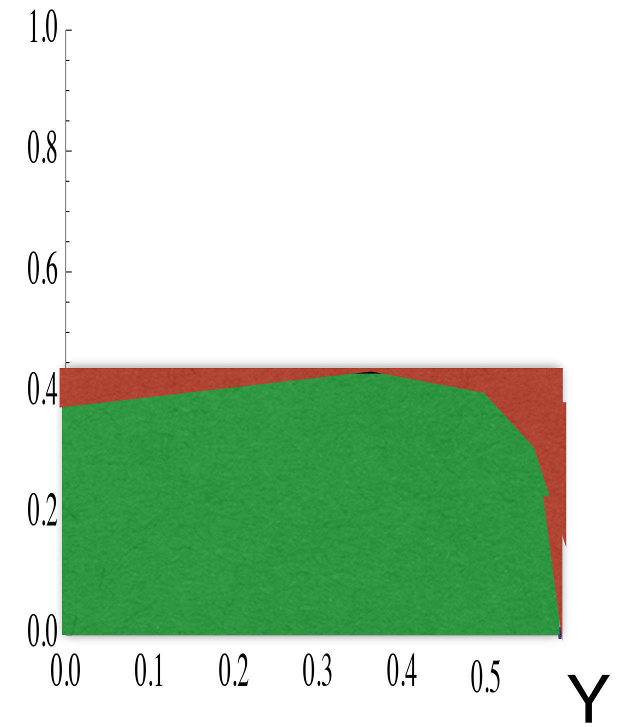

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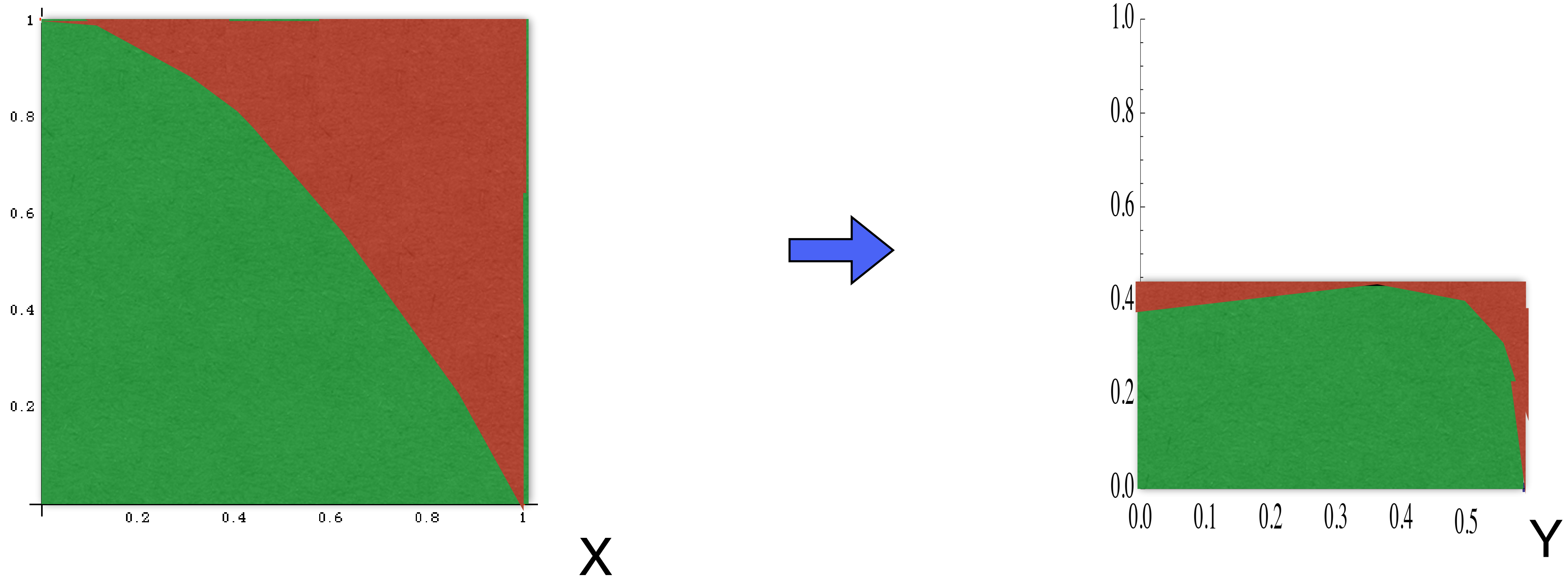
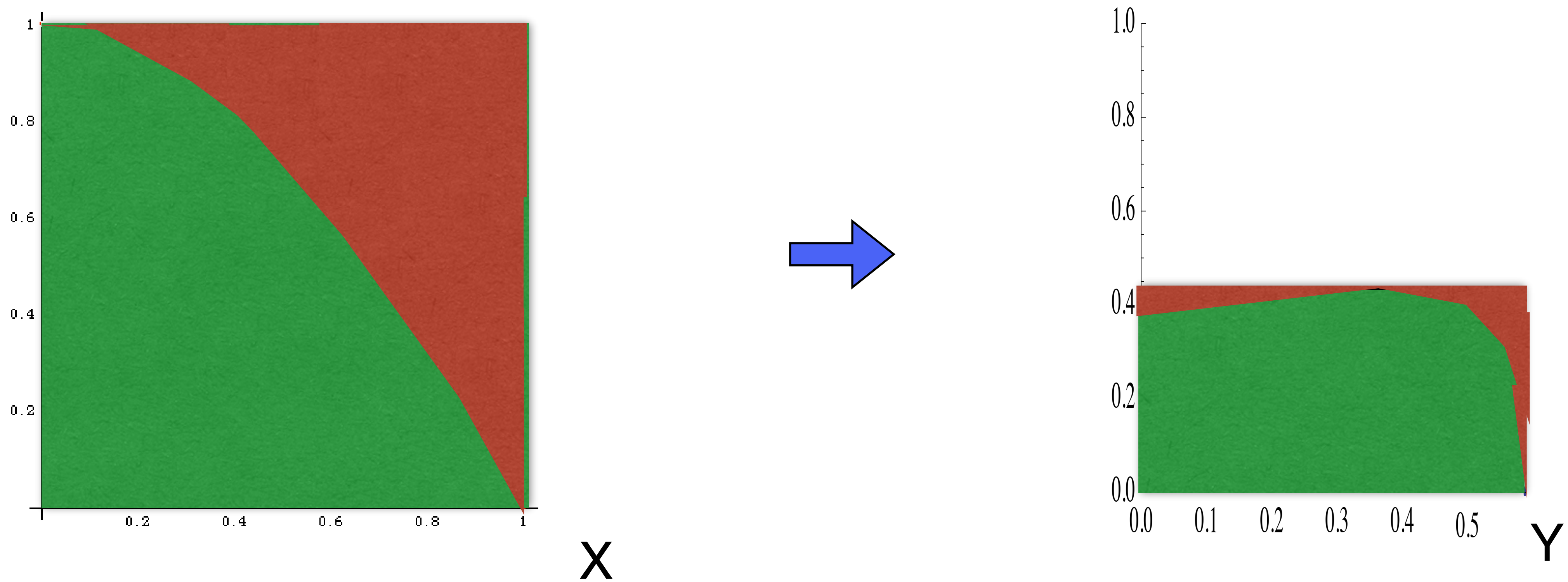


- Having smaller variance (flatter function) also allows to have better unweighting efficiency (i.e. faster code)

Importance Sampling



- Having smaller variance (flatter function) also allows to have better unweighting efficiency (i.e. faster code)

Limitations:

- Positive sign only
- The maximum should exists

Question time

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Code d'événement
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Monte-Carlo Summary

Bad Point

- Slow Convergence (especially in low number of Dimension)
- Need to know the function
  - Impact on cut

Monte-Carlo Summary

Bad Point

- Slow Convergence (especially in low number of Dimension)
- Need to know the function
  - Impact on cut

Good Point

- Complex area of Integration
- Easy error estimate
- quick estimation of the integral
- Possibility to have unweighted events

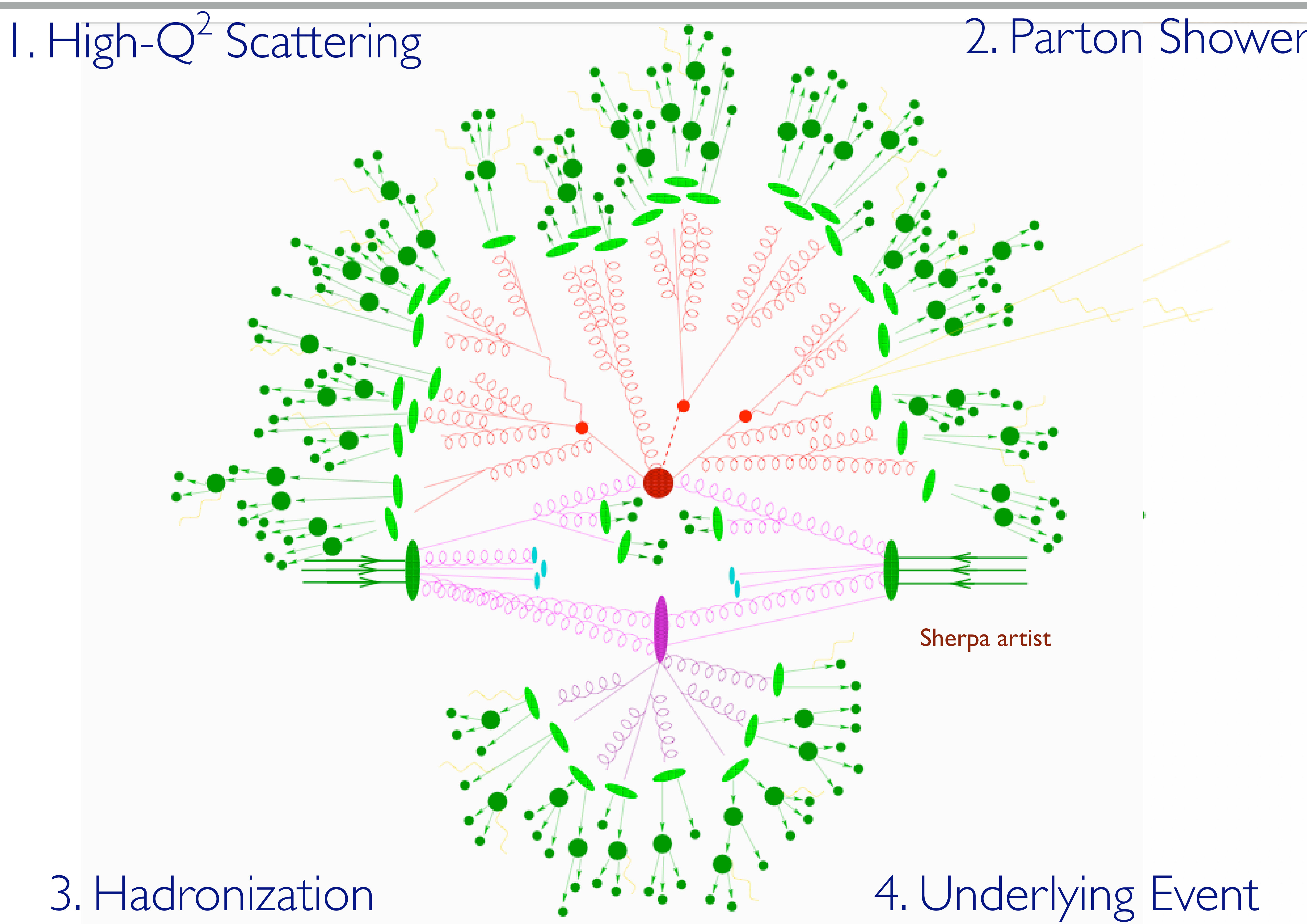
What to remember

- Analytical computation can be slower than numerical method
- Any BSM model are supported (at LO)
- Phase Space integration are slow
    - need knowledge of the function
    - cuts can be problematic
- Event generation are from free.
- All this are automated in MG5_aMC@NLO
- Important to know the physical hypothesis

Parton Shower

Olivier Mattelaer
CP3/UCLouvain

What are the MC for?



Parton shower

Goal

- We want to an **explicit** description of the SOFT radiation that are **ALREADY** included **implicitly in the LO events** (via the scale)

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Important

- Parton-Shower is **not ADDING radiation**
    - Such radiations are already included within the event-generator

Parton shower

Goal

- We want to an **explicit** description of the SOFT radiation that are **ALREADY** included **implicitly in the LO events** (via the scale)

Important

- Parton-Shower is **not ADDING radiation**
  - Such radiations are already included within the event-generator

- We need to be able to describe an arbitrarily number of parton branchings, i.e. we need to 'dress' partons with radiation
- This effect should be **unitary:** the inclusive cross section shouldn't change when extra radiation are described

Question time

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Code d'événement
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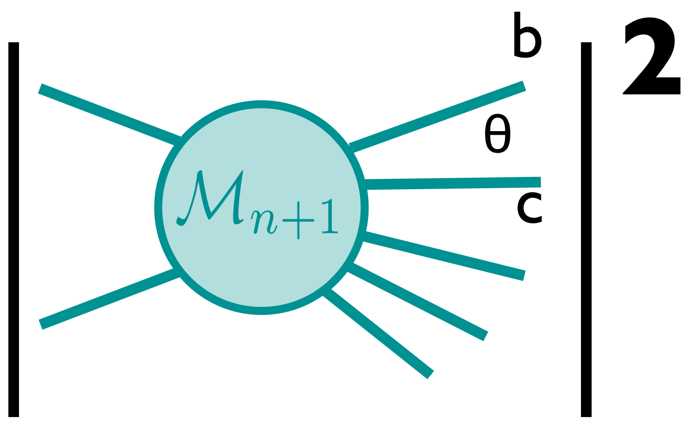
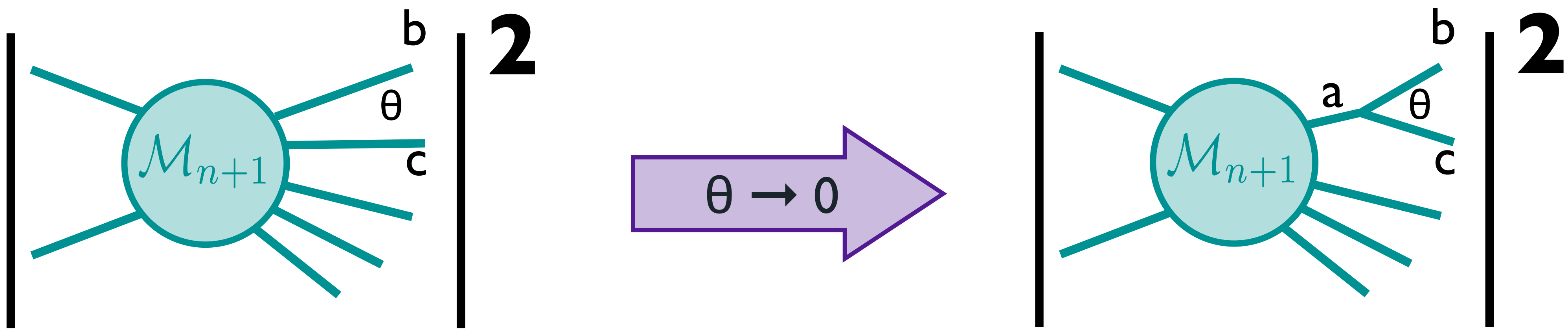
Collinear factorization



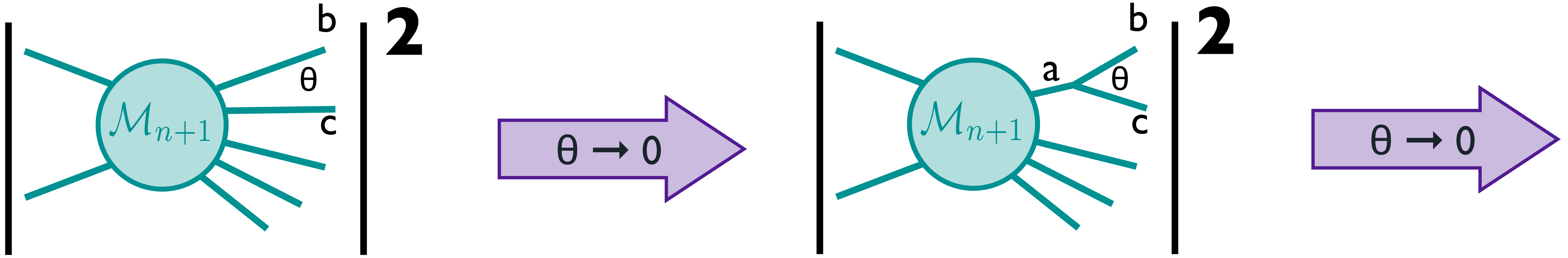
- Consider a process for which two particles are separated by a small angle θ.

Collinear factorization



- Consider a process for which two particles are separated by a small angle θ.
- In the limit of θ → 0 the contribution is coming from a single parent particle going on shell: therefore its branching is related to time scales which are very long with respect to the hard subprocess.

Collinear factorization



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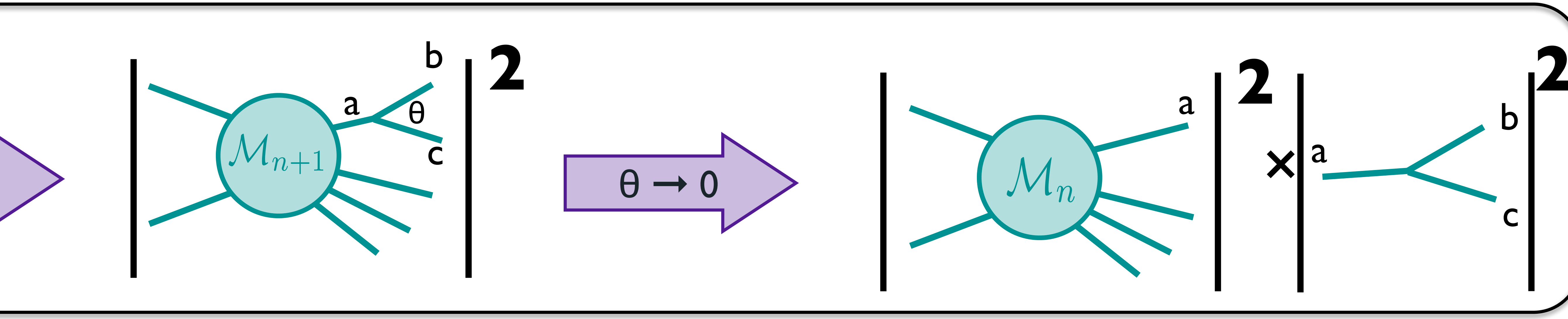

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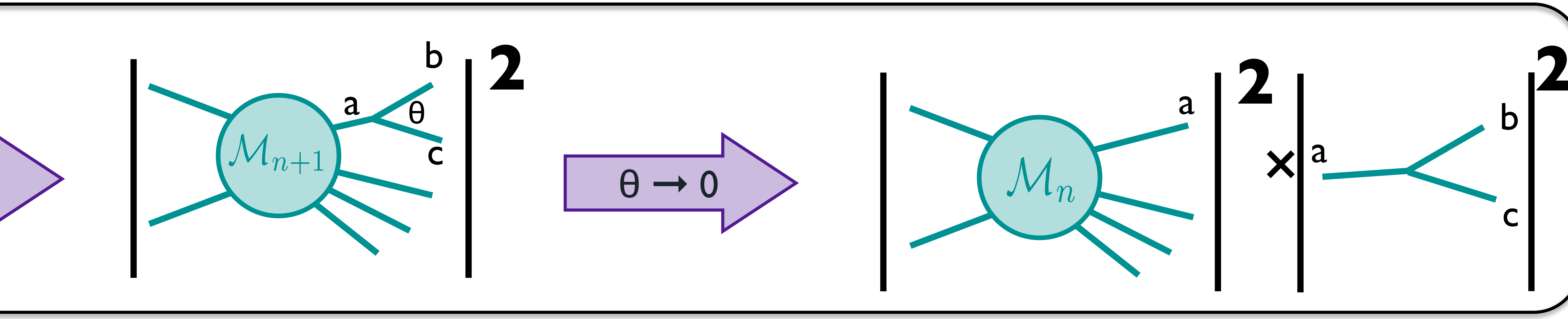
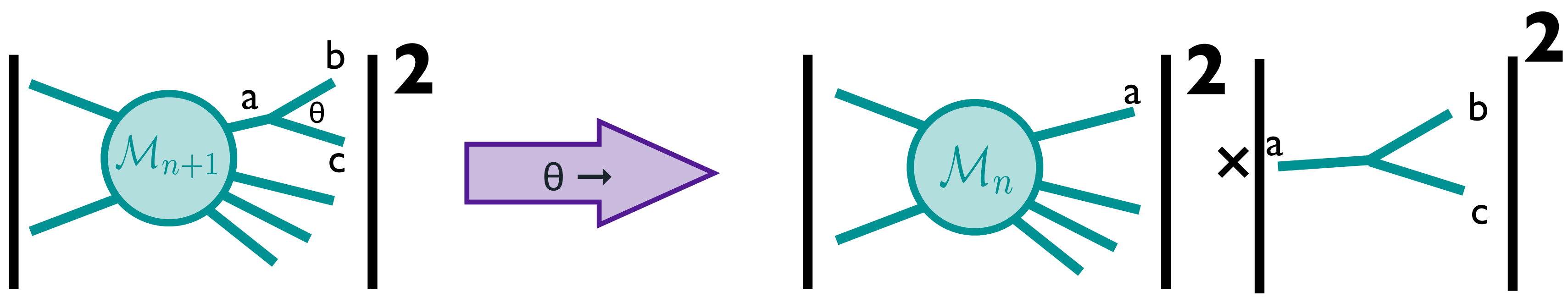


- Consider a process for which two particles are separated by a small angle θ.
- In the limit of θ → 0 the contribution is coming from a single parent particle going on shell: therefore its branching is related to time scales which are very long with respect to the hard subprocess.
- The inclusion of such a branching cannot change the picture set up by the hard process: the whole emission process must be writable in this limit as the simpler one times a branching probability.

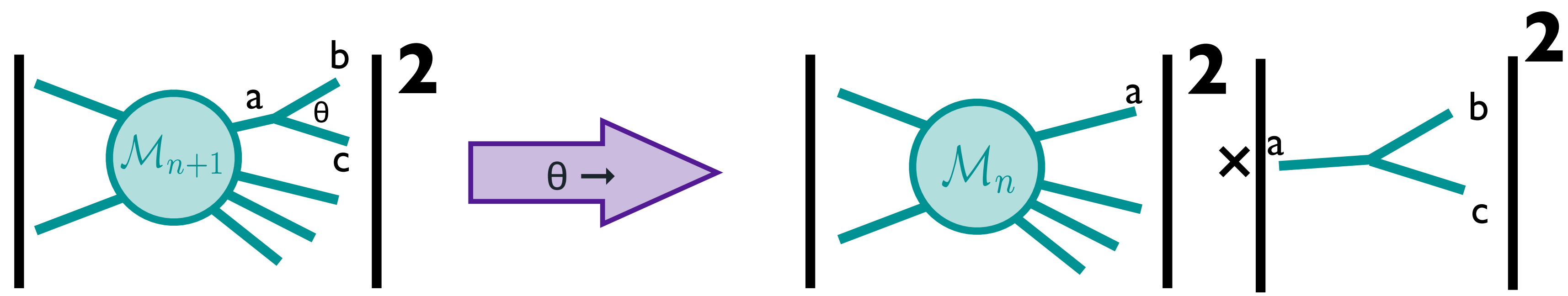
Collinear factorization



- The process factorizes in the collinear limit. This procedure it universal!

$$\frac{1}{(p_b + p_c)^2} \simeq \frac{1}{2E_b E_c (1 - \cos\theta)}$$

Collinear factorization



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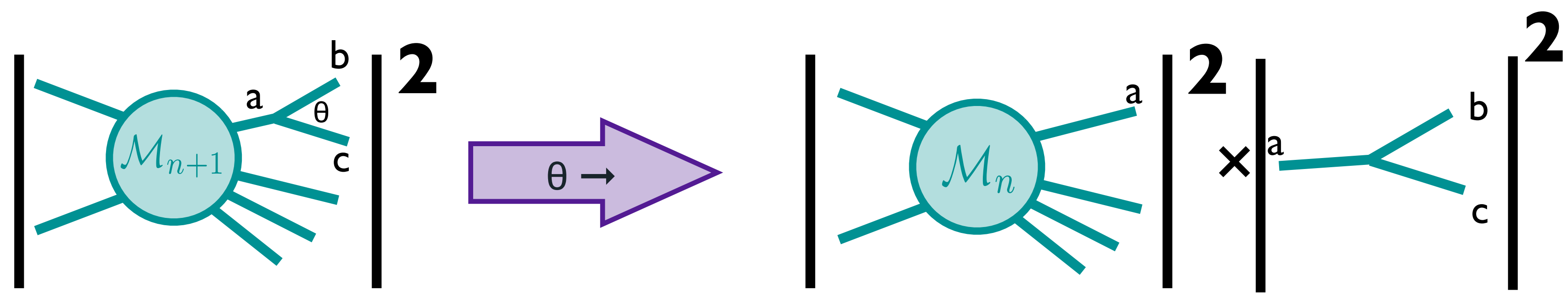
soft

Collinear factorization



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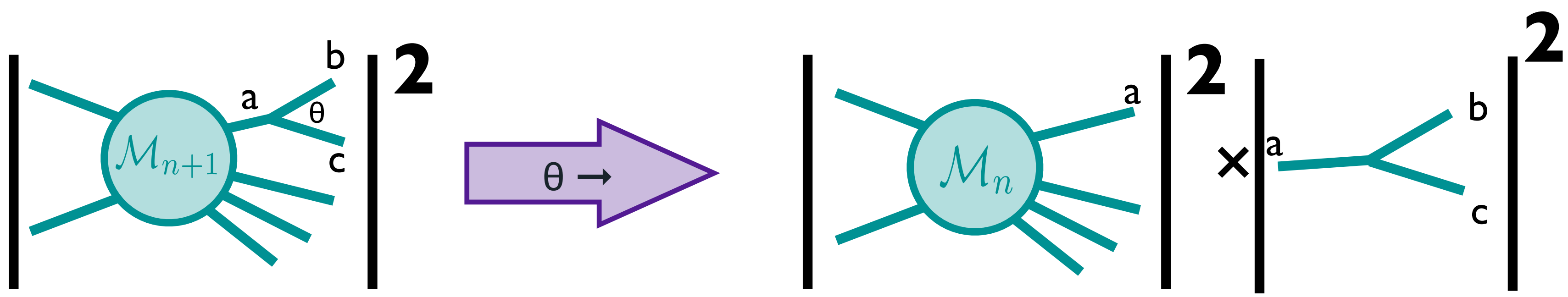
soft and collinear

divergencies

Collinear factorization



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soft and **collinear**

divergencies

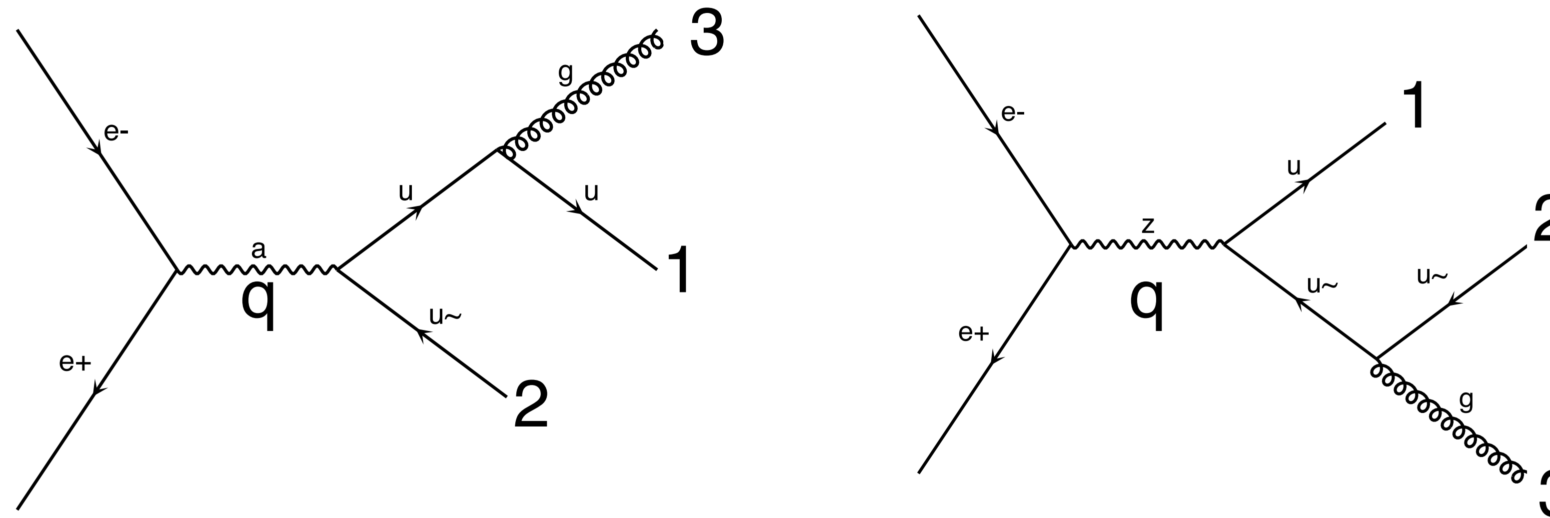
Collinear factorization:

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a\to bc}(z)$$

when θ is small.

First Example

$$e^+e^- \to q\bar{q}g$$



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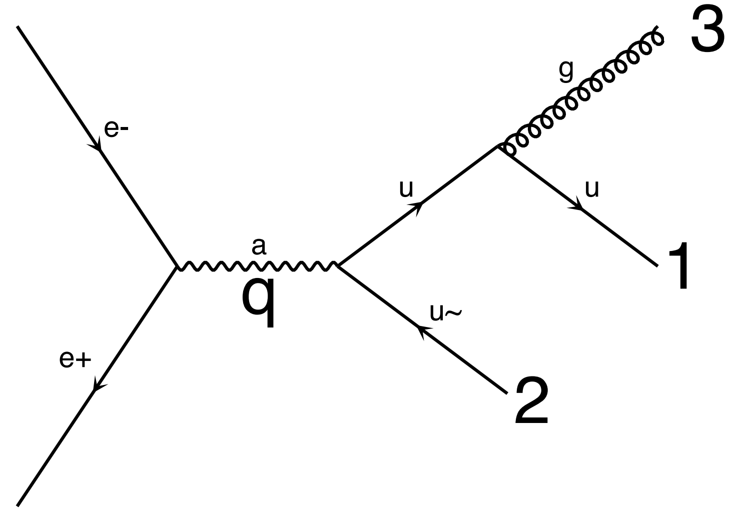

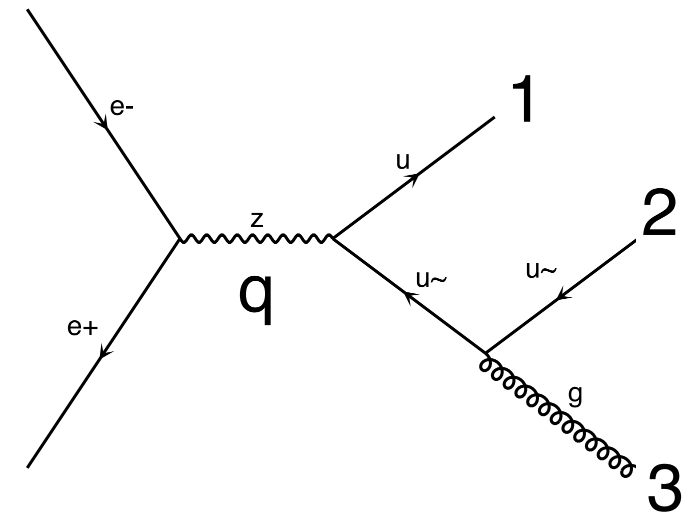

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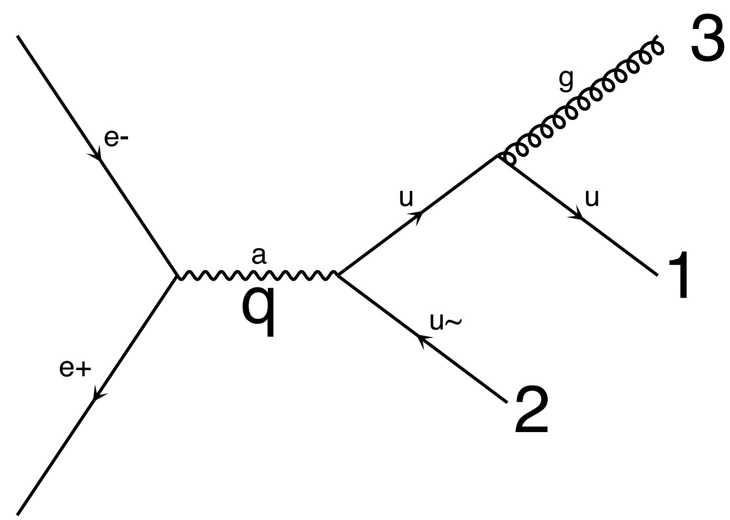

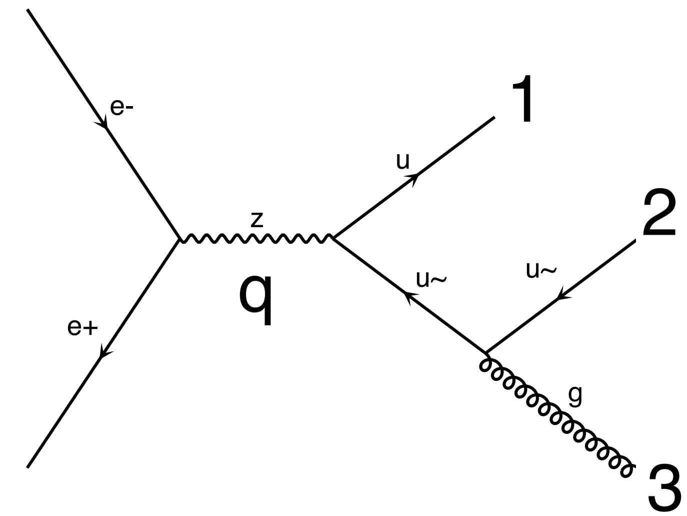

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  - Soft Divergencies
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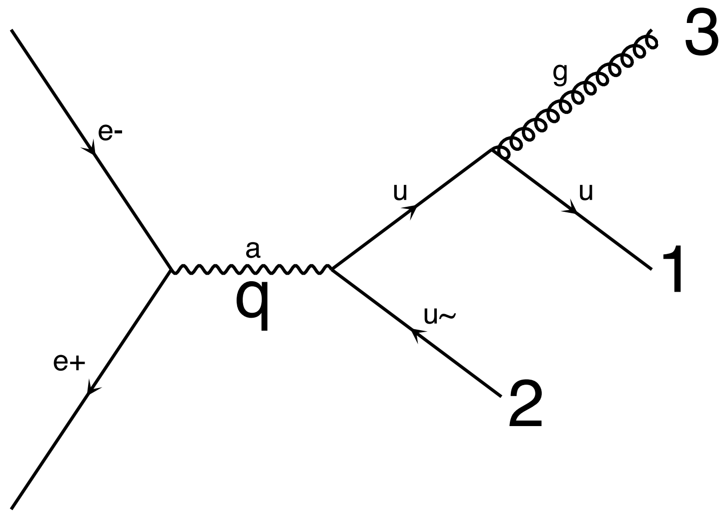

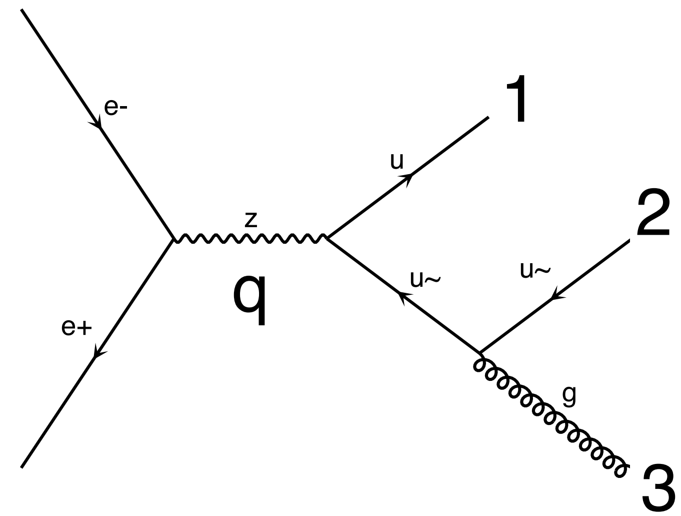

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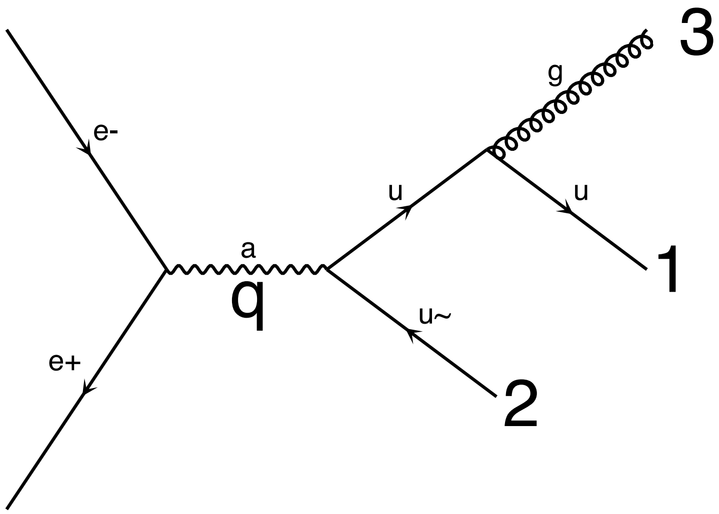

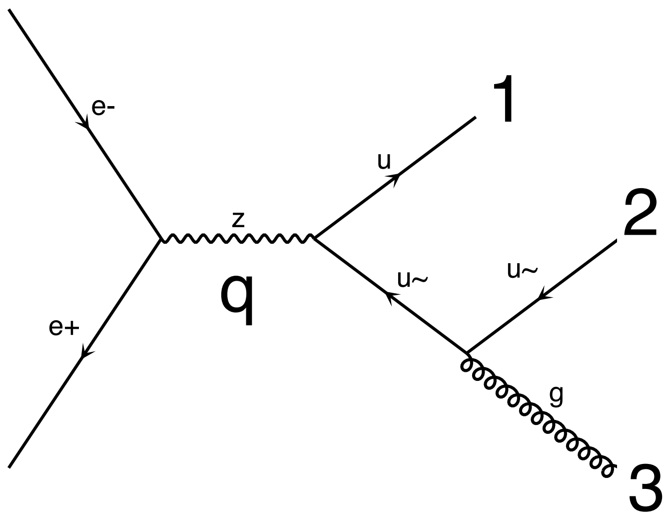

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  - Split our integral in two

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$$\approx \frac{d\theta_{13}^2}{\theta_{13}^2} + \frac{d\theta_{23}^2}{\theta_{23}^2}$$

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$$d\sigma = \sigma_0 \sum_{\text{jets}} C_F \frac{\alpha_s}{2\pi}\frac{d\theta^2}{\theta^2}\, dz \frac{1+(1-z)^2}{z}$$

☞ z fraction of energy

☞ **Generic Formula**

Parton Shower basics

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

The spin averaged (unregulated) splitting functions for the various types of branching are (Altarelli-Parisi):

$$\hat{P}_{qq}(z) = C_F\left[\frac{1+z^2}{(1-z)}\right],$$

$$\hat{P}_{gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right],$$

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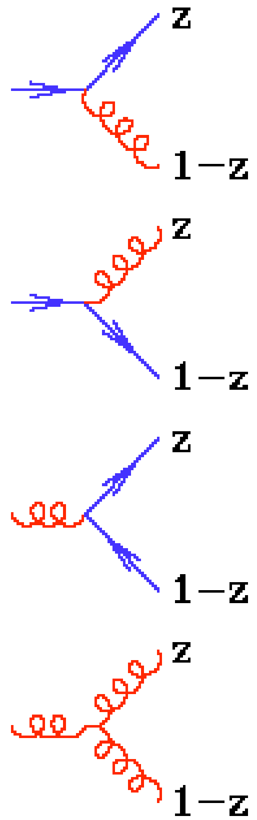

$C_F = \frac{4}{3}, C_A = 3, T_R = \frac{1}{2}.$

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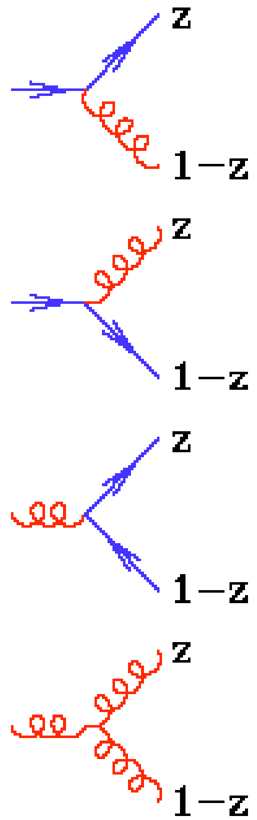

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Comments:

- * There are soft divergences in z=1 and z=0.
- * Gluons radiate the most
- * P_{qg} has no soft divergences
- * all soft divergences are gluon related.

Question time

1 Allez sur wooclap.com

2 Entrez le code d'événement dans le bandeau supérieur

Code d'événement

MADGRAPH

Activer les réponses par SMS

UCLouvain
Institut de recherche en mathématique et physique
Centre de Cosmologie, Physique des Particules et Phénoménologie

1 Go to **wooclap.com**

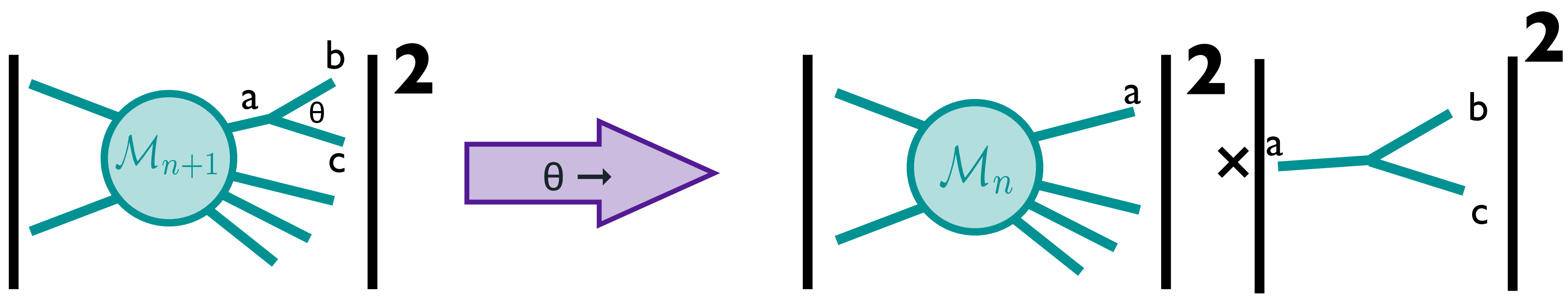
2 Enter the event code in the top banner

Event code
MADGRAPH

MonteCarlo Simulation

Olivier Mattelaer

Collinear factorization



- The process factorizes in the collinear limit. This procedure it universal!

$$\frac{1}{(p_b + p_c)^2} \simeq \frac{1}{2E_b E_c (1 - \cos\theta)}$$

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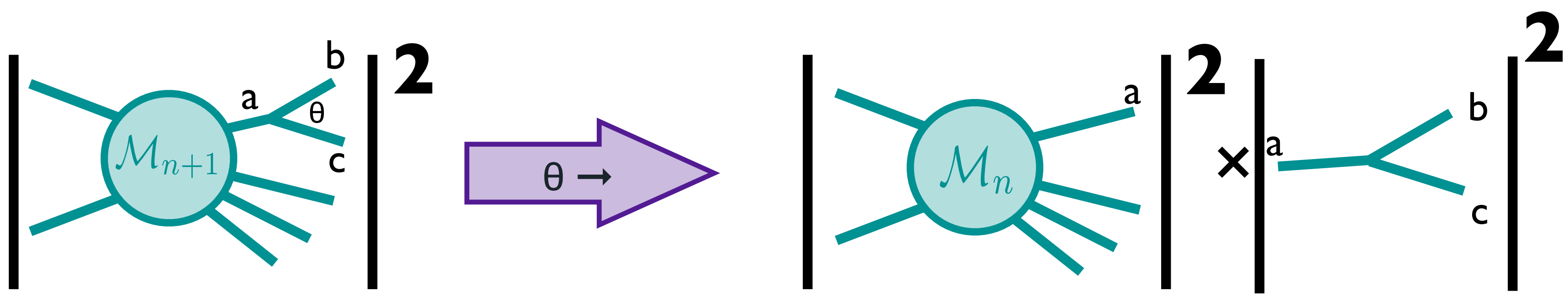

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soft

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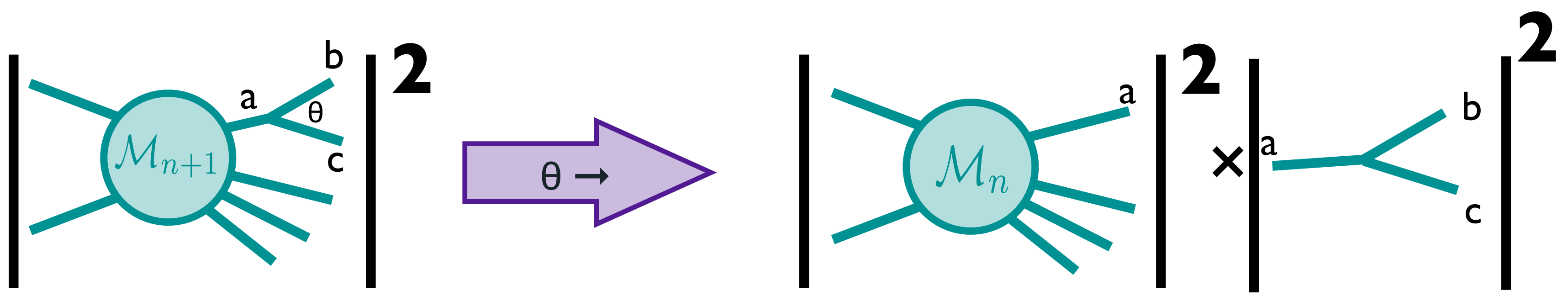

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soft and **collinear**

divergencies

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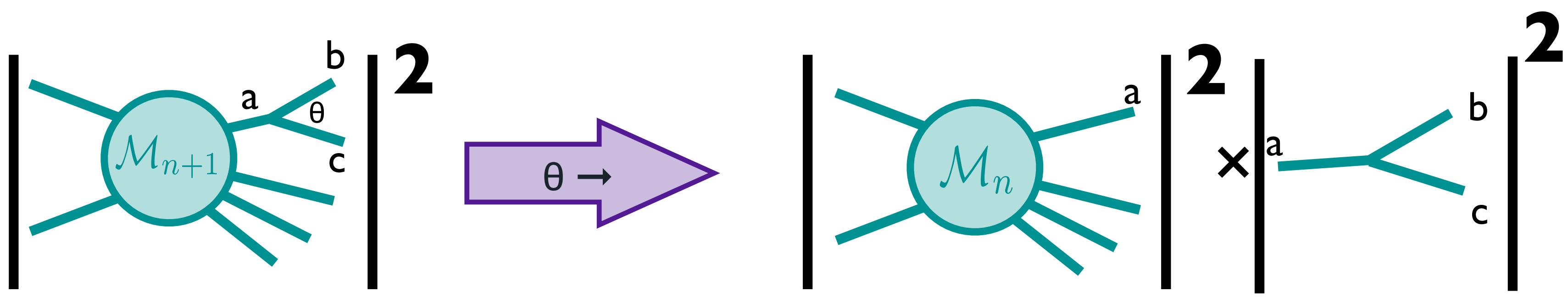

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soft and **collinear**

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when θ is small.

To Remember

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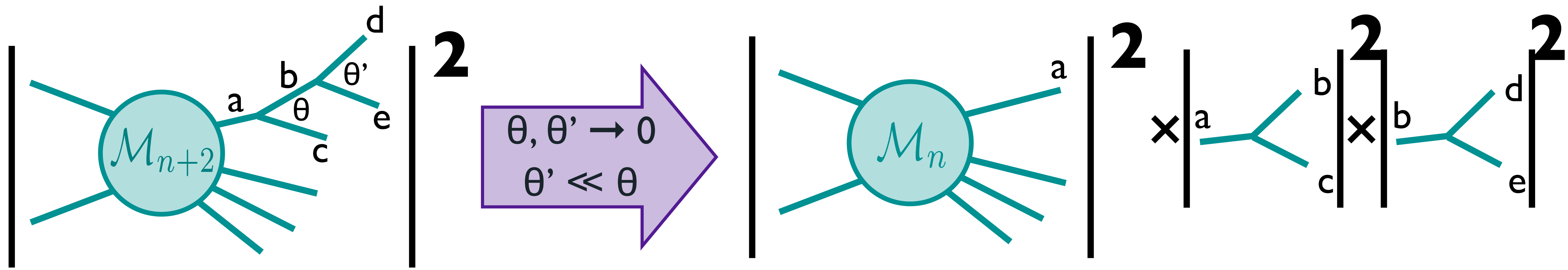
- **P** is the splitting Kernel (control the soft behaviour)

Multiple emission



- dominant contribution : θ ≫ θ' ≫ θ''...
- Collinear Factorisation does not include interference term
- Total rate for k emissions:

$$\sigma_{n+k} \propto \alpha_S^k \int_{Q_0^2}^{Q^2} \frac{dt}{t} \int_{Q_0^2}^{t} \frac{dt'}{t'} \ldots \int_{Q_0^2}^{t^{(k-2)}} \frac{dt^{(k-1)}}{t^{(k-1)}} \propto \sigma_n \left(\frac{\alpha_S}{2\pi}\right)^k \log^k(Q^2/Q_0^2)$$

- Each power of α_s comes with a logarithm. The logarithm can be easily large, and therefore it can lead to a breakdown of perturbation theory.

Sudakov Form Factor

- What is the probability of no emission?

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$$\mathcal{P}_{\mathrm{non-branching}}(t_i) = 1 - \mathcal{P}_{\mathrm{branching}}(t_i) = 1 - \frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int dz \hat{P}(z)$$

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Sudakov form factor

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$\Delta(Q^2,t)$

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➡Property: Δ(A,B) = Δ(A,C) Δ(C,B)

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Code d'événement

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Activer les réponses par SMS

Parton shower

- The Sudakov form factor is the heart of the parton shower. It gives the probability that a parton does not branch between two scales
- Using this no-emission probability the branching tree of a parton is generated.
- Define **dP_k** as the probability for k ordered splittings from leg a at given scales

$$
\begin{aligned}
dP_1(t_1) &= \Delta(Q^2, t_1)\ dp(t_1)\Delta(t_1, Q_0^2), \\
dP_2(t_1, t_2) &= \Delta(Q^2, t_1)\ dp(t_1)\ \Delta(t_1, t_2)\ dp(t_2)\ \Delta(t_2, Q_0^2)\Theta(t_1 - t_2), \\
\ldots &= \ldots \\
dP_k(t_1, \ldots, t_k) &= \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1} - t_l)
\end{aligned}
$$

- **Q_0^2** is the hadronization scale (~1 GeV). Below this scale we do not trust the perturbative description for parton splitting anymore.

Unitarity

$$dP_k(t_1, ..., t_k) \quad = \quad \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1} - t_l)$$

- The parton shower has to be unitary (the sum over all branching trees should be 1). We can explicitly show this by integrating the probability for **k** splittings:

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- Hence, the total probability is conserved

singularities

- We have shown that the showers is unitary. However, how are the IR divergences cancelled explicitly? Let's show this for the first emission:
  Consider the contributions from (exactly) 0 and 1 emissions from leg a:

$$\frac{d\sigma}{\sigma_n} = \Delta(Q^2, Q_0^2) + \Delta(Q^2, Q_0^2) \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- Expanding to first order in α_s gives

$$\frac{d\sigma}{\sigma_n} \simeq 1 - \sum_{bc} \int_{Q_0^2}^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z) + \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- Same structure of the two latter terms, with opposite signs: cancellation of divergences between the approximate virtual and approximate real emission cross sections.

- The probabilistic interpretation of the shower ensures that infrared divergences will cancel for each emission.

Final-state parton showers

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Implement your parton shower 101

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1. Start at scale t_0 and momentum fraction $z_0 = 1$

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4. Otherwise, generate $z = z_i/z_{i+1}$ with a distribution proportional to $(\alpha_s/2\pi)P(z)$, where $P(z)$ is the appropriate splitting function.
5. For each emitted particle, iterate steps 2-4 until branching stops.

To Remember

- Sudakov Form-Factor: Probability of No-emission between two scale.

$$\Delta(Q^2, t) \simeq e^{-\int_t^{Q^2} \frac{dt'}{t'} dz \frac{\alpha_S}{2\pi} \hat{P}(z)} \equiv e^{-\int_t^{Q^2} dp(t')}$$

- Parton shower is unitary (and IR save)
- Parton shower is a Markov Chain
  - ➡One emission at the time
- Each interactions has its own scale for alphas
- Various choice for the evolution parameter

aMC@NLO

Olivier Mattelaer
CP3/UCLouvain

Plan

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Perturbative expansion

$d\hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$ **Parton-level cross section**

- The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter, schematically:

$$\hat{\sigma} = \sigma^{\mathrm{Born}}\left(1+\frac{\alpha_s}{2\pi}\sigma^{(1)}+\left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)}+\left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)}+\ldots\right)$$

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LO predictions | NLO corrections | NNLO corrections | N3LO or NNNLO corrections

- Including higher corrections improves predictions and reduces theoretical uncertainties

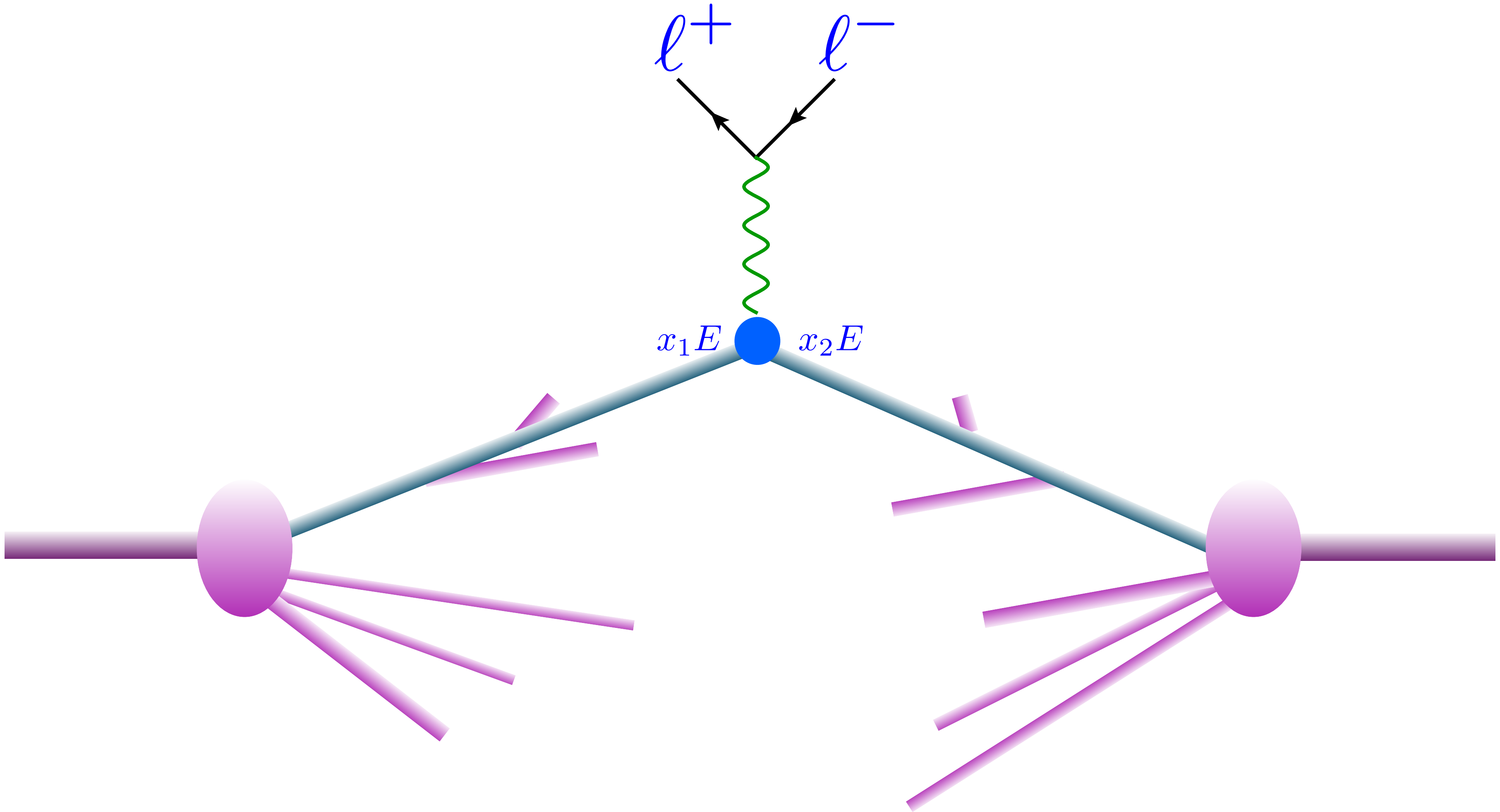
NLO corrections

- **NLO corrections have three parts:**
  - The Born contribution, i.e. the Leading order.
  - Virtual (or Loop) corrections: formed by an amplitude with a closed loop of particles interfered with the Born amplitudes
  - Real emission corrections: formed by amplitudes with one extra parton compared to the Born process
- **Both Virtual and Real emission have one power of α_s extra compared to the Born process**

$$\sigma^{\mathrm{NLO}} = \int_m d\sigma^B + \int_m d\sigma^V + \int_{m+1} d\sigma^R$$

NLO predictions

- As an example, consider Drell-Yan Z/γ* production



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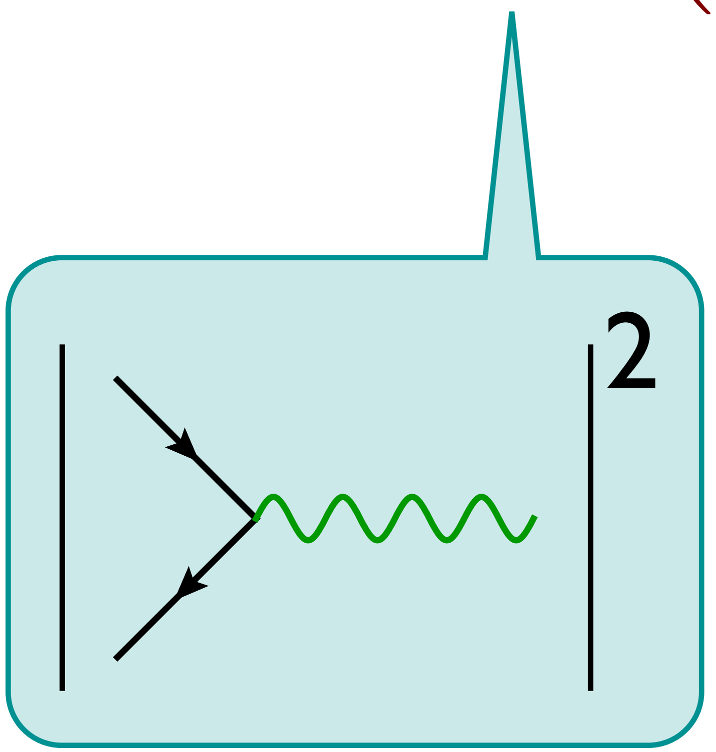

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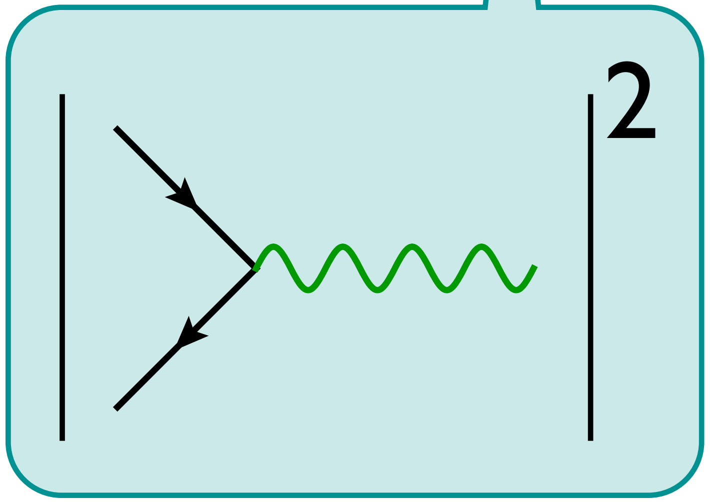

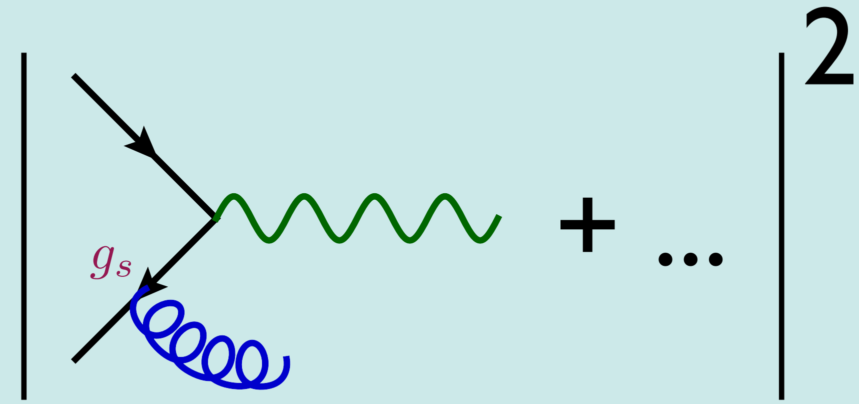

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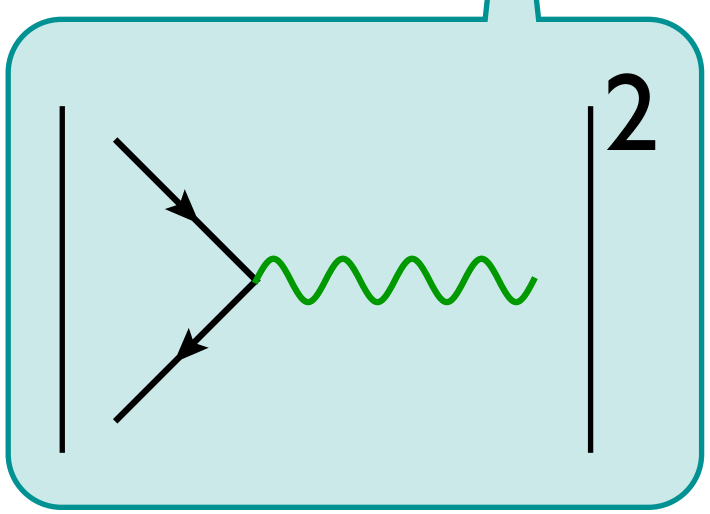

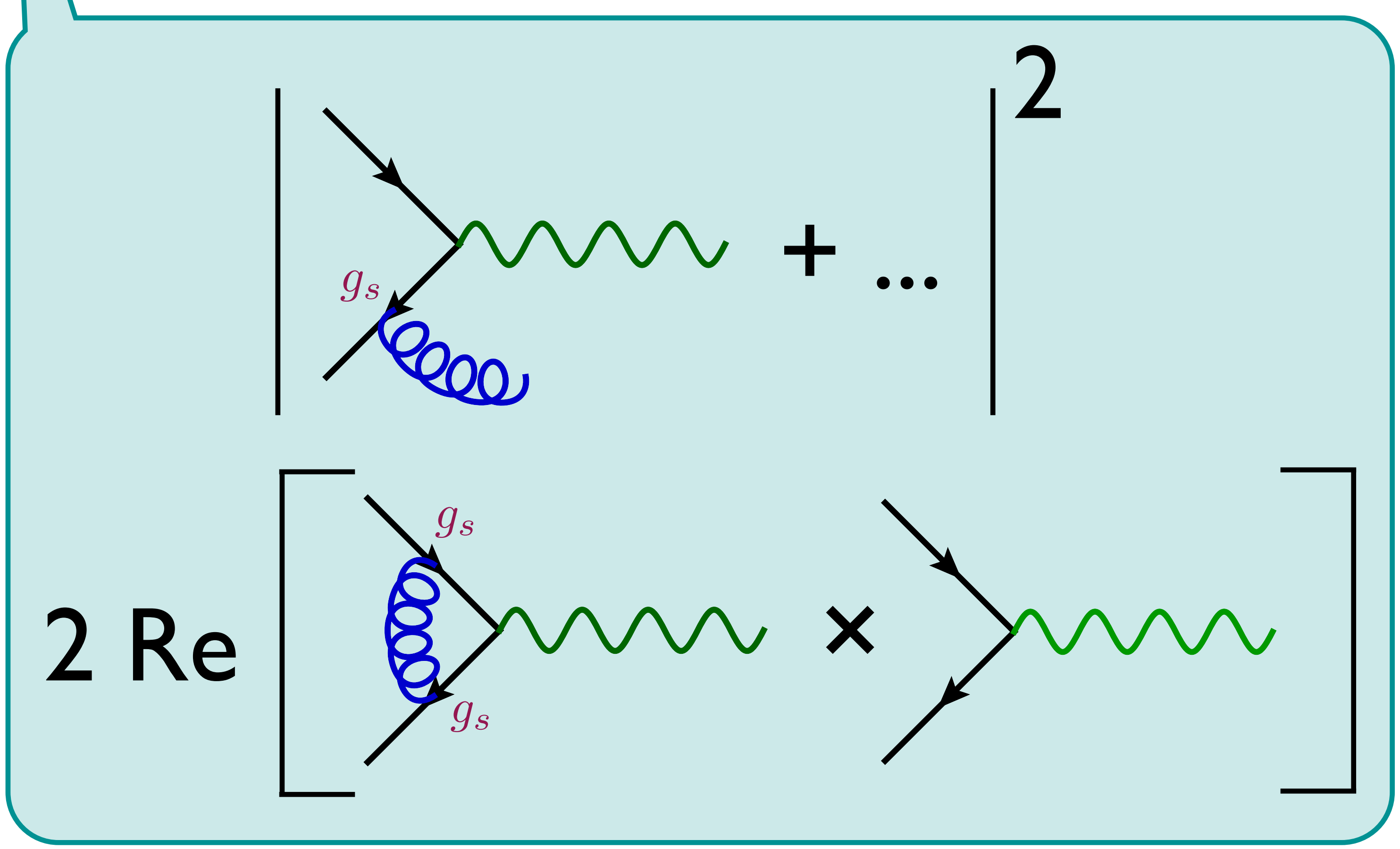

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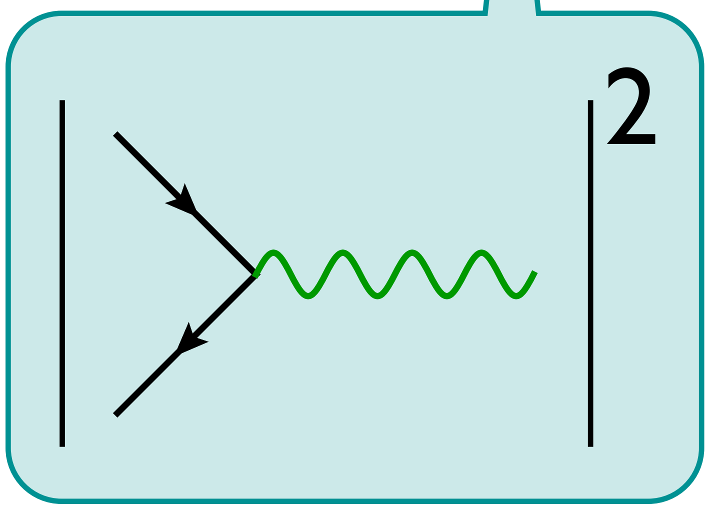
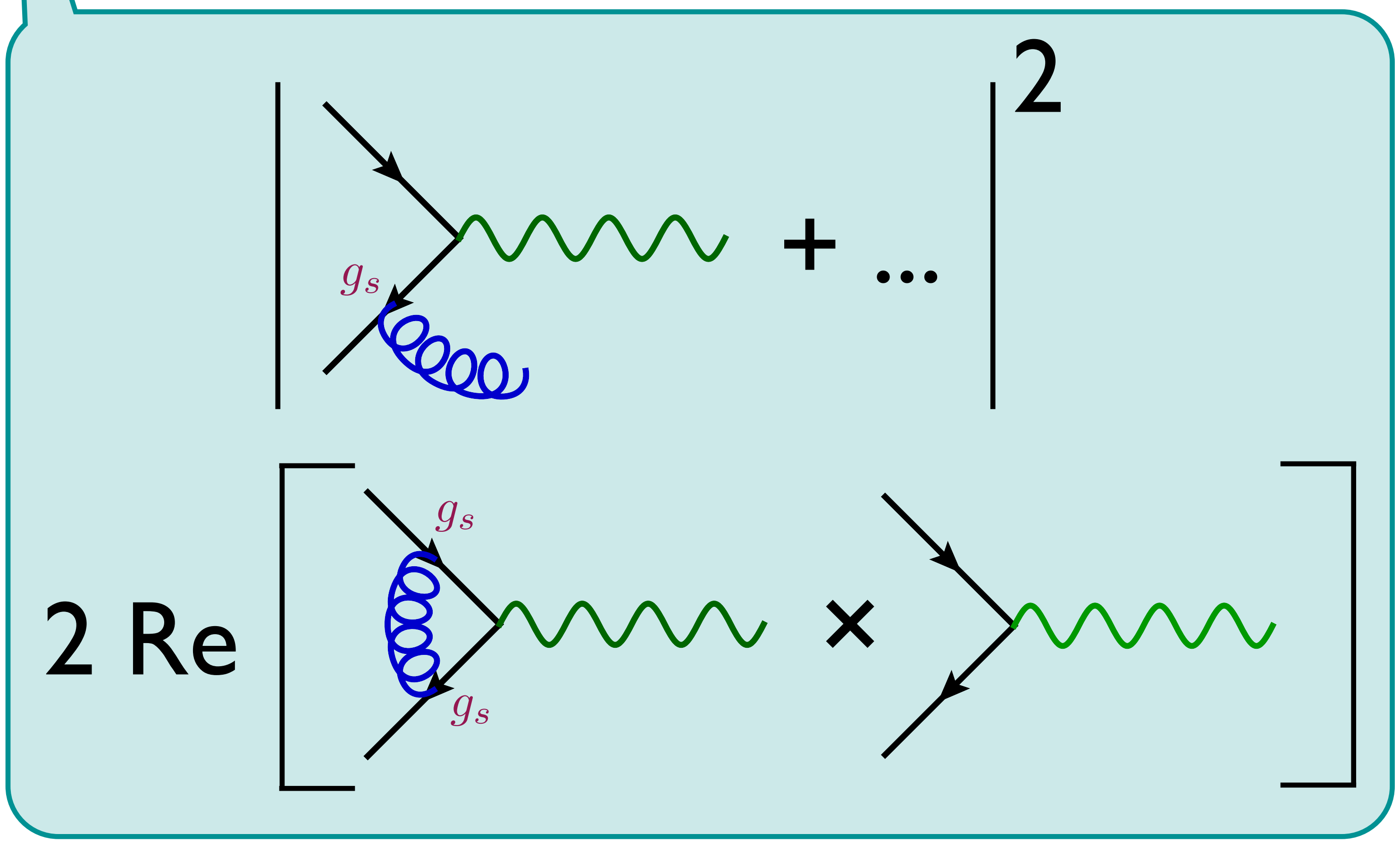




Not definite positive

Question time

1 Allez sur wooclap.com

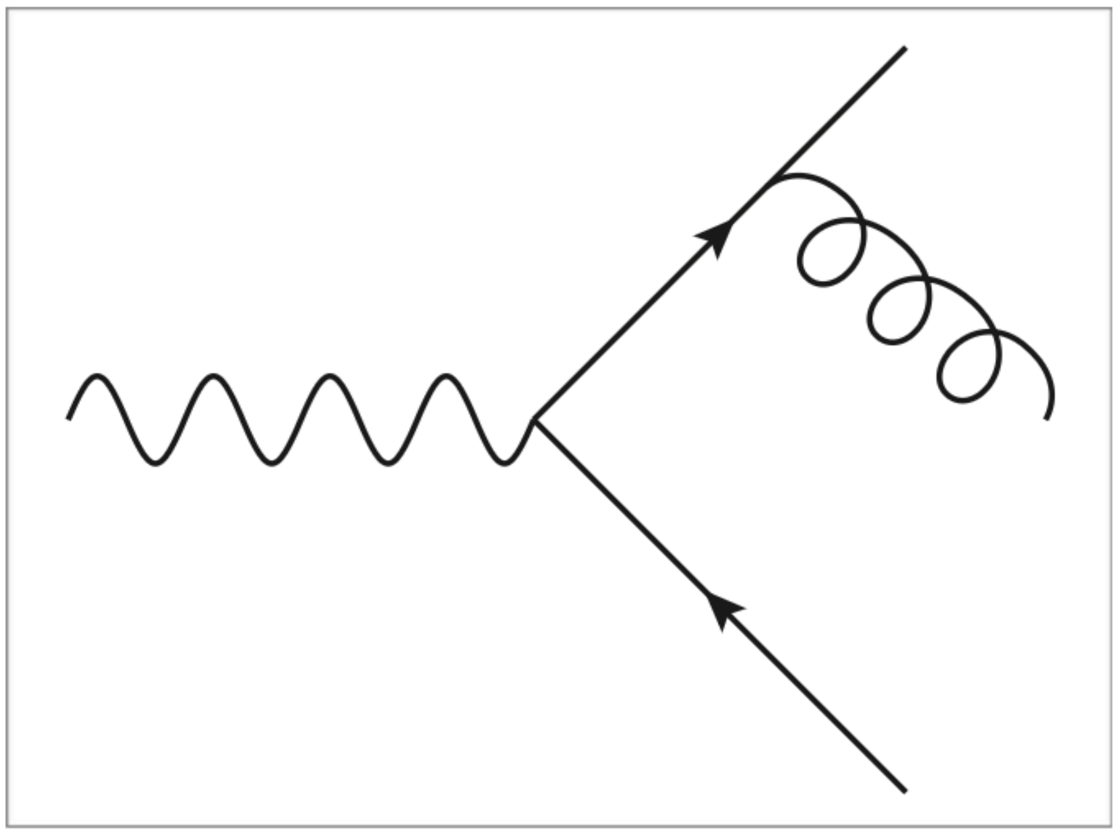
2 Entrez le code d'événement dans le bandeau supérieur

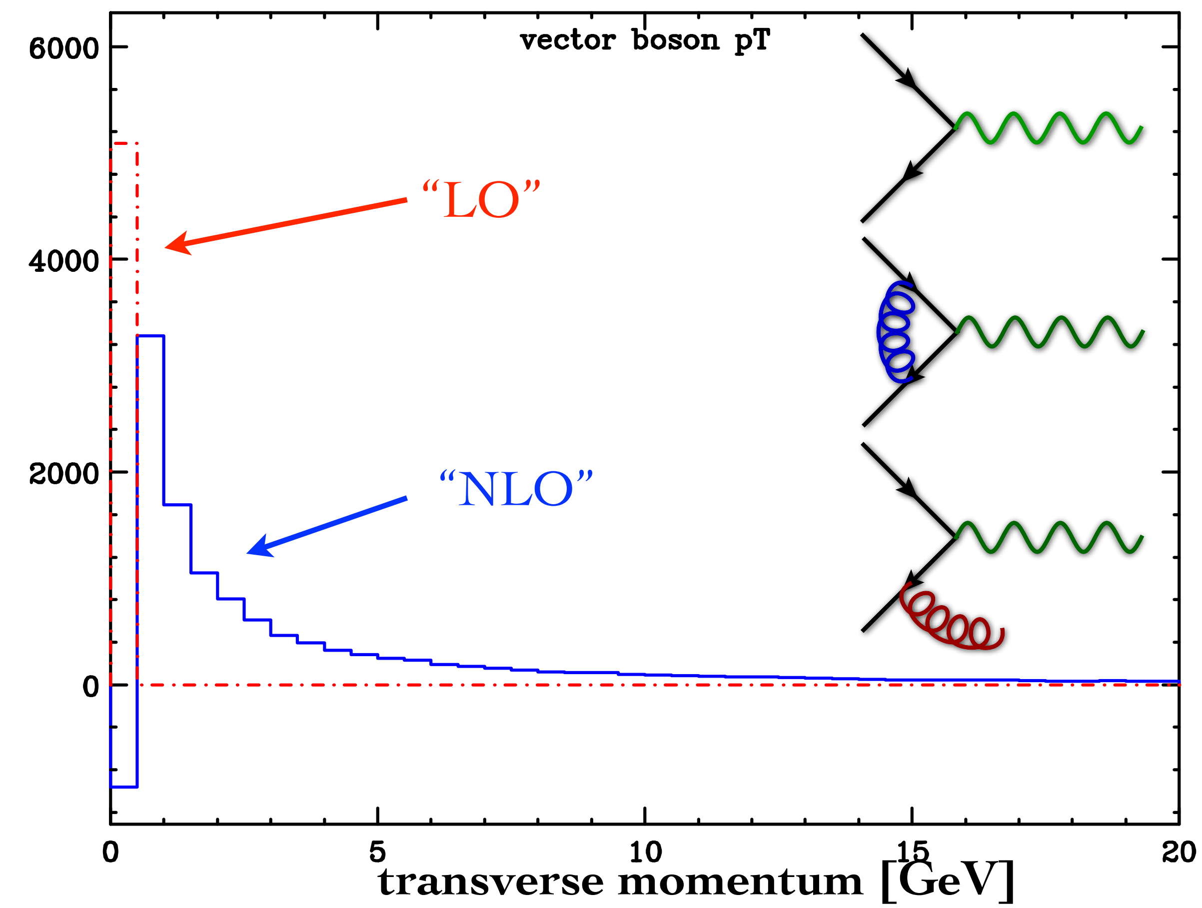
Code d'événement
MADGRAPH

Activer les réponses par SMS

Branching

$$\sigma_{NLO} = \int_n \alpha_s^b d\sigma_0 + \int_n \alpha_s^{b+1} d\sigma_V + \int_{n+1} \alpha_s^{b+1} d\sigma_R$$





$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

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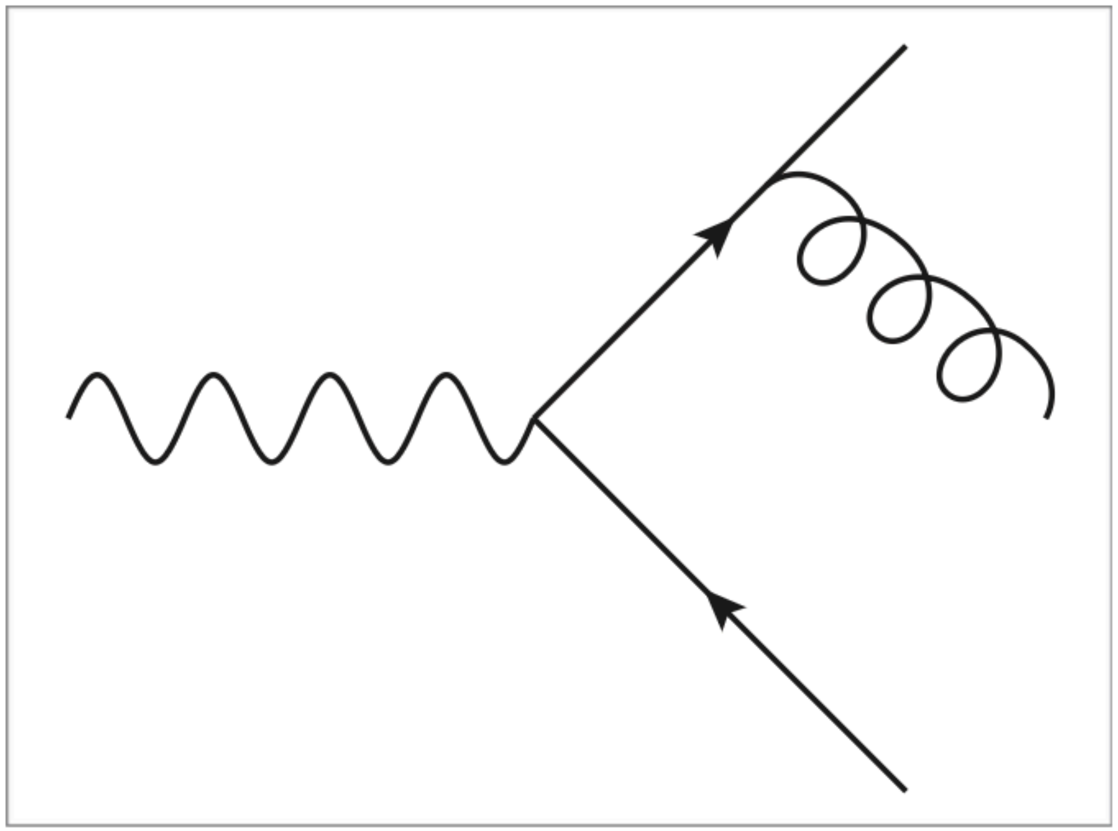

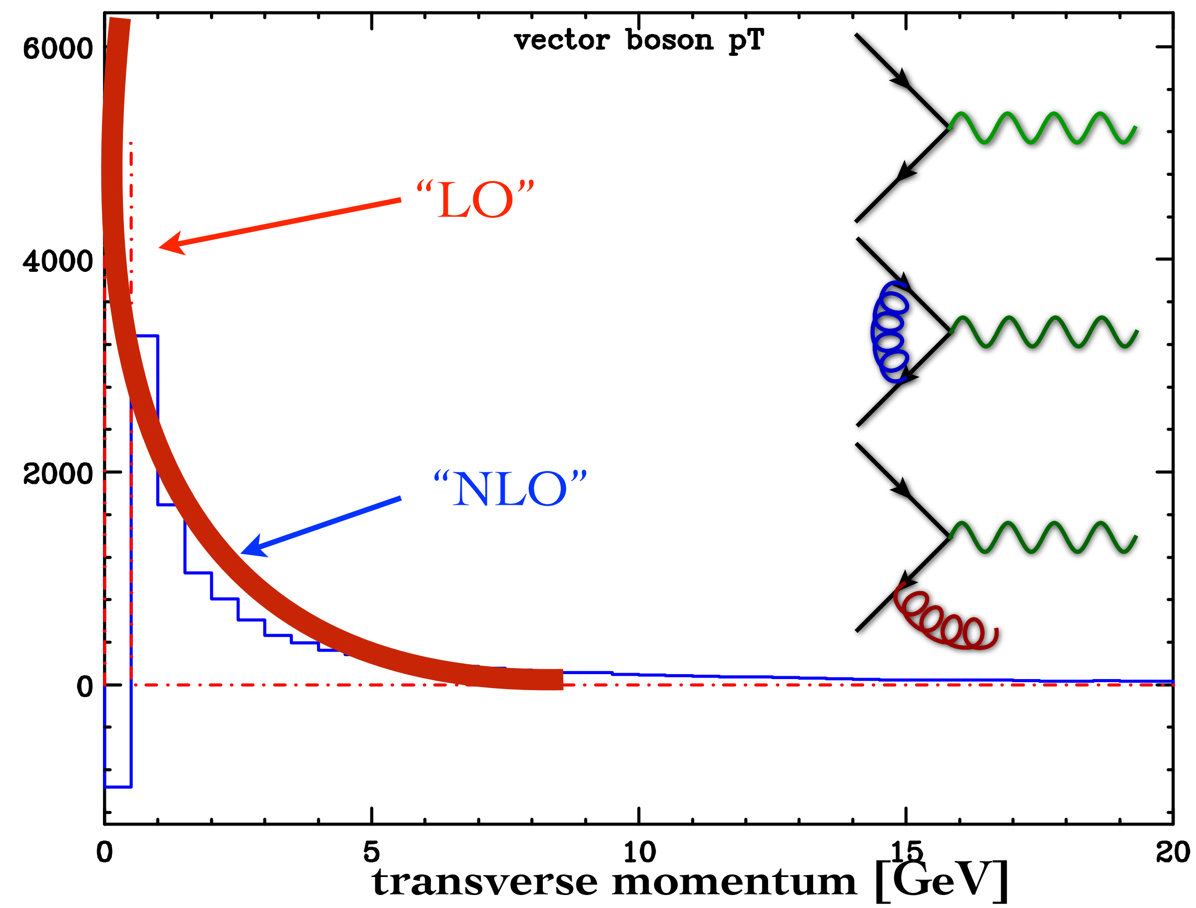

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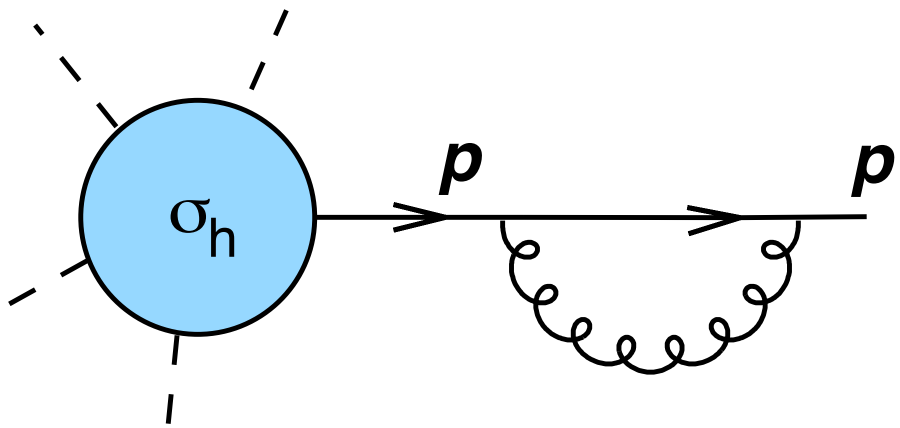
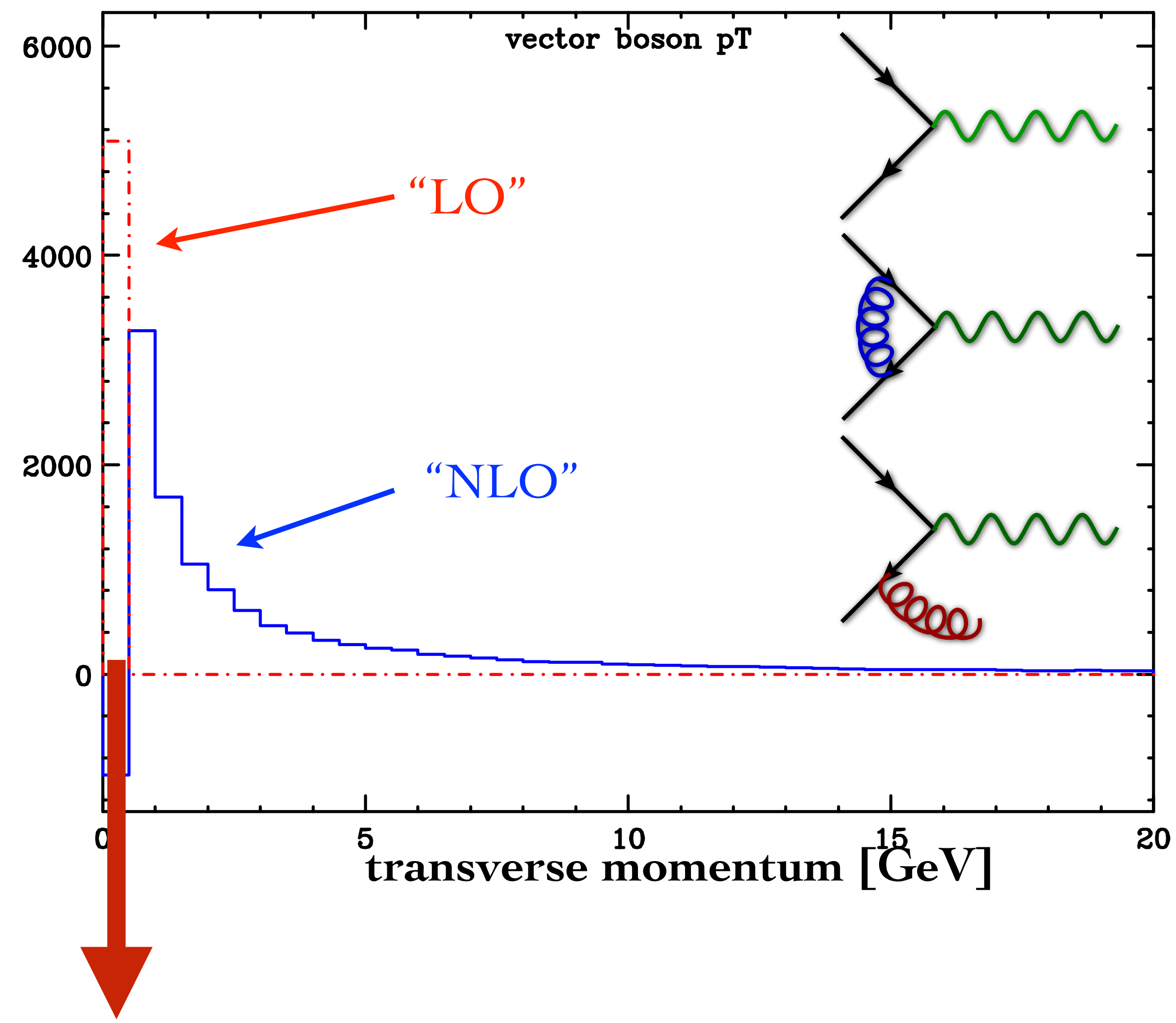


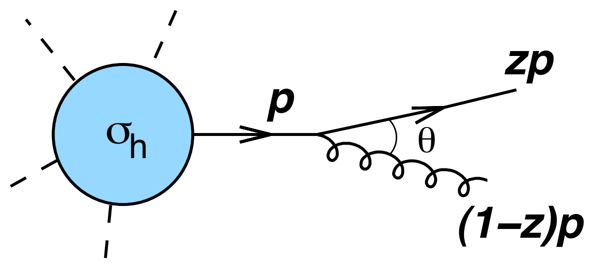
Also divergent!
- negative sign
- loop computed in dimensional regularisation
  - divergence appears as pole in the regularisation parmeters

$$\sigma_{h+V} \simeq -\sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$



Branching

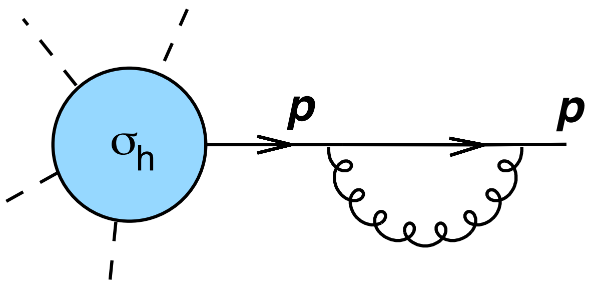
- Real:



$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

where k_t is the transverse momentum of the gluon, k_t=E sinθ.

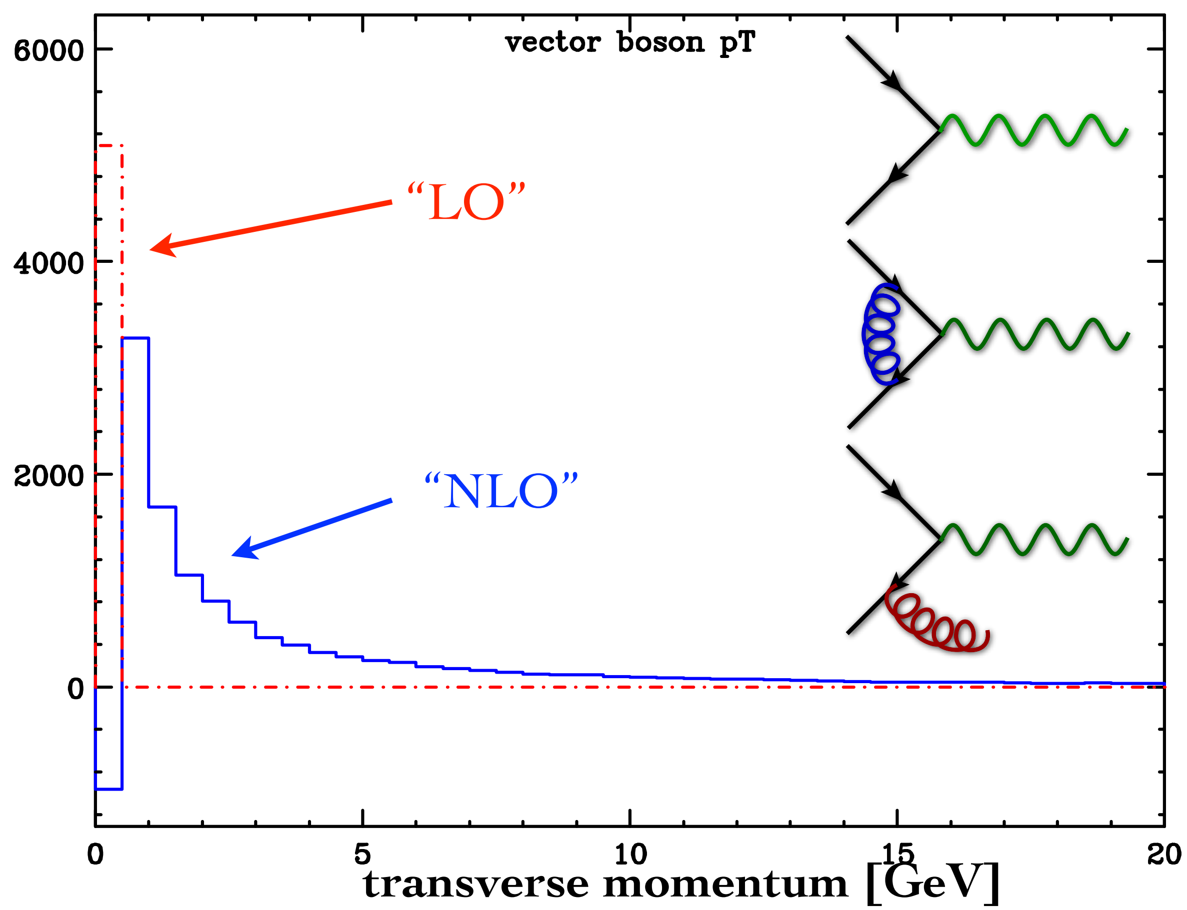
- Virtual:



$$\sigma_{h+V} \simeq -\sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

- KLN : The **sum is finite**
for observables that cannot distinguish between two collinear partons ($k_t \longrightarrow 0$); a hard and a soft parton ($z \longrightarrow 1$); and a single parton (in the virtual contributions)

Fixed Order calculations



This observables is not infra-red safe: suitable for NLO computation!

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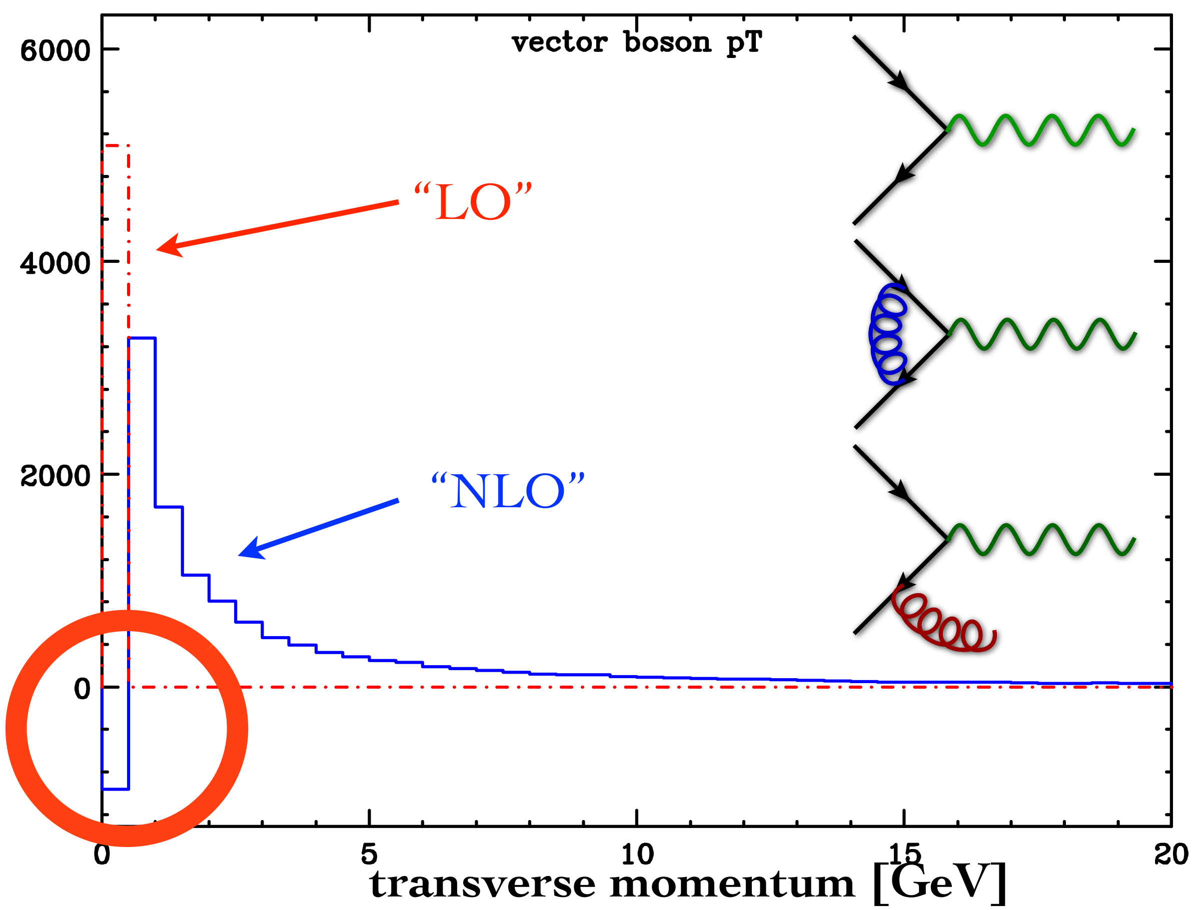


Negative contribution of the 0-bin

This observables is not infra-red safe: suitable for NLO computation!

Infrared safe observables

- For an observable to be calculable in fixed-order perturbation theory, the **observable should be infrared safe**, i.e., it should be insensitive to the emission of soft or collinear partons.
- In particular, if p_i is a momentum occurring in the definition of an observable, it most be invariant under the branching

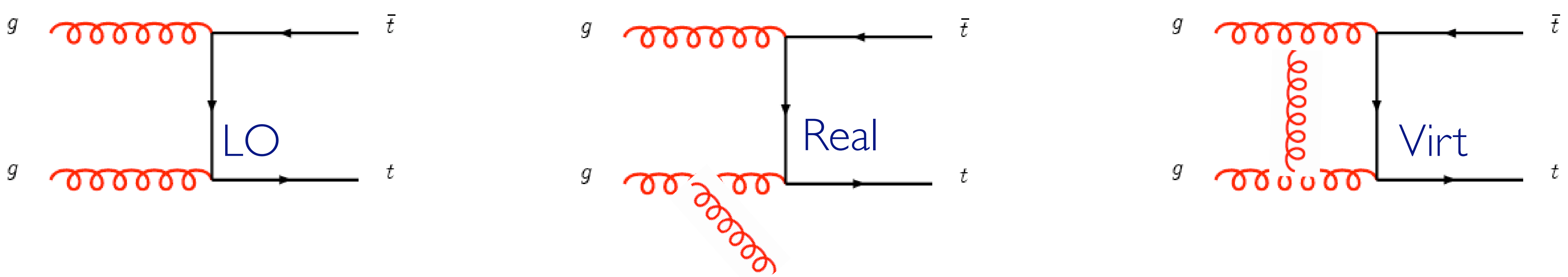
$$p_i \longrightarrow p_j + p_k,$$

whenever p_j and p_k are collinear or one of them is soft.

- Examples
  - "The number of gluons" produced in a collision is not an infrared safe observable
  - "The number of hard jets defined using the k_T algorithm with a transverse momentum above 40 GeV," produced in a collision is an infrared safe observable

NLO...?

- Are all (IR-safe) observables that we can compute using a NLO code correctly described at NLO? Suppose we have a NLO code for pp $\longrightarrow$ ttbar



LO Accuracy

NLO Accuracy

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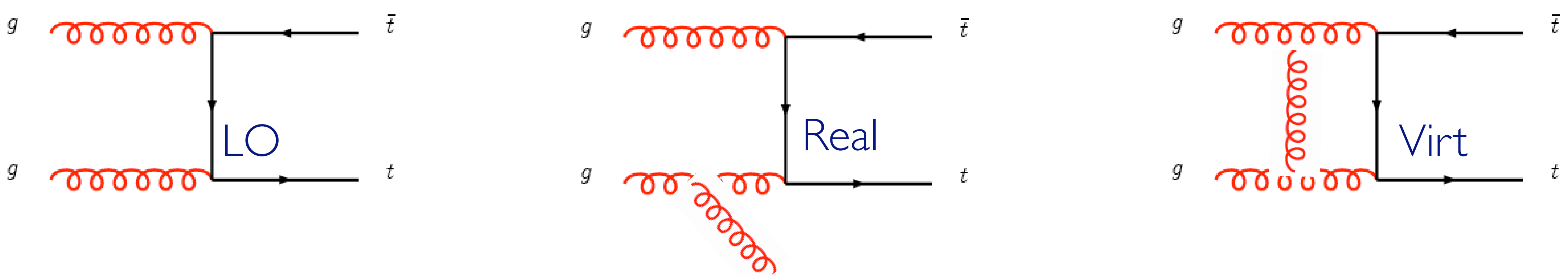

	NLO?
• Total cross section	✔
• Transverse momentum of the top quark	✔
• Transverse momentum of the top-antitop pair	✘
• Transverse momentum of the jet	✘
• Top-antitop invariant mass	✔
• Azimuthal distance between the top and anti-top	✘

Plan

- NLO Introduction
- Loop Computation
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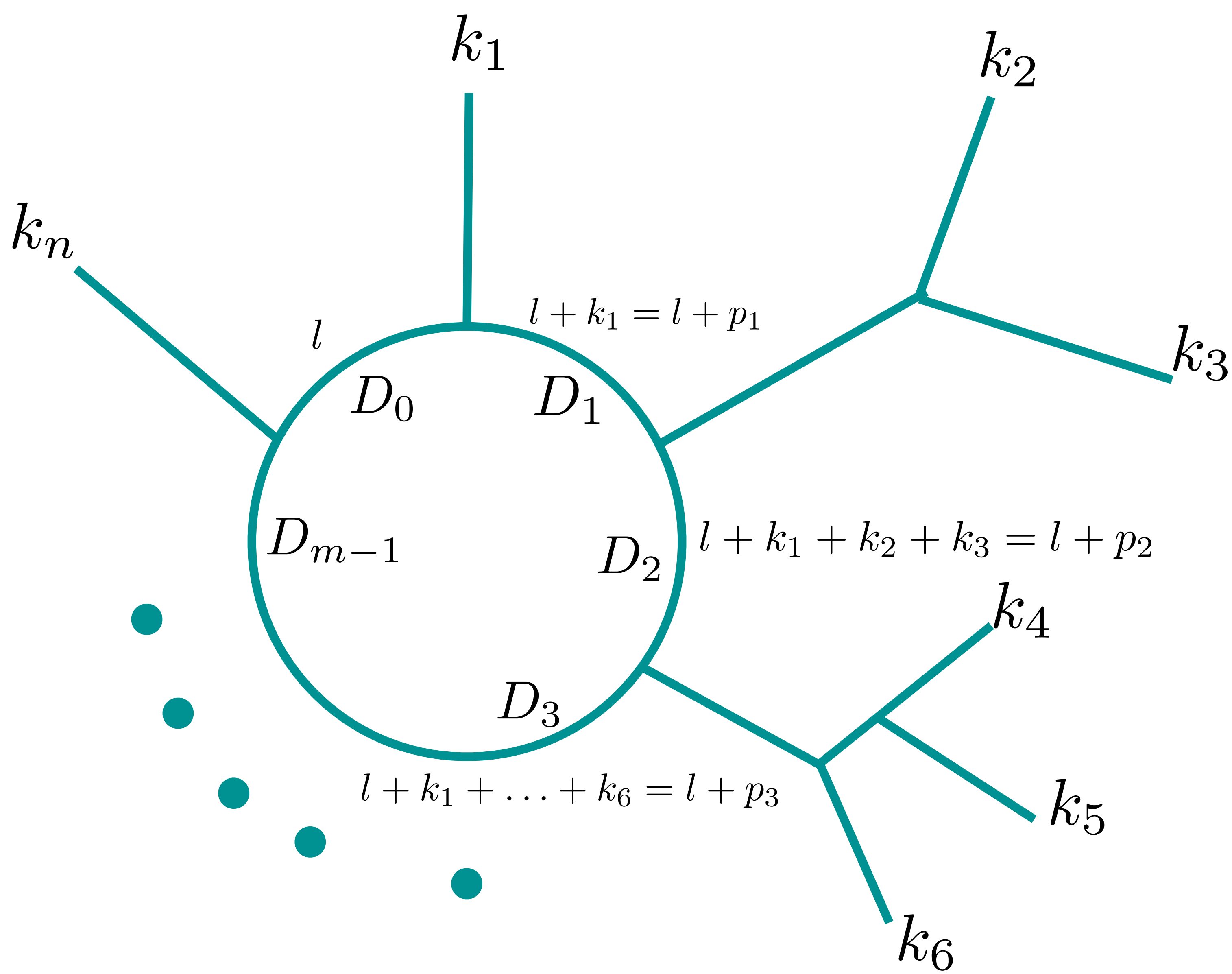
- Matching@NLO
- Live Demo

Skip this

One Loop

- Consider this *m*-point loop diagram with *n* external momenta



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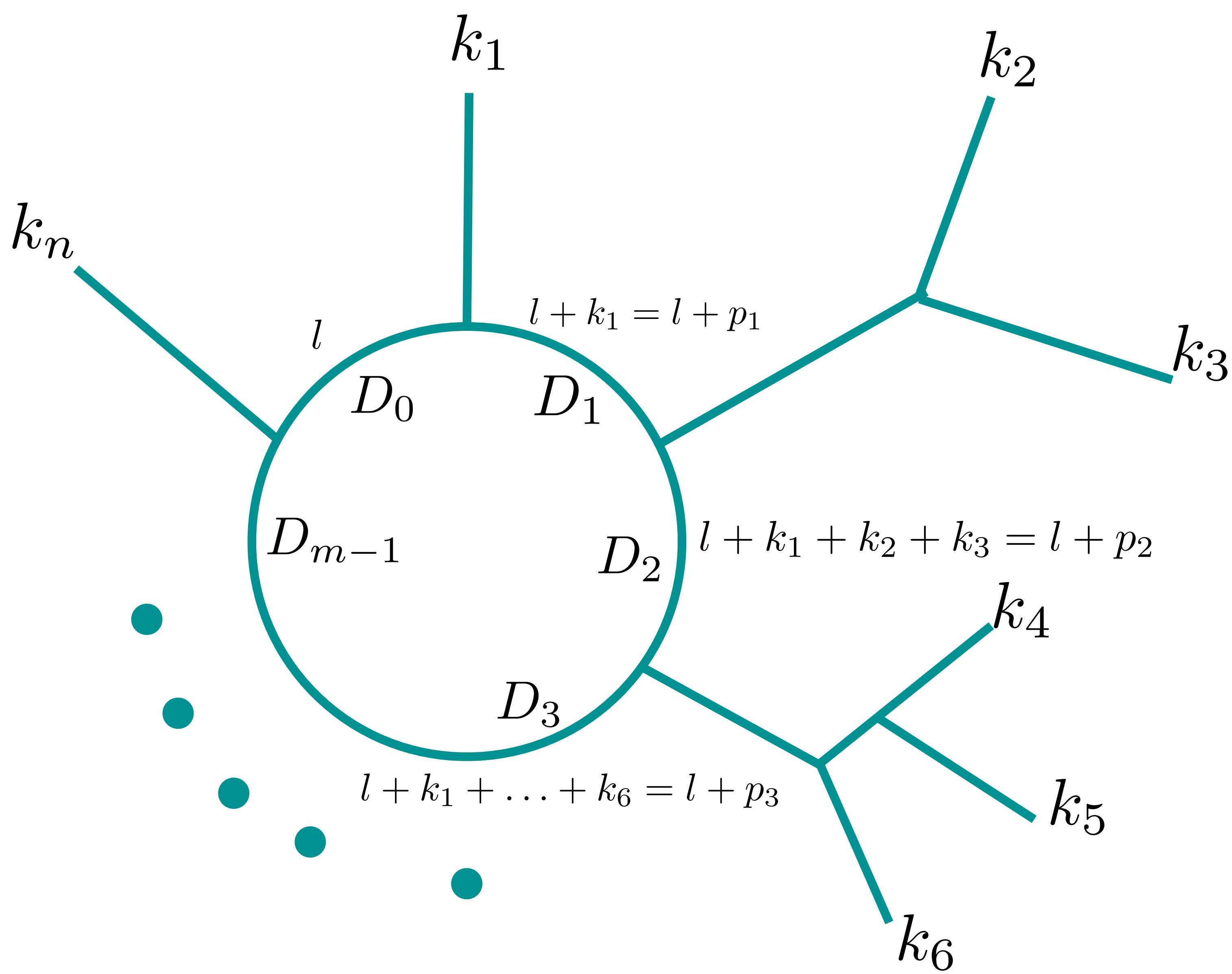

- The integral to compute is

$$\int d^d l \, \frac{N(l)}{D_0 D_1 D_2 \cdots D_{m-1}}$$

$$D_i = (l + p_i)^2 - m_i^2$$

Integrand reduction

Key Point

- Any one-loop integral can be decomposed in scalar integrals
- The task is to find these coefficients efficiently (analytically or numerically)

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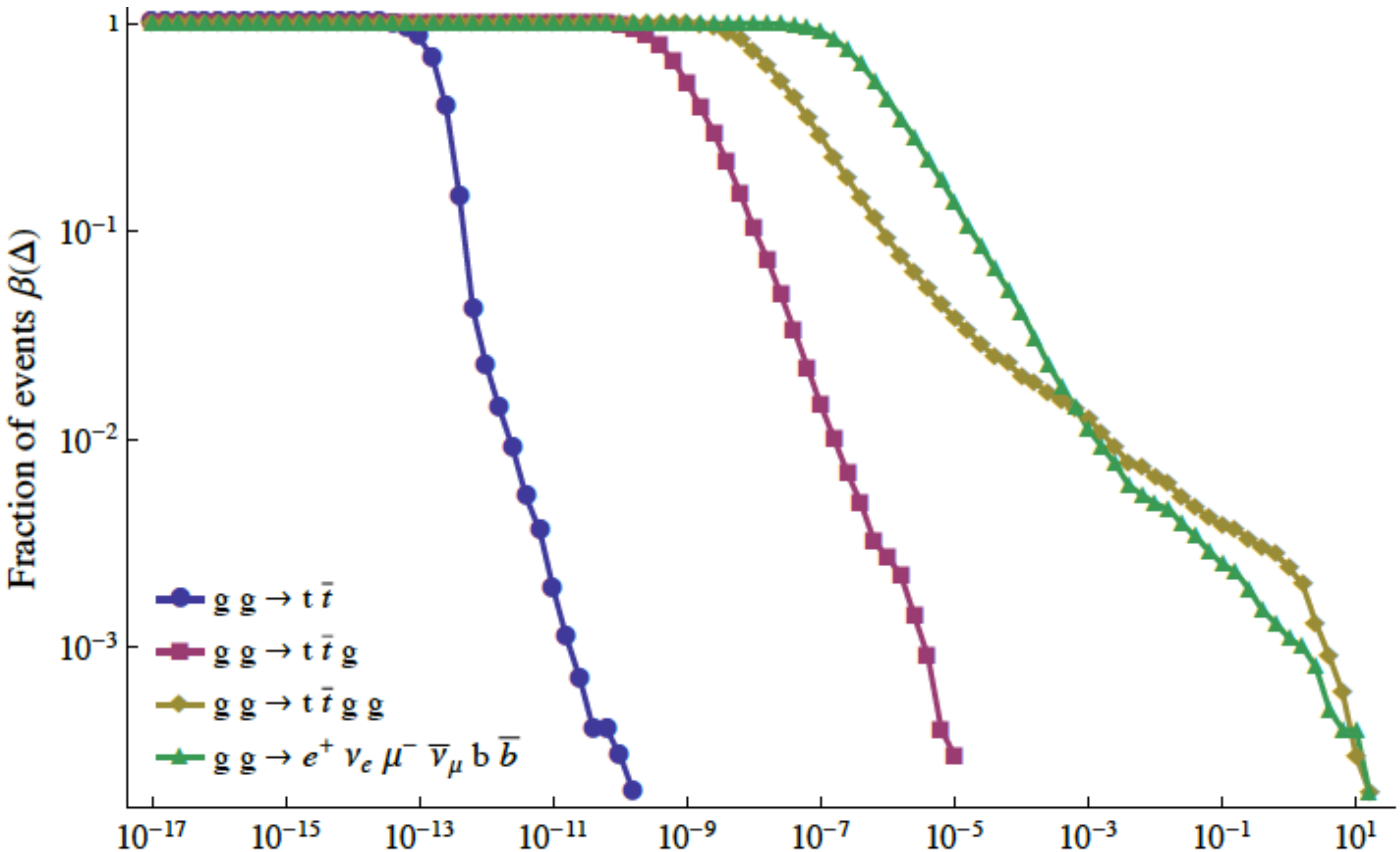
Two methods

- Passarino-Veltman
- OPP

One Tool

- MadLoop

Numerical Stability



- For 2 to 4 processes, ~7% of the Phase-space point have a precision worse than 1e-3

➡ Previous solution pass to quadruple precision (extremelly slow)

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$$\sigma^{\text{NLO}} \sim \int d^4\Phi_m\, B(\Phi_m) + \int d^4\Phi_m \int_{\text{loop}} d^d l\, V(\Phi_m) + \int d^d\Phi_{m+1}\, R(\Phi_{m+1})$$

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- Each piece is divergent
  - Cannot use a finite value for the dimensional regulator and take the limit to zero in a numerical code
- We have to cancel the divergences explicitly

phase-space integration

$$\sigma^{\text{NLO}} \sim \int d^4\Phi_m \, B(\Phi_m) + \int d^4\Phi_m \int_{\text{loop}} d^d l \, V(\Phi_m) + \int d^d\Phi_{m+1} \, R(\Phi_{m+1})$$

- This sum is Finite (KLN theory)
- Each piece is divergent
  - Cannot use a finite value for the dimensional regulator and take the limit to zero in a numerical code
- We have to cancel the divergences explicitly
- Two class of methods:

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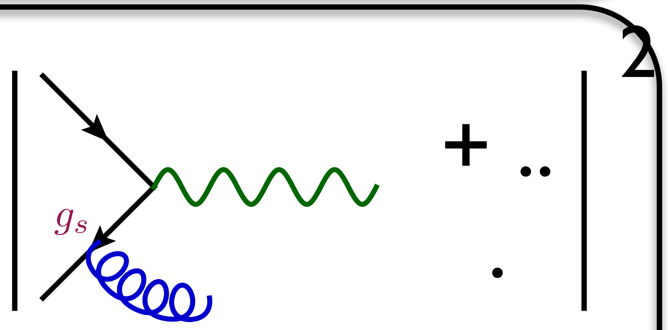
- We have to cancel the divergences explicitly
- Two class of methods:
  - Slicing and Subtraction

Example

$$\int_0^1 dx\, f(x) \qquad f(x) = \frac{g(x)}{x} \qquad g(x)$$ Finite everywhere

- Type of Divergencies of the real



- Let's introduce a regulator

$$\lim_{\epsilon \to 0} \int_0^1 dx\, \frac{g(x)}{x^{1+\epsilon}} = \lim_{\epsilon \to 0} \int_0^1 dx\, x^{-\epsilon} f(x)$$

for any non-integer non-zero value for ϵ this integral is finite

- We would like to factor out the explicit poles in ϵ so that they can be canceled explicitly against the virtual corrections

Slicing method

$$\lim_{\epsilon \to 0} \int_0^1 dx\, x^{-\epsilon} f(x) \qquad f(x) = \frac{g(x)}{x}$$

$$\lim_{\epsilon \to 0} \int_0^1 dx \frac{g(x)}{x^{1-\epsilon}} = \lim_{\epsilon \to 0} \left(\int_0^\delta dx \frac{g(x)}{x^{1-\epsilon}} + \int_\delta^1 dx \frac{g(x)}{x^{1-\epsilon}} \right)$$

Slicing method

$$\lim_{\epsilon \to 0} \int_0^1 dx\, x^{-\epsilon} f(x) \qquad f(x) = \frac{g(x)}{x}$$

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$$= \lim_{\epsilon\to 0}\frac{\delta^\epsilon}{\epsilon} g(0) + \int_\delta^1 dx \frac{g(x)}{x}$$

$$= \lim_{\epsilon\to 0}\left(\frac{1}{\epsilon} + log(\delta)\right) g(0) + \int_\delta^1 dx \frac{g(x)}{x}$$

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Subtraction method

$$\lim_{\epsilon\to 0}\int_0^1 dx\, x^{-\epsilon} f(x) \qquad f(x)=\frac{g(x)}{x}$$

$$\lim_{\epsilon\to 0}\int_0^1 dx\, x^{-\epsilon} f(x) = \lim_{\epsilon\to 0}\int_0^1 dx\, x^{-\epsilon}\left[\frac{g(0)}{x} + f(x) - \frac{g(0)}{x}\right]$$

Subtraction method

$$\lim_{\epsilon \to 0} \int_0^1 dx\, x^{-\epsilon} f(x) \qquad f(x) = \frac{g(x)}{x}$$

- Add and subtract the same term

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Same pole !

Subtraction method

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$$= \lim_{\epsilon\to 0}\frac{-1}{\epsilon}g(0) + \int_0^1 dx\,\frac{g(x)-g(0)}{x}$$

Same pole ! Numerically integrable.

Slicing vs Subtraction

- In both cases the pole is extracted and we end up with a finite remainder:

$$g(0)\log\delta + \int_\delta^1 dx\,\frac{g(x)}{x}$$

$$\int_0^1 dx\,\frac{g(x)-g(0)}{x}$$

- Subtraction acts like a plus distribution
- Slicing works only for small δ: δ-independence of cross section and distributions must be proven; subtraction is exact
- Both methods have cancelations between large numbers. If for a given observable $\lim_{x\to 0} O(x) \neq O(0)$ or we choose a too small bin size, instabilities will arise (we cannot ask for an infinite resolution)
- Subtraction is in general more flexible: good for automation

Slide from M. Zaro

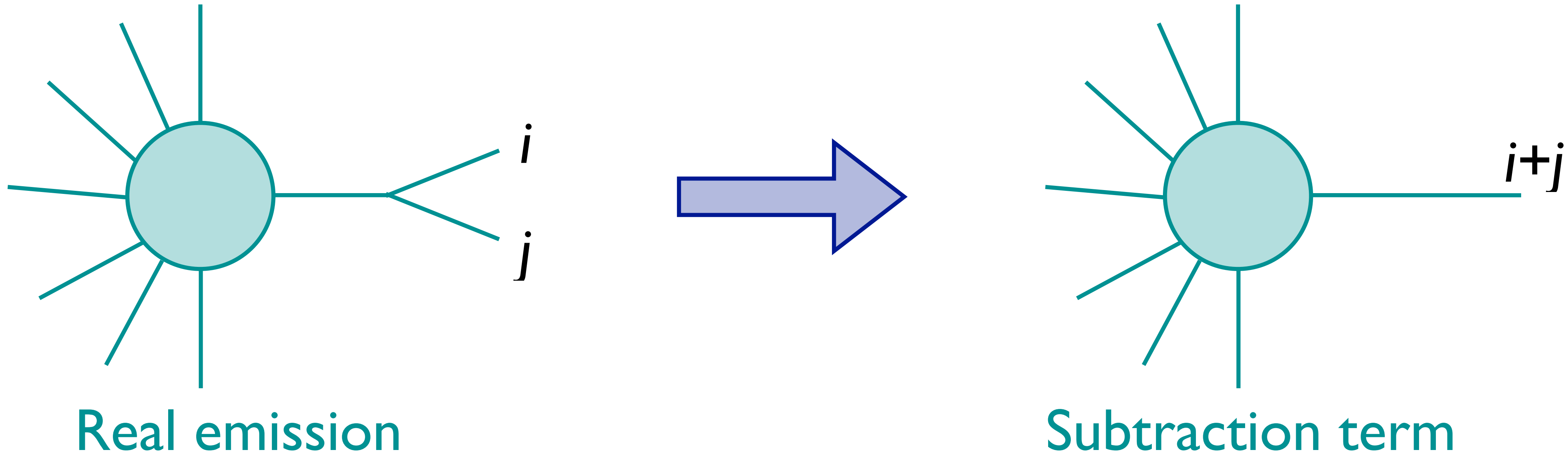
Subtraction

$$\sigma_{NLO} = \int d^4\Phi_n \mathcal{B} + \int d^4\Phi_n \mathcal{V} + \int d^4\Phi_{n+1} \mathcal{R}$$

Doing subtractions:

$$\sigma_{NLO} = \int d^4\Phi_n \mathcal{B}$$
$$+ \int d^4\Phi_n \left(\mathcal{V} + \int d^d\Phi_1 \mathcal{C} \right)_{\varepsilon\to 0}$$
$$+ \int d^4\Phi_{n+1} \left(\mathcal{R} - \mathcal{C} \right)$$

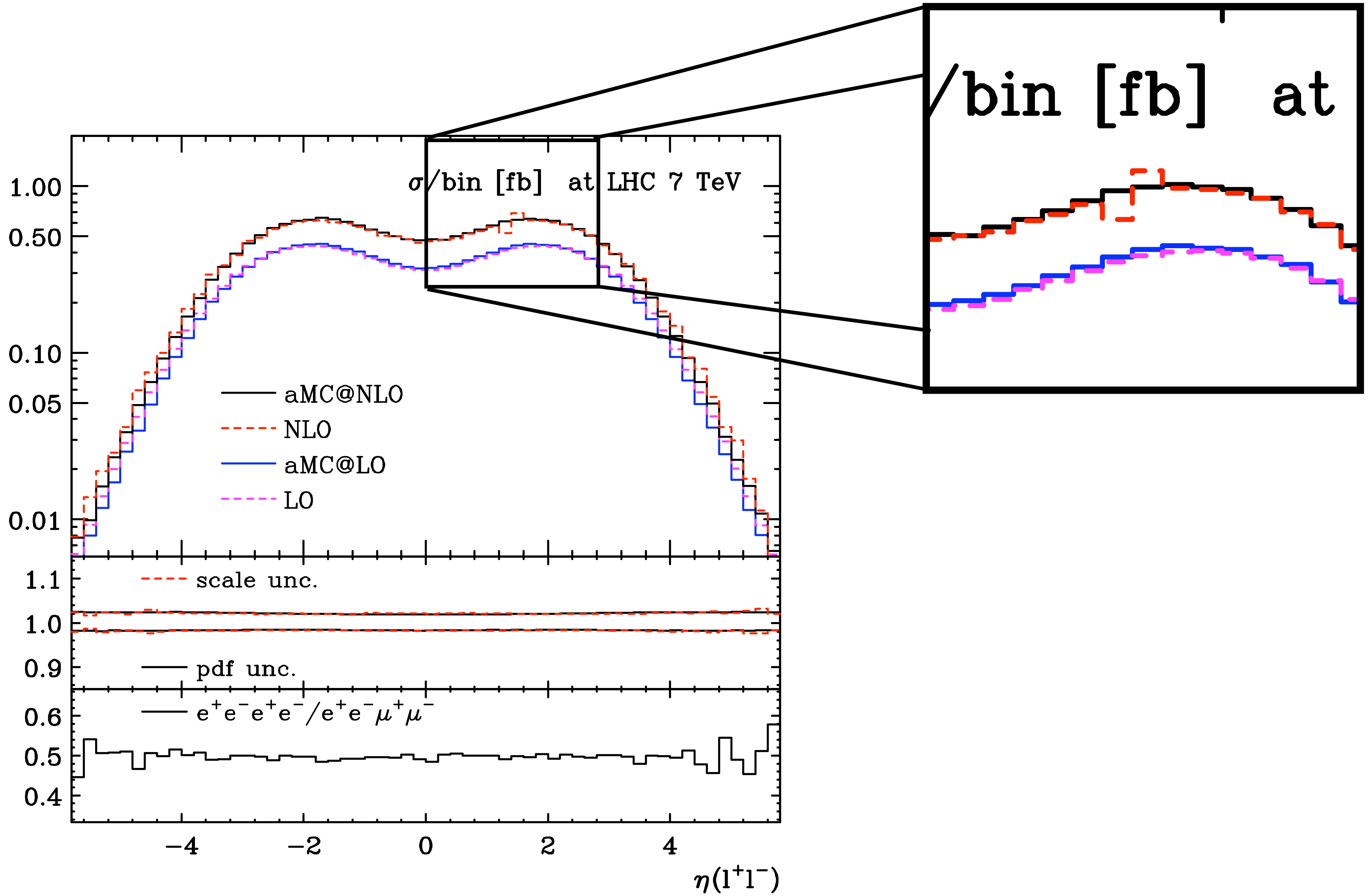
counter events



- This is not possible without changing any of the other momenta in the process
- When applying cuts or making plots, events and counter events might end-up in different bins
  - Use IR-safe observables and don't ask for infinite resolution! (KLN theorem)

4 charged lepton

- The NLO results shows a typical peak-dip structure that hampers fixed order calculations



Counter-Events

Dipole subtraction

Catani, Seymour, hep-ph/9602277 & hep-ph/9605323

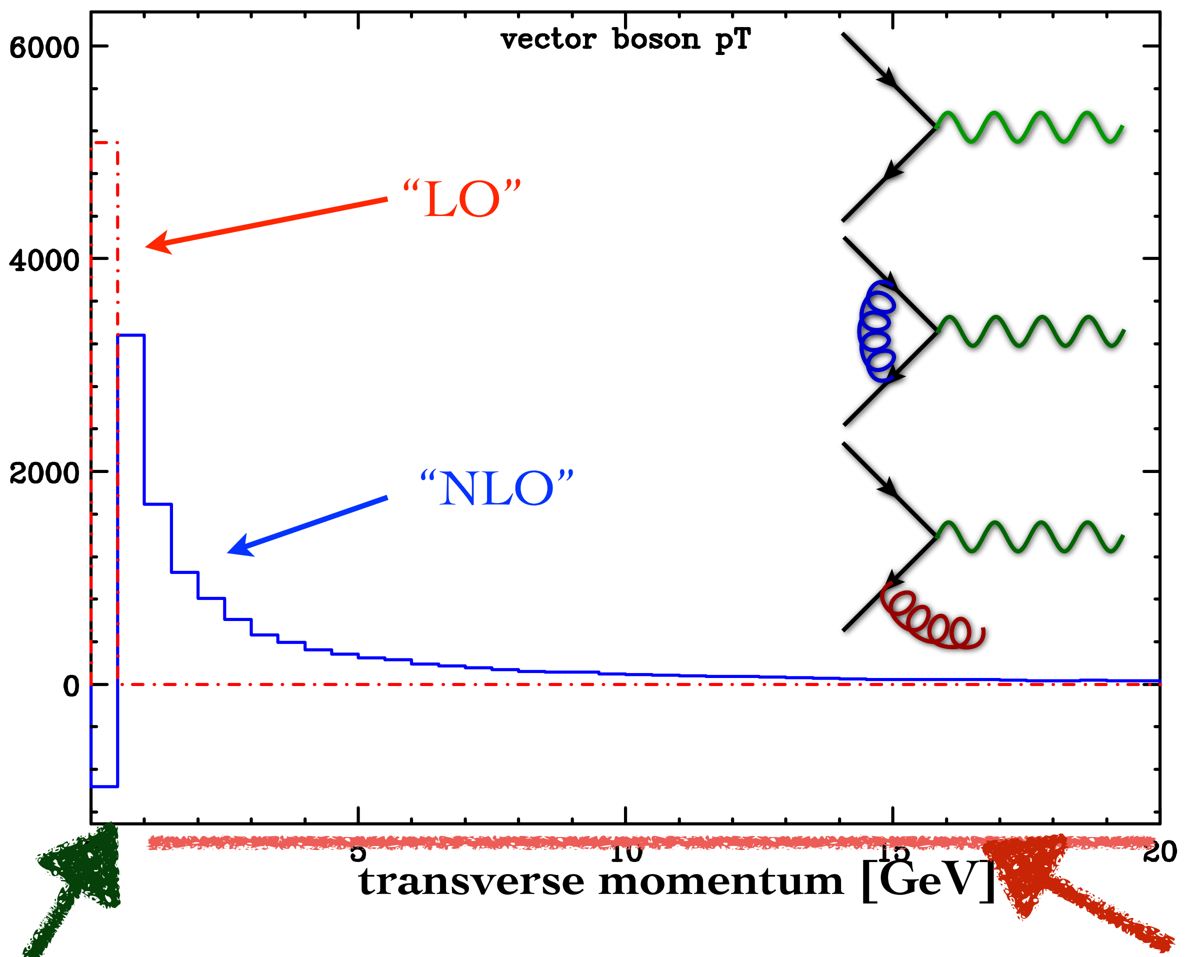
- Recoil taken by one parton $\rightarrow N^3$ scaling
- Method evolves from cancelation of soft divergences
- Proven to work for simple and complicated processes
- Automated in MadDipole, AutoDipole, Sherpa, Helac-NLO, …

FKS subtraction

Frixione, Kunszt, Signer, hep-ph/9512328

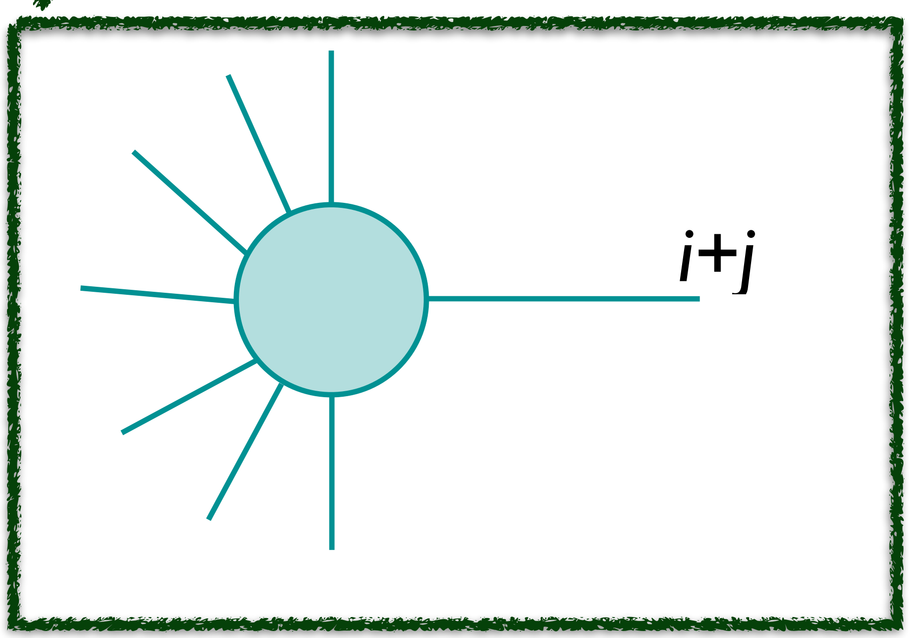
- Recoil distributed among all particles $\rightarrow N^2$ scaling
- Method evolves from cancelation of collinear divergences
- Proven to work for simple and complicated processes
- Automated in MadGraph5_aMC@NLO and in the Powheg box/Powhel

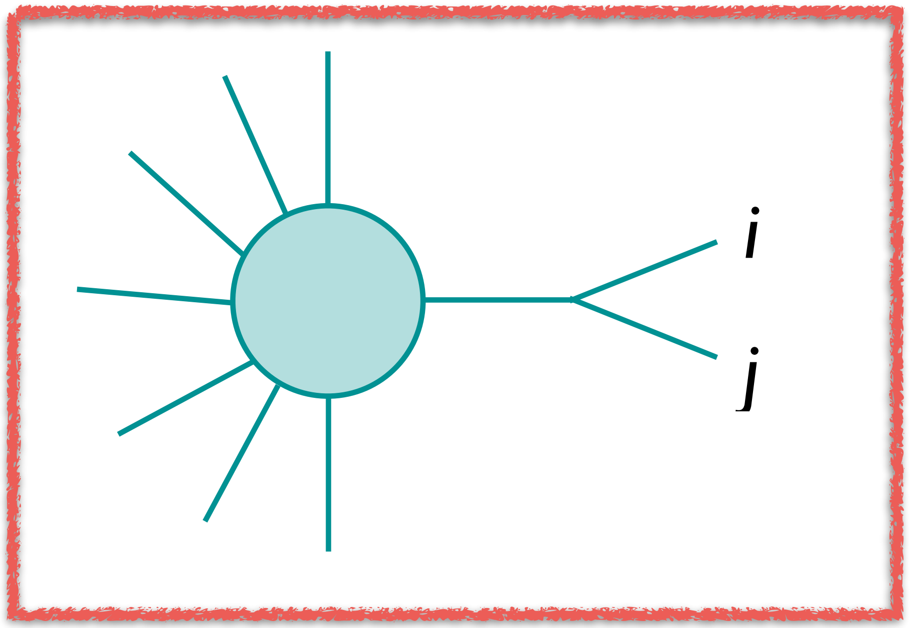
No sample of events (FNLO)



Issue to generate (unweighted) events
Since two different kinematic (and they are not bounded independently

Need to fill histograms (on the flight)





Question time

1. Allez sur wooclap.com
2. Entrez le code d'événement dans le bandeau supérieur

Code d'événement
MADGRAPH

Activer les réponses par SMS

To Remember

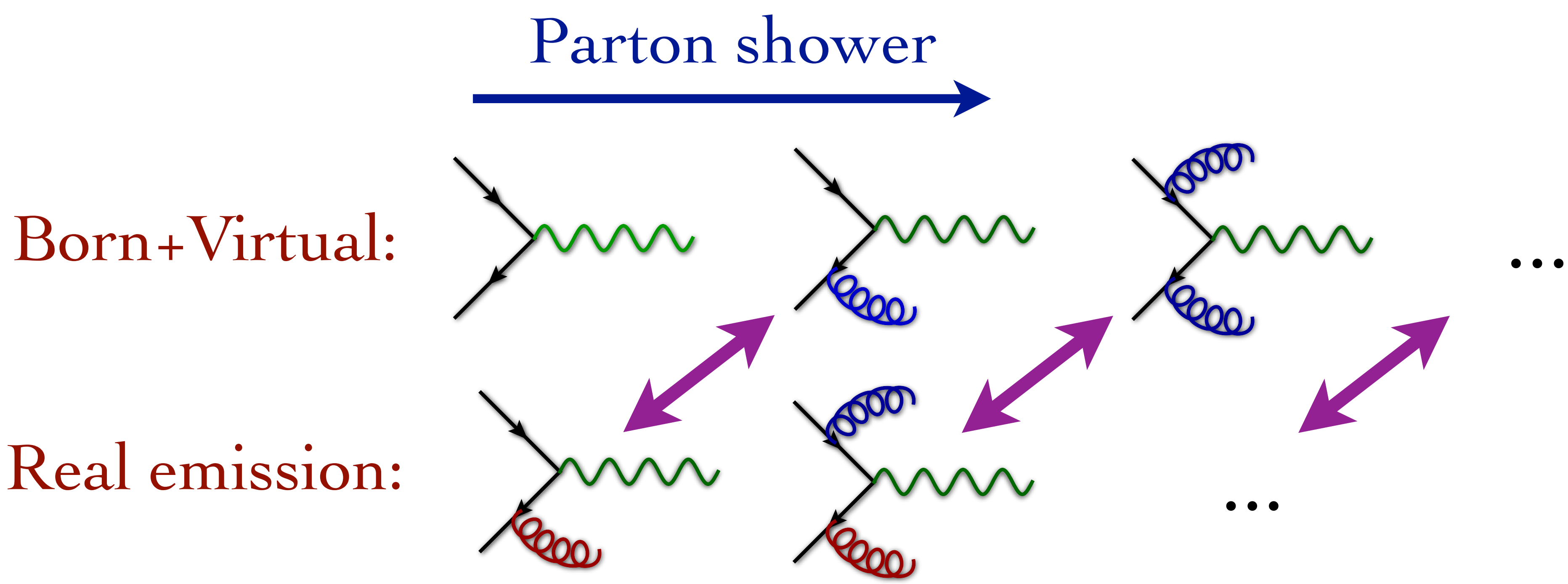
- Virtual and real matrix element are not finite, but their sum is. Subtraction methods can be used to extract divergences for real-emission matrix elements and cancel explicitly the poles from the virtuals
- Event and counterevents have different kinematics. Unweighting is not possible, we need to fill plots on-the-fly with weighted events
- For plots, only IR-safe observable with finite resolution must be used!

Plan

- NLO Introduction
- Loop Computation
- Dealing with Singularities
- Matching@NLO

Matching NLO

- At NLO one faces even more severe double-counting issues:



- And also part of the virtual contribution is double counted through the definition of the Sudakov factor Δ

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Code d'événement

MADGRAPH

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Naive implementation

- Let us *assume* we can generate events separately for Born, virtuals and real emissions, and that we pass them to a parton shower
$$\frac{d\sigma_{\text{``}NLO+PS\text{''}}}{dO} = [\mathcal{B}+\mathcal{V}]\, d\Phi_n\, I^{n}_{MC}(O) + d\Phi_{n+1}\mathcal{R}\, I^{n+1}_{MC}(O)$$
- Do we get the NLO cross section?
  - Let us expand the shower operator at order α_S (0 or 1 emission)

$$I_{MC} = \Delta_a\left(Q, Q_0\right) + \Delta_a\left(Q, Q_0\right) d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

$$\Delta_a\left(Q, Q_0\right) = \exp\left[-\int d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}\right] \simeq 1 - \int d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

$$I_{MC} \simeq 1 - \int d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc} + d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

$$\frac{d\sigma_{\text{“}NLO+PS\text{”}}}{dO} = [\mathcal{B}+\mathcal{V}]\, d\Phi_n\, I^n_{MC}(O) + d\Phi_{n+1}\mathcal{R}\, I^{n+1}_{MC}(O)$$

- At order α_S we get

$$\frac{d\sigma_{\text{“}NLO+PS\text{”}}}{dO} = [\mathcal{B}+\mathcal{V}]\, d\Phi_n + d\Phi_{n+1}\mathcal{R}$$
$$- \mathcal{B}\, d\Phi_n \int d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc} + \mathcal{B}\, d\Phi_n d\Phi_1 \frac{\alpha_s(t)}{2\pi} P_{a\to bc}$$

- Which is not the NLO

MC@NLO procedure

[Frixione & Webber (2002)]

- To remove the double counting, we can add and subtract the same term to the *m* and *m*+1 body configurations

$$\frac{d\sigma_{\mathrm{NLOwPS}}}{dO} = \left[d\Phi_m (B + \int_{\mathrm{loop}} V + \int d\Phi_1 MC) \right] I_{\mathrm{MC}}^{(m)}(O) + \left[d\Phi_{m+1}(R - MC) \right] I_{\mathrm{MC}}^{(m+1)}(O)$$

MC@NLO procedure

[Frixione & Webber (2002)]

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- Where the *MC* are defined to be the contribution of the parton shower to get from the *m* body Born final state to the *m*+1 body real emission final state

MC@NLO procedure

[Frixione & Webber (2002)]

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$$\frac{d\sigma_{\rm NLOwPS}}{dO} = \left[d\Phi_m \left(B + \int_{\rm loop} V + \int d\Phi_1 MC\right)\right] I_{\rm MC}^{(m)}(O) + \left[d\Phi_{m+1}(R - MC)\right] I_{\rm MC}^{(m+1)}(O)$$

- Where the *MC* are defined to be the contribution of the parton shower to get from the *m* body Born final state to the *m*+1 body real emission final state

$$MC = \left|\frac{\partial\left(t^{MC}, z^{MC}, \phi\right)}{\partial\Phi_1}\right| \frac{1}{t^{MC}} \frac{\alpha_s}{2\pi} \frac{1}{2\pi} P\left(z^{MC}\right) \mathcal{B}$$

MC@NLO procedure

[Frixione & Webber (2002)]

- To remove the double counting, we can add and subtract the same term to the *m* and *m*+1 body configurations

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Double counting avoided

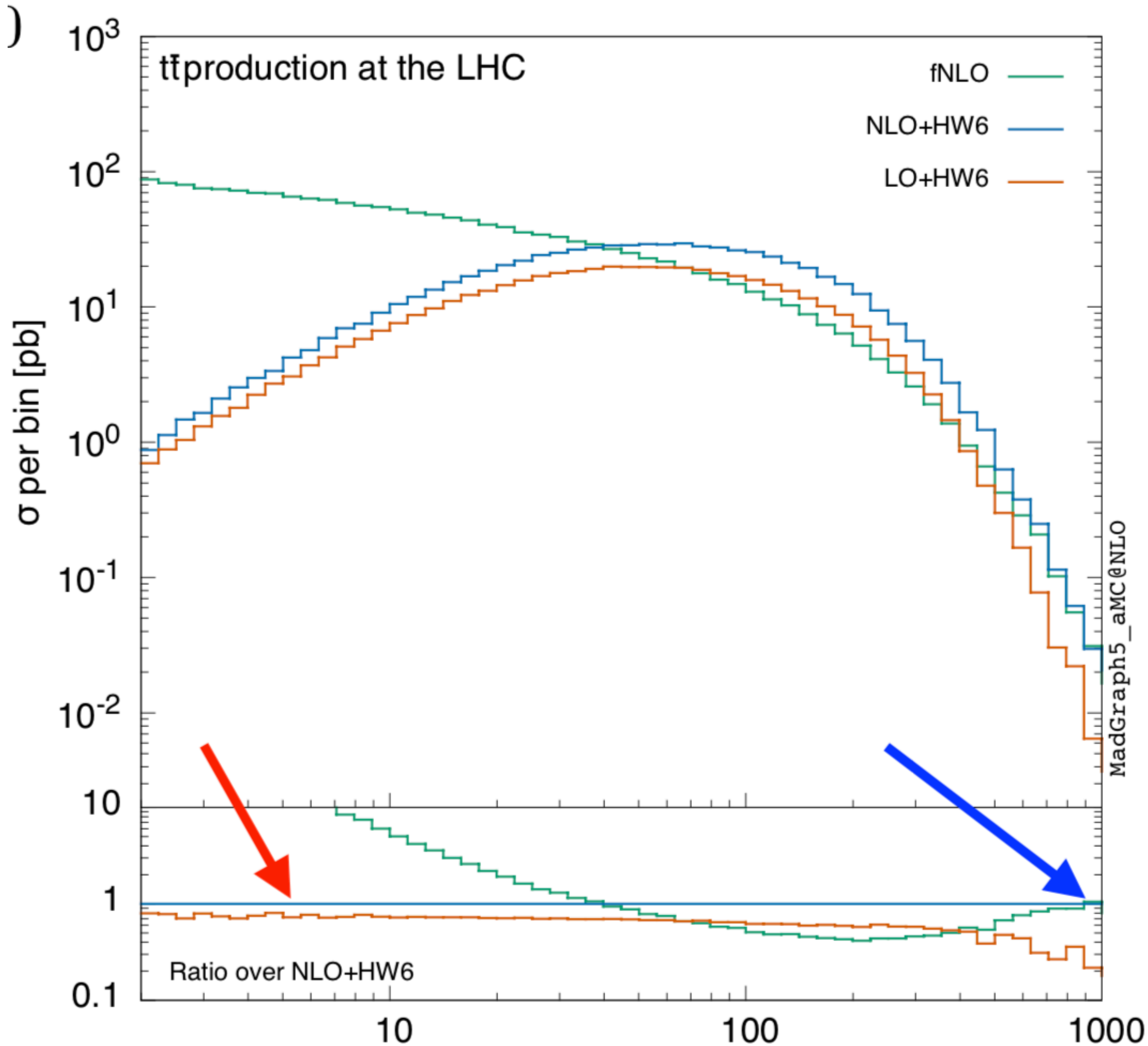
$$\frac{d\sigma_{\rm NLOwPS}}{dO} = \left[d\Phi_m (B + \int_{\rm loop} V + \int d\Phi_1 MC)\right] I_{\rm MC}^{(m)}(O)$$
$$+ \left[d\Phi_{m+1}(R - MC)\right] I_{\rm MC}^{(m+1)}(O)$$

- Expanded at NLO

$$I_{\rm MC}^{(m)}(O) dO = 1 - \int d\Phi_1 \frac{MC}{B} + d\Phi_1 \frac{MC}{B} + \dots$$

$$\frac{d\sigma_{\text{“}MC@NLO\text{”}}}{dO} = \left[\mathcal{B} + \mathcal{V} + \int d\Phi_1 MC\right] d\Phi_n + d\Phi_{n+1}\left[\mathcal{R} - MC\right]$$
$$+ \mathcal{B}\left[- \int d\Phi_1 MC + d\Phi_1 MC\right] d\Phi_n$$

Smooth matching



Stability & unweighting

$$\frac{d\sigma_{\rm NLOwPS}}{dO} = \left[d\Phi_m(B + \int_{\rm loop} V + \int d\Phi_1 MC)\right] I_{\rm MC}^{(m)}(O) + \left[d\Phi_{m+1}(R - MC)\right] I_{\rm MC}^{(m+1)}(O)$$

- The MC subtraction terms are defined to be what the shower does to get from the m to the m+1 body matrix elements. Therefore the cancellation of singularities is exact in the (R - MC) term

- The integral is bounded all over phase-space; we can therefore generate unweighted events!
  - "S-events" (which have m body kinematics)
  - "H-events" (which have m+1 body kinematics)

MC@NLO properties

- Good features of including the subtraction counter terms

1. **Double counting avoided**: The rate expanded at NLO coincides with the total NLO cross section

2. **Smooth matching**: MC@NLO coincides (in shape) with the parton shower in the soft/collinear region, while it agrees with the NLO in the hard region

3. **Stability:** weights associated to different multiplicities are separately finite. The *MC* term has the same infrared behavior as the real emission (there is a subtlety for the soft divergence)

Negative weights

$$\frac{d\sigma_{\text{NLOwPS}}}{dO} = \left[d\Phi_m\left(B + \int_{\text{loop}} V + \int d\Phi_1 MC\right)\right] I_{\text{MC}}^{(m)}(O) + \left[d\Phi_{m+1}(R - MC)\right] I_{\text{MC}}^{(m+1)}(O)$$

- We generate events for the two terms between the square brackets (S- and H-events) separately
- There is no guarantee that these contributions are separately positive (even though predictions for infra-red safe observables should always be positive!)
- Therefore, when we do event unweighting we can only unweight the events **up to a sign**. These signs should be taken into account when doing a physics analysis (i.e. making plots etc.)
- The events are only physical when they are showered.

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Code d'événement

MADGRAPH

Activer les réponses par SMS

What we have learned

- That all observable of an NLO computation are not NLO accurate.
- How to evaluate the loop
- NLO computation done with counter-events
  - ➡ bin miss-match possible
- NLO+PS generation allow event generation
  - ➡ Events Physical only after the Parton-Shower.
  - ➡ The Events should be generated for a given shower (in MC@NLO)