The Standard Model of Particle Physics

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Elementary Particles (before above EW symmetry breakdown)

					# of real fields
Fermions (spin = 1/2)	$Q_k = \begin{pmatrix} u_{Lk} \\ d_{Lk} \end{pmatrix}$	3	2	$\frac{1}{6}$	$6 \times 2 \times 2$
	u_{Rk}	3	1	$\frac{2}{3}$	$3 \times 2 \times 2$
$k = 1, 2, 3$ ↓ (generation, family, flavor)	d_{Rk}	3	1	$-\frac{1}{3}$	$3 \times 2 \times 2$
	$L_k = \begin{pmatrix} \nu_{Lk} \\ l_{Lk} \end{pmatrix}$	1	2	$-\frac{1}{2}$	$2 \times 2 \times 2$
	l_{Rk}	1	1	-1	$1 \times 2 \times 2$ ↕ spin
Gauge bosons (spin = 1)		A^a_μ $a = 1 \dots 8$	W^i_μ $i = 1, 2, 3$	B_μ	$(8+3+1) \times 2$
Higgs boson (spin = 0)	ϕ	1	2	$-\frac{1}{2}$	$2 \times 2 = 4$

$SU(3)_c \times SU(2)_L \times U(1)_Y$: gauge symmetry

All the above elementary particles are <u>massless</u> above the EW symmetry breaking <u>except</u> the Higgs boson.

Elementary particles (below EW symmetry breakdown scale : $v = 246\,\mathrm{GeV}$)

Fermions (spin = 1/2)	$u,\ c,\ t$	3	2/3	quarks
	$d,\ s,\ b$	3	−1/3	
	$\nu_e,\ \nu_\mu,\ \nu_\tau$	1	0	leptons*
	$e,\ \mu,\ \tau$	1	−1	
Gauge bosons (spin = 1)	A_μ^a $(a=1\ldots 8)$	8	0	gluons
	$W_\mu^\pm$	1	±1	weak bosons
	Z_μ	1	0	
	A_μ	1	0	photon
Higgs boson (spin = 0)	H	1	0	Higgs boson
		$SU(3)_c$	$\times\ U(1)_{EM}$	

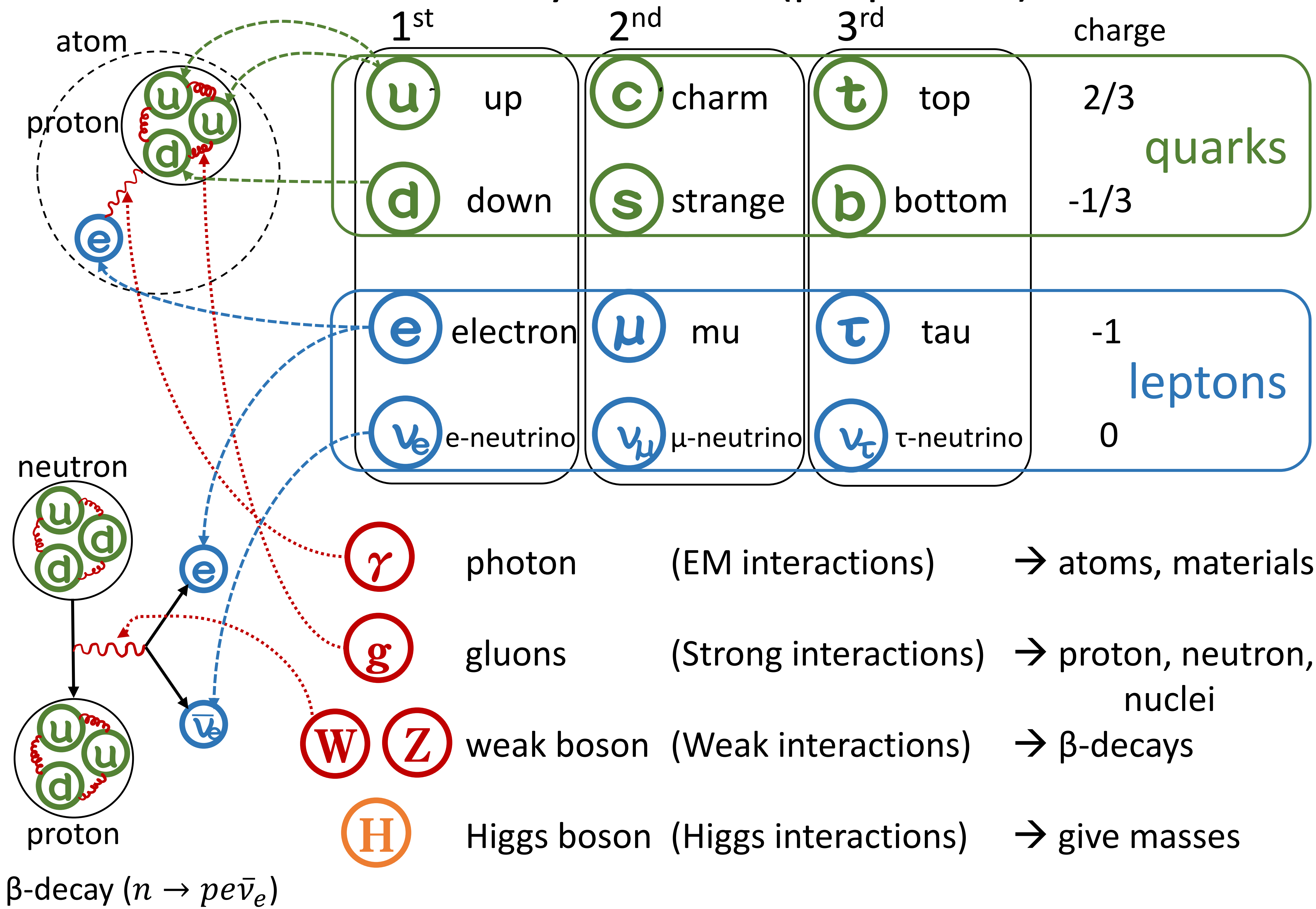
* ν-mixing appears @EFT

Elementary particles (below the QCD confinement scale : $(1\,\mathrm{fm})^{-1} \approx 200\,\mathrm{MeV}$)

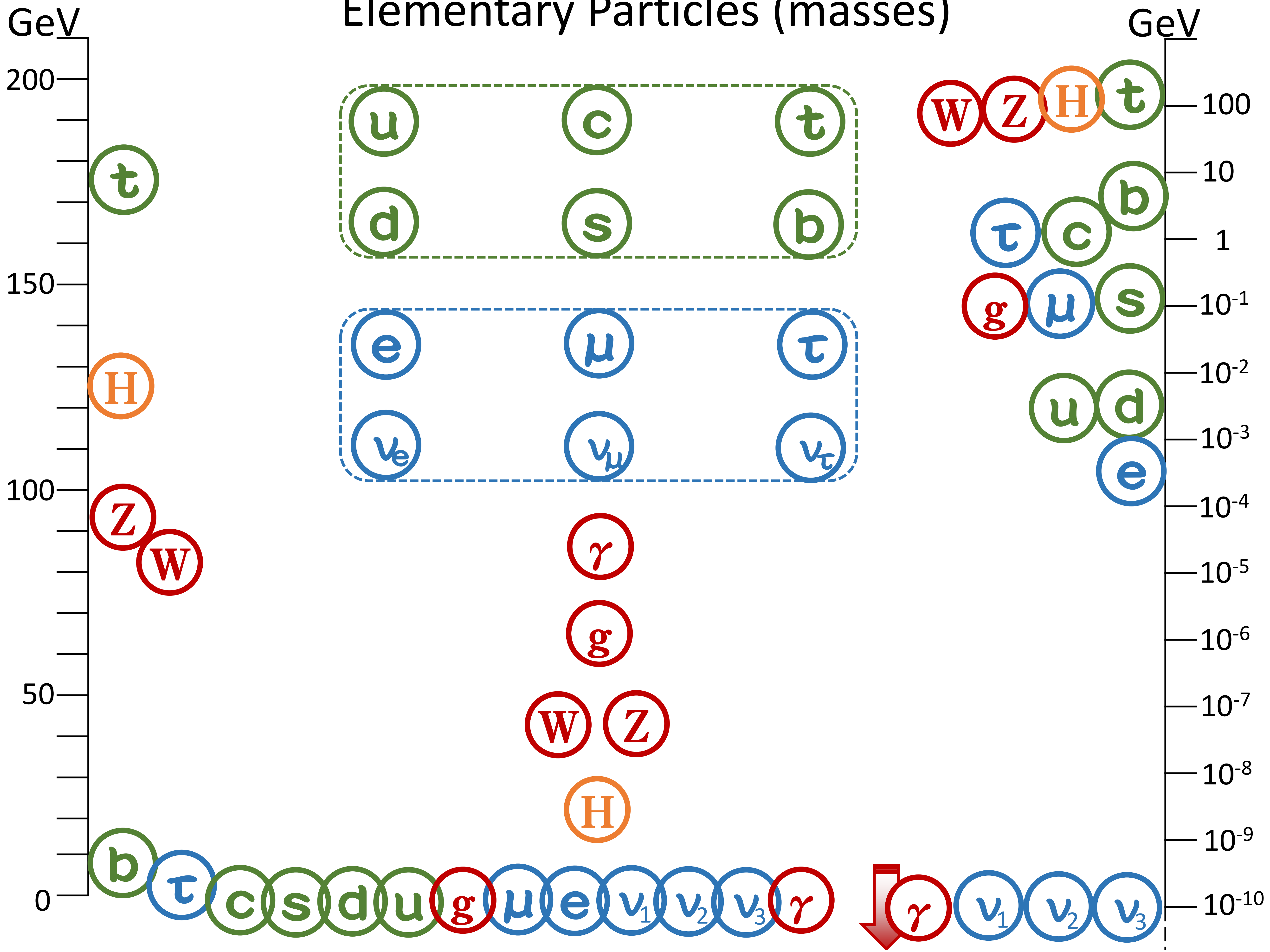
Hadrons (color singlets)	Baryons (qqq)	spin = 1/2	$p(uud)$, $n(udd)$, $\Lambda(uds)$, $\Lambda_c(udc)$, $\Lambda_b(udb)$, …
		spin = 3/2	$\Delta^{++}(uuu)$, $\Delta^{+}(uud)$, $\Delta^{0}(udd)$, $\Delta^{-}(ddd)$, …
	Mesons ($q\bar{q}$)	spin = 0	$\pi^+(u\bar{d})$, $\pi^-(\bar{u}d)$, $\pi^0(u\bar{u}+d\bar{d})$, $K^+(u\bar{s})$, …, $\eta_c(c\bar{c})$, $\eta_b(b\bar{b})$, …
		spin = 1	$\rho^+(u\bar{d})$, $\rho^-(\bar{u}d)$, $\rho^0(u\bar{u}+d\bar{d})$, $K^{*+}(u\bar{s})$, $\phi(s\bar{s})$, $J/\psi(c\bar{c})$, $\Upsilon(b\bar{b})$, …
Leptons	$Q=-1$	spin = 1/2	e, μ, τ
	$Q=0$	spin = 1/2	ν_e, ν_μ, ν_τ
Gauge bosons (spin 1)		$Q=\pm1$	$W^\pm$
		$Q=0$	Z, γ
Higgs boson (spin 0)		$Q=0$	H

These are the particles which are ~~produced and~~ detected by collider experiments.

Elementary Particles (properties)



Elementary Particles (masses)



Quantum Field Theory (QFT) is the language with which the particles are described. ⑦

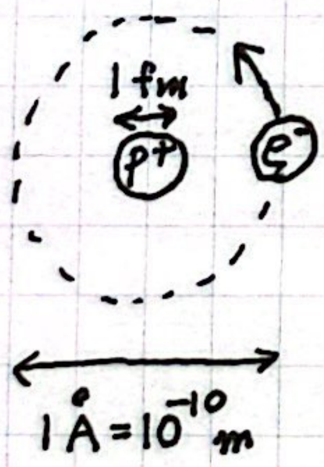
$\approx$ Quantum Mechanics (QM) + Special Relativity

19 th Century : Classical Physics (Newton + Maxwell)

$\Downarrow$ $\Downarrow$ $\leftarrow$ Rutherford probed atom by α beam [$c = \hbar = 1$ unit]

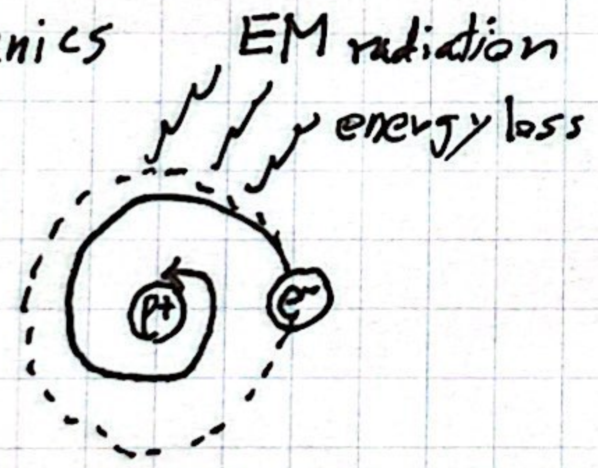
20 th Century : Quantum Mechanics

atom



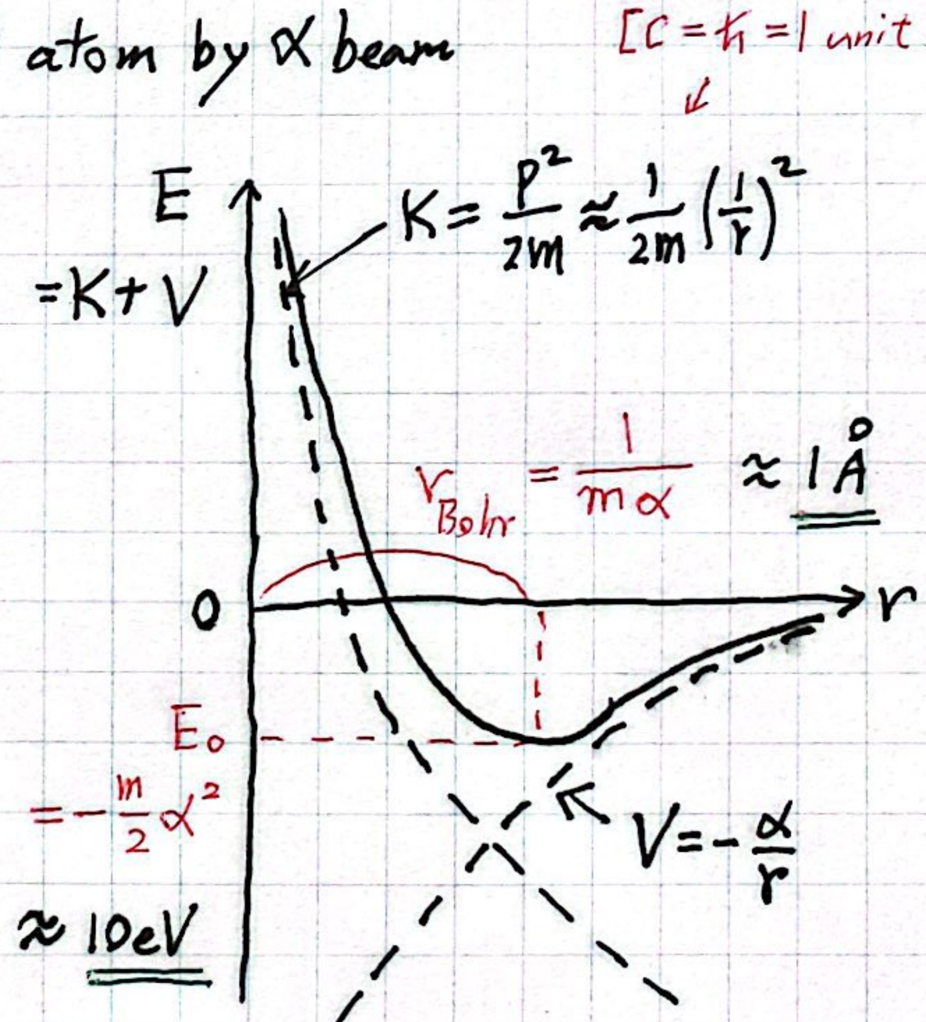
1 fm $= 10^{-15}$ m

Classical Physics $\Longrightarrow$



All atoms are instantly dead!

But we are alive!



QM : e^- momentum ($\vec{P}$) and location ($\vec{x}$) cannot be determined simultaneously, because $[x, p_x] = i\hbar$ ($\hbar$ is the quantum of $\int dx\, p$)

$$\underset{\text{Energy}}{H}\,\psi(x) = \Big[\underset{\text{Kinetic}}{\frac{\vec{p}^{\,2}}{2m}} + \underset{\text{Potential}}{V(r)}\Big]\psi(x) = \Big[\underbrace{\frac{(i\vec{\nabla})^2}{2m}}_{\text{blows up at } r \approx 0} + V(r)\Big]\psi(x) = E\,\psi(x)$$

$|\psi(x)|^2$ determins the probability of finding e^- at $\vec{x}$.

experiments!

QFT 2 : Special Relativity

Illusion : time and space are independent and absolute (idea, phillosophy, ... based on ...)

Reality : mearure of time and space depends on the relative velocity of the (Lorentz) frames.

Fact = Reality check : The velocity of light, c, is exactly the same in all frames ⇐ Michelson-Morley

Partulate (Einstein) : Physics should be the same in all frames whose relative velocity is a constant.

(~~no~~ acceleration. ⇒ General Relativity)

Poincare : Physics should be the same under

$$\begin{cases} \text{Translation} : & x^\mu \to x'^\mu = x^\mu + a^\mu \\ \text{Lorentz transformation} : & x^\mu \to x'^\mu = L^\mu{}_\nu x^\nu \end{cases}$$

4 generators : P^μ ($\mu = 0,1,2,3$) ⇒ Energy-Momentum conservation

6 generators : $J_x, J_y, J_z, K_x, K_y, K_z$

⇒ Angular Momentum conservation + ...

$$\Rightarrow \quad E^2 - |\vec{p}|^2 = m^2 \quad \text{is invariant !}$$

$$E = \sqrt{m^2 + |\vec{p}|^2} \approx mc^2$$

⇒ particles are labeled by their invariant mass and spin because they have the same values in all Lorentz frames.

QFT 3 : What happens when QM and Special Relativity are combined ?

particles are created and annihilated when $E \gg mc^2$!

particle numbers are not conserved. $\Leftrightarrow$ QM studies one or two particle system when $v \ll c$.

$$\gamma\gamma \longleftrightarrow e^+e^- \quad \text{when } E_{tot} \gtrsim 2m_e c^2$$

QM : $\psi(t,\vec{x})$ is the wave function of an electron $\Rightarrow |\psi(t,\vec{x})|^2$ measures the probability of finding e at...

QFT : $$\psi_i(t,\vec{x}) = \sum_h \int \frac{d^3\vec{k}}{(2\pi)^3 2E} \left[\underbrace{a(\vec{k},h)}_{\text{annihilate } e^-} u_i(\vec{k},h) e^{-i(Et-\vec{k}\cdot\vec{x})} + \underbrace{b^\dagger(\vec{k},h)}_{\text{creat } e^+} v_i(\vec{k},h) e^{i(Et-\vec{k}\cdot\vec{x})} \right]_{E=\sqrt{m^2+|\vec{k}|^2}}$$

($i = 1,2,3,4$)

$\Rightarrow$ annihilation & creation are quantized : $[a(\vec{k},h), a^\dagger(\vec{k}',h')] = [b(\vec{k},h), b^\dagger(\vec{k}',h')] = (2\pi)^3 2E\, \delta^3(\vec{k}-\vec{k}')$

$$\Rightarrow \begin{cases} \text{one } e^- \text{ state} & \langle 0|\, \psi(t,\vec{x})\, a^\dagger(\vec{k},h)\, |0\rangle = u(\vec{k},h)\, e^{-i(Et-\vec{k}\cdot\vec{x})} \\ \text{one } e^+ \text{ state} & \langle 0|\, \psi^\dagger(t,\vec{x})\, b^\dagger(\vec{k},h)\, |0\rangle = v(\vec{k},h)\, e^{-i(Et-\vec{k}\cdot\vec{x})} \end{cases}$$

positive energy !

$\Rightarrow$ spin !

Dirac found that e^- wave functions should have 4 components, which should mix under Lorentz transformation.

Many physicists contribute to make QFT to solve the negative energy problem ! $\Rightarrow$ e^+ is predicted !

QFT 4 : Perturbative calculation of scattering amplitudes

$$\mathcal{L}(\phi_K(x), \partial^\mu \phi_K(x)) = \mathcal{L}^{(0)}(\phi_K(x), \partial^\mu \phi_K(x)) + \mathcal{L}_{int}(x)$$

solve E.O.M. $\partial^\mu \frac{\delta \mathcal{L}}{\delta \partial^\mu \phi_K} + \frac{\delta \mathcal{L}}{\delta \phi_K} = 0$

plane wave solution : $\phi_K(x) = \int \frac{d^3\vec{p}}{(2\pi)^3 2E} \left[a_K(\vec{p}) \epsilon_K(\vec{p}) e^{-ipx} + b_K^+(\vec{p}) \epsilon_K^*(\vec{p}) e^{ipx} \right]_{p^0 = E = \sqrt{m^2 + |\vec{p}|^2}}$

if $\phi_K(x)$ has spin, then under Lorentz transformation : $\begin{cases} \phi_K(x) \to \phi'_{K'}(x) = L_{K'K} \phi_K(L^{-1}x) \\ \epsilon_K(\vec{p}) \to \epsilon'_{K'}(\vec{p}) = L_{K'K} \epsilon_K(L^{-1}p) \end{cases}$

$\Rightarrow$ obtain the Green's function of the E.O.M. operator for free (plain wave) solution.

propagator

Time ordering

$$\langle f | S | i \rangle = \langle f | (1 + iT) | i \rangle = \langle f | T e^{i\int d^4x \mathcal{L}_{INT}(x)} | i \rangle = \langle f | \sum_{n=0}^{\infty} T \left(i \int d^4x \mathcal{L}_{INT}(x) \right)^n | i \rangle$$

$$= i (2\pi)^4 \delta^4 \left(\sum_{in} p_K^\mu - \sum_{out} p_K^\mu \right) M_{fi}$$

$|i\rangle = a^+(\vec{p}_1)\, b^+(\vec{p}_2)\, |0\rangle$

$|f\rangle = a^+(\vec{p}_3)\, b^+(\vec{p}_4) \cdots |0\rangle$

$\Rightarrow |M_{fi}|^2$ gives cross section ($2 \to n$) & decay width ($1 \to n$)

(Fermi's Golden Rule for plane-wave normalization.)

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{gauge\,fix} + \mathcal{L}_{Higgs} + \mathcal{L}_{fermion} + \mathcal{L}_{Yukawa} = \mathcal{L}^{(0)}_{SM} + \mathcal{L}_{INT}^{(x)}$$

↳ determins free field E.O.M. and its solutions (wave functions) and hence also Green's function.

$$\mathcal{L}_{gauge} = -\frac{1}{4} A^a_{\mu\nu} A^{a\mu\nu} - \frac{1}{4} W^k_{\mu\nu} W^{k\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$a = 1$ to 8 $\quad$ $k = 1$ to 3

$$\mathcal{L}_{Higgs} = (D_\mu \phi)^\dagger (D^\mu \phi) - \Big[\underbrace{\frac{\lambda}{4}(\phi^\dagger \phi)^2 + \mu^2 (\phi^\dagger \phi)}_{V(\phi)} \Big]$$

$$\sigma^\mu_\pm = (1, \pm \vec{\sigma})$$

$$\mathcal{L}_{fermion} = \sum_{i=1}^{3} \left\{ Q_i^\dagger \sigma_-^\mu iD_\mu Q_i + u_{Ri}^\dagger \sigma_+^\mu iD_\mu u_{Ri} + d_{Ri}^\dagger \sigma_+^\mu iD_\mu d_{Ri} + L_i^\dagger \sigma_-^\mu iD_\mu L_i + l_{Ri}^\dagger \sigma_+^\mu iD_\mu l_{Ri} \right\}$$

$$\mathcal{L}_{Yukawa} = \sum_{i=1}^{3} \sum_{j=1}^{3} \left\{ y^u_{ij} Q_i^\dagger \phi u_{Rj} + y^d_{ij} Q_i^\dagger \phi^c d_{Rj} + y^l_{ij} L_i^\dagger \phi^c l_{Rj} + h.c. \right\}$$

$SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge invariance of the above Lagrangian is almost manifest:

$$\phi_k \to \phi'_k = U_{kk'} \phi_{k'} \quad , \quad D_\mu \to D'_\mu = U D_\mu U^{-1}$$

Lorentz invariance is far more difficult to prove. $\partial^\mu, D^\mu, A^{a\mu}, W^{k\mu}, B^\mu$ all transforms as $V^\mu = L^\mu{}_\nu V^\nu$

$$\left.\begin{aligned} \psi_L \to \psi'_L &= S_L \psi_L = e^{-i \sum_{k=1}^{3} \frac{\sigma^k}{2}(\theta_k - i\eta_k)} \psi_L \\ \psi_R \to \psi'_R &= S_R \psi_R = e^{-i \sum_{k=1}^{3} \frac{\sigma^k}{2}(\theta_k + i\eta_k)} \psi_R \end{aligned}\right\} \Rightarrow S_L^\dagger S_R = S_R^\dagger S_L = 1$$

$\psi_L^\dagger \psi_R, \psi_R^\dagger \psi_L$ are invariant

$\psi_L^\dagger \sigma_-^\mu \psi_L, \psi_R^\dagger \sigma_+^\mu \psi_R$ as V^μ. ///

Green's function method of calculating scattering amplitudes at tree and at higher orders

scalar : $\mathcal{L}^{(0)} = \frac{1}{2}(\partial^\mu\phi)(\partial_\mu\phi) - \frac{1}{2}m^2\phi^2 \Rightarrow$ E.O.M. $(\partial^\mu\partial_\mu + m^2)\phi(x) = 0$

$$(p^2 - m^2)\hat{\phi}(p) = 0$$

$$(p^2 - m^2)D(p^2) = i \Rightarrow D(p^2) = \frac{i}{p^2 - m^2}$$

fermion : $\mathcal{L}^{(0)} = \bar{\psi}(i\partial^\mu\gamma_\mu - m)\psi \Rightarrow$ E.O.M. $(i\partial^\mu\gamma_\mu - m)\psi = 0$

$$(p^\mu\gamma_\mu - m)u(p) = 0$$

$$(p^\mu\gamma_\mu - m)G(p) = i \Rightarrow G(p) = \frac{i}{p^\mu\gamma_\mu - m} = \frac{i(p^\mu\gamma_\mu + m)}{p^2 - m^2}$$

gaugeboson : $\mathcal{L}^{(0)} = -\frac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) \Rightarrow$ E.O.M. $(g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu)A_\nu = 0$

$$\underbrace{(g^{\mu\nu}p^2 - p^\mu p^\nu)}\hat{A}_\nu(p) = 0$$

det = 0 because $p_\mu(g^{\mu\nu}p^2 - p^\mu p^\nu) = 0$

$\Rightarrow$ No Green's function $\Rightarrow$ No amplitudes.

$\Rightarrow$ Not all $A^\mu(x)$ are independent.

$\Rightarrow$ constraints : $\partial_\mu A^\mu(x) = 0$ ← covariant gauge

$n_\mu A^\mu(x) = 0$ ← axial/L.C. gauges

$\Rightarrow \partial^2 A^\mu(x) = 0$

$p^2 \tilde{A}^\mu(p) = 0$

depends on the constraints.

$p^2 D^{\mu\nu}(p) = i \quad \Rightarrow D^{\mu\nu}(p) = \dfrac{i \sum_\lambda \varepsilon^\mu(p,\lambda)\varepsilon^\nu(p,\lambda)^*}{p^2}$

$\mathcal{L}_{G.F.}$ method : $\mathcal{L}^{(0)} + \mathcal{L}_{G.F.}$

QED : $\mathcal{L}^{(0)} + \mathcal{L}_{G.F.} = -\frac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) - \frac{1}{2\xi}(\partial^\mu A_\mu)^2$ covariant ξ-gauge

$\Rightarrow$ E.O.M.

all 4 contributes

$\Rightarrow$ Green's function $D^{\mu\nu}(p) = \dfrac{i}{p^2}\left[-g^{\mu\nu} + (1-\xi)\dfrac{p^\mu p^\nu}{p^2}\right]$

$\xi = 1$ Feynman gauge

$\xi = 0$ Landau gauge 3pol

LC gauge in QED : $\mathcal{L}^{(0)} + \mathcal{L}_{G.F.} = -\frac{1}{4}(\partial^\mu A^\nu - \partial^\nu A^\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) - \frac{1}{2\xi}(n_\mu A^\mu)^2$ $\quad n^2 = 0$

$\Rightarrow$ E.O.M. : $\left(g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu - \frac{n^\mu n^\nu}{\xi}\right)A_\nu(x) = 0$

$$\left(g^{\mu\nu}p^2 - p^\mu p^\nu + \frac{n^\mu n^\nu}{\xi}\right)\tilde{A}_\nu(p) = 0$$

$$\left(g^{\mu\nu}p^2 - p^\mu p^\nu + \frac{n^\mu n^\nu}{\xi}\right)D_{\nu\rho}(p) = i\,\delta^\mu{}_\rho \quad : \text{Green's function}$$

$$D^{\mu\nu}(p) = \frac{i}{p^2}\left(-g^{\mu\nu} + \underbrace{\frac{n^\mu p^\nu + n^\nu p^\mu}{n\cdot p}}\right)$$

If we choose $n^\mu = (1, -\vec{p}/|\vec{p}|)$, we find that

$$-g^{\mu\nu} + \frac{n^\mu p^\nu + p^\mu n^\nu}{n\cdot p} = \sum_{h=\pm1}\varepsilon^\mu(p,h)\varepsilon^\nu(p,h)^* + \tilde{\varepsilon}^\mu(p,h=0)\tilde{\varepsilon}^\nu(p,h=0)$$

- smooth transition to the on-shell limit $(Q \to 0)$ $\quad \tilde{\varepsilon}^\mu(p,h=0) = \varepsilon^\mu(p,h=0) - \frac{q^\mu}{Q}$ $\quad Q = \sqrt{|q^2|}$
- e^{-y} at high energies, because $n\cdot q = q^0 + |\vec{q}|$ $\quad = \frac{Q}{n\cdot q}n^\mu$

$\Rightarrow$ All unphysical 'gauge cancellation' among ~~Feynman diagram~~ FD gauge - disappears.

KH, J.Kanzaki, K.Mawatari [arXiv:2003.03003]

LC gauge in EW theory: $\mathcal{L}^{(0)} + \mathcal{L}_{G.F.} = -\frac{1}{4}(\partial^\mu Z^\nu - \partial^\nu Z^\mu)(\partial_\mu Z_\nu - \partial_\nu Z_\mu) + \underline{\frac{1}{2} m^2 Z_\mu Z^\mu} + \underline{\frac{1}{2}(\partial^\mu \pi)^2} - \underline{m(\partial^\mu Z_\mu)\pi}$

$$- \frac{1}{2\xi}(n^\mu Z_\mu)^2$$

$\Rightarrow$ E.O.M.: $$\begin{pmatrix} (\partial^2 + m^2) g^\mu{}_\nu - \partial^\mu \partial_\nu - \frac{n^\mu n_\nu}{\xi} & m\partial^\mu \\ -m\partial_\nu & -\partial^2 \end{pmatrix} \begin{pmatrix} Z^\nu(x) \\ \pi(x) \end{pmatrix} = 0$$

$$\begin{pmatrix} (-q^2 + m^2) g^\mu{}_\nu + q^\mu q_\nu - \frac{n^\mu n_\nu}{\xi} & -imq^\mu \\ imq_\nu & q^2 \end{pmatrix} \begin{pmatrix} \tilde{Z}^\nu(q) \\ \tilde{\pi}(q) \end{pmatrix} = 0$$

$$\begin{pmatrix} -g^\rho{}_\mu + \frac{n^\rho q_\mu + q^\rho n_\mu}{n\cdot q} + \xi(q^2-m^2)\frac{q^\rho q_\mu}{(n\cdot q)^2} & i\frac{mn^\rho}{n\cdot q} + i\xi(q^2-m^2)\frac{mq^\rho}{(n\cdot q)^2} \\ -i\frac{mn_\mu}{n\cdot q} - i\xi(q^2-m^2)\frac{mq_\mu}{(n\cdot q)^2} & 1 + \xi(q^2-m^2)\frac{m^2}{(n\cdot q)^2} \end{pmatrix} \begin{pmatrix} (-q^2+m^2)g^\mu{}_\nu + q^\mu q_\nu - \frac{n^\mu n_\nu}{\xi} & -imq^\mu \\ imq_\nu & q^2 \end{pmatrix} \begin{pmatrix} \tilde{Z}^\nu(q) \\ \tilde{\pi}(q) \end{pmatrix}$$

$$= (q^2 - m^2) \begin{pmatrix} \tilde{Z}^\nu(q) \\ \tilde{\pi}(q) \end{pmatrix} = 0$$

$\Rightarrow$ Green's function $(\xi = 0)$: $$D^R{}_M(q) = \frac{i}{q^2 - m^2} \begin{pmatrix} -g^\mu{}_\nu + \frac{n^\mu q_\nu + q^\mu n_\nu}{n\cdot q} & im\frac{n^\mu}{n\cdot q} \\ -im\frac{n_\nu}{n\cdot q} & 1 \end{pmatrix}$$

5×5 FD gauge propagator of massive gauge boson.

J. Chen, KH, J. Kanzaki, K. Mawatari, Y. Zheng [arXiv; 2211.14562].