

Flavor vs. Collider

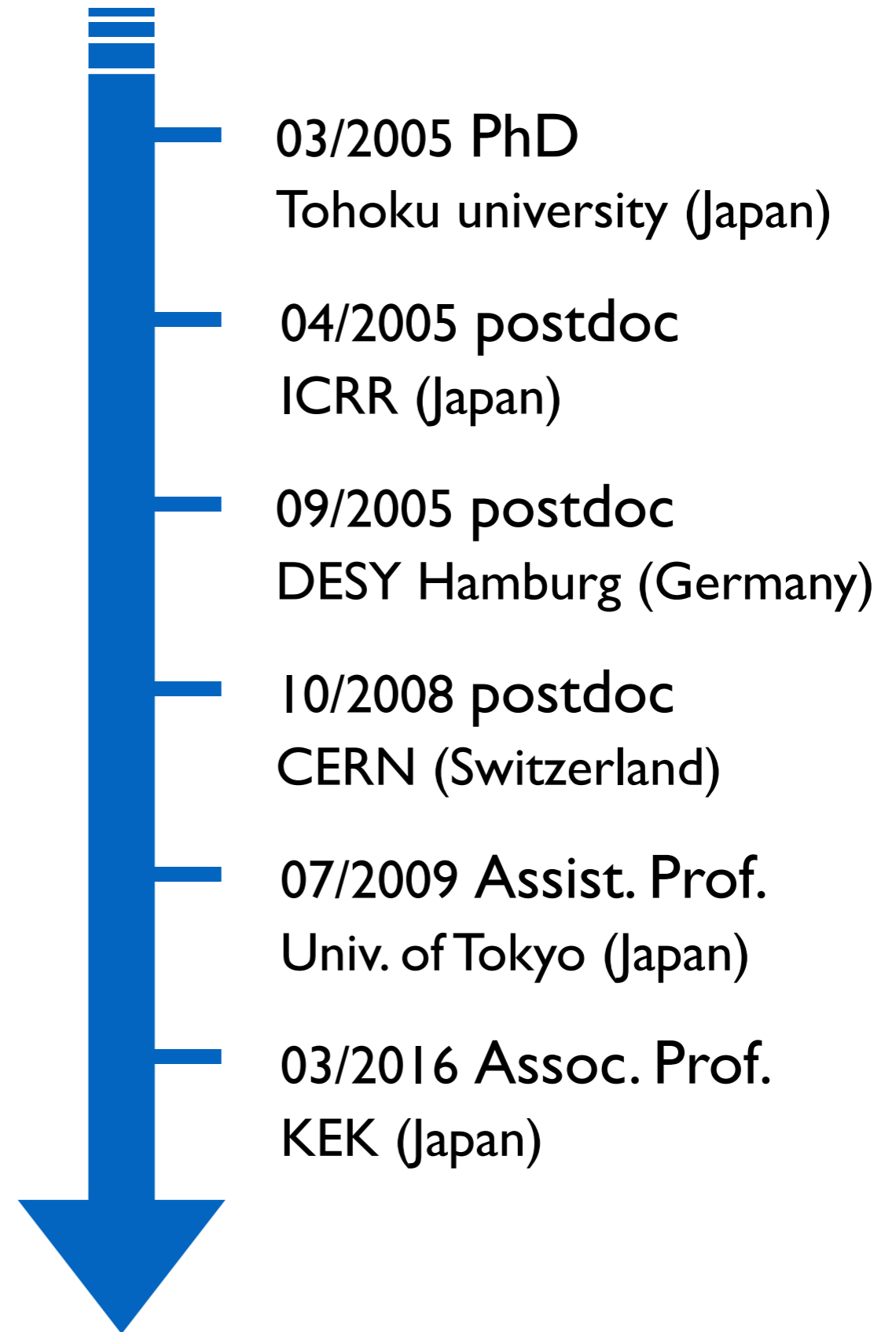
Motoi Endo (KEK)

5th Iwate Collider School, Iwate, 2026.3.5

Who am I?

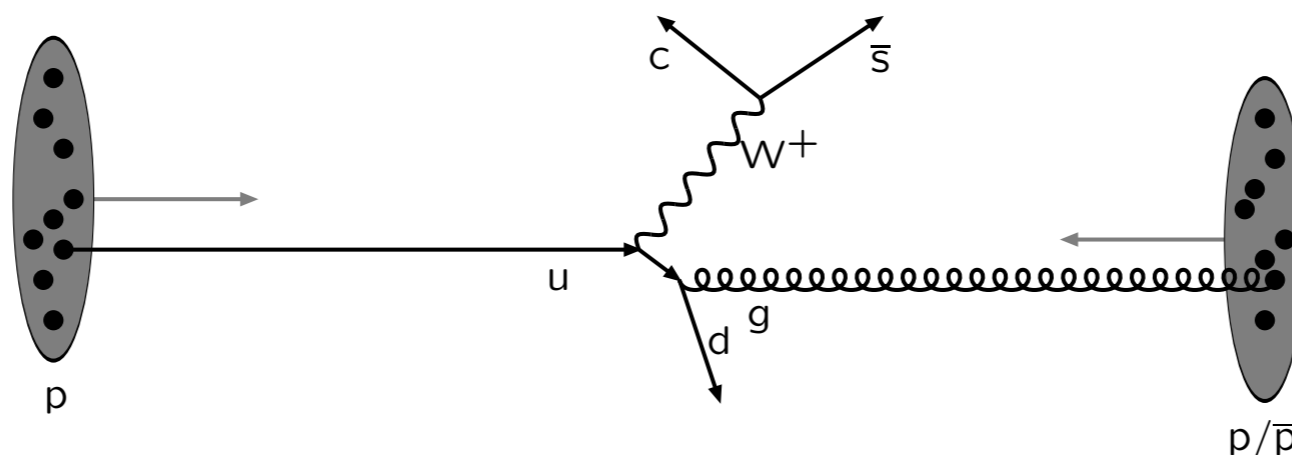
Professor at KEK
Theory center

Particle phenomenology
& particle cosmology
Recently, flavor physics

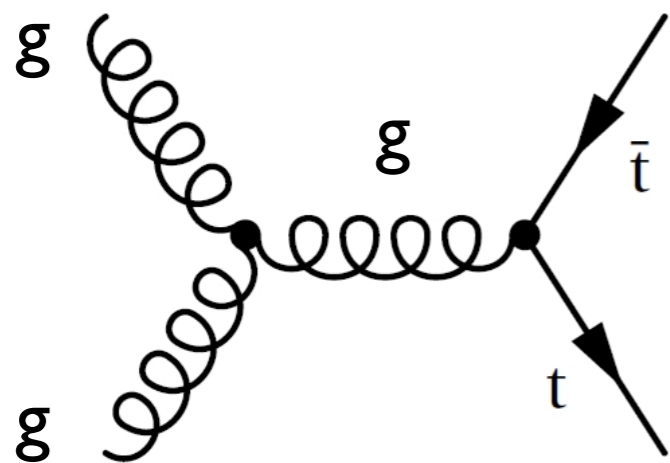


“Direct” search for new particles at *colliders*

LHC: proton-proton collision at center-of-energy of 7-14TeV



New particle productions e.g., via $pp \rightarrow \chi\chi$ (cf. $gg \rightarrow t\bar{t}$)

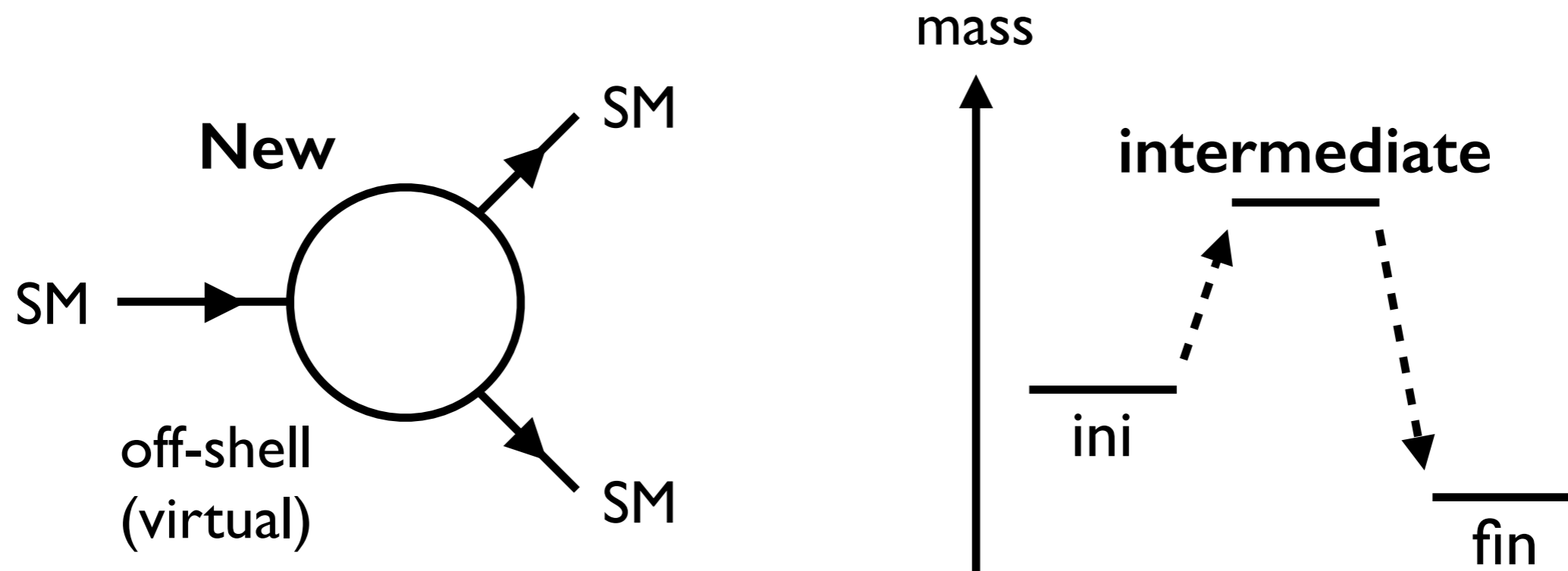


New heavy particles can be produced **directly** by collisions at high energy.

Sensitivity is limited by beam energy.

How to probe BSM in higher energy scale?

“Indirect” search at *flavor* experiments



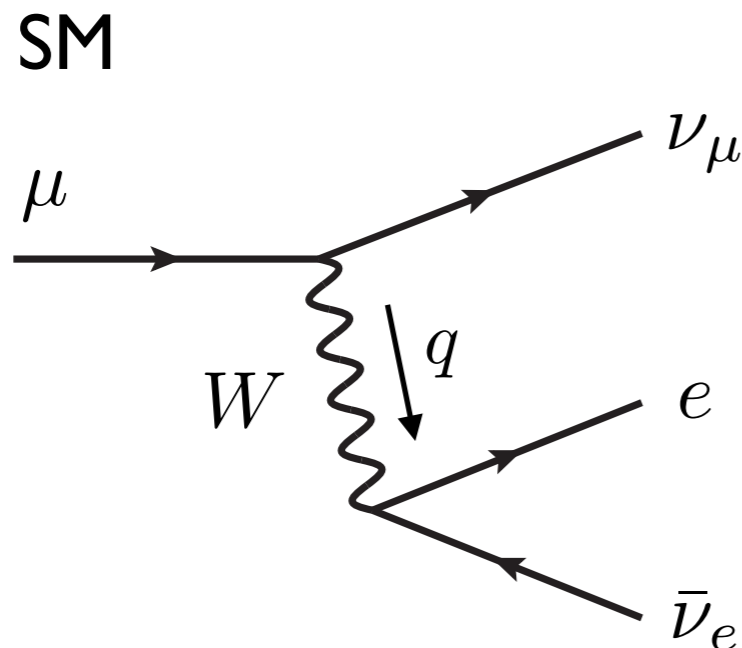
Processes with initial and final states composed by SM particles.

Heavy particles can appear **indirectly**, *i.e.*, through radiative corrections.

Quantum principle allows heavy new particles to contribute as **intermediate** (off-shell/virtual) states.

Example of decay induced by heavy particle

Muon decay: $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$



$$\mathcal{A} \propto \frac{g^2}{q^2 - M_W^2} (\bar{\nu}_\mu \gamma^\mu P_L \mu) (\bar{e} \gamma_\mu P_L \nu_e)$$

$$\approx - \frac{g^2}{M_W^2} (\bar{\nu}_\mu \gamma^\mu P_L \mu) (\bar{e} \gamma_\mu P_L \nu_e)$$

Fermi coupling const. $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$

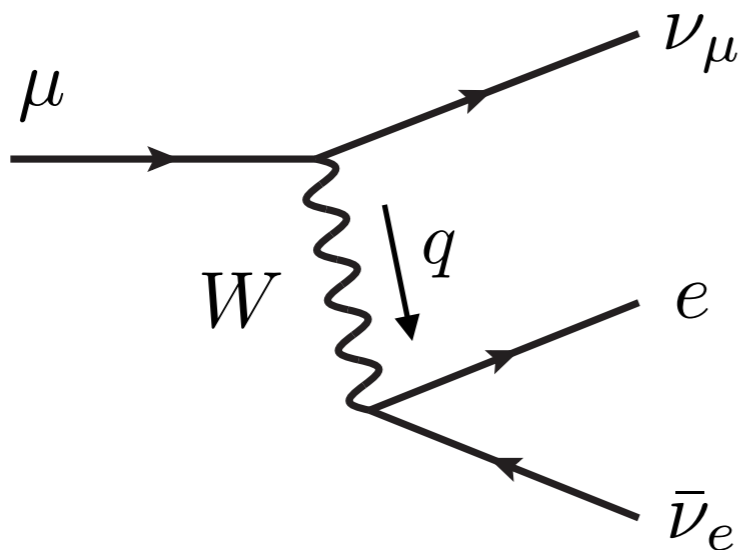
Muon decay proceeds via off-shell exchange of W -boson even though W boson is much heavier than muon.

Muon lifetime (inverse of total decay rate of muon) has been used to determine Fermi coupling constant.

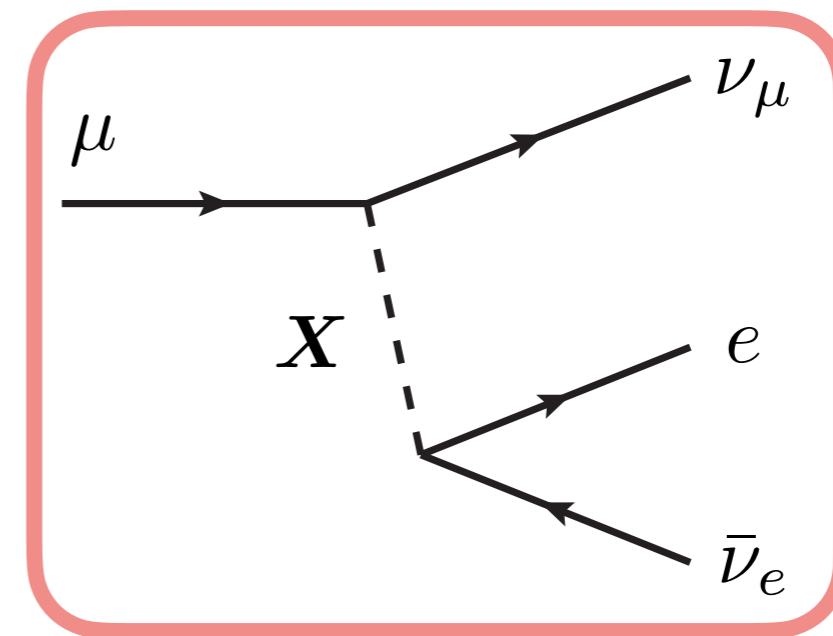
Correction from heavy new particle

Muon decay: $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$

SM



Physics beyond SM (BSM)



$$G_F \rightarrow G_F^{\text{SM}} + \delta G_F$$

Introduce new heavy particles (X) in off-shell states.

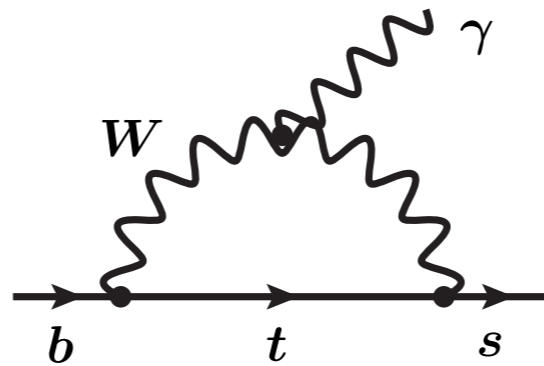
Such contributions are observed as deviations from SM predictions.

Sensitivity to BSM is determined by precision rather than beam energy.

Which process is sensitive to BSM?

Quark *flavor* violation

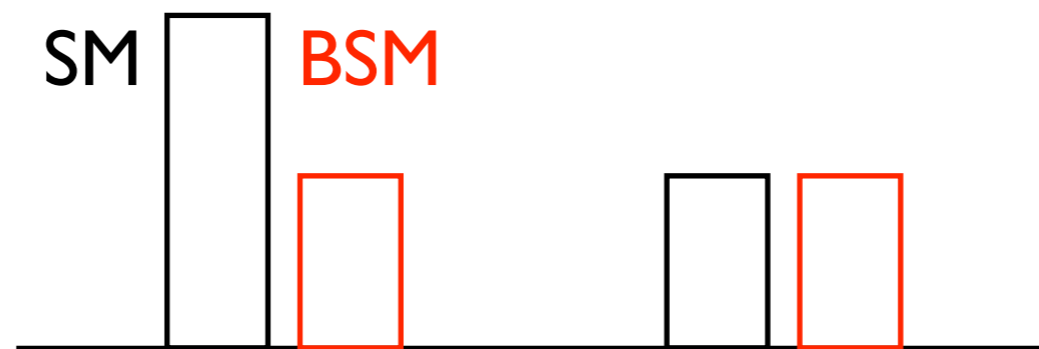
Processes where a quark changes its type = *flavor* (e.g., bottom to strange), violating the conservation of individual quark identity.



High sensitivity to (off-shell) contributions of new particles:

SM contributions are highly suppressed or forbidden at the tree level.

$$\text{Exp} - \text{SM} = \text{BSM}$$



Outline

- What is 'collider' & 'flavor'? — Done
- Quark flavor violation
- Hint of physics beyond SM (BSM)

Lecture
& Review

- How to test BSM contributions?
 - Test with collider observables
 - Test with flavor observables

Advanced

- Summary

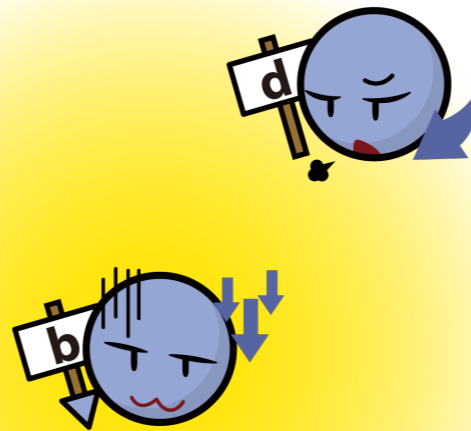
B meson

Quarks are not isolated (cf. confinement), but dressed by quarks and gluons.

B meson consists of b quark w/. light u,d quark ($m_b \gg m_{u,d}$).

$$B^0 : (\bar{b}d)$$

$$\bar{B}^0 : (b\bar{d})$$



Pseudo scalar

$$J^P = 0^-$$

b quark decays into lighter quarks by exchanging W boson in SM.

Many decay channels — Targets at Belle II, LHCb, future exp, e.g., Z factory

LHCb experiment

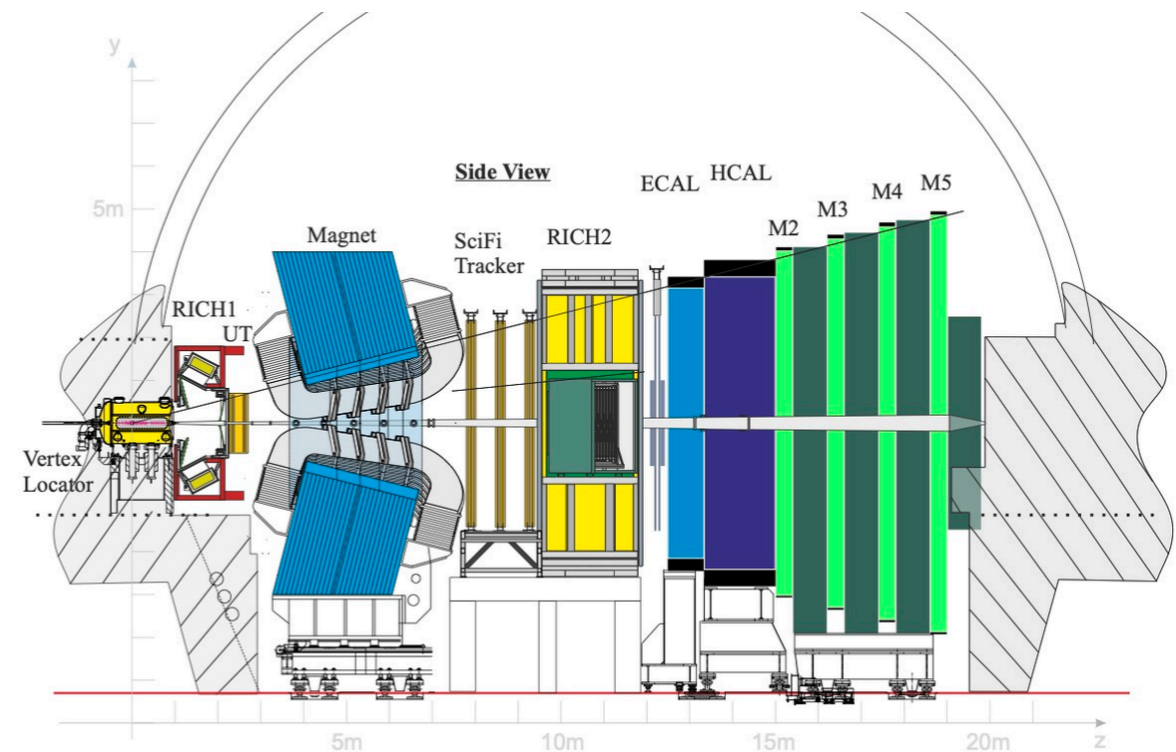
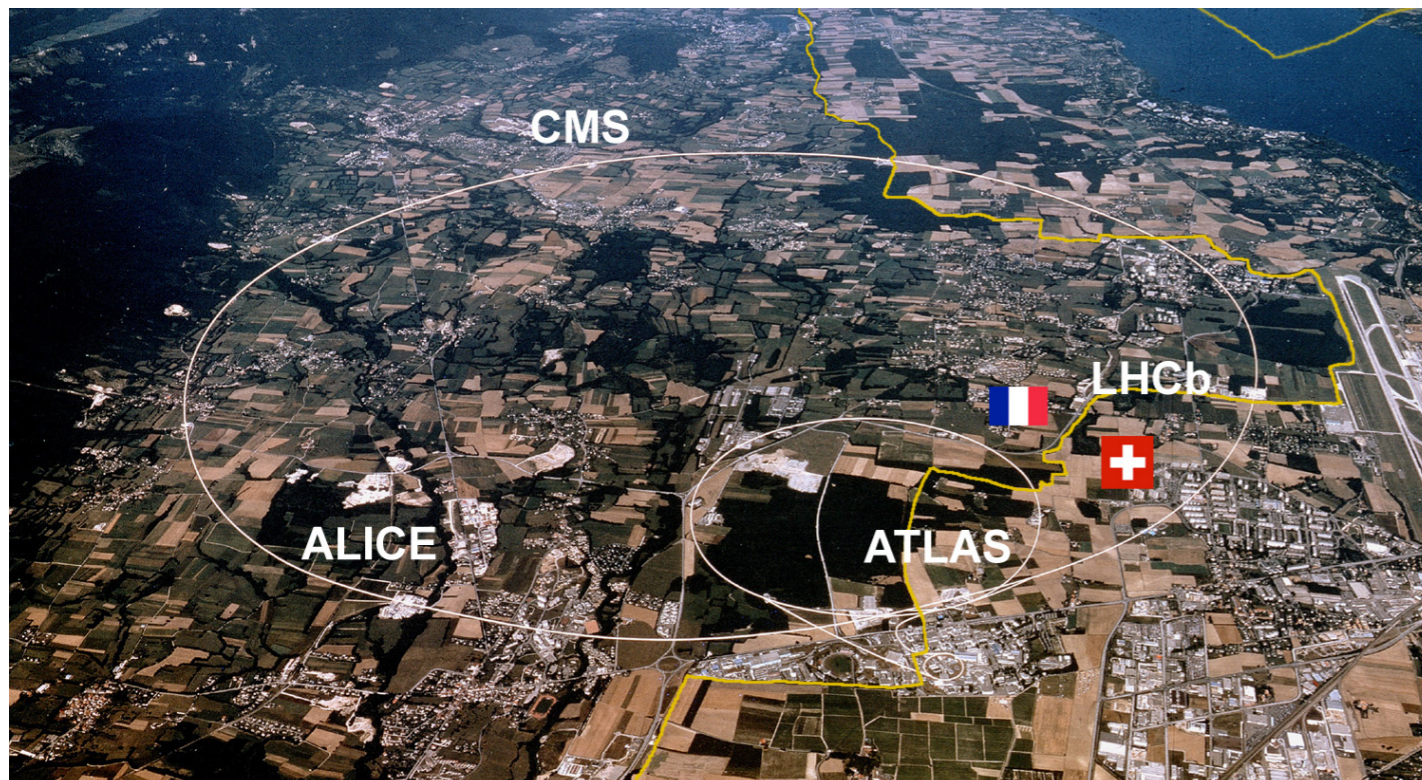
one of four LHC experiments, 2008-

pp collision at $\sqrt{s}=7-14\text{TeV}$, producing b quarks in forward region.

Target integrated luminosity: $270-300\text{fb}^{-1}$ (~ 2041)

→ $B^0: 3 \times 10^{13}$, $B^\pm: 3 \times 10^{13}$. Moreover, $B_s^0: 1 \times 10^{13}$, $B_c^\pm: 1 \times 10^{11}$, $\Lambda_b: 2 \times 10^{13}$

Hadron collider: huge statistics, but challenging to reconstruct events.



Belle II experiment

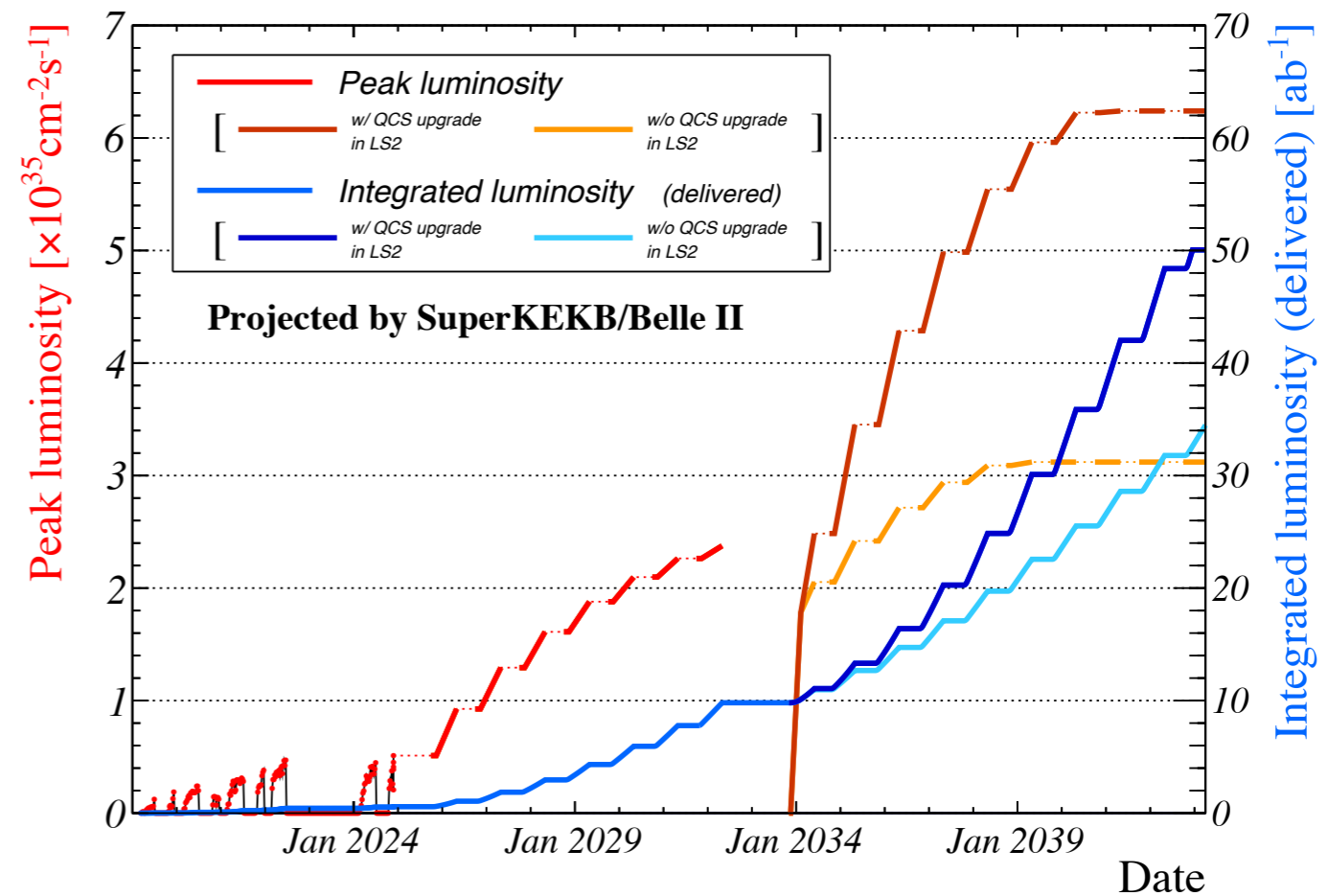
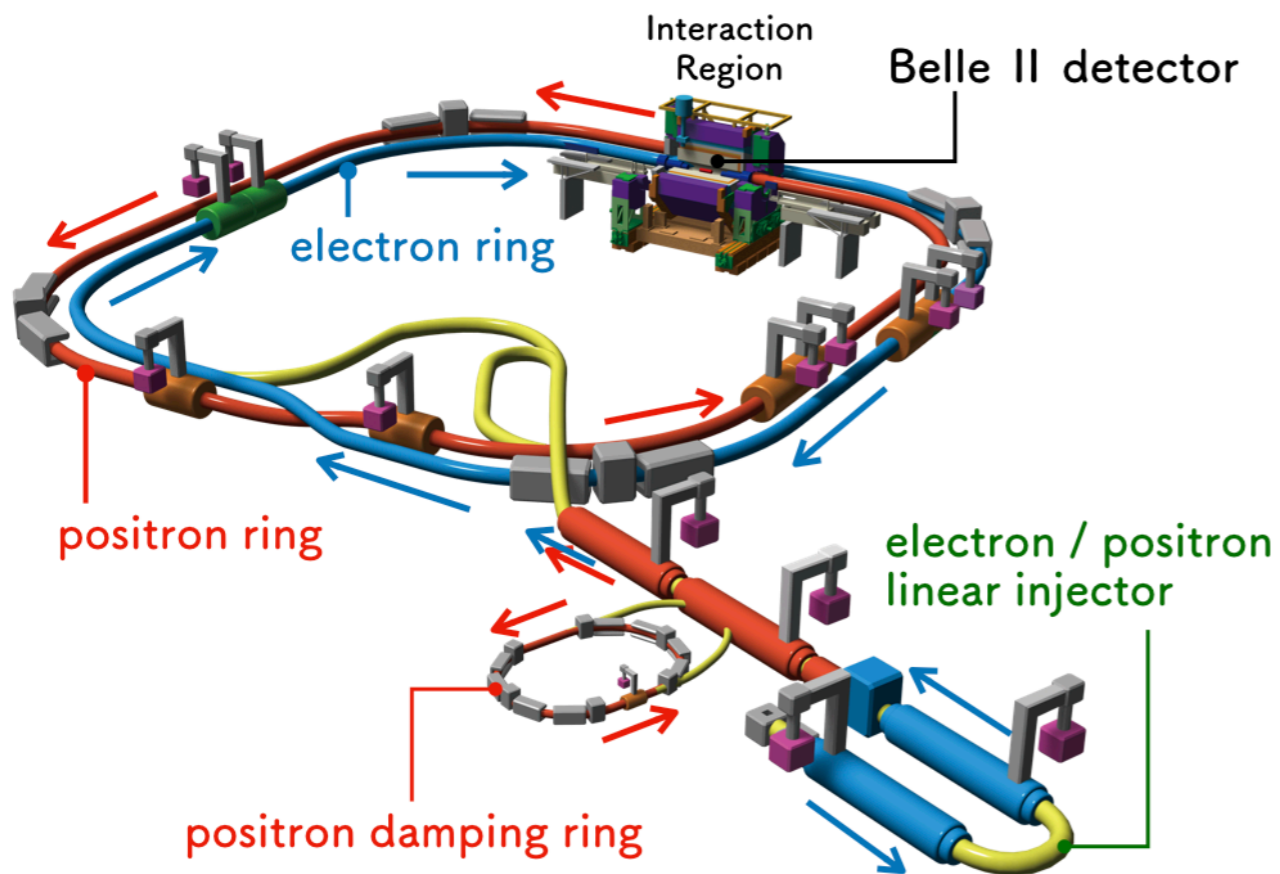
cf. Belle: 1999-2010, Belle II: 2018-

$e^- (7\text{GeV}) e^+ (4\text{GeV}) \rightarrow Y(4S) \rightarrow B\bar{B}$ (also $D\bar{D}, \tau^+\tau^-, \dots$)

Target integrated luminosity: 50ab^{-1} ($\sim 2040\text{s}$)

$\rightarrow B^0: 5.4 \times 10^{10}, B^\pm: 5.7 \times 10^{10}$ (also B_s^0 via $Y(5S)$ production)

Lepton collider: clean environment allows us to reconstruct events.



Z factory project

e^+e^- collision at $\sqrt{s} \sim 90\text{GeV}$ (around Z-boson mass)

→ Experiment of Z-boson factory cf. Higgs factory

(i) proposals of future “Higgs Factories”

\sqrt{s} / GeV $\int L / \text{ab}^{-1}$	91	160	240- 250	350- 365	550	1000- 3000	Beam Polarization
LCF	0.1	0.5*	3	0.4	8	8	(80%, 30%)
CEPC	100	6.9	21.6	1*			(0, 0)
FCC-ee	205	19.2	10.8	3.16			(0, 0)

(+ emerging Muon Colliders, μ Tristen, HELEN, HALHF, ReLiC, ALEGRO, XCC, etc)

LCF: Linear Collider Facility @ CERN; identical to scenarios proposed by LC Vision Team up to 550 GeV; representing ILC, CLIC, C³

Z factory project

e^+e^- collision at $\sqrt{s} \sim 90\text{GeV}$ (around Z-boson mass)

→ Experiment of Z-boson factory cf. Higgs factory

→ **B-hadron factory** cf. $\text{Br}(Z \rightarrow b\bar{b}) = 15.12\%$

Lepton collider: clean environment allows us to reconstruct events.

All B hadrons are produced.

“Tera-Z” = 10^{12} Z

Particle	BESIII	Belle II (50 ab^{-1} on $\Upsilon(4S)$)	LHCb (300 fb^{-1})	CEPC ($4 \times \text{Tera-Z}$)
B^0, \bar{B}^0	-	5.4×10^{10}	3×10^{13}	4.8×10^{11}
B^\pm	-	5.7×10^{10}	3×10^{13}	4.8×10^{11}
B_s^0, \bar{B}_s^0	-	6.0×10^8 (5 ab^{-1} on $\Upsilon(5S)$)	1×10^{13}	1.2×10^{11}
B_c^\pm	-	-	1×10^{11}	7.2×10^8
$\Lambda_b^0, \bar{\Lambda}_b^0$	-	-	2×10^{13}	1×10^{11}

Ai et.al, Flavor Physics at CEPC: a General Perspective, 2412.19743

FCC-ee: $\sim 5 \times$ “Tera-Z”

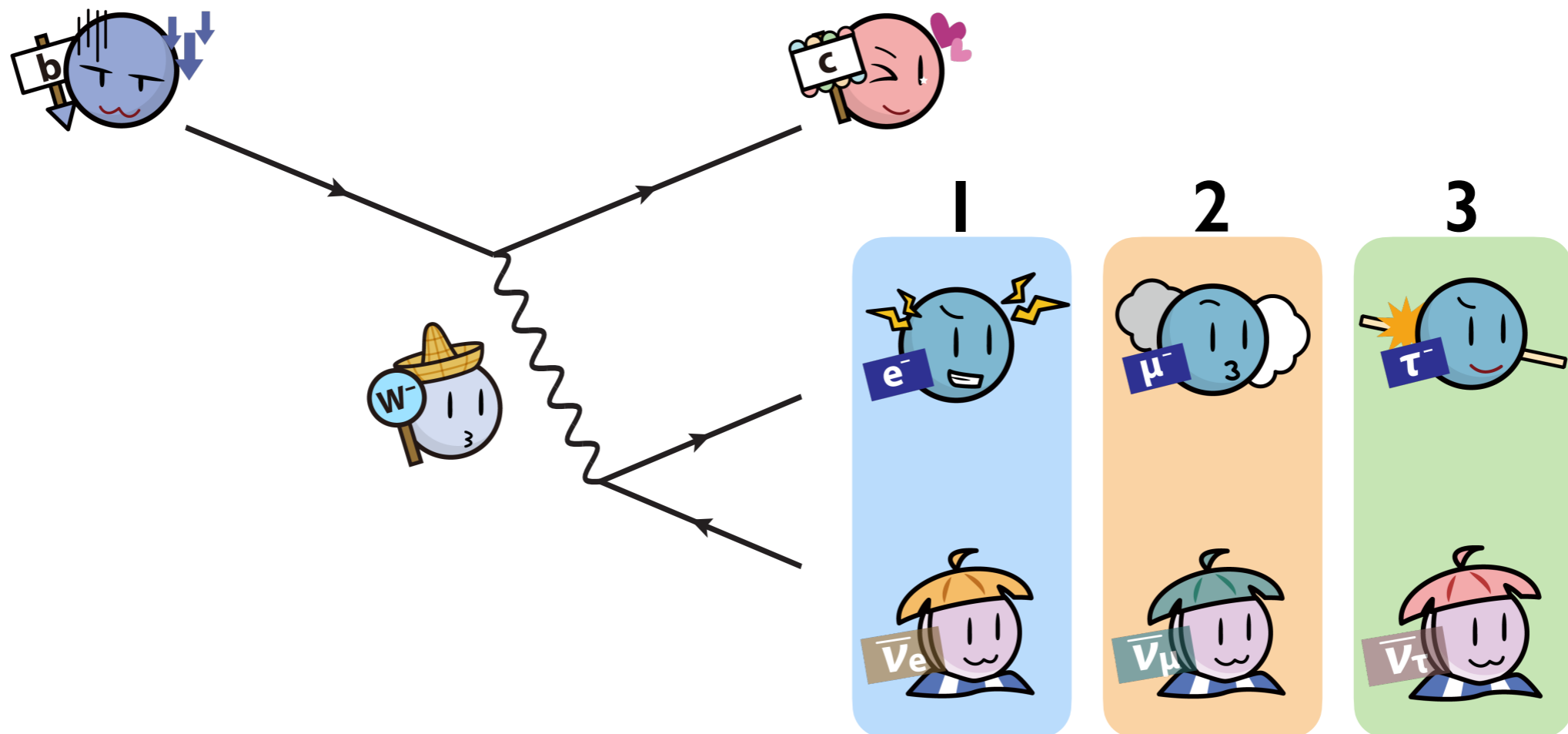
* LCF also proposes Giga-Z projects

Semileptonic decay of b quark

Electron, muon, and tau lepton have same gauge charges.

In SM, quark decay rates into leptons become same (but mass effects).

— lepton-flavor universality



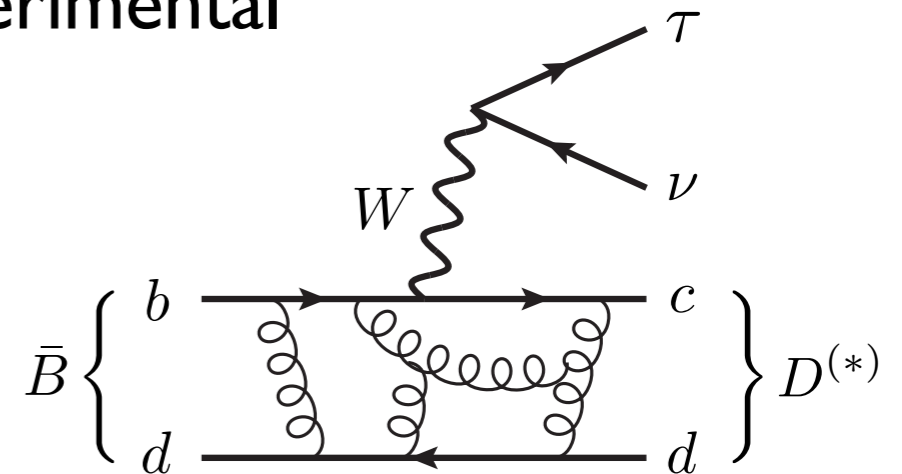
Lepton-flavor universality violation

$$B, D : J^P = 0^-$$

$$D^* : J^P = 1^-$$

Making ratios benefits from cancellations of experimental and theoretical uncertainties (hadrons and V_{cb}).

$$R_{D^{(*)}} = \frac{\Gamma(B \rightarrow D^{(*)} \tau^- \bar{\nu}_\tau)}{\Gamma(B \rightarrow D^{(*)} \ell^- \bar{\nu}_\ell)} \quad \ell = e, \mu$$



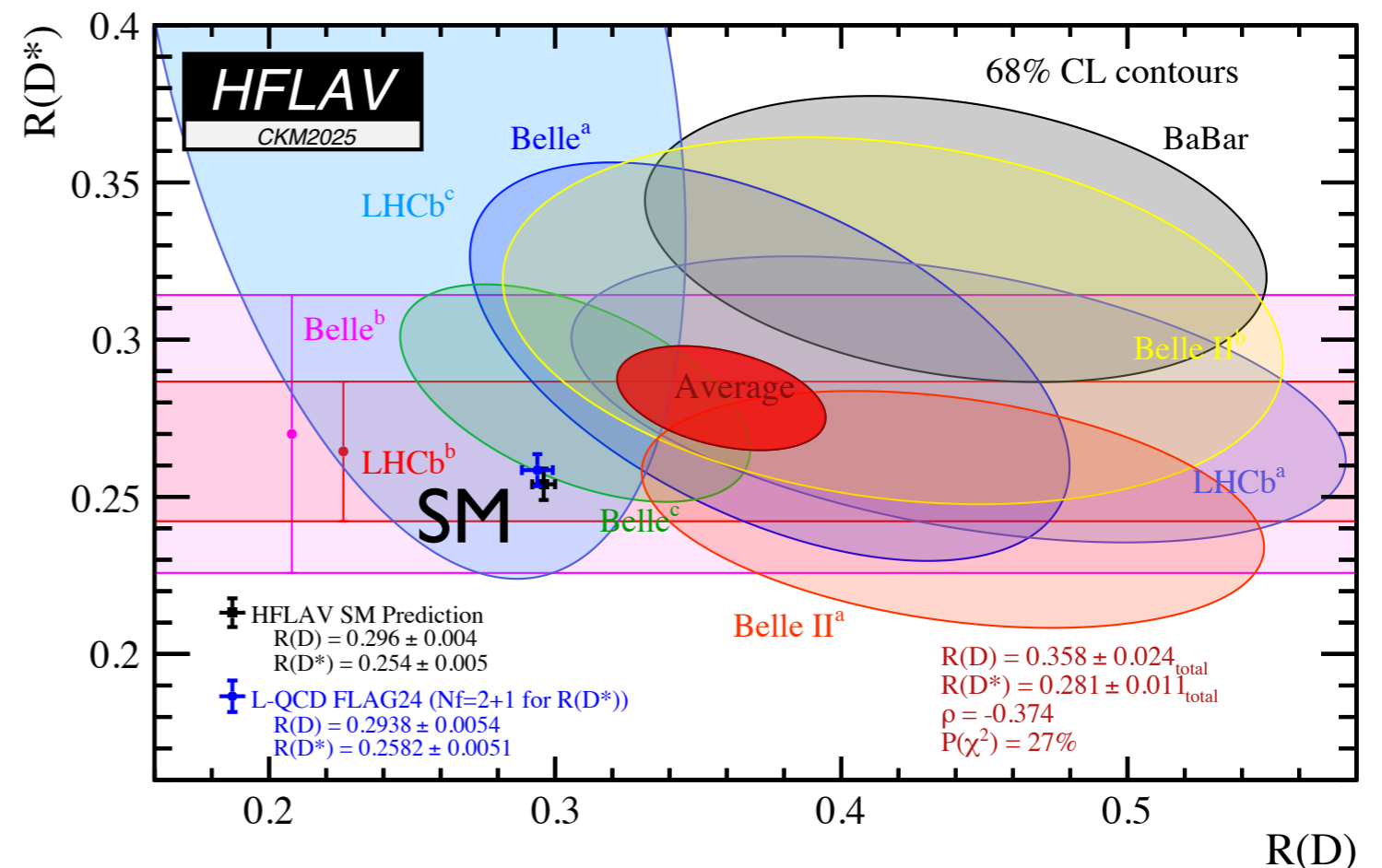
Current status:

3.8 σ tension

from SM

BSM contributions to $B \rightarrow D^{(*)} \ell \nu$ are limited.

Imply BSM in $b \rightarrow c \tau \bar{\nu}$.



Theoretical prediction

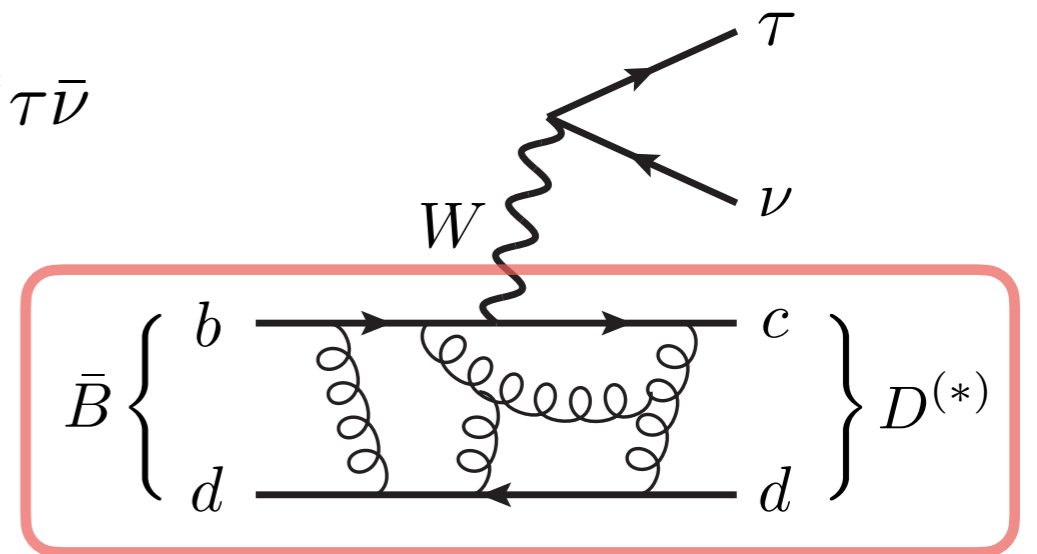
In SM, b quark decay proceeds via W boson exchange.

b quark is not isolated, but is dressed by quark and gluons:

$$b \rightarrow c\tau\bar{\nu} \Rightarrow \bar{B} \rightarrow D\tau\bar{\nu}, \bar{B} \rightarrow D^*\tau\bar{\nu}$$

Transition amplitude:

$$\mathcal{A} = -\frac{4G_F}{\sqrt{2}} V_{cb} \langle D^{(*)}\tau\bar{\nu} | (\bar{c}_L\gamma^\mu b_L)(\bar{\tau}_L\gamma_\mu\nu_\tau) | \bar{B} \rangle$$



Introduce hadron matrix elements including **form factors**.

$$\langle D | \bar{c}\gamma^\mu b | \bar{B} \rangle = f_+(q^2)(p_B + p_D)^\mu + f_-(q^2)(p_B - p_D)^\mu$$

$$\langle D | \bar{c}\gamma^\mu\gamma_5 b | \bar{B} \rangle = 0$$

Hadron matrix element parametrization

1. List all dynamical variables, e.g., momentum, polarization vector

$$p_B, p_D$$

2. Matrix elem. consists of all possible linear combinations of variables in '1' which match the Lorentz structures and symmetries (e.g., C, P, T)

$$\langle D | \bar{c} \gamma^\mu b | \bar{B} \rangle : p_B^\mu, p_D^\mu \rightarrow (p_B + p_D)^\mu, (p_B - p_D)^\mu$$

$$\langle D | \bar{c} \gamma^\mu \gamma_5 b | \bar{B} \rangle = 0 \quad \text{because of Parity}$$

Parity: odd odd odd

3. Form factors are Lorentz scalar ($q = p_B - p_D$)

$$p_B^2 = m_B^2, p_D^2 = m_D^2, p_B \cdot p_D (\rightarrow q^2)$$

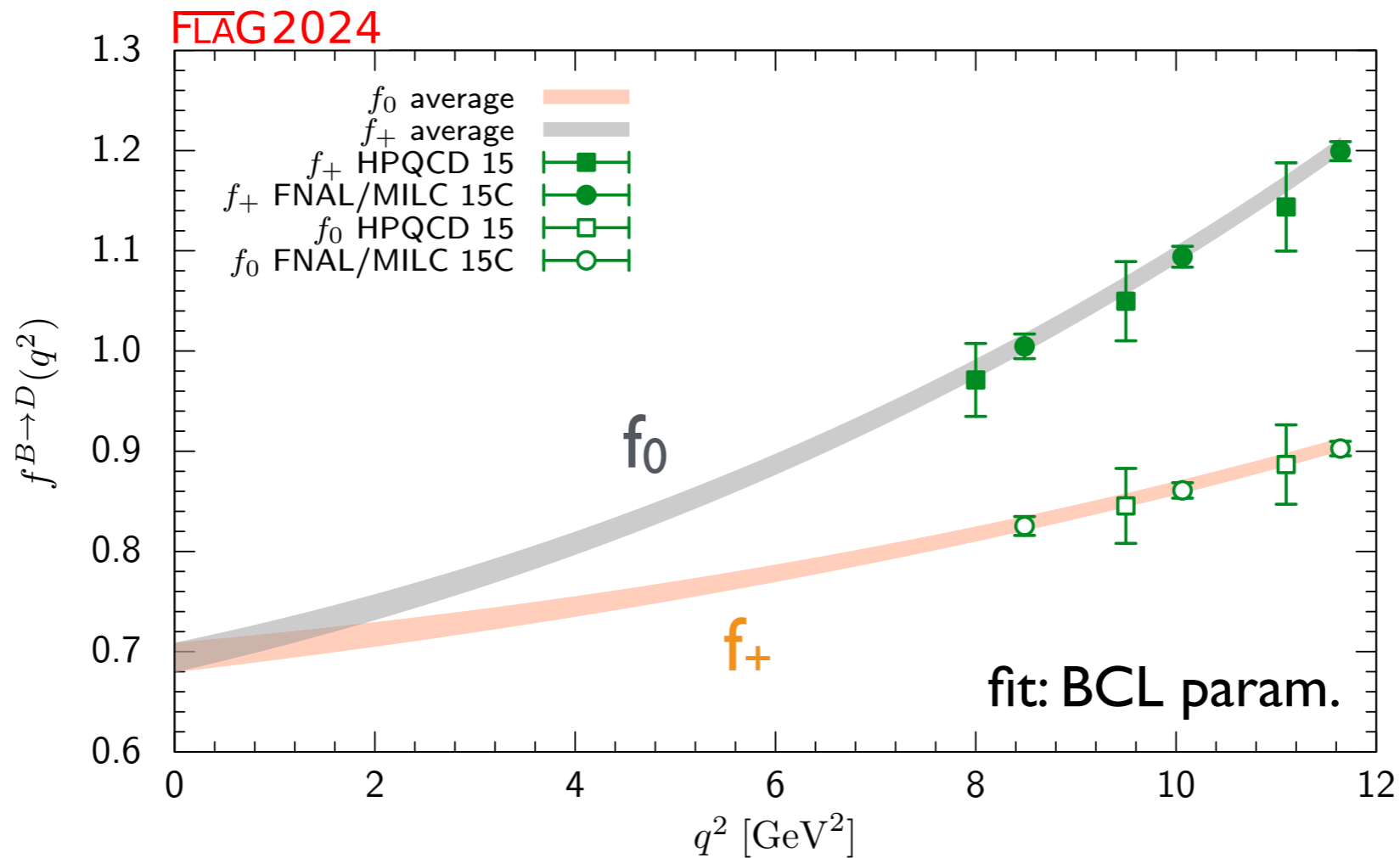
$$\rightarrow \langle D | \bar{c} \gamma^\mu b | \bar{B} \rangle = f_+(q^2) (p_B + p_D)^\mu + f_-(q^2) (p_B - p_D)^\mu$$

4. Determine form factors by using lattice, experimental inputs, etc.

→ Literature search

Form factor from lattice

$$\begin{aligned} \langle D | \bar{c} \gamma^\mu b | \bar{B} \rangle &= f_+(q^2) (p_B + p_D)^\mu + f_-(q^2) q^\mu \\ &= f_+(q^2) (p_B + p_D)^\mu + [f_0(q^2) - f_+(q^2)] \frac{m_B^2 - m_D^2}{q^2} q^\mu \end{aligned}$$



$$q = p_B - p_D$$

Form factor for BSM

Scalar operator: $q = p_B - p_D = p_b - p_c$

$$q_\mu \langle D | \bar{c} \gamma^\mu b | \bar{B} \rangle = \langle D | \bar{c} (\not{p}_b - \not{p}_c) b | \bar{B} \rangle = (m_b - m_c) \langle D | \bar{c} b | \bar{B} \rangle$$

$$\begin{aligned} q_\mu \langle D | \bar{c} \gamma^\mu b | \bar{B} \rangle &= q_\mu \left[f_+(q^2) (p_B + p_D)^\mu + [f_0(q^2) - f_+(q^2)] \frac{m_B^2 - m_D^2}{q^2} q^\mu \right] \\ &= (m_B^2 - m_D^2) f_0(q^2) \end{aligned}$$

$$\therefore \langle D | \bar{c} b | \bar{B} \rangle = \frac{m_B^2 - m_D^2}{m_b - m_c} f_0(q^2)$$

Here, Dirac equation in momentum space representation: $\not{p}\psi = m\psi$

Also, $\langle D | \bar{c} \gamma_5 b | \bar{B} \rangle = 0$

Tensor operator: new form factor $f_T(q^2)$

$$\langle D | \bar{c} \sigma^{\mu\nu} b | \bar{B} \rangle = -\frac{2i f_T(q^2)}{m_B + m_D} (p_B^\mu p_D^\nu - p_B^\nu p_D^\mu)$$

Decay rate

Transition amplitude

$$\begin{aligned}
 \mathcal{A} &= -\frac{4G_F}{\sqrt{2}} V_{ub} \langle D\ell^+ \bar{\nu} | (\bar{c}_L \gamma^\mu b_L) (\bar{\tau}_L \gamma_\mu \nu_L) | \bar{B} \rangle && \begin{cases} P = p_B + p_D \\ q = p_B - p_D = p_\tau + p_\nu \end{cases} \\
 &= -\frac{4G_F}{\sqrt{2}} V_{ub} \langle D(k) | \bar{c}_L \gamma^\mu b_L | \bar{B}(p) \rangle \bar{u}(p_\tau) \gamma_\mu P_L v(p_\nu) \\
 &= -\sqrt{2} G_F V_{ub} \langle D(k) | \bar{c} \gamma^\mu b | \bar{B}(p) \rangle \bar{u}(p_\tau) \gamma_\mu P_L v(p_\nu) && \leftarrow P_L = \frac{1 - \gamma_5}{2} \\
 &= -\sqrt{2} G_F V_{ub} [f_+(q^2) P^\mu + f_-(q^2) q^\mu] \bar{u}(p_\tau) \gamma_\mu P_L v(p_\nu) \\
 &= -\sqrt{2} G_F V_{ub} f_+(q^2) \bar{u}(p_\nu) \not{P} P_L v(p_e) + \underbrace{(m_\tau \text{ term})}_{\text{ignore for simplicity}} && \leftarrow \text{EOM for leptons}
 \end{aligned}$$

Spin-averaged/summed amplitude squared:

$$\overline{\sum_{\text{spin}} |\mathcal{M}|^2} = 2G_F^2 |V_{ub}|^2 f_+(q^2)^2 (\lambda_B - u^2)$$

where $u = -2k(p_\tau - p_\nu)$, $\lambda_B = \lambda(m_B^2, m_D^2, q^2)$

$$[\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx]$$

Decay rate

Integration of 3-body phase space (m_τ is neglected)

$$\frac{d\Gamma(\bar{B} \rightarrow D\tau\bar{\nu})}{dq^2} = \frac{1}{256\pi^3 m_B^3} \int_{-\lambda_B^{1/2}}^{+\lambda_B^{1/2}} du \overline{\sum_{\text{spin}} |\mathcal{M}|^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} f_+(q^2)^2 \lambda_B^{3/2}$$

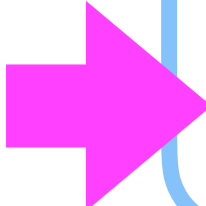
Including m_τ (w/o derivation \rightarrow Let's try!)

$$\begin{aligned} \frac{d\Gamma(\bar{B} \rightarrow D\tau\bar{\nu})}{dq^2} &= \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \lambda_B^{1/2} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \\ &\times \left[\left(1 + \frac{m_\tau^2}{2q^2}\right) \lambda_B f_+(q^2)^2 + \frac{3m_\tau^2}{2q^2} (m_B^2 - m_D^2)^2 f_0(q^2)^2 \right] \\ & \qquad \qquad \qquad m_\tau^2 < q^2 < (m_B - m_D)^2 \end{aligned}$$

* Instead of q^2 , w is also used in literature:

$$w = \frac{m_B^2 + m_D^2 - q^2}{2m_B m_D}$$

Outline

- 
- What is ‘collider’ & ‘flavor’? — Done
 - Quark flavor violation
 - Hint of physics beyond SM (BSM)

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& Review

- How to test BSM contributions?
 - Test with collider observables
 - Test with flavor observables

Advanced

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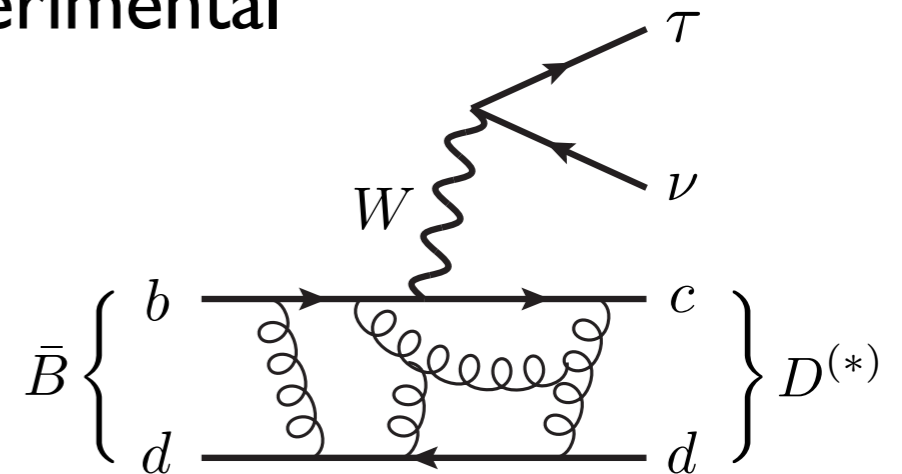
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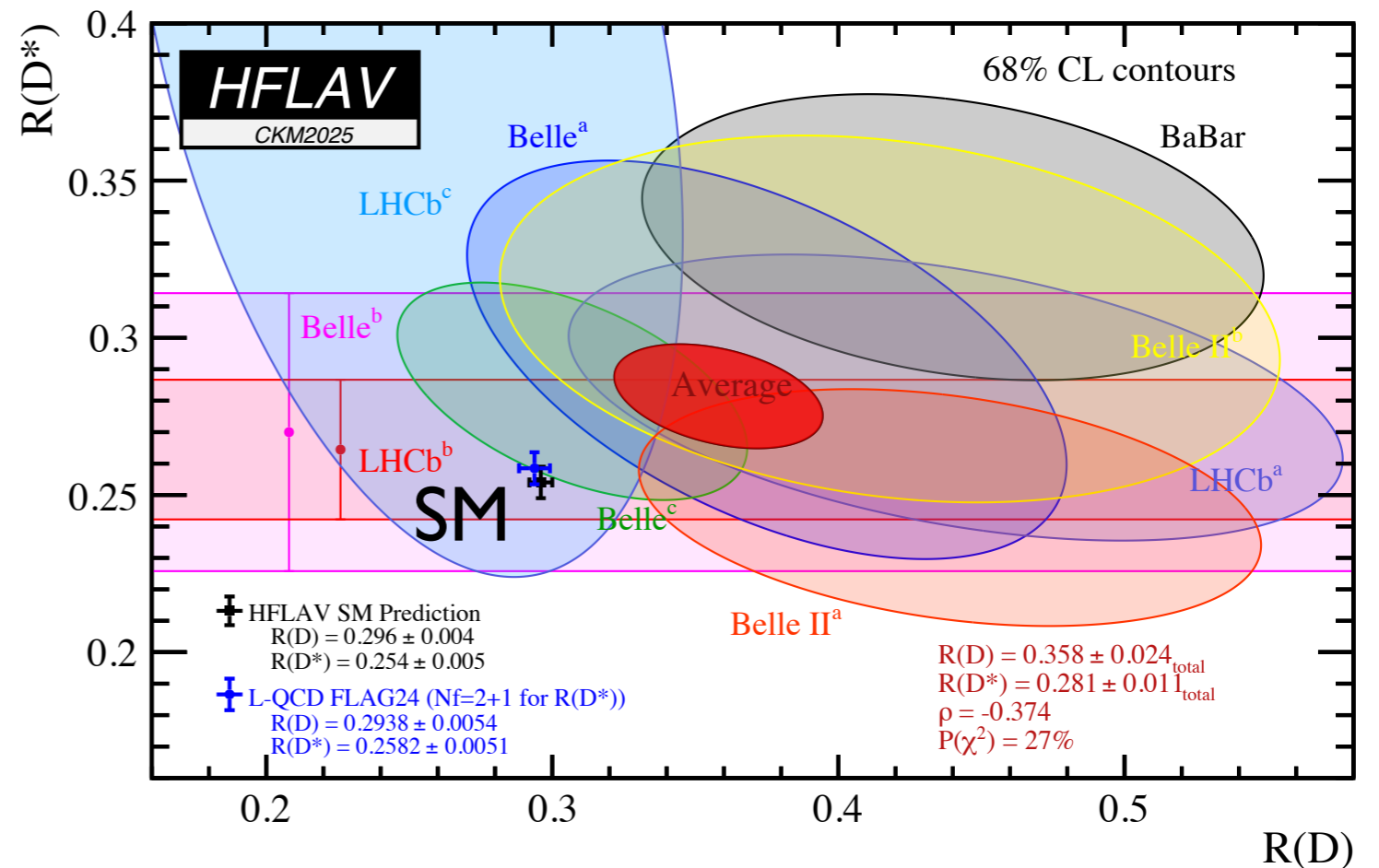
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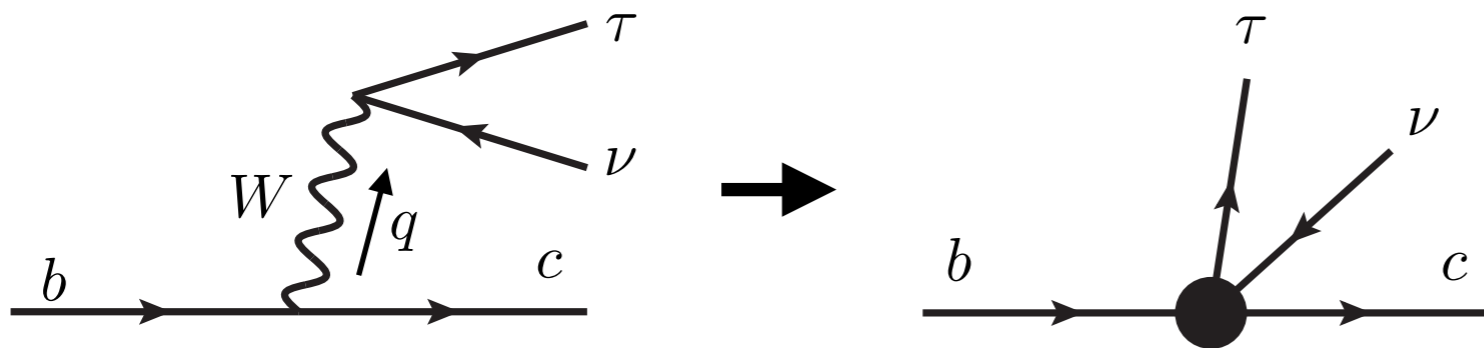


Physics beyond SM: low-energy effective theory

SM contribution is obtained by integrating out heavy W-boson propagation, leading to effective interaction (\rightarrow low-energy effective theory).

Similarly, by integrating out new heavy particles, BSM contributions are expressed by effective interactions.

Standard Model

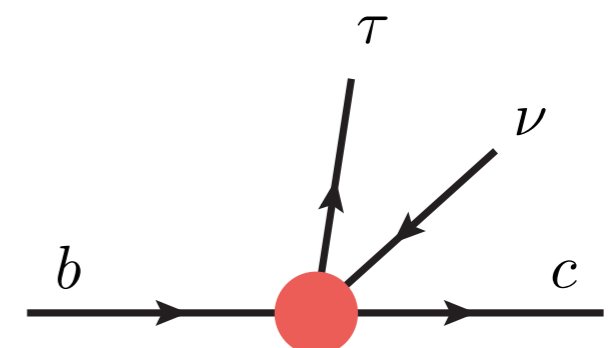


$$\frac{g^2}{q^2 - M_W^2} (\bar{c}_L \gamma^\mu b_L) (\bar{\tau}_L \gamma_\mu \nu_L) \approx \frac{-g^2}{M_W^2} (\bar{c}_L \gamma^\mu b_L) (\bar{\tau}_L \gamma_\mu \nu_L)$$

W-boson

effective

New heavy particle



$$\frac{1}{M^2} (\bar{c} \Gamma b) (\bar{\tau} \Gamma \nu)$$

effective

Physics beyond SM: low-energy effective theory

BSM contributes only to tau channel.

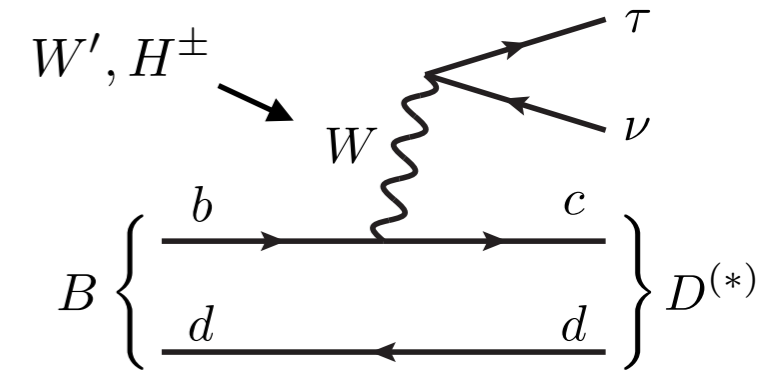
$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} \left[\underbrace{(1 + C_{V_L})}_{\text{SM}} O_{V_L} + C_{S_L} O_{S_L} + C_{S_R} O_{S_R} + C_T O_T \right]$$

Fit to R_D and R_{D^*} results

	operator	Wilson coefficient	
V_L	$(\bar{c}\gamma^\mu P_L b)(\bar{\tau}\gamma_\mu P_L \nu)$	$C_{V_L} \approx 0.08$	} BSM scale O(1)TeV
S_L	$(\bar{c}P_L b)(\bar{\tau}P_L \nu)$	$C_{S_L} \approx -0.57 \pm i 0.86$	
S_R	$(\bar{c}P_R b)(\bar{\tau}P_L \nu)$	$C_{S_R} \approx 0.18$	
T	$(\bar{c}\sigma^{\mu\nu} P_L b)(\bar{\tau}\sigma_{\mu\nu} P_L \nu)$	$C_T \approx 0.02 \pm i 0.13$	

Constraints/correlations depend on UV models.

BSM models in UV scale



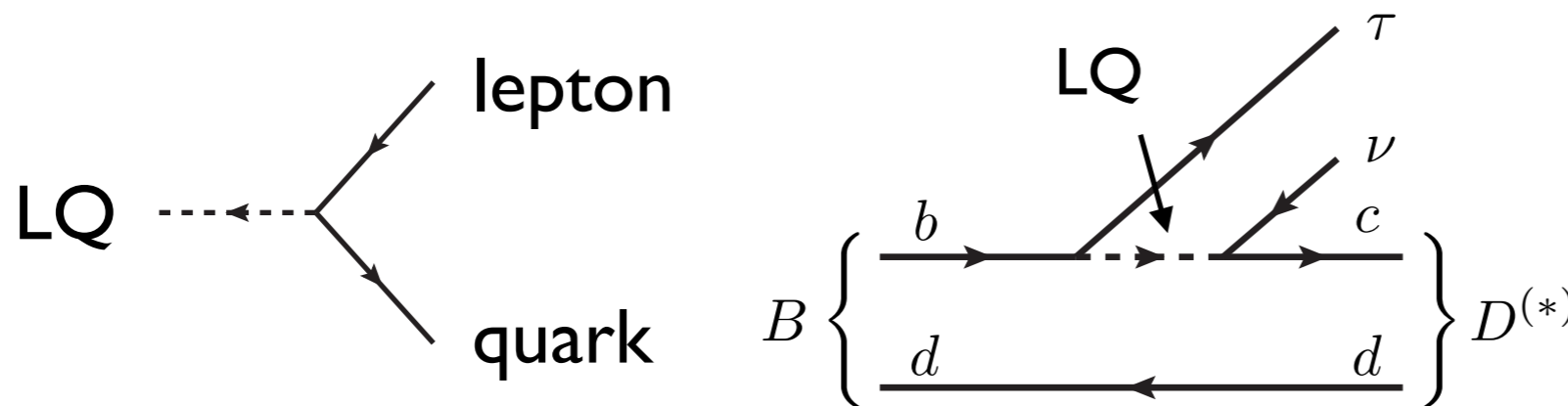
W' is ruled out by flavor constraints on Z' and colliders.

Charged Higgs boson in Type-II two-Higgs doublet model (2HDM) cannot explain R_D and R_{D^*} excesses simultaneously.

Two major scenarios: **charged Higgs boson (non Type-II) & leptoquark.**

Leptoquark (LQ): spin 0 or 1 boson, carry both lepton & baryon #, couple to lepton and quark simultaneously.

R_2, S_1, U_1 can explain both R_D, R_{D^*} excesses.



LQ charge

$(SU(3), SU(2), U(1))$	Spin	Symbol
$(\mathbf{3}, \mathbf{3}, 1/3)$	0	S_3
$(\mathbf{3}, \mathbf{2}, 7/6)$	0	R_2
$(\mathbf{3}, \mathbf{2}, 1/6)$	0	\tilde{R}_2
$(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$	0	\tilde{S}_1
$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$	0	S_1
$(\bar{\mathbf{3}}, \mathbf{1}, -2/3)$	0	\bar{S}_1
$(\mathbf{3}, \mathbf{3}, 2/3)$	1	U_3
$(\bar{\mathbf{3}}, \mathbf{2}, 5/6)$	1	V_2
$(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$	1	\tilde{V}_2
$(\mathbf{3}, \mathbf{1}, 5/3)$	1	\tilde{U}_1
$(\mathbf{3}, \mathbf{1}, 2/3)$	1	U_1
$(\mathbf{3}, \mathbf{1}, -1/3)$	1	\bar{U}_1

Fit to R_D and R_{D^*} in LQ models

LQ contribution after LQ decoupling: $C_i \sim \frac{g_{LQ}^2}{M_{LQ}^2}$

$$R_2 : C_{S_L} = 4C_T$$

$$S_1 : C_{V_L}, C_{S_L} = -4C_T$$

$$U_1 : C_{S_R} = -2e^{i\phi}C_{V_L} \quad [\text{UV: U(2)-flavored model}]$$

Include QCD corrections, and then, fit to R_D and R_{D^*} exp. data

$$R_2 : C_{S_L} = 8.4 C_T = -0.09 \pm i 0.56$$

$$S_1 : C_{S_L} = -8.9 C_T = 0.18 \quad (C_{V_L} = 0)$$

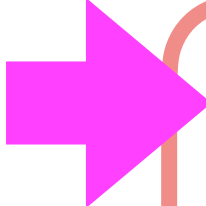
$$U_1 : C_{V_L} = 0.08, \phi = 0.5\pi \quad (C_{S_R} = -3.7e^{i\phi}C_{V_L})$$

All are consistent with experimental constraints such as LHC.

Outline

- What is 'collider' & 'flavor'? — Done
- Quark flavor violation
- Hint of physics beyond SM (BSM)

Lecture
& Review

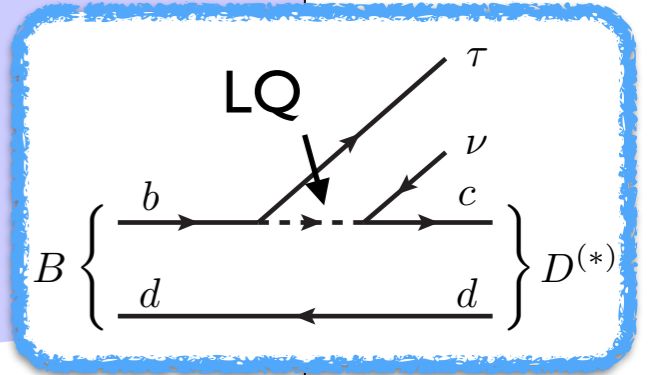
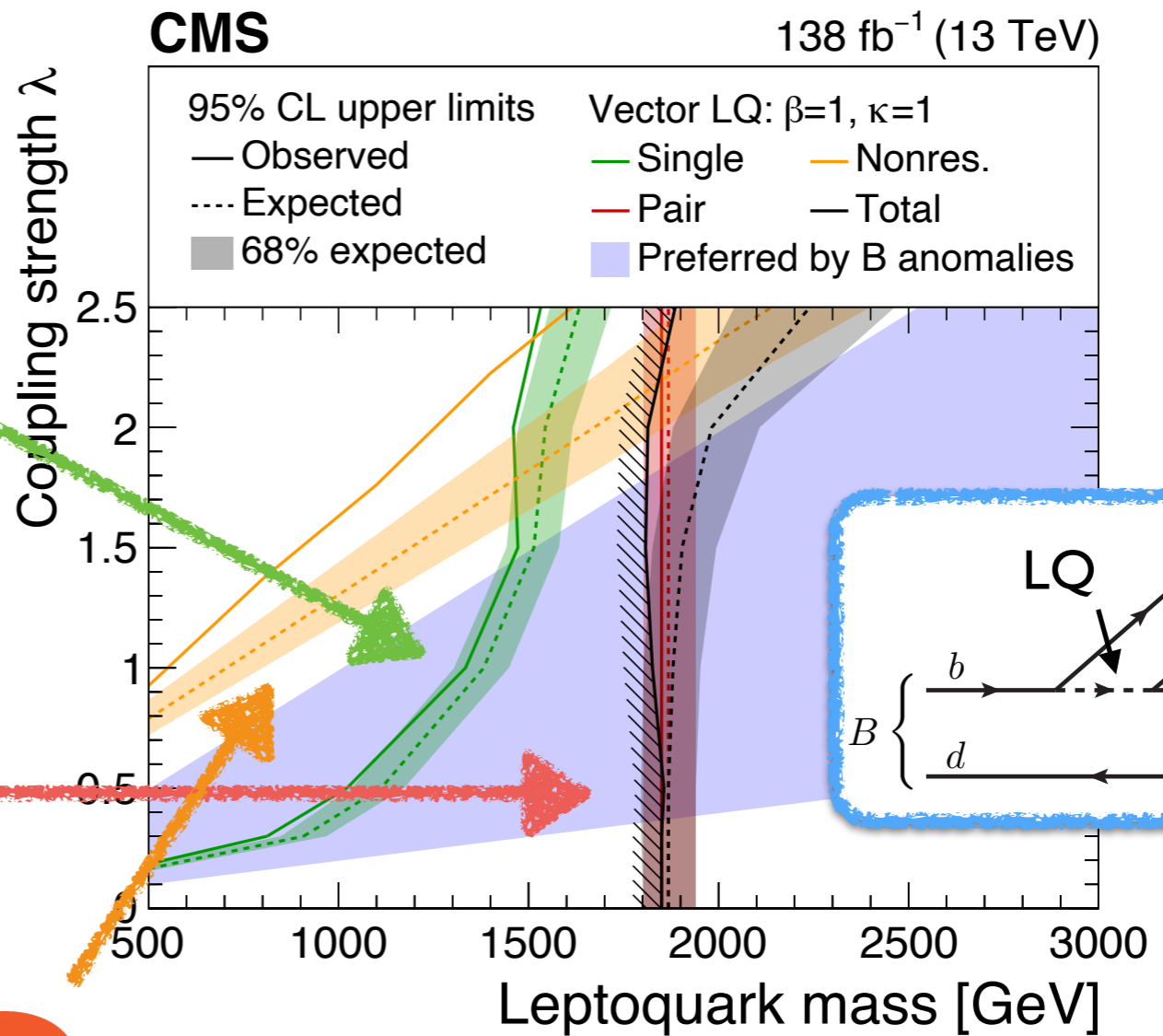
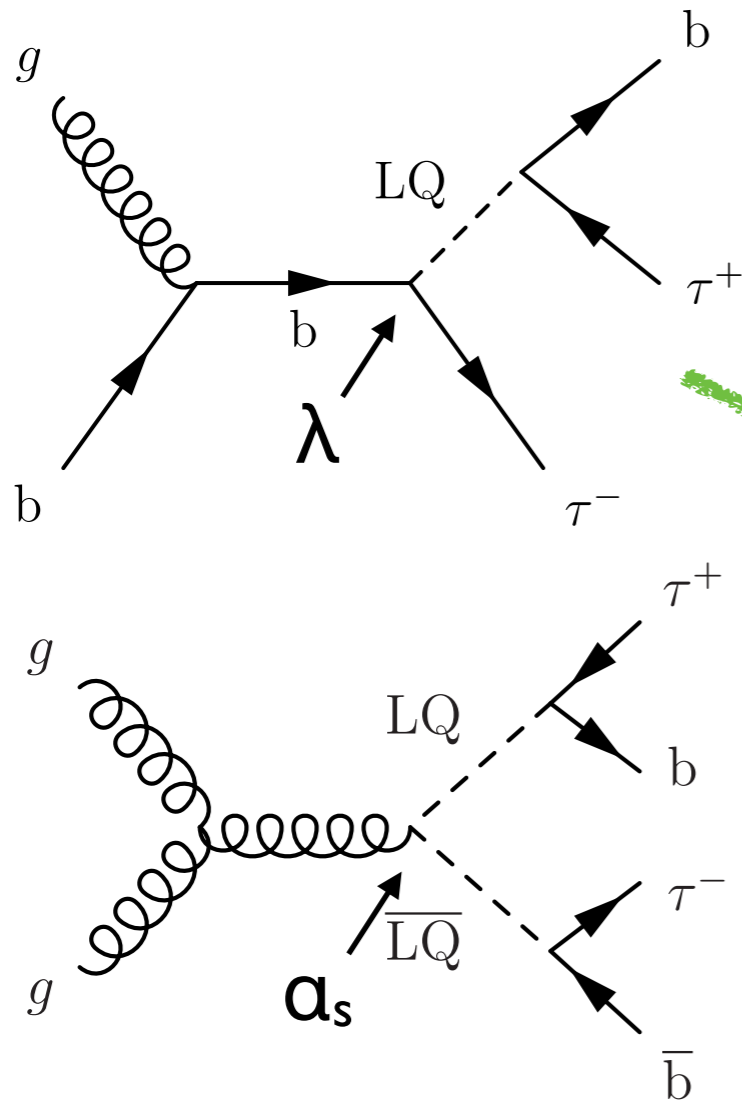
- 
- How to test BSM contributions?
 - Test with collider observables
 - Test with flavor observables

Advanced

- Summary

Direct search for new particle at colliders

Direct (on-shell) productions of new particle \rightarrow LQ mass $> \sim 1.8\text{TeV}$



Indirect search at colliders

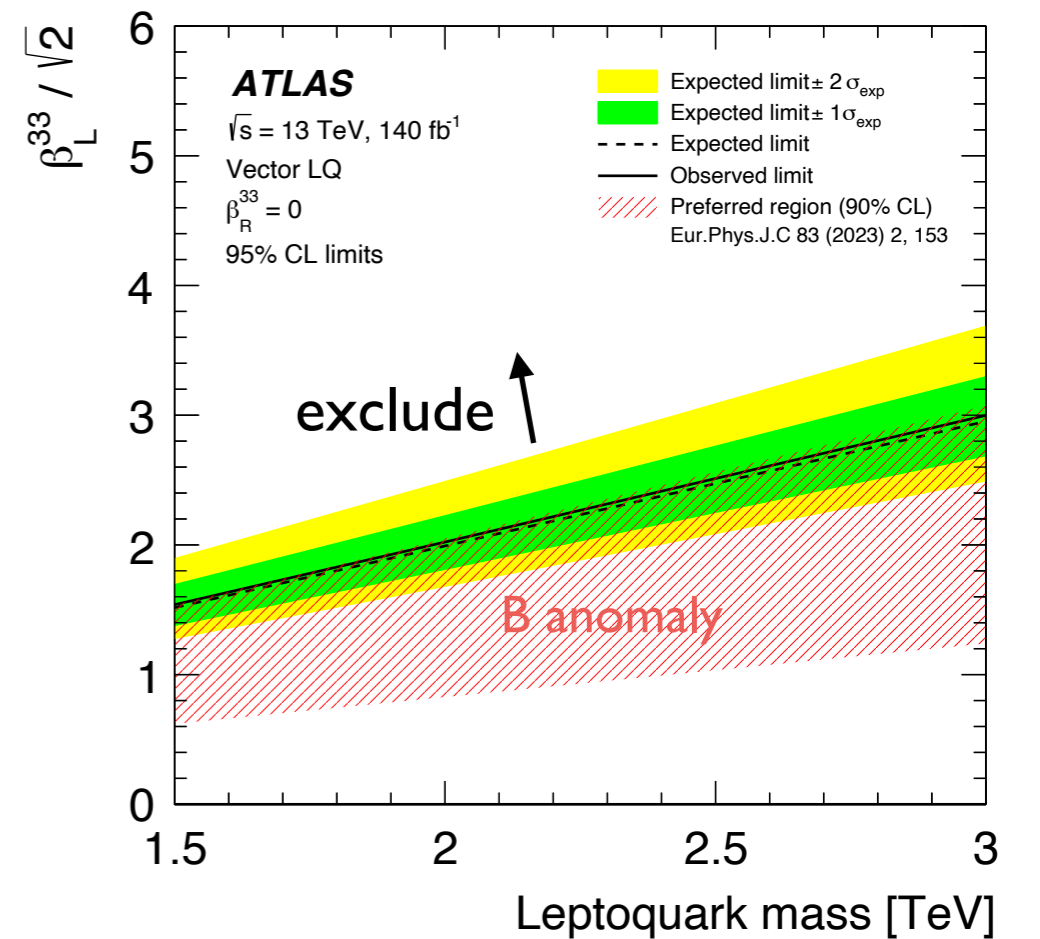
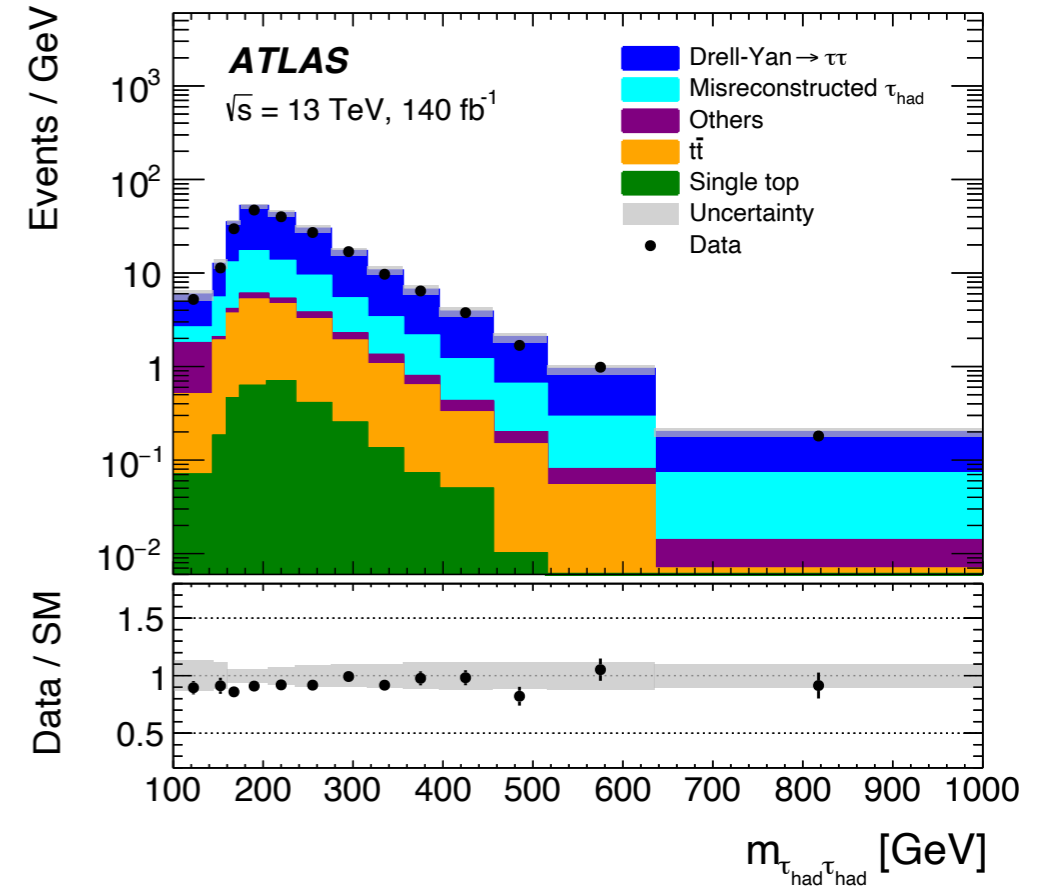
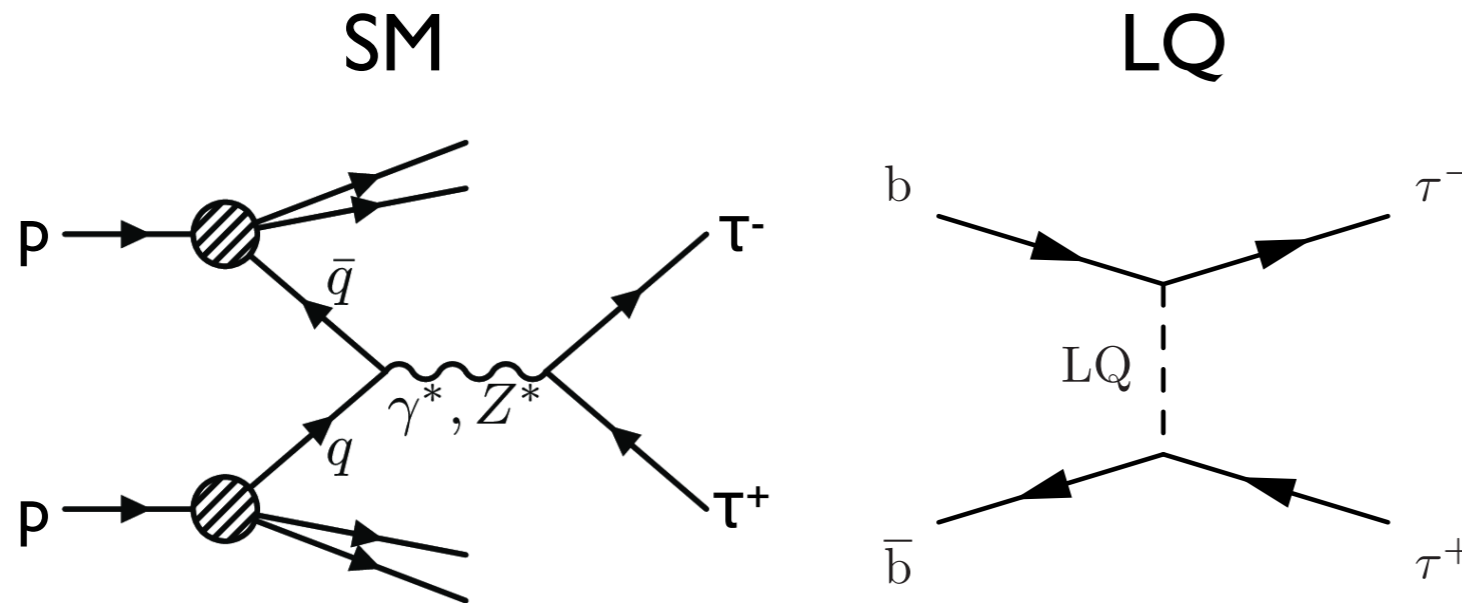
$b\bar{b} \rightarrow \tau^+\tau^-$ scattering via LQ exchange.

τ pair production in SM: γ, Z exchange

$\tau^+\tau^-$ invariant mass distribution

$$m_{\tau\tau}^2 \equiv (p_{\tau^+} + p_{\tau^-})^2$$

LQ predicts events w/. large $m_{\tau\tau} \leftrightarrow$ SM

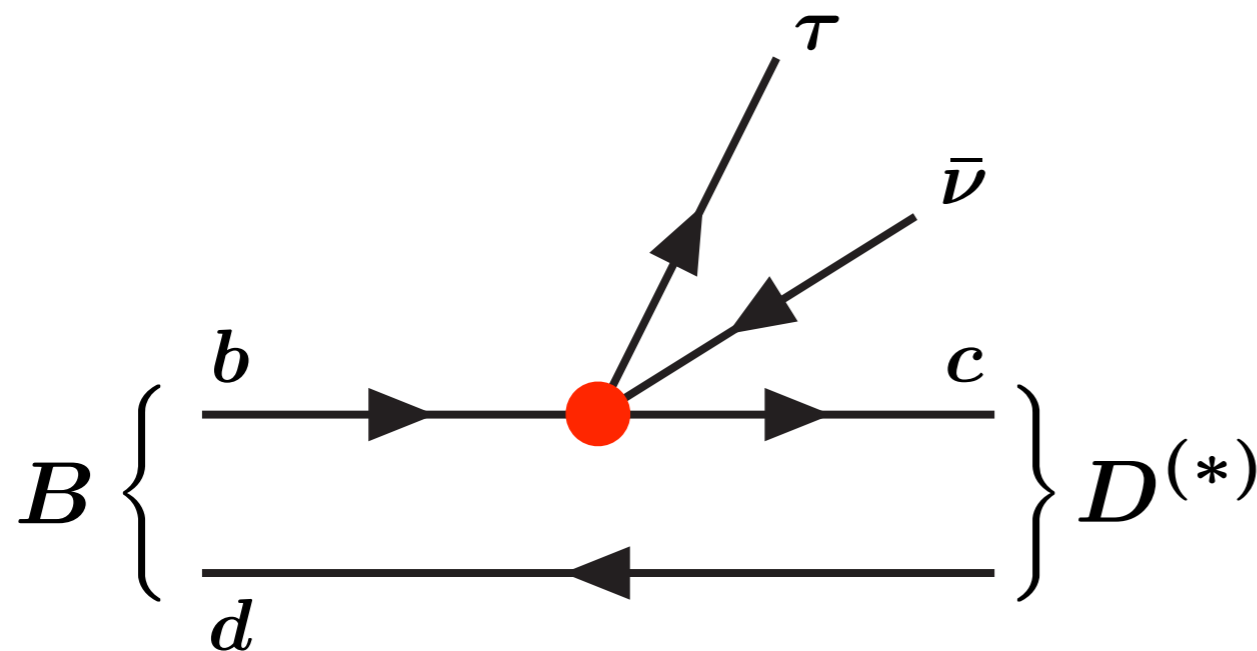


Crossing symmetry

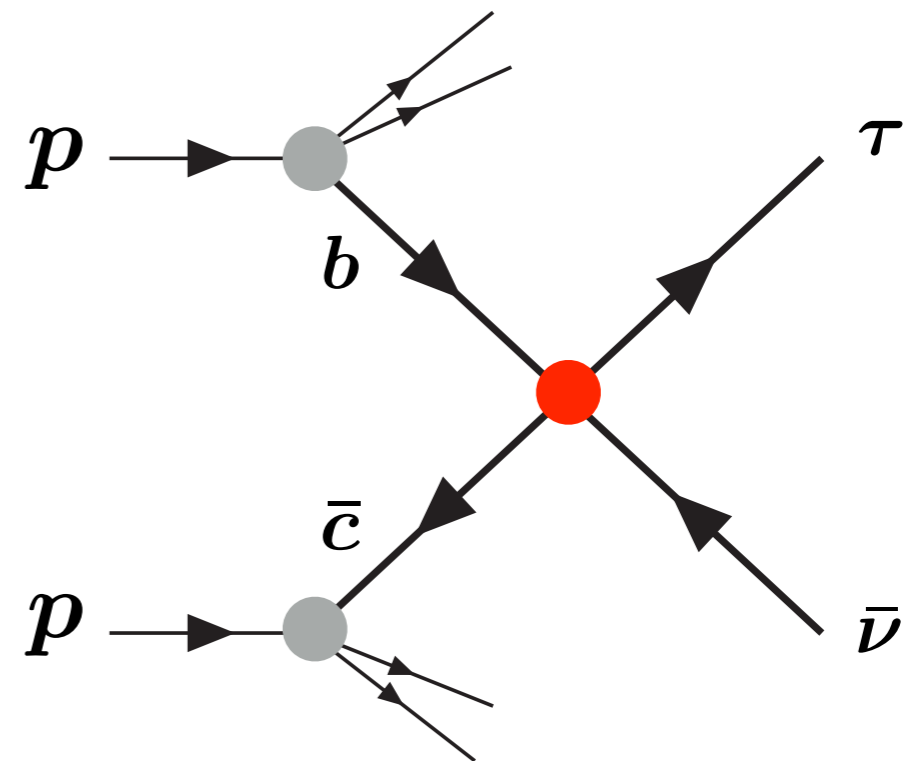
$bb \rightarrow \tau\tau$ depends only on b - τ coupling of LQ.

Generally contributes to $b\bar{c} \rightarrow \tau\bar{\nu}$ scattering due to crossing sym.

B meson decay



LHC



LHC search

ATLAS, PRD 100 (2019) 5, 052013

CMS, PLB 792 (2019) 107

Search for events with energetic (“hard”) τ with large missing transverse energy (“MET”).

In LQ models, effective interactions are generated by t-channel exchange of LQ.

$$\text{LQ} \quad \mathcal{A} \propto \frac{1}{t - M_{\text{LQ}}^2} \quad (t < 0)$$

$$\text{EFT} \quad \rightarrow \frac{-1}{M_{\text{LQ}}^2} \quad \text{if } |t| \ll M_{\text{LQ}}^2$$

Not always the case for $M_{\text{LQ}} \sim 1 \text{ TeV}$ because S/B is large in the region with *hard* τ and *large* MET.

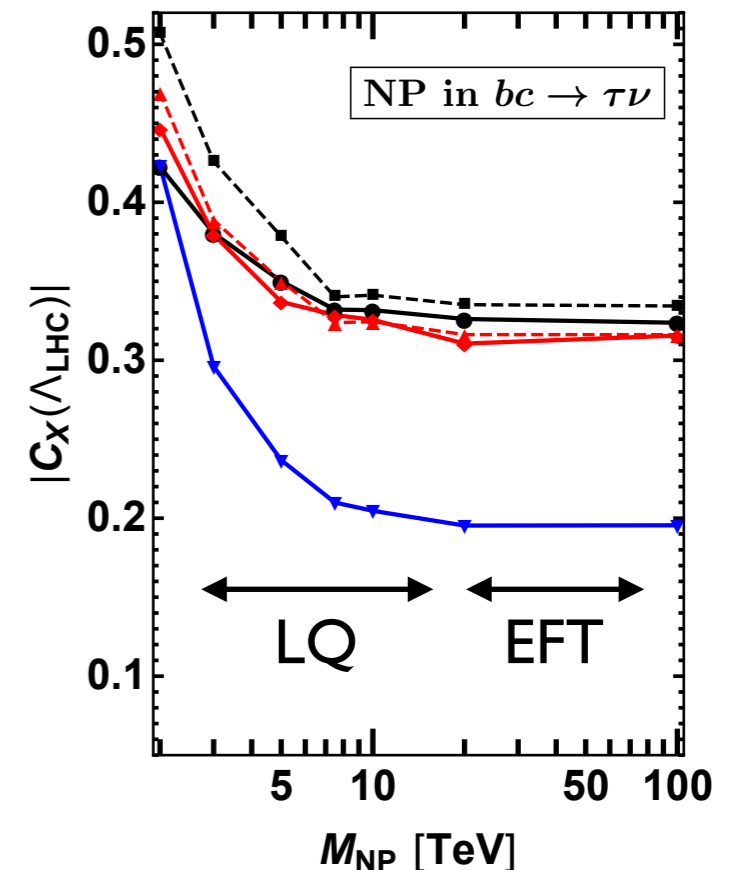
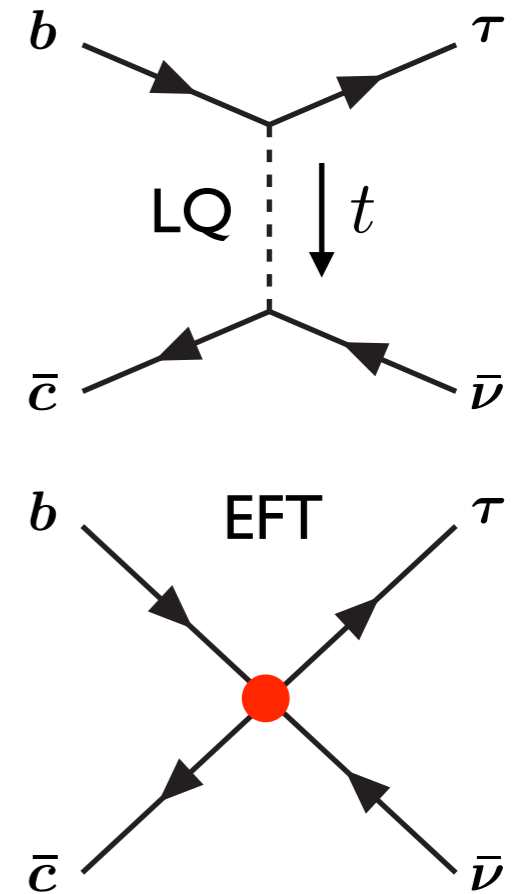


Fig. 95% upper limit on Wilson coefficient
Iguero, Takeuchi, Watanabe, EPJC 81 (2021) 5, 406

Additional b-jet in final state

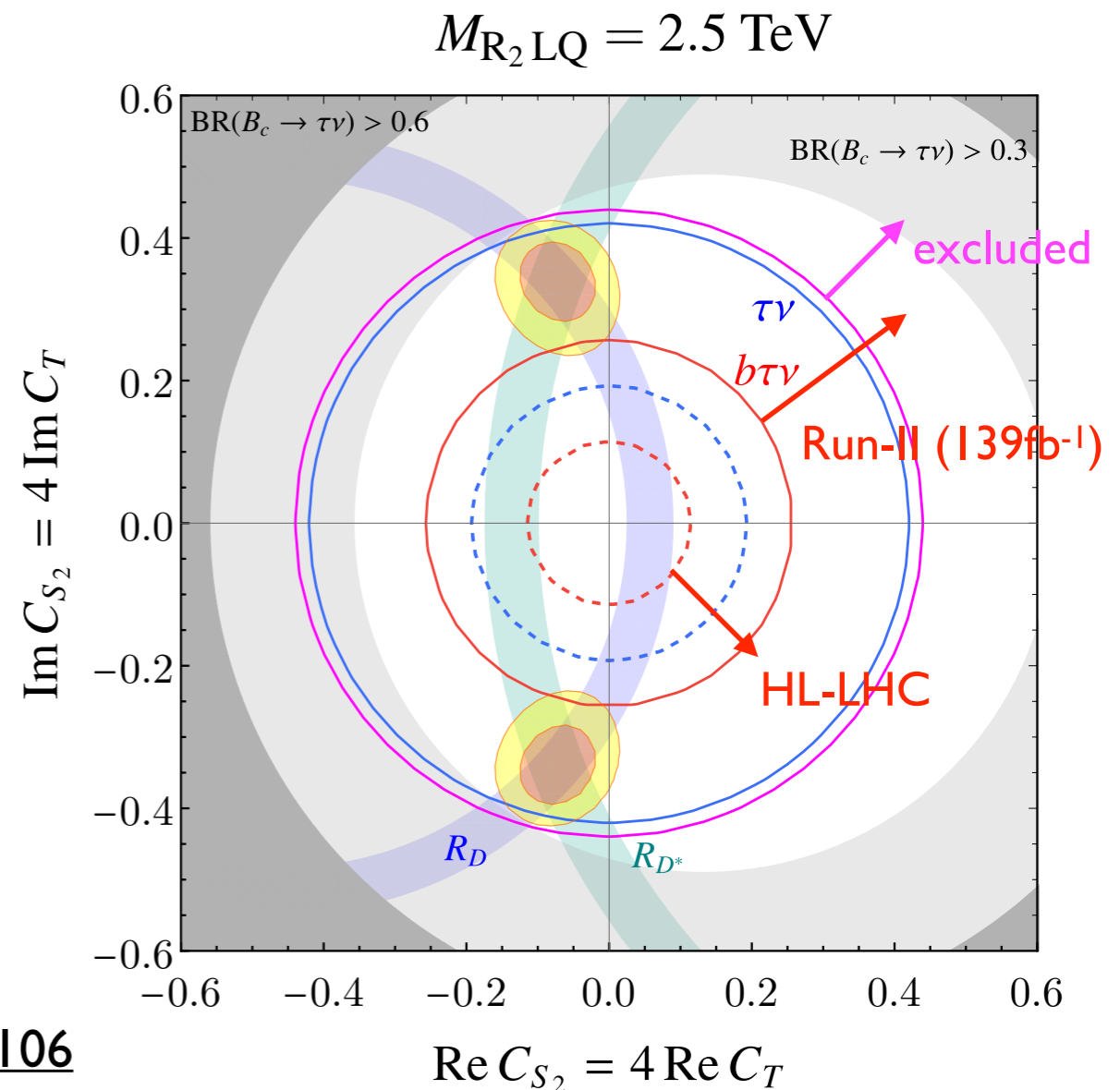
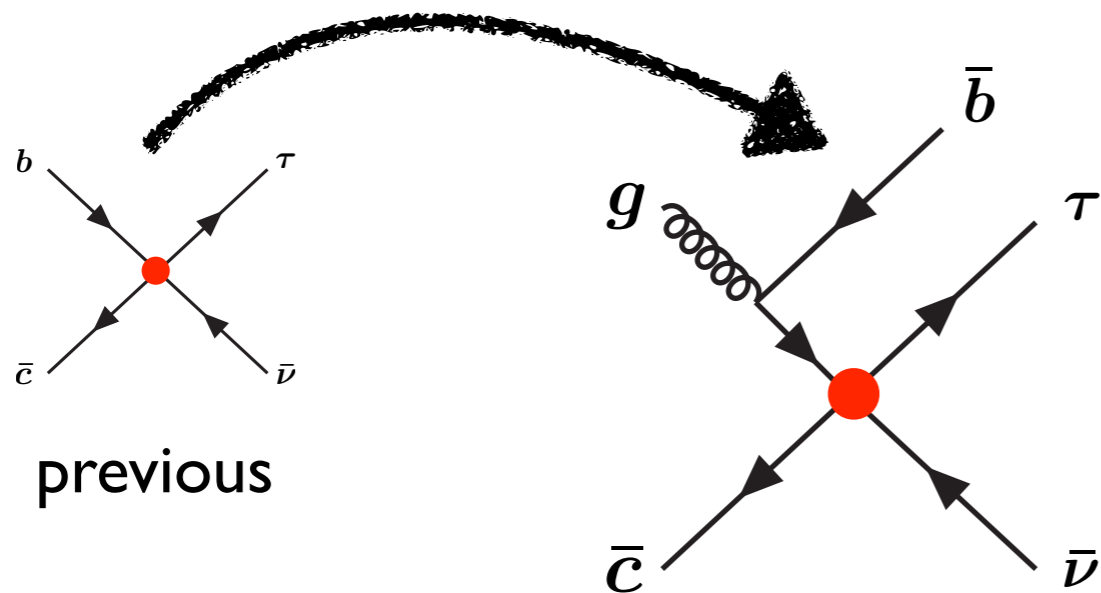
Assign b quark in the final state.

Advantage: parton distribution function & SM background events.

Disadvantage: three-body final state.

→ Better sensitivity to BSM.

Most of $R_{D^{(*)}}$ -favored parameter regions are accessible at HL-LHC.



Outline

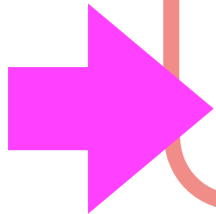
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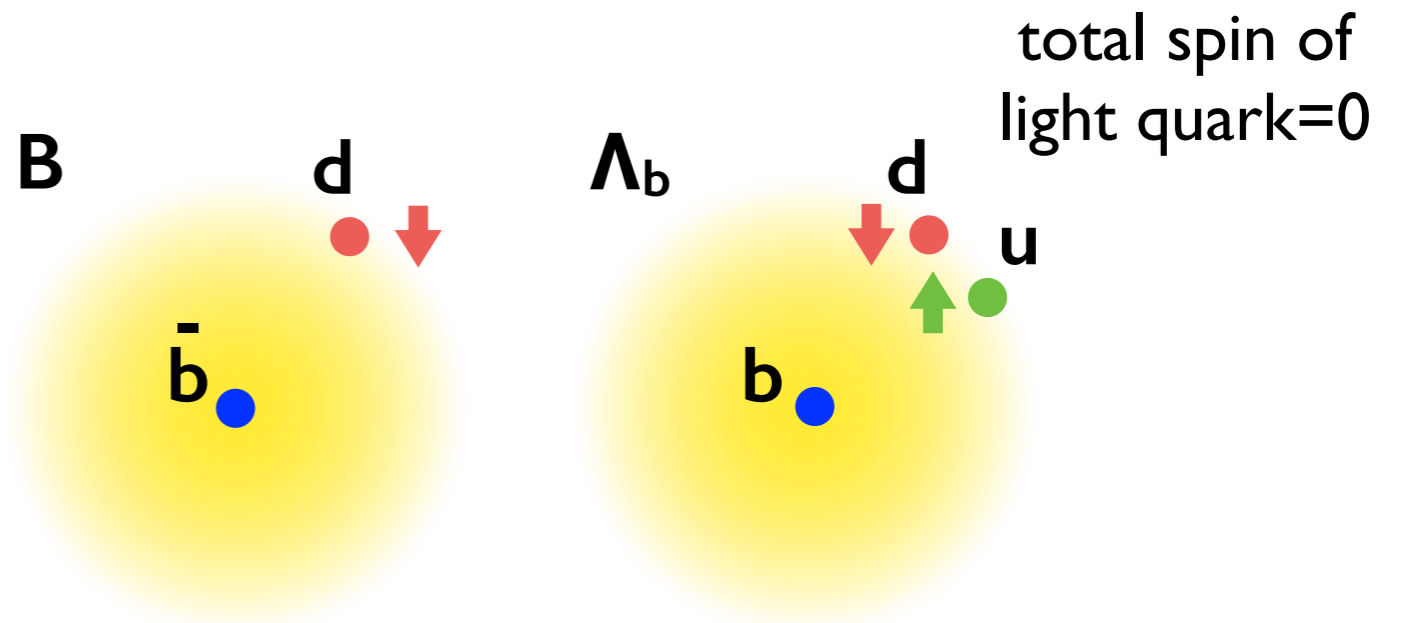


Q. How to test R_D and R_{D^*} excess?

Other decays proceed via $b \rightarrow c \tau \nu$. [cf., Λ_Q baryon made of (udQ)]

$$R_{\Lambda_c} = \frac{\mathcal{B}(\Lambda_b \rightarrow \Lambda_c \tau \bar{\nu})}{\mathcal{B}(\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu})}$$

$$\begin{cases} R_{\Lambda_c}^{\text{exp}} = 0.285 \pm 0.073 \\ R_{\Lambda_c}^{\text{SM}} = 0.324 \pm 0.004 \end{cases}$$



Consistent with SM prediction ($\leftrightarrow R_D, R_{D^*}$).

BSM structure or shortcomings in experimental results/SM values?

Heavy quark limit and zero recoil limit

Hadron wave functions are factorized in the heavy quark limit ($m_{b,c} \rightarrow \infty$).

$$\begin{aligned} \mathcal{M}(H_b \rightarrow H_c \tau \bar{\nu}) &= \frac{4G_F}{\sqrt{2}} V_{cb} \sum_X \mathcal{C}_X \langle H_c | \bar{c} \Gamma_X b | H_b \rangle \langle \tau \bar{\nu} | \bar{\tau} \Gamma_X \nu | 0 \rangle \\ &= \sum_{\text{spin}} \underbrace{\langle s_c, \lambda_c; J_{\ell_c}, \lambda | J_{H_c}, \lambda_{H_c} \rangle \langle s_b, \lambda_b; J_{\ell_b}, \lambda | J_{H_b}, \lambda_{H_b} \rangle}_{\text{Clebsch-Gordan}} \\ &\quad \times \underbrace{M_{\lambda_c, \lambda_b}}_{\text{heavy-quark transition}} \underbrace{\langle \phi_c(J_{\ell_c}, \lambda) | \phi_b(J_{\ell_b}, \lambda) \rangle}_{\text{light sector: form factor (Isgur-Wise ftn)}} \end{aligned}$$

heavy-quark transition light sector: form factor (Isgur-Wise ftn)

$$M_{\lambda_c, \lambda_b} = \frac{4G_F}{\sqrt{2}} V_{cb} \sum_X \mathcal{C}_X \bar{u}_c(\lambda_c) \Gamma_X u_b(\lambda_b) \langle \tau \bar{\nu} | \bar{\tau} \Gamma_X \nu | 0 \rangle$$

Form factor dependencies are reduced in the zero-recoil limit ($m_c \rightarrow m_b$).

Consequently, the following sum rule holds exactly in these limits:

$$\frac{R_{\Lambda_c}}{R_{\Lambda_c}^{\text{SM}}} = \frac{1}{4} \frac{R_D}{R_D^{\text{SM}}} + \frac{3}{4} \frac{R_{D^*}}{R_{D^*}^{\text{SM}}}$$

note: $R_{H_c} = \frac{\Gamma(H_b \rightarrow H_c \tau \bar{\nu}_\tau)}{\Gamma(H_b \rightarrow H_c \ell \bar{\nu}_\ell)}$ $\ell = e, \mu$

Sum rule with experimental results

Corrections arising from violating the limits are negligible.

Compared with experimental results:

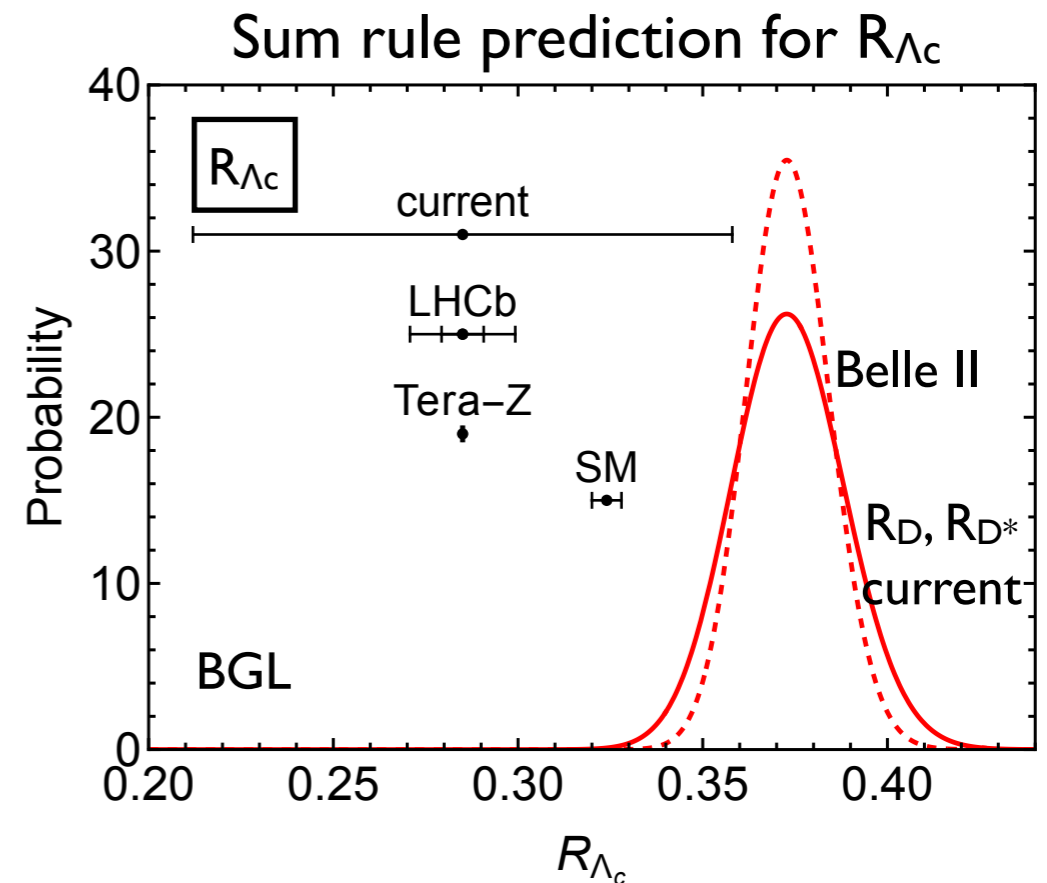
$$\frac{R_{\Lambda_c}}{R_{\Lambda_c}^{\text{SM}}} \approx \frac{1}{4} \frac{R_D}{R_D^{\text{SM}}} + \frac{3}{4} \frac{R_{D^*}}{R_{D^*}^{\text{SM}}} \quad \leftarrow \text{exp in } R_D \text{ and } R_{D^*}$$

Conclusion:

Current experimental results satisfy the sum rule.

R_{Λ_c} prediction will become more precise by Belle II (\rightarrow form factor).

R_D and R_{D^*} excess can be tested by LHCb and Tera-Z.



Summary

Flavor (and CP) violations are key probes for BSM at high energy scales.

Sensitivity to BSM can reach above ~ 1 TeV.

We reviewed the semileptonic decays of B mesons.

Currently, potential BSM signals have been reported in $B \rightarrow D^{(*)} \tau \nu$.

We provided an overview of BSM interpretations.

Could be tested in the near future through interplay of flavor & collider.

Backup

Form factor for anti-quark

In the previous slides, $\langle D | \bar{c} \Gamma b | \bar{B} \rangle$ is $\langle D_d^+ | \bar{c} \Gamma b | \bar{B}_d^0 \rangle$ or $\langle \bar{D}_u^0 | \bar{c} \Gamma b | B_u^- \rangle$, where the spectator quark is 'd' or 'u'.

Interchange of $d \leftrightarrow u$ makes no difference because of isospin symmetry.
(\rightarrow isospin violation)

Sign depends on operators for **quarks \leftrightarrow anti-quarks** (\rightarrow next slide)

$$\langle D^- | \bar{b} \gamma^\mu c | B^0 \rangle = - \langle D^+ | \bar{c} \gamma^\mu b | \bar{B}^0 \rangle$$

$$\langle D^- | \bar{b} c | B^0 \rangle = \langle D^+ | \bar{c} b | \bar{B}^0 \rangle$$

$$\langle D^- | \bar{b} \sigma^{\mu\nu} c | B^0 \rangle = - \langle D^+ | \bar{c} \sigma^{\mu\nu} b | \bar{B}^0 \rangle$$

for $CP | B(p) \rangle = - | \bar{B}(\tilde{p}) \rangle$, $CP | D(p) \rangle = - | \bar{D}(\tilde{p}) \rangle$ under CP transformation.

Four momentum: $p = (E, \vec{p})$, $\tilde{p} = (E, -\vec{p})$

Form factor for anti-quark: derivation

CP transformation (see, e.g., QFT textbook by Peskin&Schroeder)

$$(CP) \bar{\psi}_1(p_1) \gamma^\mu \psi_2(p_2) (CP) = -(-)^\mu \bar{\psi}_2(\tilde{p}_2) \gamma^\mu \psi_1(\tilde{p}_1)$$

$$(CP) \bar{\psi}_1(p_1) \psi_2(p_2) (CP) = \bar{\psi}_2(\tilde{p}_2) \psi_1(\tilde{p}_1)$$

$$(CP) \bar{\psi}_1(p_1) \sigma^{\mu\nu} \psi_2(p_2) (CP) = -(-)^\mu (-)^\nu \bar{\psi}_2(\tilde{p}_2) \sigma^{\mu\nu} \psi_1(\tilde{p}_1)$$

$$[(-)^\mu = 1 \text{ for } \mu = 0 \text{ and } -1 \text{ for } \mu = 1, 2, 3]$$

Vector: $\langle D^- | \bar{b} \gamma^\mu c | B^0 \rangle = \langle D^- | (CP)^2 \bar{b} \gamma^\mu c (CP)^2 | B^0 \rangle \quad \leftarrow (CP)^2 = 1$

$$= [-\langle D^+ |] \cdot [-(-)^\mu \bar{c} \gamma^\mu b] \cdot [-|B^0 \rangle] \Big|_{p \rightarrow \tilde{p}} \quad \leftarrow (CP) |M(p)\rangle = -|\bar{M}(\tilde{p})\rangle$$

$$= -(-)^\mu \langle D^+ | \bar{c} \gamma^\mu b | \bar{B}^0 \rangle \Big|_{p \rightarrow \tilde{p}}$$

$$= -(-)^\mu \left[f_+(\tilde{q}^2) (\tilde{p}_B + \tilde{p}_D)^\mu + [f_0(\tilde{q}^2) - f_+(\tilde{q}^2)] \frac{m_B^2 - m_D^2}{\tilde{q}^2} \tilde{q}^\mu \right]$$

$$= - \left[f_+(q^2) (p_B + p_D)^\mu + [f_0(q^2) - f_+(q^2)] \frac{m_B^2 - m_D^2}{q^2} q^\mu \right]$$

$$= -\langle D^+ | \bar{c} \gamma^\mu b | \bar{B}^0 \rangle$$

$\leftarrow (CP) \bar{\psi}_1(p_1) \gamma^\mu \psi_2(p_2) (CP) = -(-)^\mu \bar{\psi}_2(\tilde{p}_2) \gamma^\mu \psi_1(\tilde{p}_1)$

$$\tilde{p}^\mu = (-)^\mu p^\mu$$

$$\tilde{q}^2 = q^2$$

Form factor for anti-quark: derivation

Scalar: CP transformation or apply equation of motion:

$$q_\mu \langle D^- | \bar{b} \gamma^\mu c | B^0 \rangle = \langle D^- | \bar{b} (\not{p}_b - \not{p}_c) c | B^0 \rangle = -(m_b - m_c) \langle D^- | \bar{b} c | B^0 \rangle$$

$$q_\mu \langle D^- | \bar{b} \gamma^\mu c | B^0 \rangle = q_\mu \left[-f_+(q^2) (p_B + p_D)^\mu - [f_0(q^2) - f_+(q^2)] \frac{m_B^2 - m_D^2}{q^2} q^\mu \right]$$

$$= -(m_B^2 - m_D^2) f_0(q^2)$$

$$\therefore \langle D^- | \bar{b} c | B^0 \rangle = \frac{m_B^2 - m_D^2}{m_b - m_c} f_0(q^2) = \langle D^+ | \bar{c} b | \bar{B}^0 \rangle$$

$\langle D^- | \bar{b} \gamma^\mu c | B^0 \rangle = -\langle D^+ | \bar{c} \gamma^\mu b | \bar{B}^0 \rangle$

Dirac equation for *anti-quark* in momentum space repr.: $\not{p}\psi = -m\psi$

Tensor: CP transformation

$$\langle D^- | \bar{b} \sigma^{\mu\nu} c | B^0 \rangle = -\langle D^+ | \bar{c} \sigma^{\mu\nu} b | \bar{B}^0 \rangle$$

$$(CP) \bar{\psi}_1(p_1) \sigma^{\mu\nu} \psi_2(p_2) (CP) = -(-)^\mu (-)^\nu \bar{\psi}_2(\tilde{p}_2) \sigma^{\mu\nu} \psi_1(\tilde{p}_1)$$

cf. Relative sign between operators is independent of phase convention.