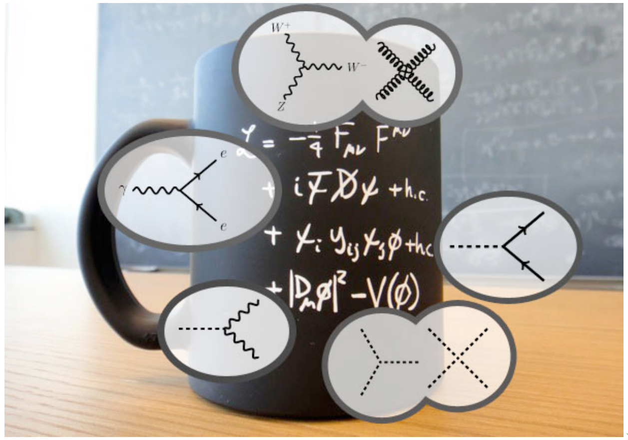
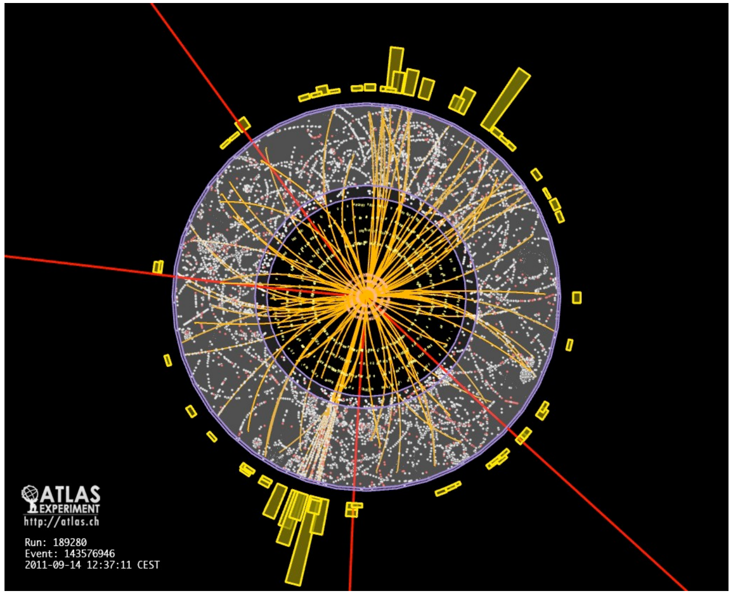
UCLouvain
Institut de recherche en mathématique et physique
Centre de Cosmologie, Physique des Particules et Phénoménologie
MCnet

MonteCarlo Simulation

Olivier Mattelaer



Monte-Carlo Physics

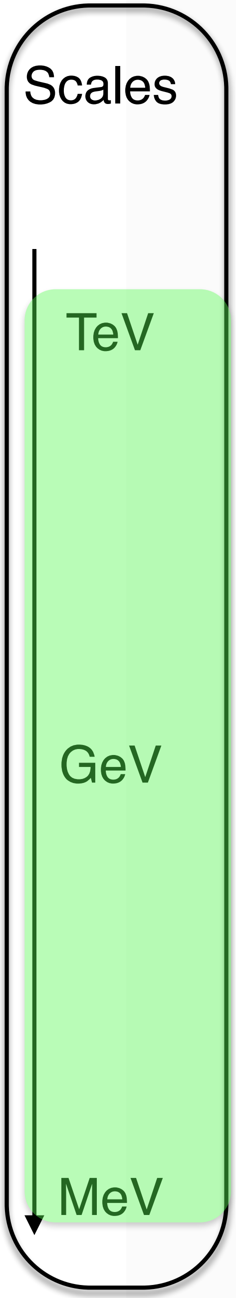


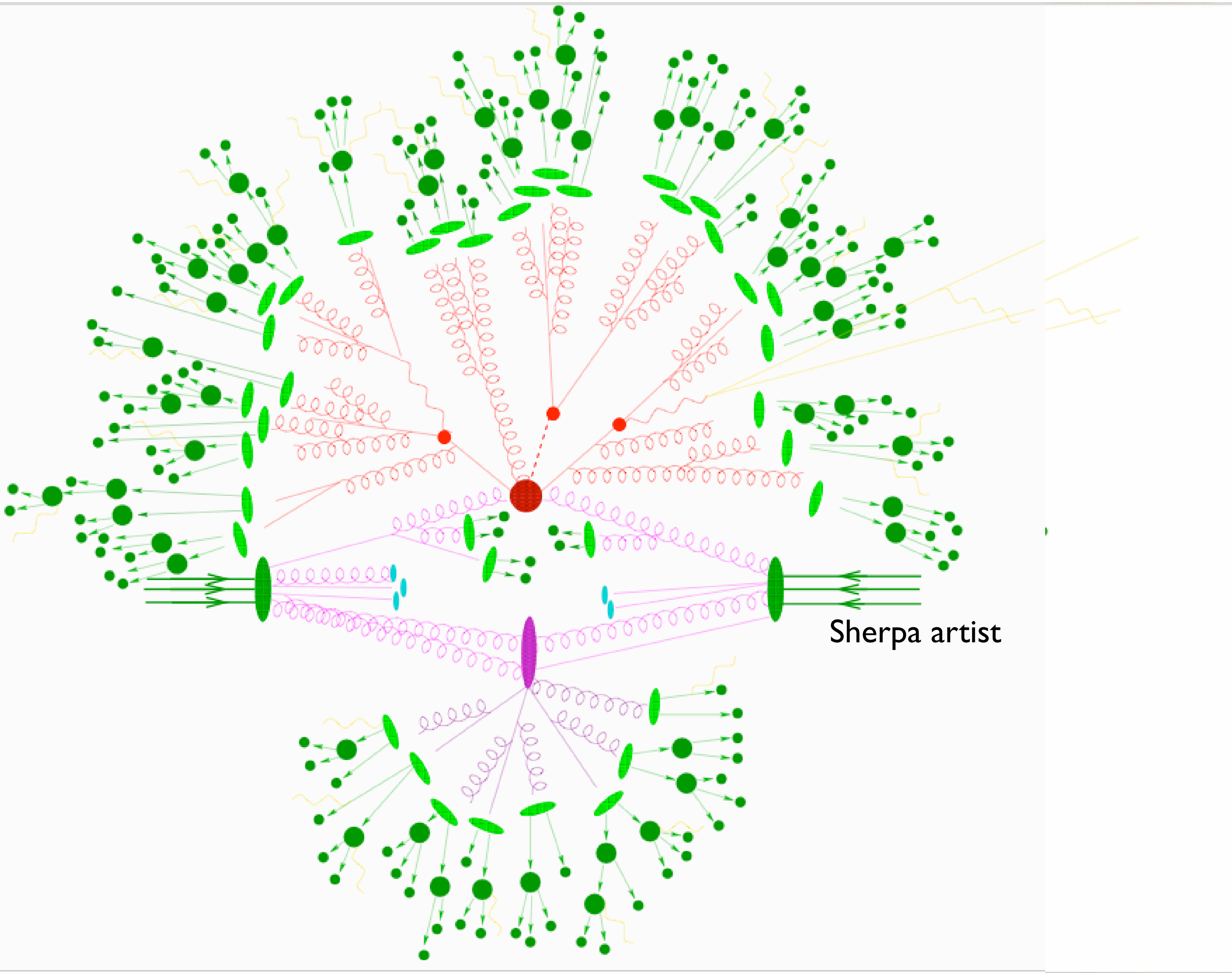
Our goal

- Cross-section
- Differential cross-section
- Un-weighted events

Simulation of collider events

What are the MC for?





What are the MC for?

Scales

TeV

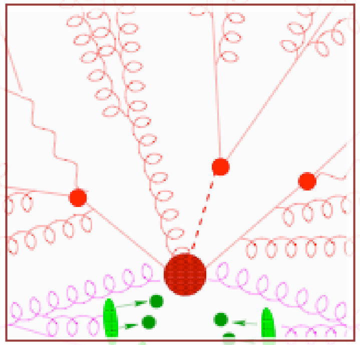
GeV

MeV

1. High-Q^2 Scattering

2. Parton Shower

☞ where BSM physics lies



☞ process dependent

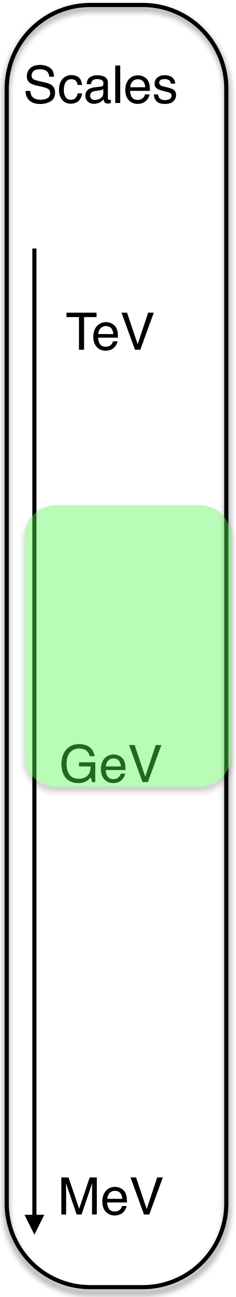
☞ first principles description

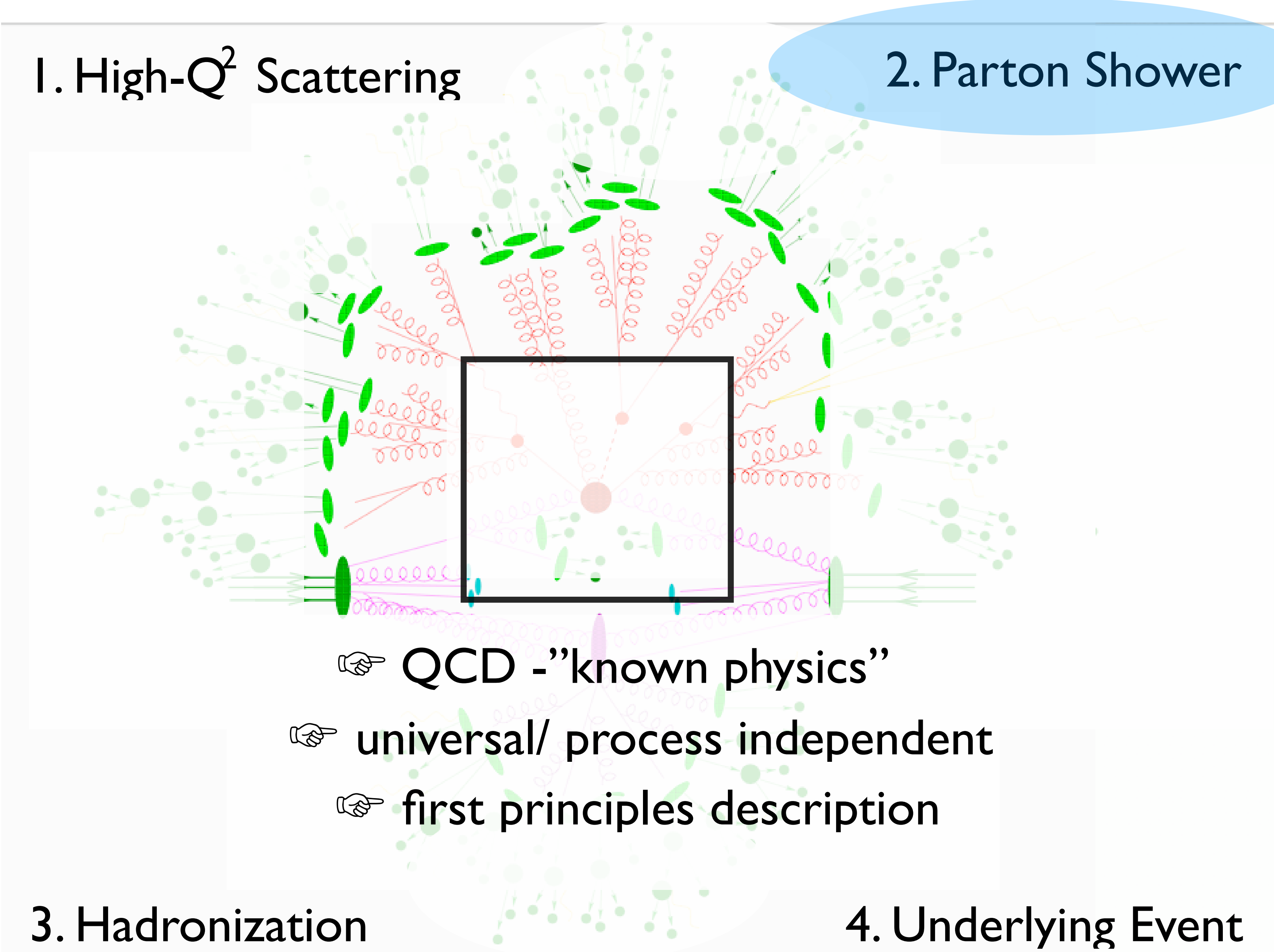
☞ it can be systematically improved

3. Hadronization

4. Underlying Event

What are the MC for?



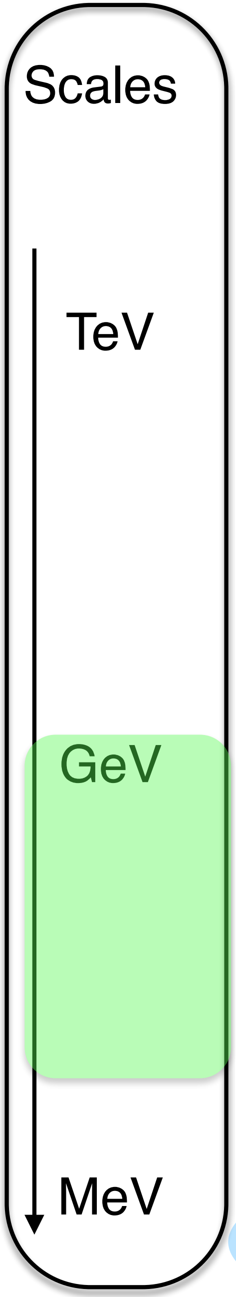


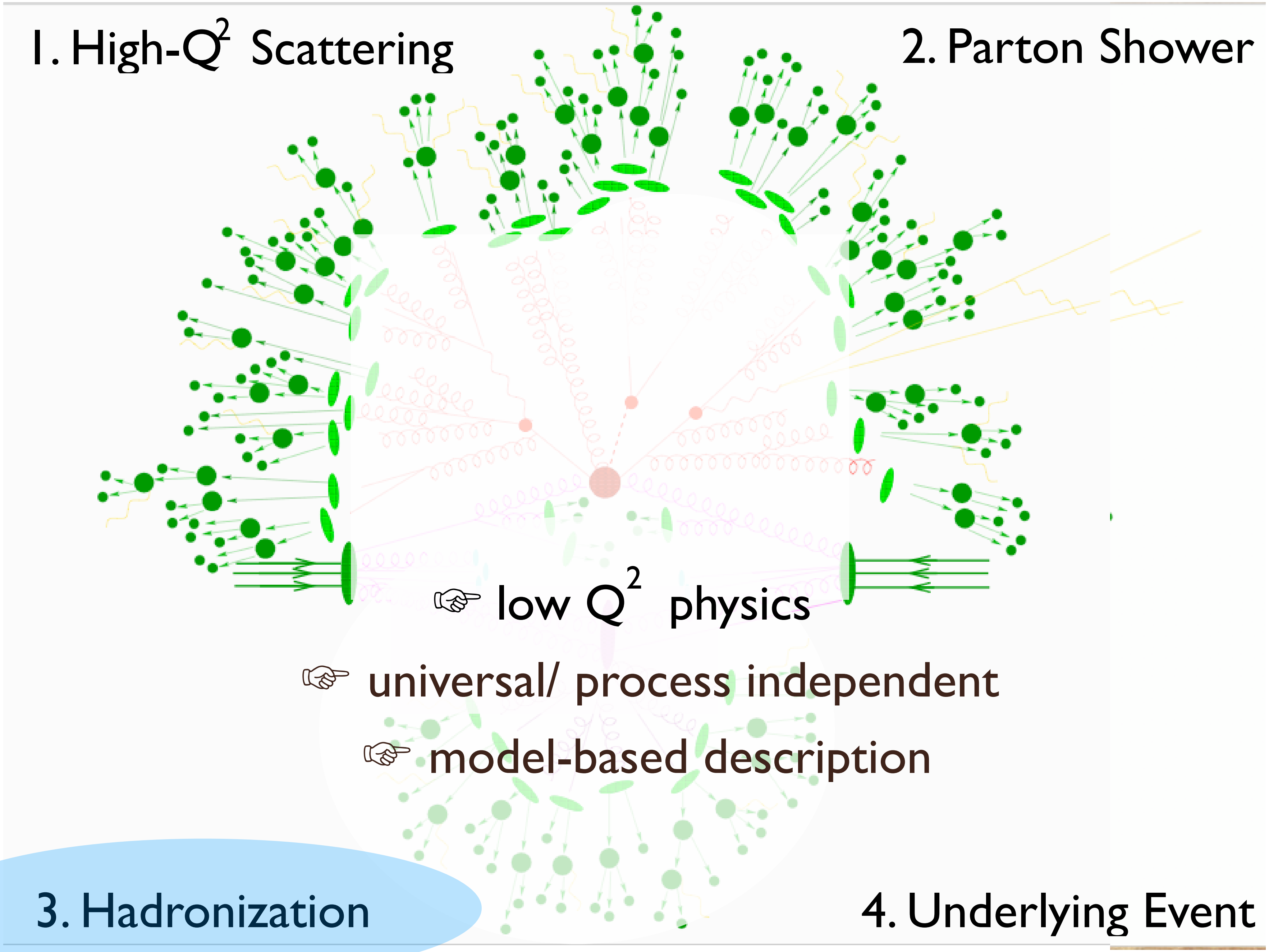
☞ QCD -"known physics"

☞ universal/ process independent

☞ first principles description

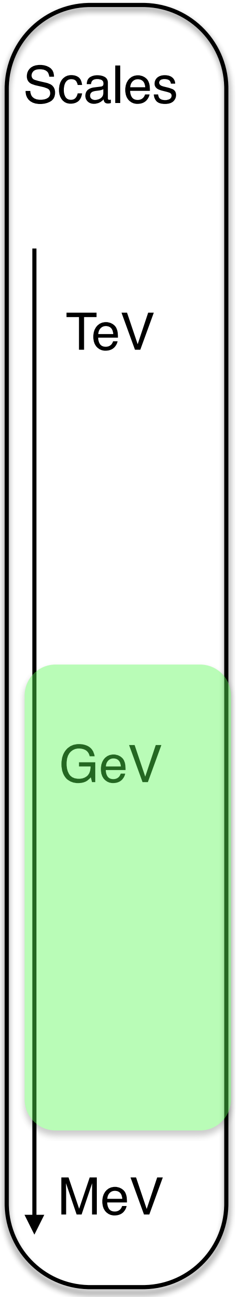
What are the MC for?

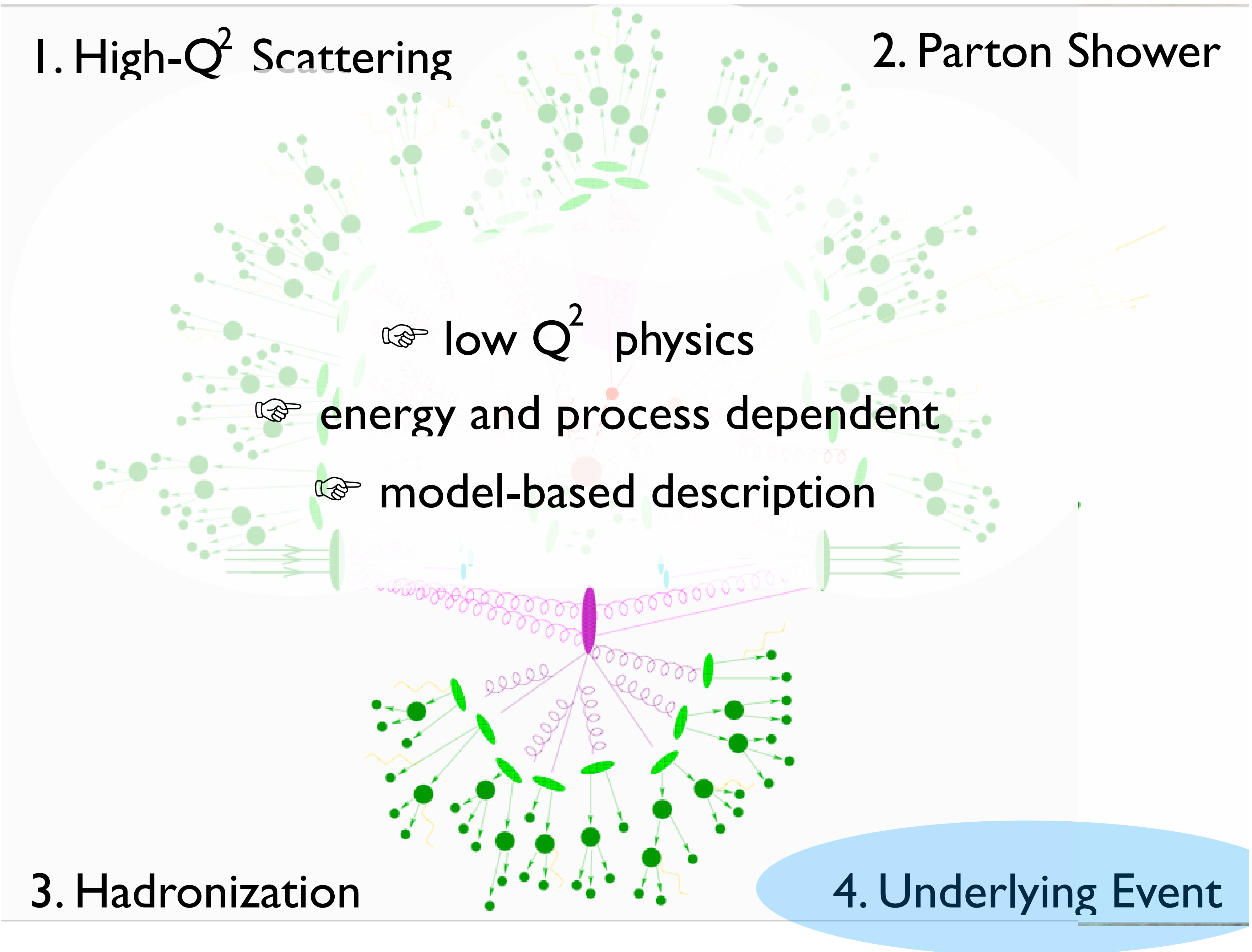




- ☞ low Q^2 physics
- ☞ universal/ process independent
- ☞ model-based description

What are the MC for?





- ☞ low Q^2 physics
- ☞ energy and process dependent
- ☞ model-based description

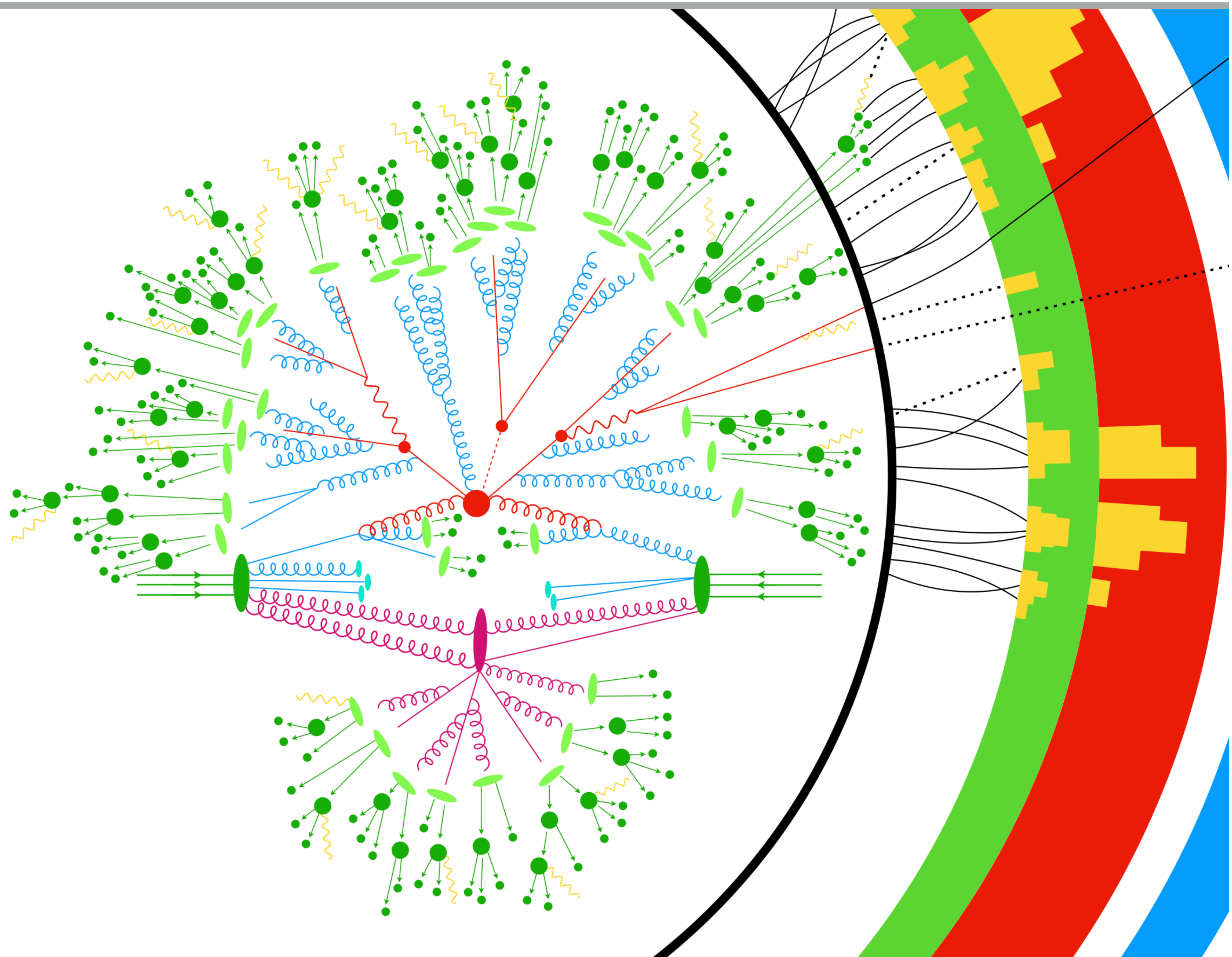
What are the MC for?

Scales

TeV

GeV

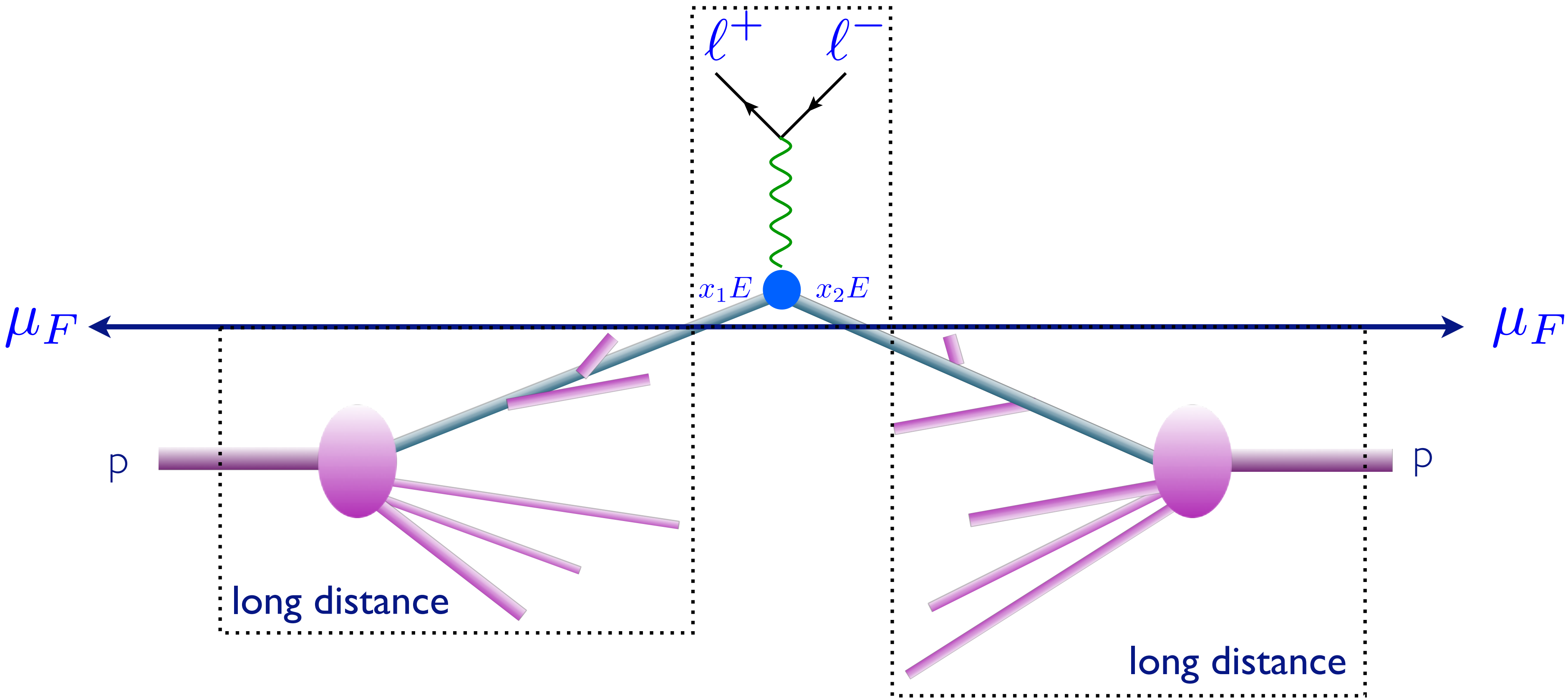
MeV



To Remember

- Multi-scale problem
  - ➡ New physics visible only at High scale
  - ➡ Problem split in different scale
    - ❑ Factorisation theorem

Master formula for the LHC



$$\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

Parton-level cross section

Perturbative expansion

$d\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$ **Parton-level cross section**

- The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter, schematically:

$$\hat{\sigma} = \sigma^{\text{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \ldots\right)$$

LO predictions	NLO corrections	NNLO corrections	N3LO or NNNLO corrections

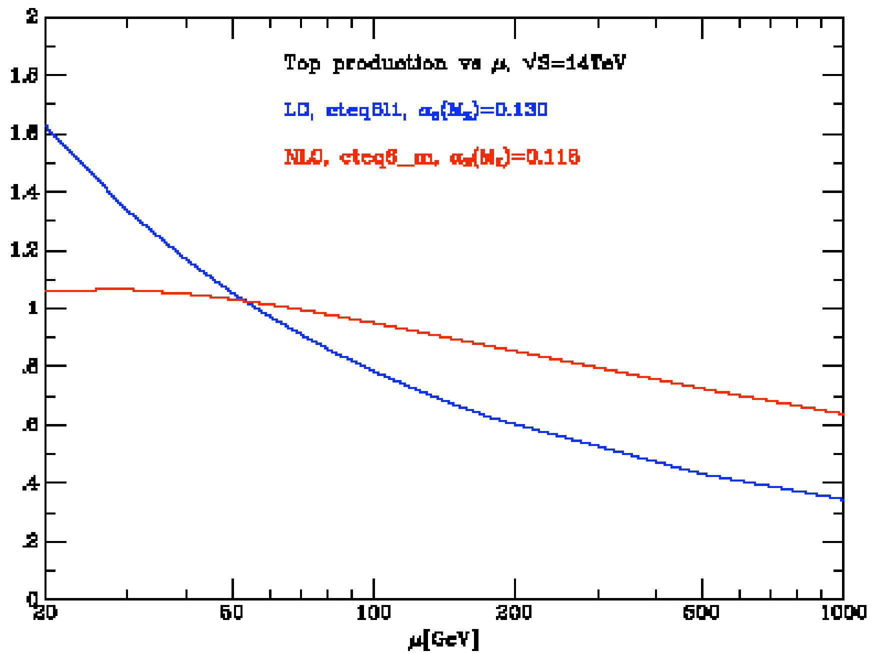
- Including higher corrections improves predictions and reduces theoretical uncertainties

Improved predictions

$$d\sigma = \sum_{a,b} \int dx_1 dx_2 \;\; f_a(x_1, \mu_F) f_b(x_2, \mu_F) \, d\hat{\sigma}_{ab \to X}(\hat{s}, \mu_F, \mu_R)$$

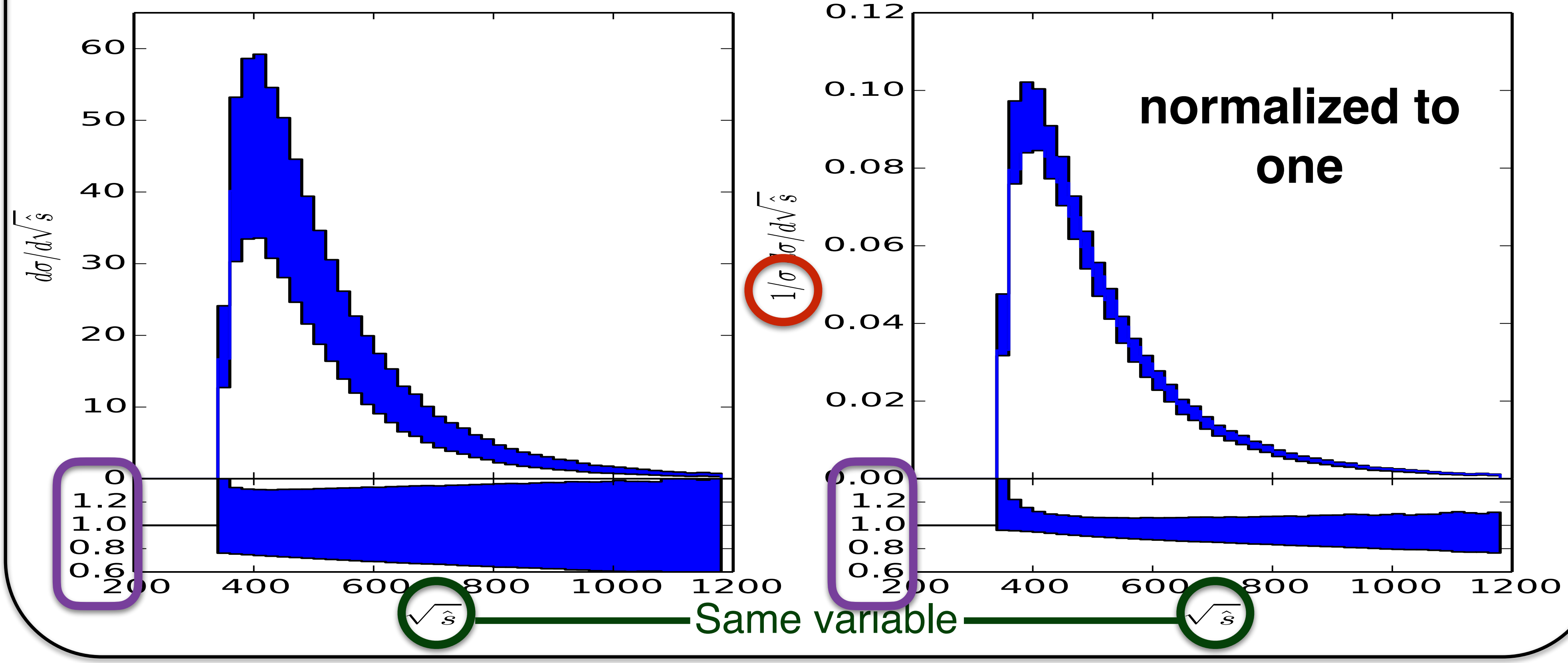
$$\hat{\sigma} = \sigma^{\text{Born}} \left(1 + \frac{\alpha_s}{2\pi} \sigma^{(1)} + \left(\frac{\alpha_s}{2\pi} \right)^2 \sigma^{(2)} + \left(\frac{\alpha_s}{2\pi} \right)^3 \sigma^{(3)} + \ldots \right)$$

- Leading Order predictions can depend strongly on the renormalization and factorization scales
- Including higher order corrections reduces the dependence on these scales



LO

LO computation (top quark pair)



At LO:

- Large scale uncertainty
- but mainly in the Normalisation
- LO is good for shape

To Remember

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$

Phase-space integral

Parton density functions

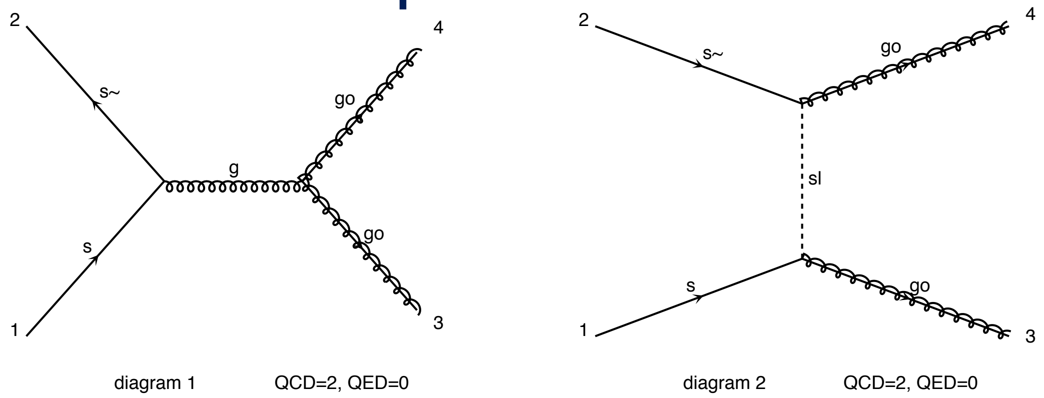
Parton-level cross section

- PDF: content of the proton
  - ➡ Define the physics/processes that will dominate on your accelerator
- LO: good for shape
- NLO/NNLO: Reduce scale uncertainty
- Computation are inclusive (+ any jet) due to renormalization/factorization scale

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



- Evaluate the matrix-element

$|\mathcal{M}|^2$ ➡Need Feynman Rules!

- Phase-Space Integration

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

Easy enough

Hard

Tommorow

Very Hard (in general)

Now

Monte Carlo Integration

Monte Carlo Integration

Calculations of cross section or decay widths involve integrations over high-dimension phase space of very peaked functions:

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

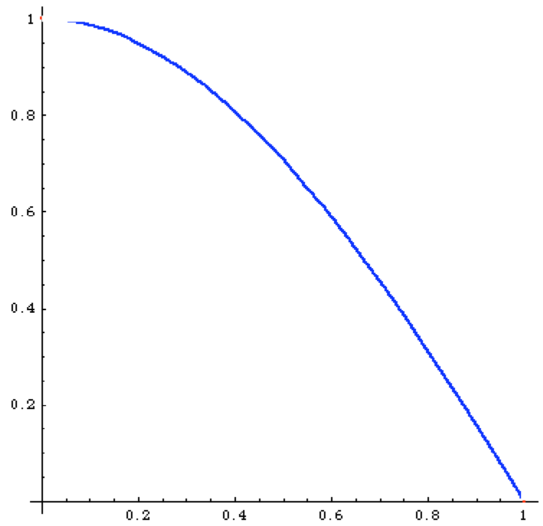
$$Dim[\Phi(n)] \sim 3n$$

General and flexible method is needed

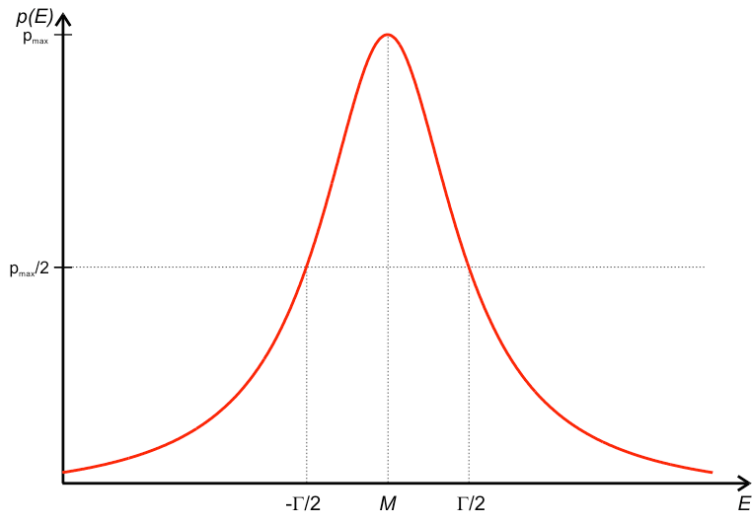
Not only integrating but also **generates events**

Integration

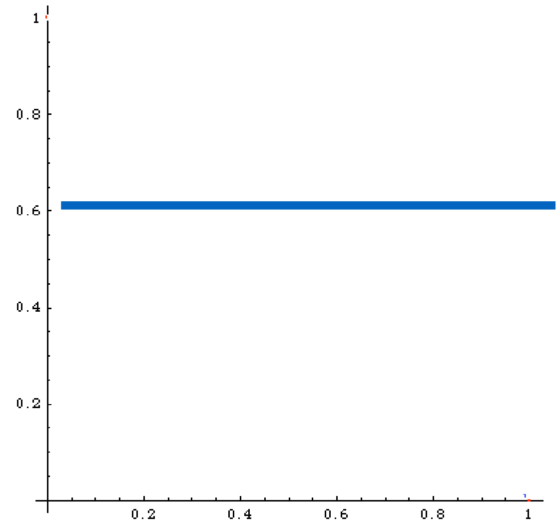
$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$



$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$



$$\int dx\, C$$



	simpson	**MC**
3	0,638	0,3
5	0,6367	0,8
20	0,63662	0,6
100	0,636619	0,65
1000	0,636619	0,636

Method of evaluation

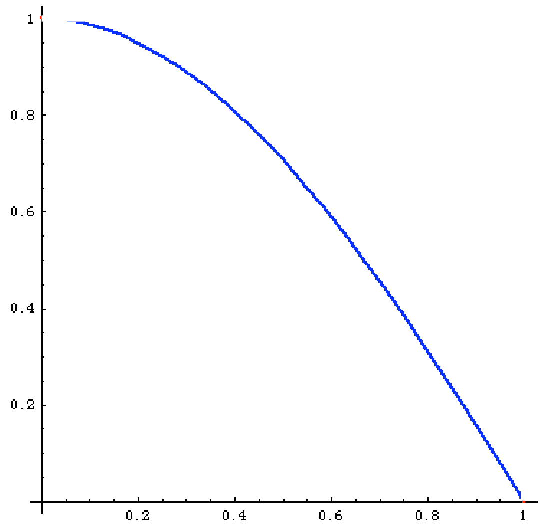
- MonteCarlo $1/\sqrt{N}$
- Trapezium $1/N^2$
- Simpson $1/N^4$

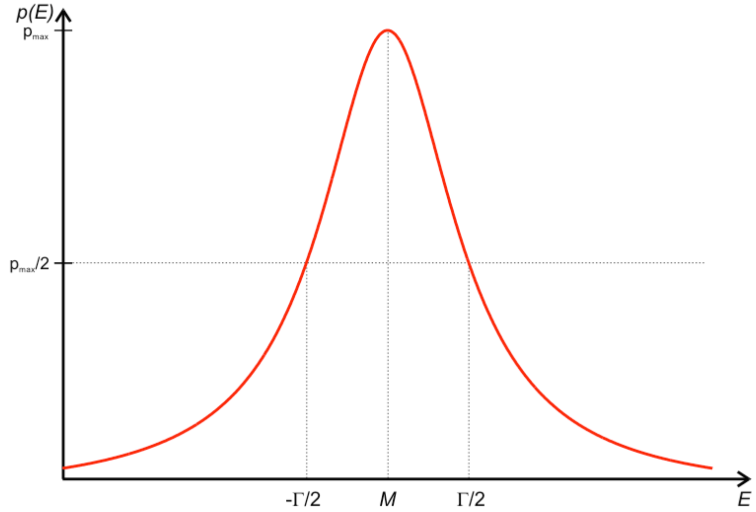
Integration

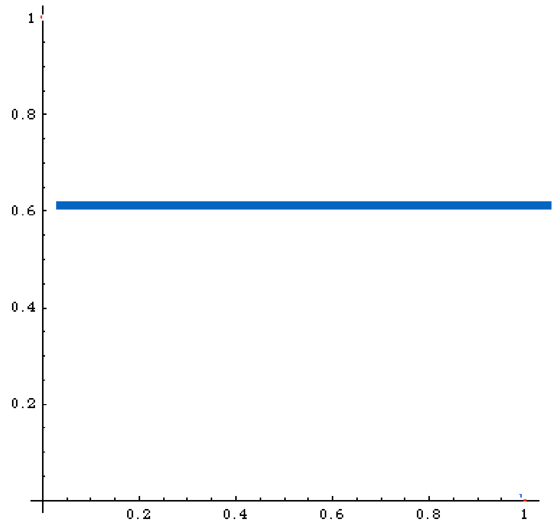
$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\int dx\, C$$







Method of evaluation

- MonteCarlo $1/\sqrt{N}$
- Trapezium $1/N^2$
- Simpson $1/N^4$

More Dimension →

$1/\sqrt{N}$

$1/N^{2/d}$

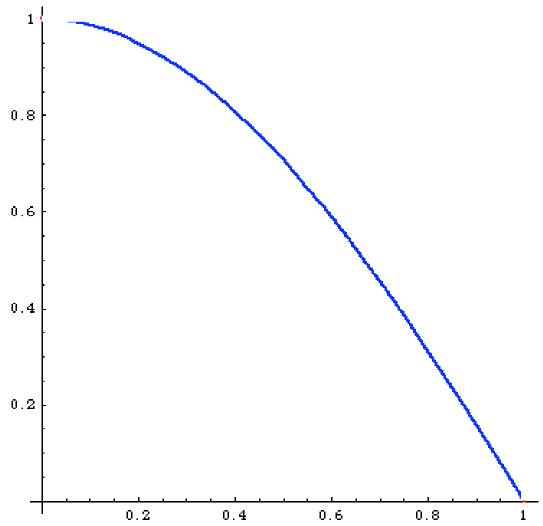
$1/N^{4/d}$

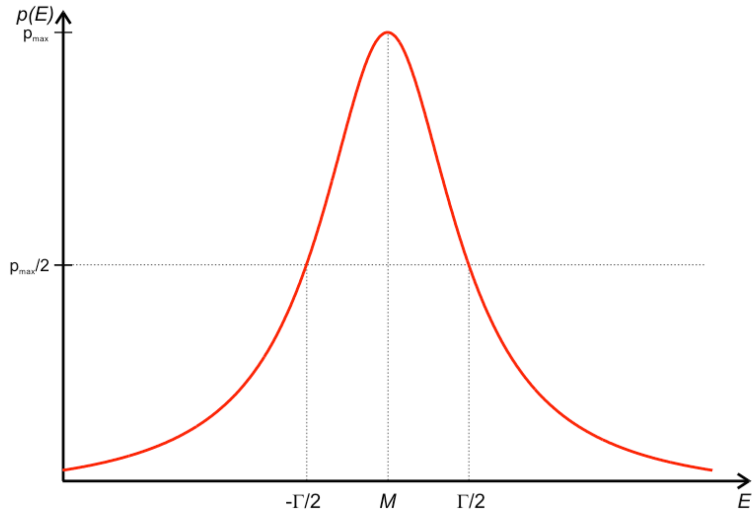
Integration

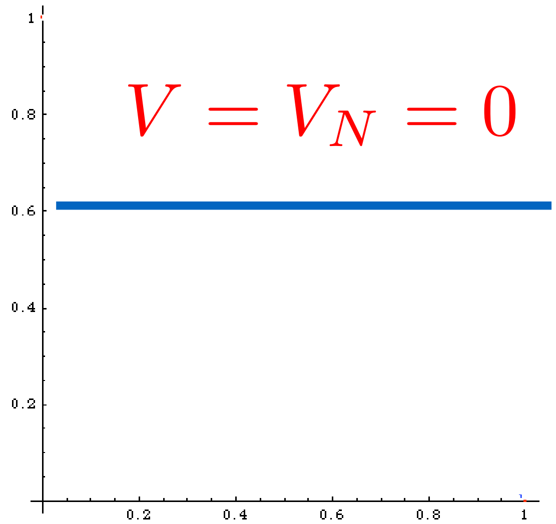
$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\int dx\, C$$

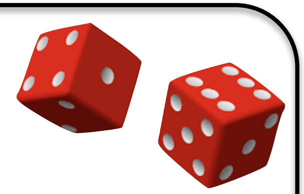






$$V = V_N = 0$$

$$I = \int_{x_1}^{x_2} f(x)dx \quad \Rightarrow \quad I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x)$$

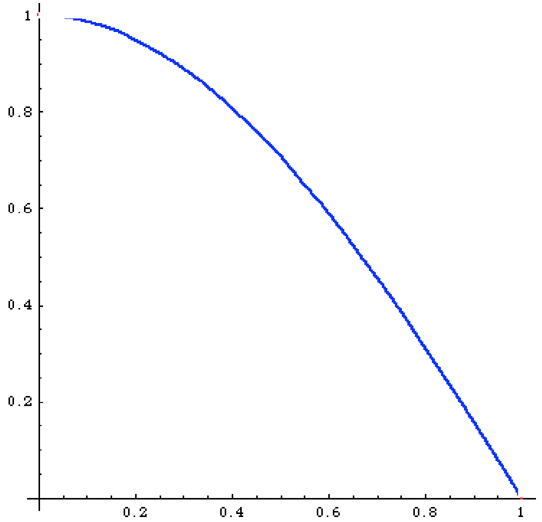


$$V = (x_2 - x_1)\int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \quad \Rightarrow \quad V_N = (x_2 - x_1)^2 \frac{1}{N}\sum_{i=1}^{N}[f(x)]^2 - I_N^2$$

$$I = I_N \pm \sqrt{V_N/N}$$

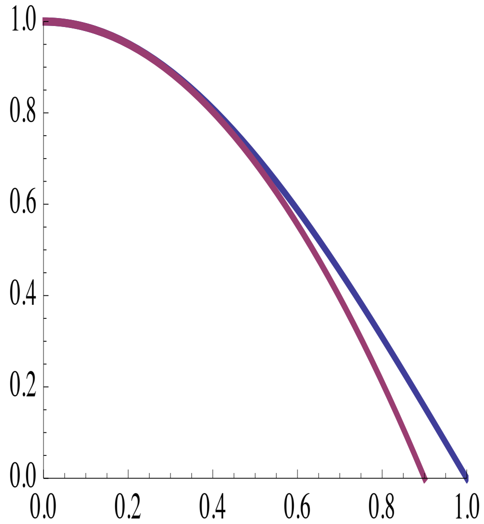
Can be minimized!

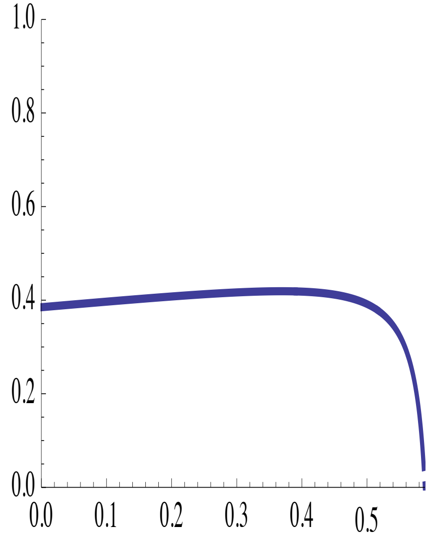
Importance Sampling



$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$I_N = 0.637 \pm 0.307/\sqrt{N}$$





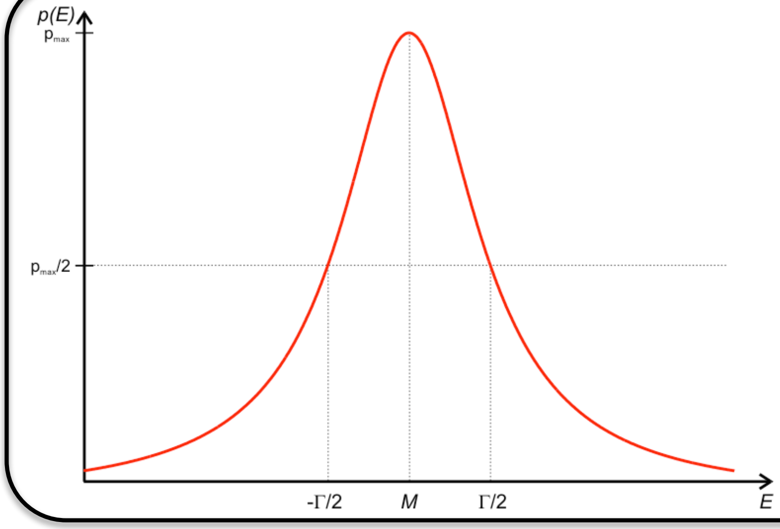
$$I = \int_0^1 dx (1 - cx^2) \frac{\cos\left(\frac{\pi}{2} x\right)}{(1 - cx^2)} = \int_{\xi_1}^{\xi_2} d\xi \frac{\cos \frac{\pi}{2} x[\xi]}{1 - x[\xi]^2 c}$$

$$\simeq 1$$

$$I_N = 0.637 \pm 0.031/\sqrt{N}$$

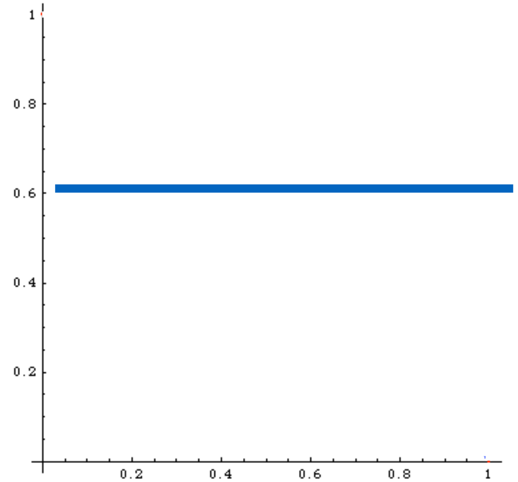
The Phase-Space parametrization is important to have an efficient computation!

Importance Sampling

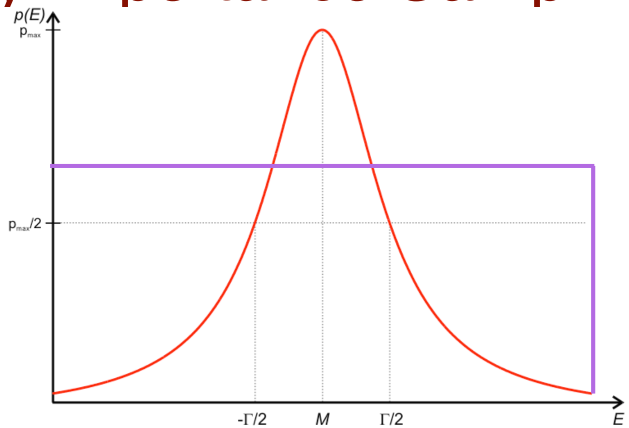


$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

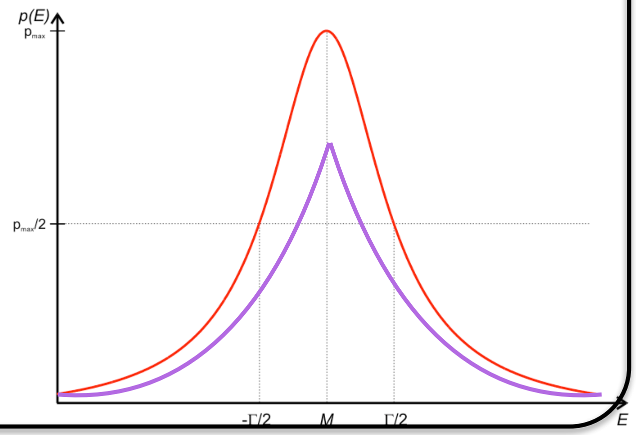
$$\xi = arctan\left(\frac{q^2 - M^2}{\Gamma M}\right)$$



Why Importance Sampling?



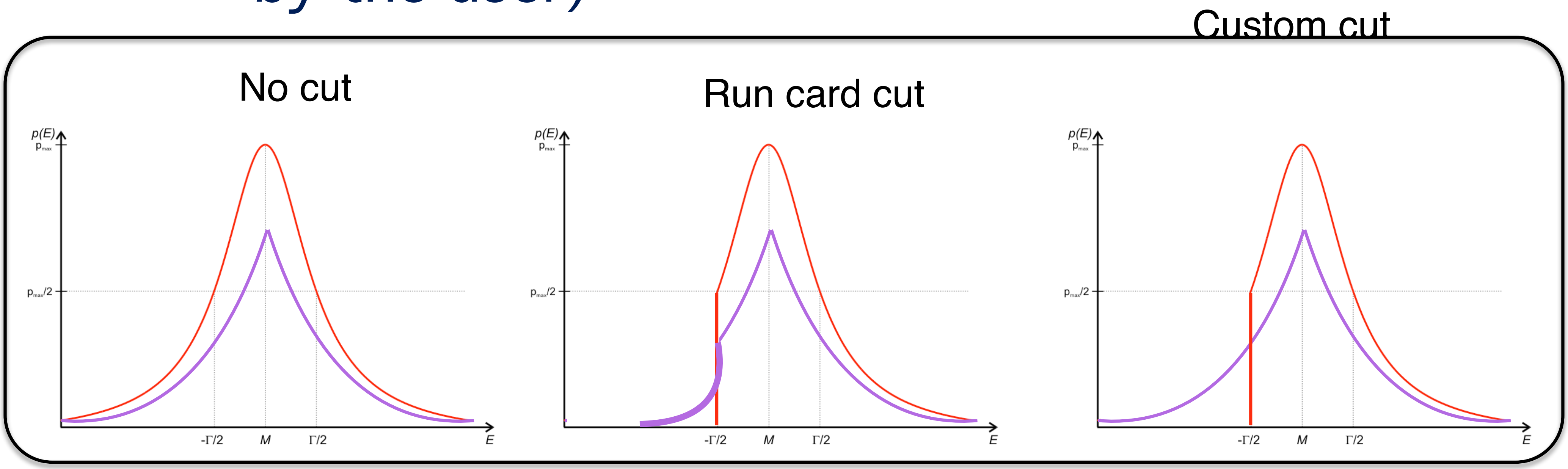
Probability of using that point p(x)



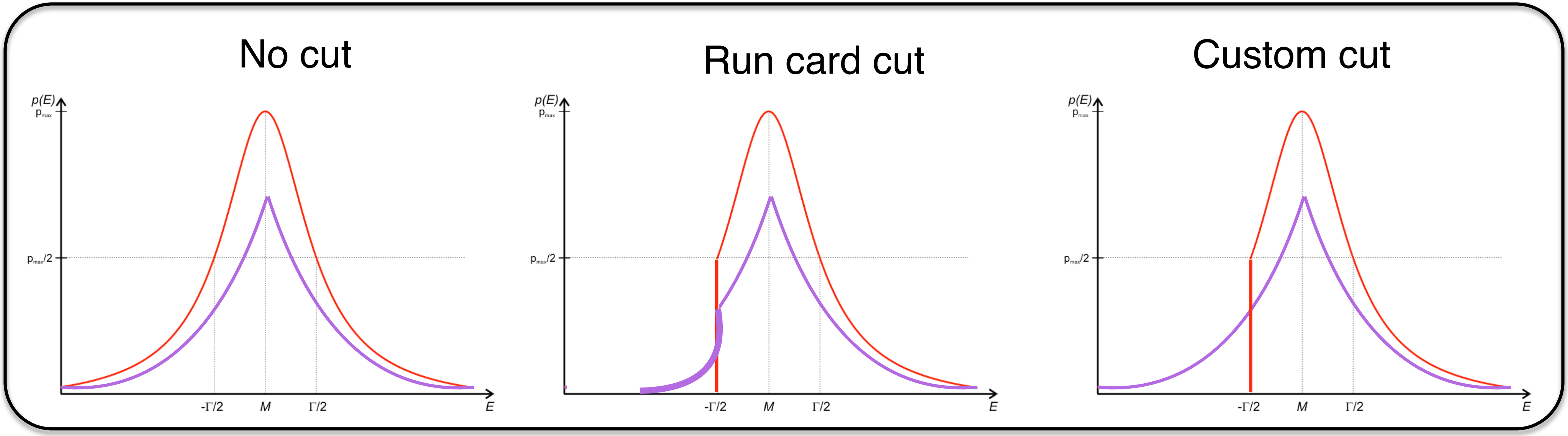
The change of variable ensure that the evaluation of the function is done where the function is the largest!

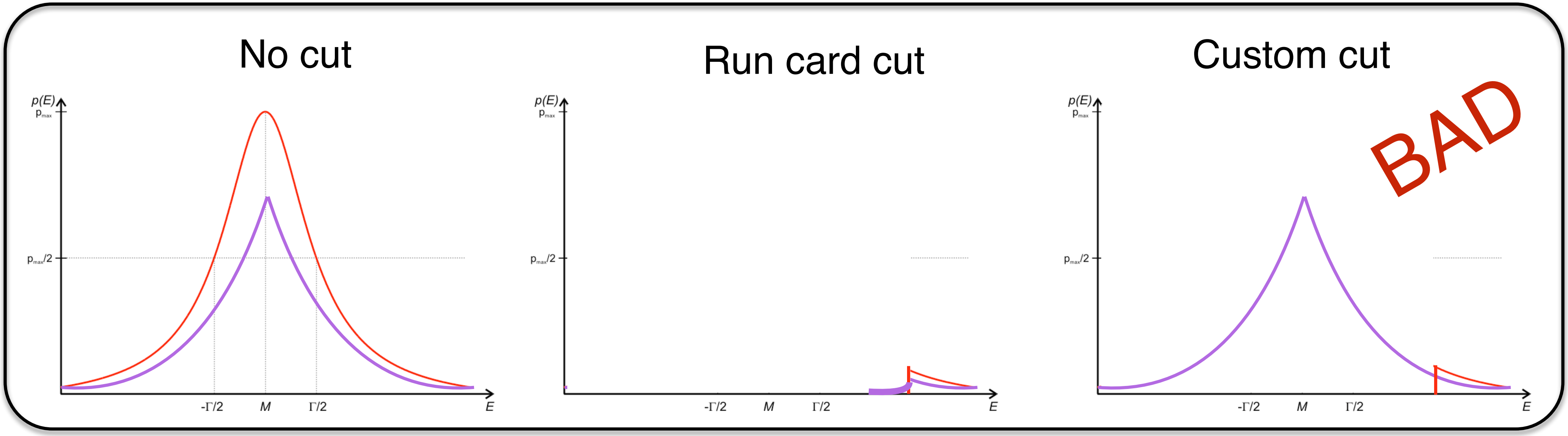
Cut Impact

- Events are generated according to our best knowledge of the function (with importance sampling
  - ➡ Adding a cut needs to modified the phase-space integrator
  - ➡ Not possible for custom cut (hardcoded by the user)



Cut Impact





Might miss the contribution and think it is just zero.

Importance Sampling

Key Point

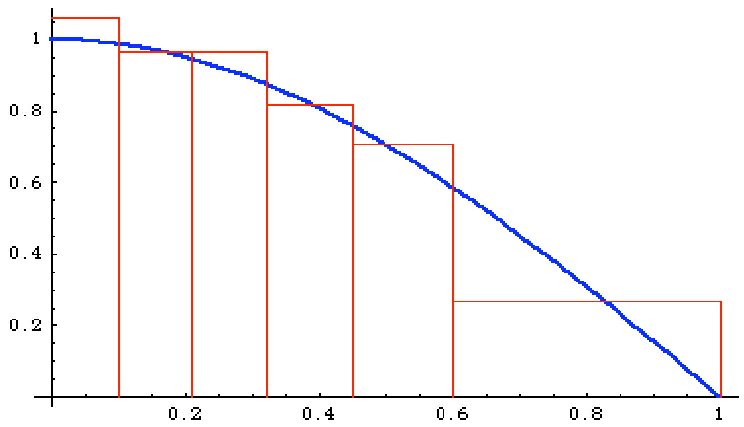
- Generate the random point in a distribution which is close to the function to integrate.
- This is a change of variable, such that the function is flatter in this new variable.
- Needs to know an approximate function.

Adaptative Monte-Carlo

- Create an approximation of the function on the flight!

Adaptative Monte-Carlo

- Create an approximation of the function on the flight!



Algorithm

1. Creates bin such that each of them have the same contribution.

   ➡Many bins where the function is large

2. Use the approximate for the importance sampling method.

VEGAS

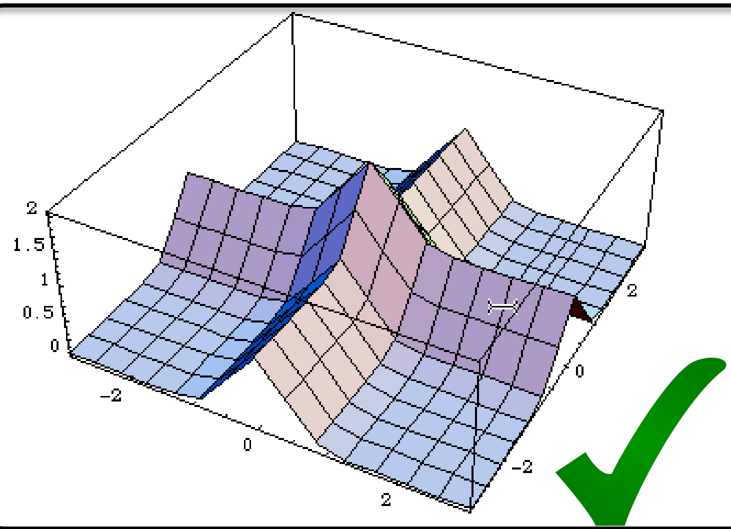
More than one Dimension

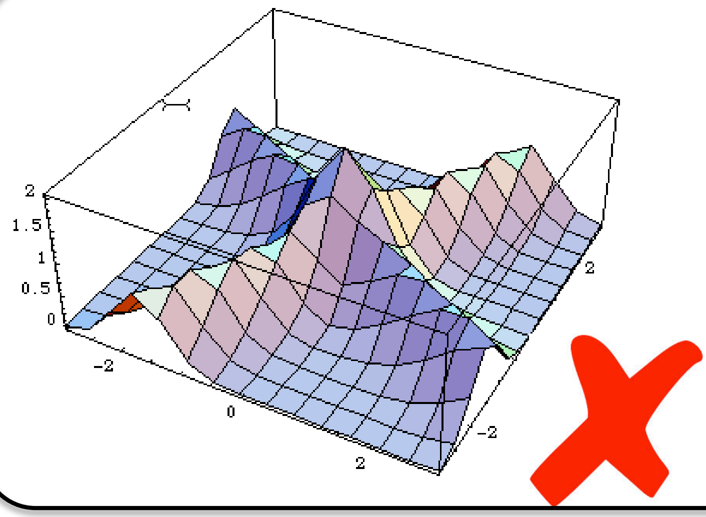
- VEGAS works only with 1(few) dimension
  - ➡memory problem

Solution

- Use projection on the axis

$$\vec{p}(x) = p(x) \cdot p(y) \cdot p(z) \ldots$$



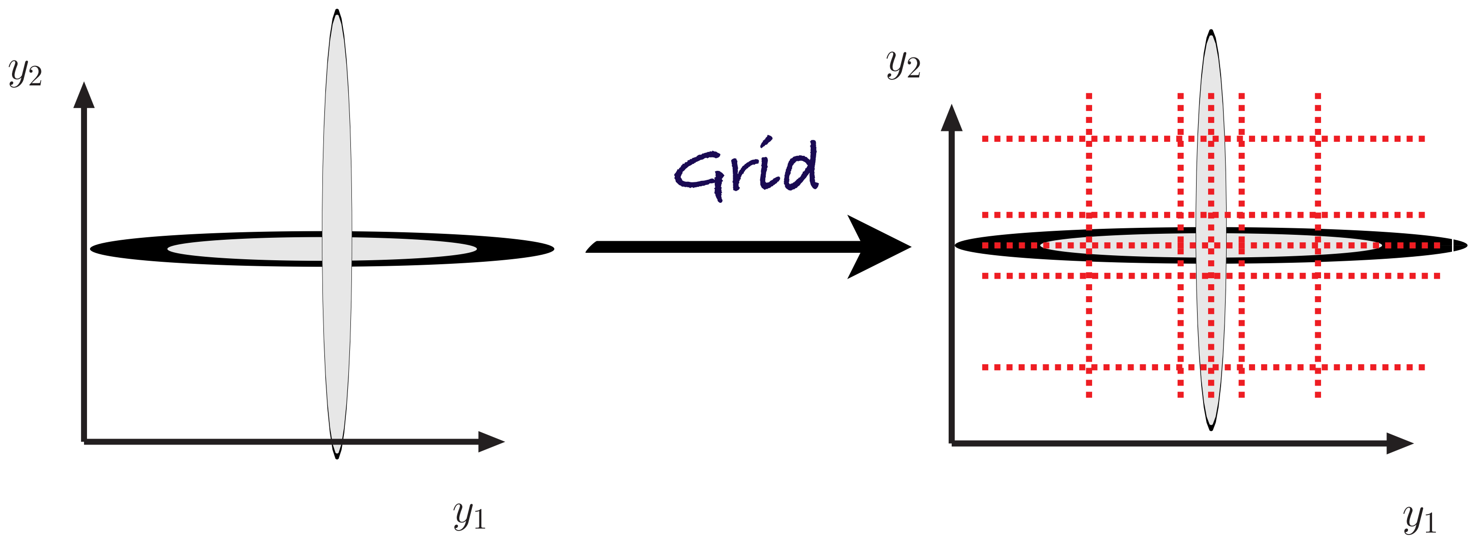


- We need to ensure the factorization !

➡Additional change of variable

Monte-Carlo Integration

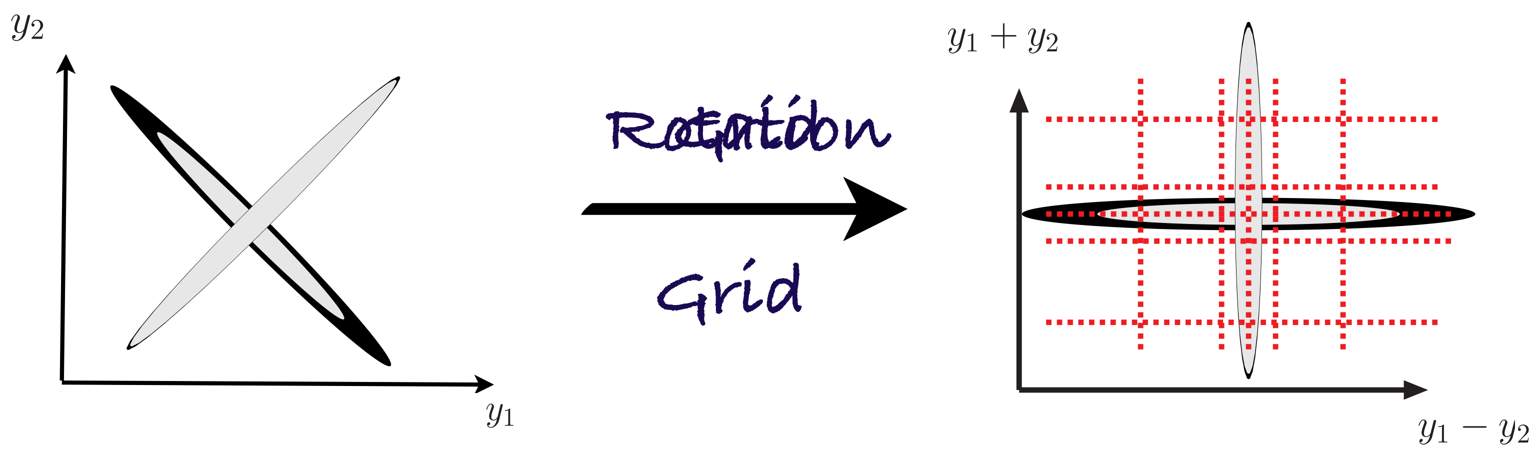
- The choice of the parameterisation has a strong impact on the efficiency



- ☐ The adaptive Monte-Carlo Technique picks point in interesting areas
  → The technique is efficient

Monte-Carlo Integration

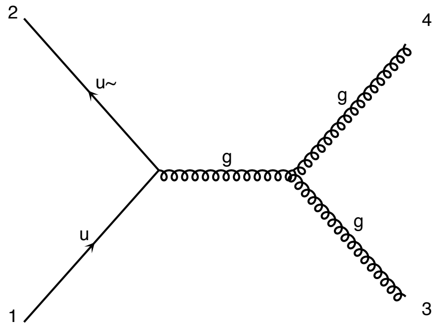
- The choice of the parametrization has a strong impact on the efficiency

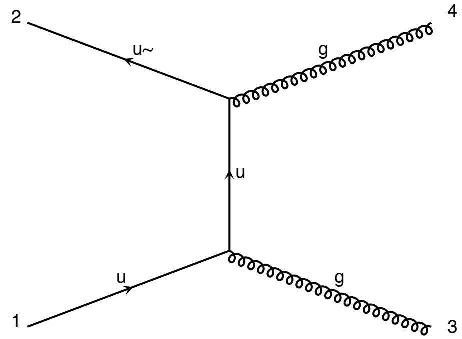


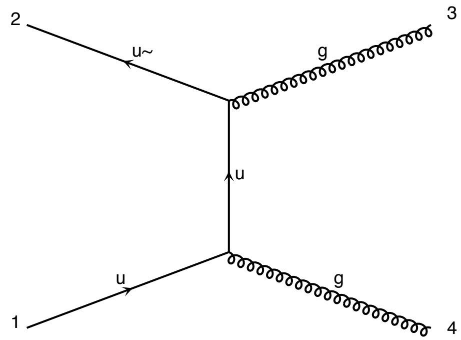
- The adaptive Monte-Carlo Techniques picks point
  point interesting areas

  → The technique is efficient / The integral converges slowly

Example: QCD 2 → 2







$$\propto \frac{1}{\hat{s}} = \frac{1}{(p_1 + p_2)^2} \qquad \propto \frac{1}{\hat{t}} = \frac{1}{(p_1 - p_3)^2} \qquad \propto \frac{1}{\hat{u}} = \frac{1}{(p_1 - p_4)^2}$$

Three very different pole structures contributing to the same matrix element.

*Method used in MadGraph

Trick in MadEvent: Split the complexity

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

≈ 1

- Any single diagram is “easy” to integrate (pole structures/suitable integration variables known from the propagators)
- Divide integration into pieces, based on diagrams
- All other peaks taken care of by denominator sum

N Integral

- Errors add in quadrature so no extra cost
- “Weight” functions already calculated during $|M|^2$ calculation
- Parallel in nature

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

P1_qq_wpwm

s= 725.73 ± 2.07 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G2.2	377.6	1.67	142.285	7941.0	21
G3	239	1.16	220.04	10856.0	45.5
G1	109.1	0.378	70.88	3793.0	34.8

P1_gg_wpwm

s= 20.714 ± 0.332 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G1.2	20.71	0.332	7.01	373.0	18

term of the above sum.

each term might not be gauge invariant

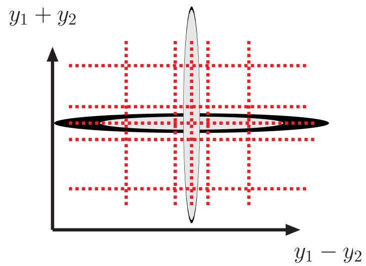
To Remember

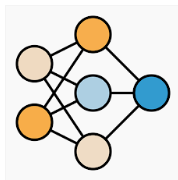
- Phase-Space integration is difficult
- We need to know the function
  - ➡ Be careful with cuts
- MadGraph split the integral in different contribution linked to the Feynman Diagram
  - ➡ Those are not the contribution of a given diagram

Can we do Better?

- Importance sampling/VEGAS is learning a function
  - ➡ HOT TOPIC: Machine Learning
  - ➡ Lot of work in progress

multi-channel VEGAS → MADNIS (Last week number)





Setup	Channel	Integral I [pb]	σ/I
VEGAS	1	2.057(4)	0.98
$\eta = 3.9\%$	3	108.4(3)	1.46
	4	31.54(7)	1.20
	7	73.4(2)	[illegible]
	sum	215.4(4)	1.39

W2j

→

Setup	Channel	Integral I [pb]	σ/I
VEGAS-Flow	1	0.0059(3)	0.24
(trained α,	3	100.27(6)	0.37
stratified)	4	10.86(2)	0.55
$\eta = 14.3\%$	7	104.16(8)	[illegible]
	sum	215.30(10)	0.47

(Preliminary)

Variance reduce by a factor 3 (so convergence 9 times faster)

Event generation also three times more efficient

UCLouvain
Institut de recherche en mathématique et physique
Centre de Cosmologie, Physique des Particules et Phénoménologie

MadGraph5_aMC@NLO

Olivier Mattelaer

What are the MC for?

Scales

TeV

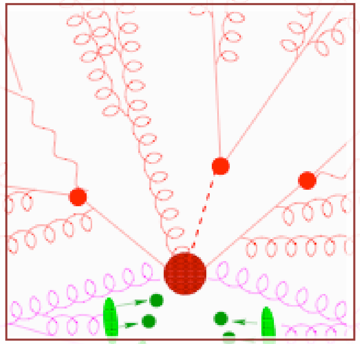
GeV

MeV

1. High-Q^2 Scattering

2. Parton Shower

☞ where BSM physics lies



☞ process dependent

☞ first principles description

☞ it can be systematically improved

3. Hadronization

4. Underlying Event

To Remember

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$

Phase-space integral

Parton density functions

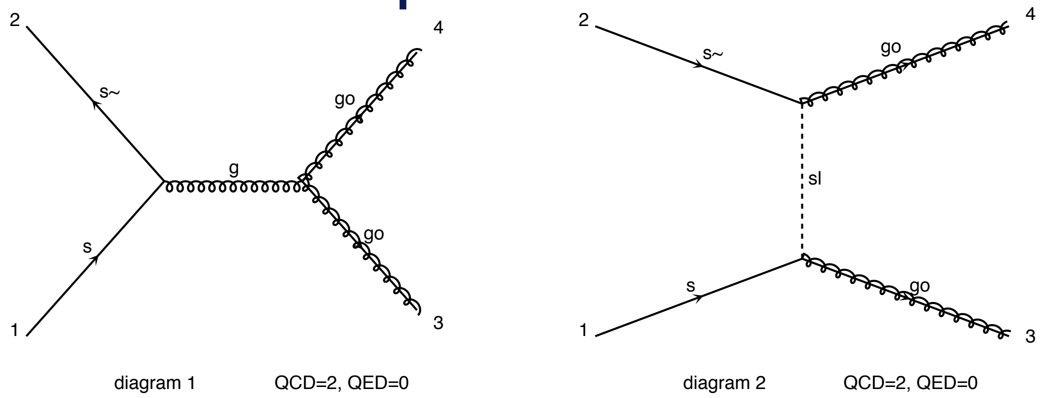
Parton-level cross section

- PDF: content of the proton
  - ➡ Define the physics/processes that will dominate on your accelerator
- LO: good for shape
- NLO/NNLO: Reduce scale uncertainty
- Computation are inclusive (+ any jet) due to renormalization/factorization scale

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



- Evaluate the matrix-element

$$|\mathcal{M}|^2$$ ➡Need Feynman Rules!

- Phase-Space Integration

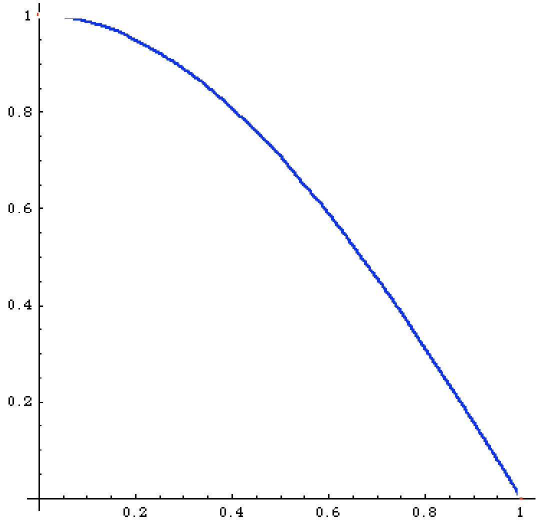
$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

Easy enough

Hard

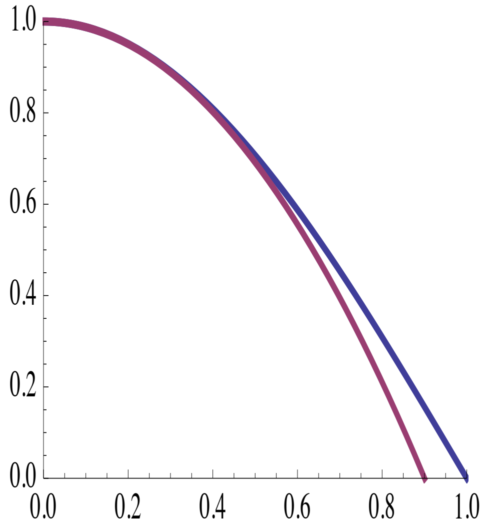
Very Hard (in general)

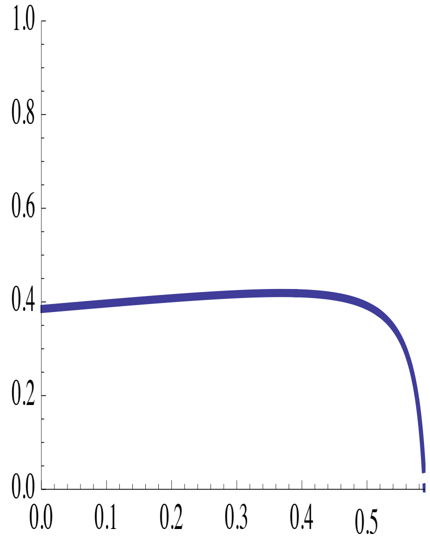
Importance Sampling



$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$

$I_N = 0.637 \pm 0.307/\sqrt{N}$





$$I = \int_0^1 dx(1-cx^2)\frac{\cos\left(\frac{\pi}{2}x\right)}{(1-cx^2)} = \int_{\xi_1}^{\xi_2} d\xi \frac{\cos\frac{\pi}{2}x[\xi]}{1-x[\xi]^2 c}$$

$\simeq 1$

$I_N = 0.637 \pm 0.031/\sqrt{N}$

The Phase-Space parametrization is important to have an efficient computation!

To Remember

- Phase-Space integration is difficult
- We need to know the function
  - ➡ Be careful with cuts
- MadGraph split the integral in different contribution linked to the Feynman Diagram
  - ➡ Those are not the contribution of a given diagram

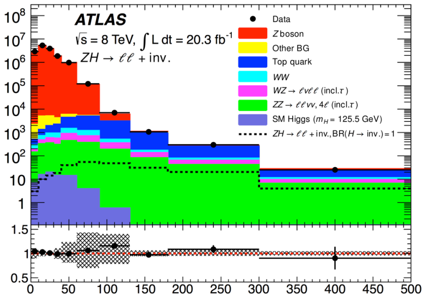
Goal of today

- Event Generation
- Learn how we evaluate (tree-level) matrix-element
- Learn Narrow-width Approximation

Event Generation

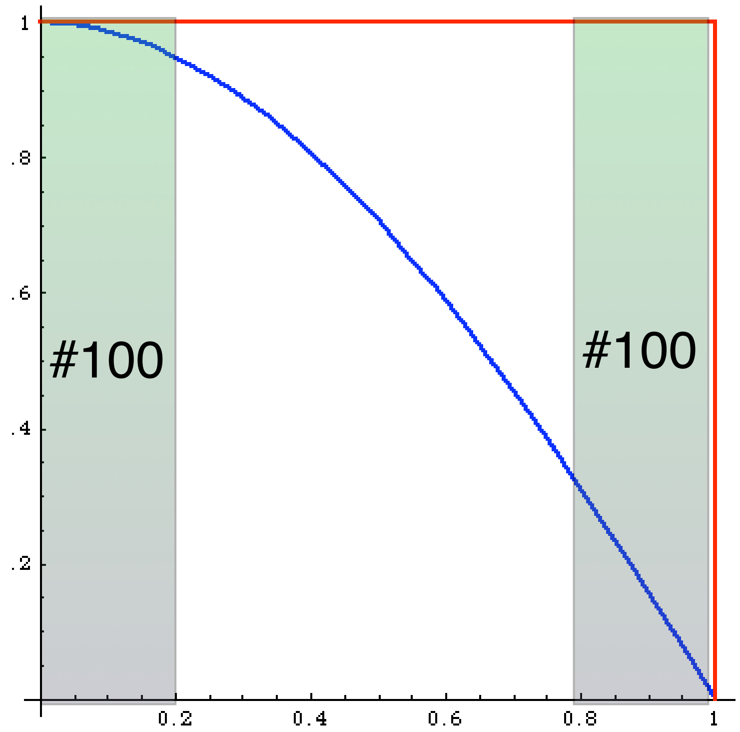
What is the goal?

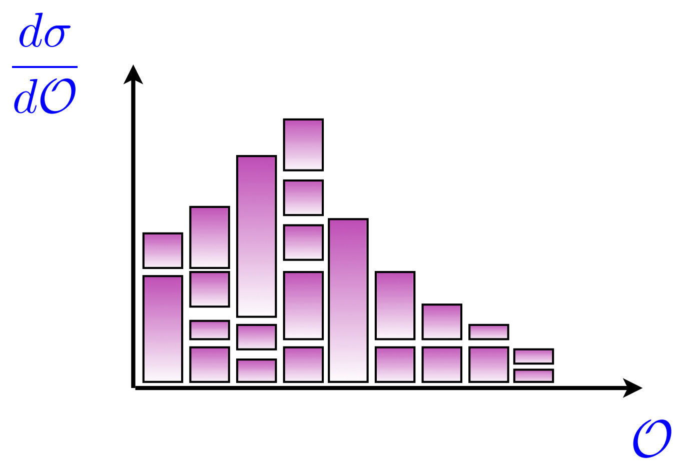
- Cross-section
  - But large theoretical uncertainty



- Differential Cross-Section
  - Provided as sample of events
  - Sample size is problematic
    - Those events will need to have full detector simulation

How to get sample?

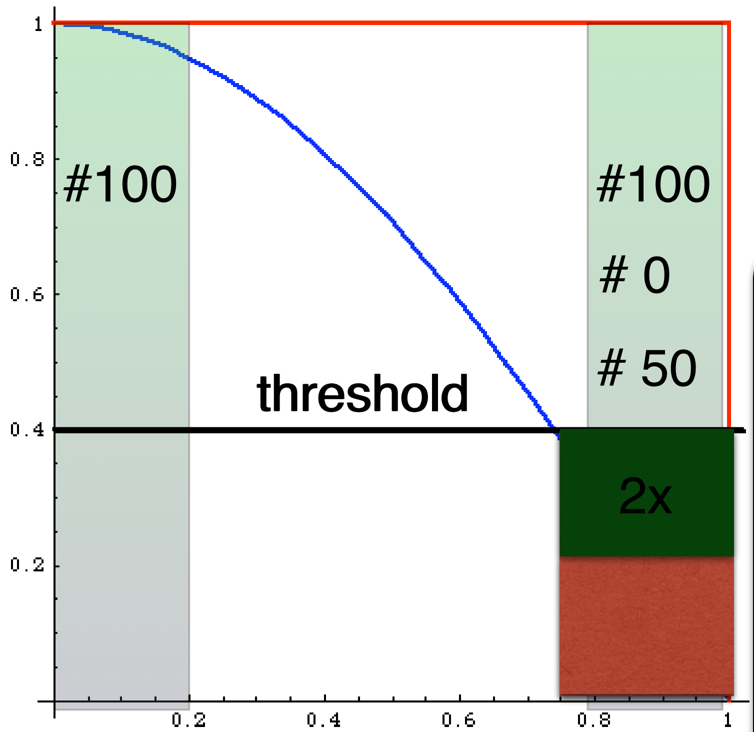


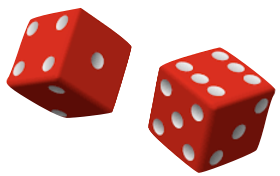


- Monte-Carlo integration use random points
  - We can keep those
    - (Uncorrelated) sample

- Points not distributed as the real function
- Need to keep track of the importance of each point (weight)
- Typically a lot of event have low information

Do we need to keep small weight?

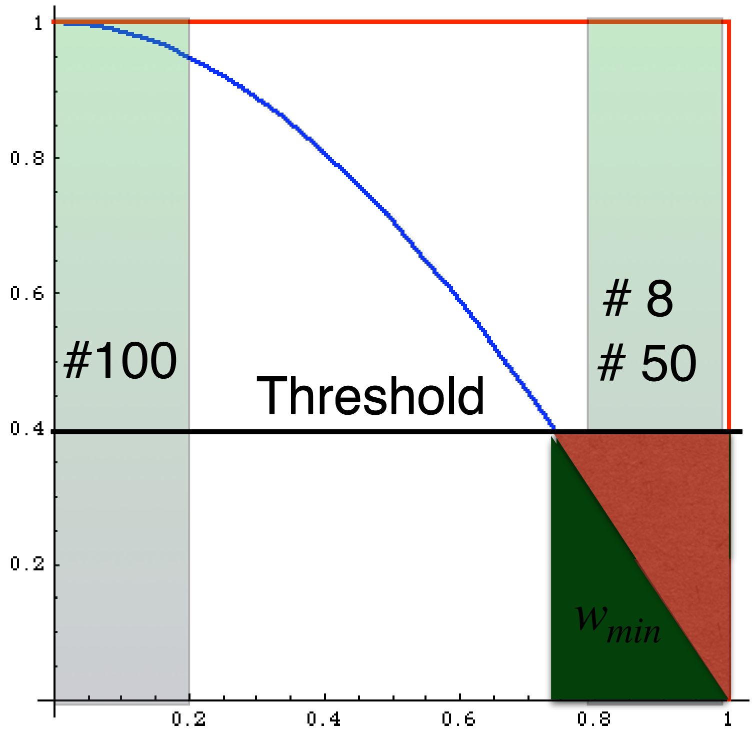




- Let's put a minimum
  - Discard events below the minimum
  - NO! We loose cross-section/ bias ourself

- Let's put a minimum
  - But keep 50% of the events below
  - Multiply the weight of each event by 2 (preserve cross-section)
  - We loose information
  - But we gain in file size

Do we need to keep small weight?

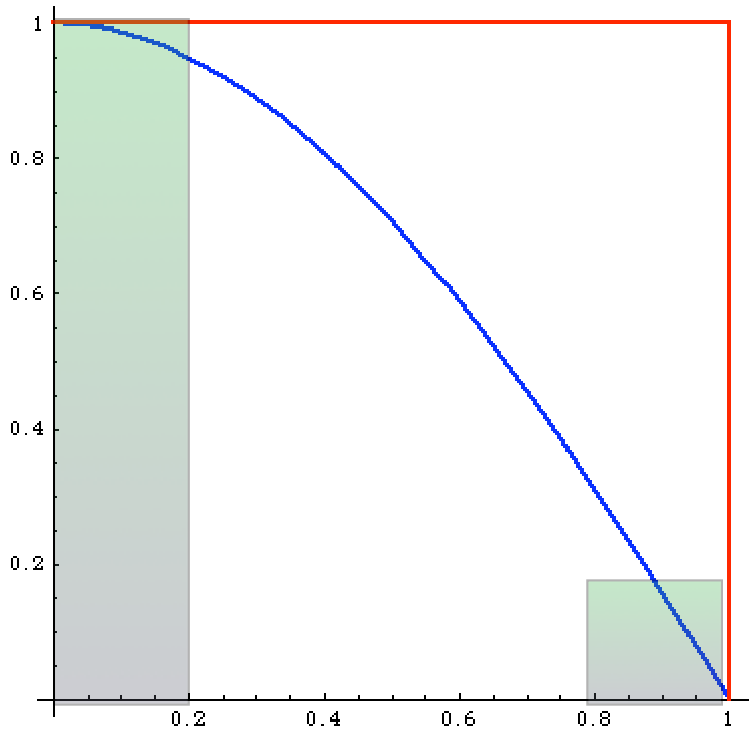


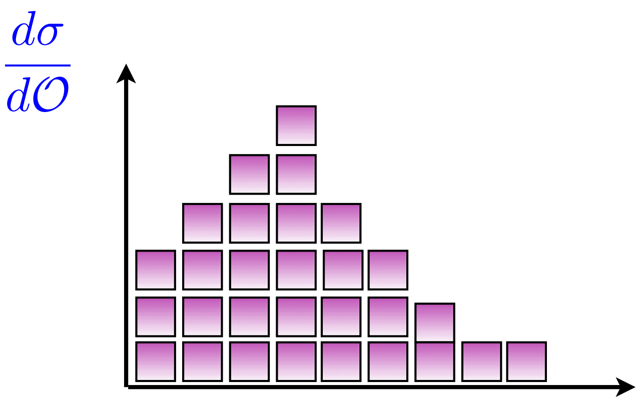
- Let's put a threshold
  - But keep 50% of the events below
  - Multiply the weight of each event by 2 (preserve cross-section)

- Let's improve
  - Let's make the threshold proportional to the weight
  - Keep each event with $\frac{100w}{w_{thres}}\%$ probability
  - If kept multiply his weight by $\frac{w_{thres}}{w}$
  - So the new weight is w_{thres}

Unweighted events

Events distributed as in nature

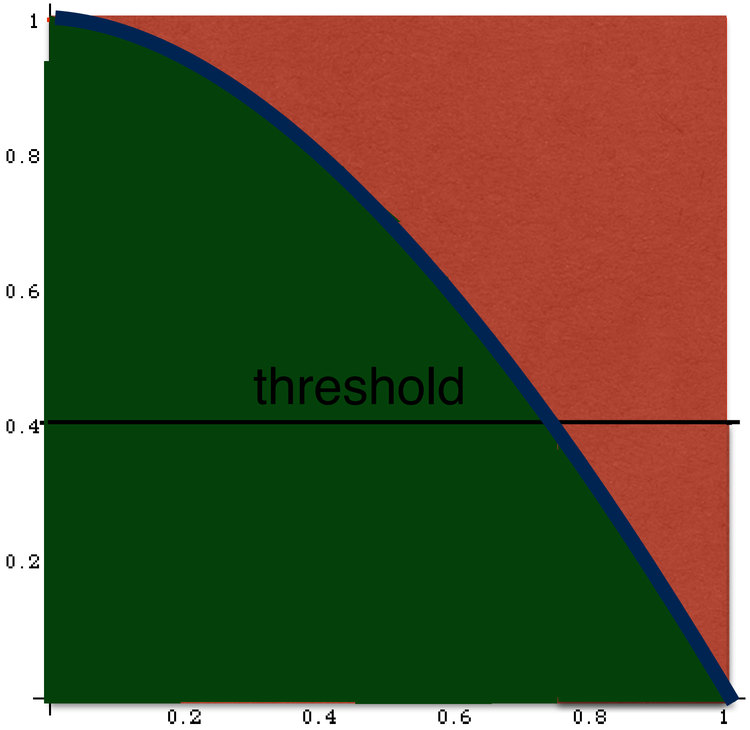




- All bins should event event proportional to their cross-section (Up to Poisson distribution)
- All events should have the same weight

- This correspond to the smallest file size or maximum compression

Do we need to keep small weight?

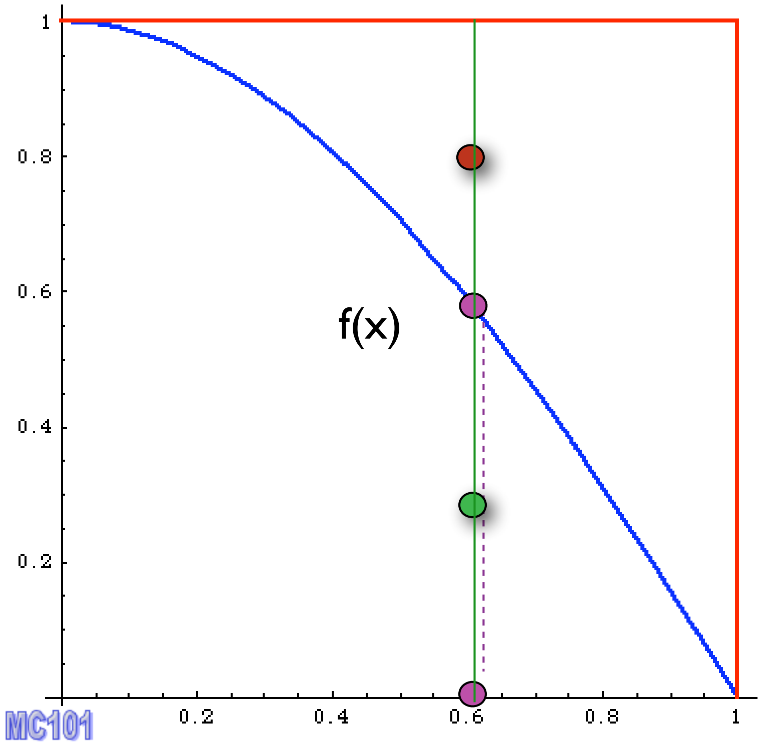


- Let’s improve
  - Let’s make the threshold proportional to the weight
  - So the new weight is w_{thres}

- Let’s all event have the same weight
  - So set $w_{thres} > \max(w)$
- Maximal compression

Event generation

$$\int f(x)dx = \frac{1}{N}\sum_{i=1}^{N} f(x_i) = \frac{1}{N}\sum_{i=1}^{N} \frac{f(x_i)}{w_{thres}} w_{thres}$$

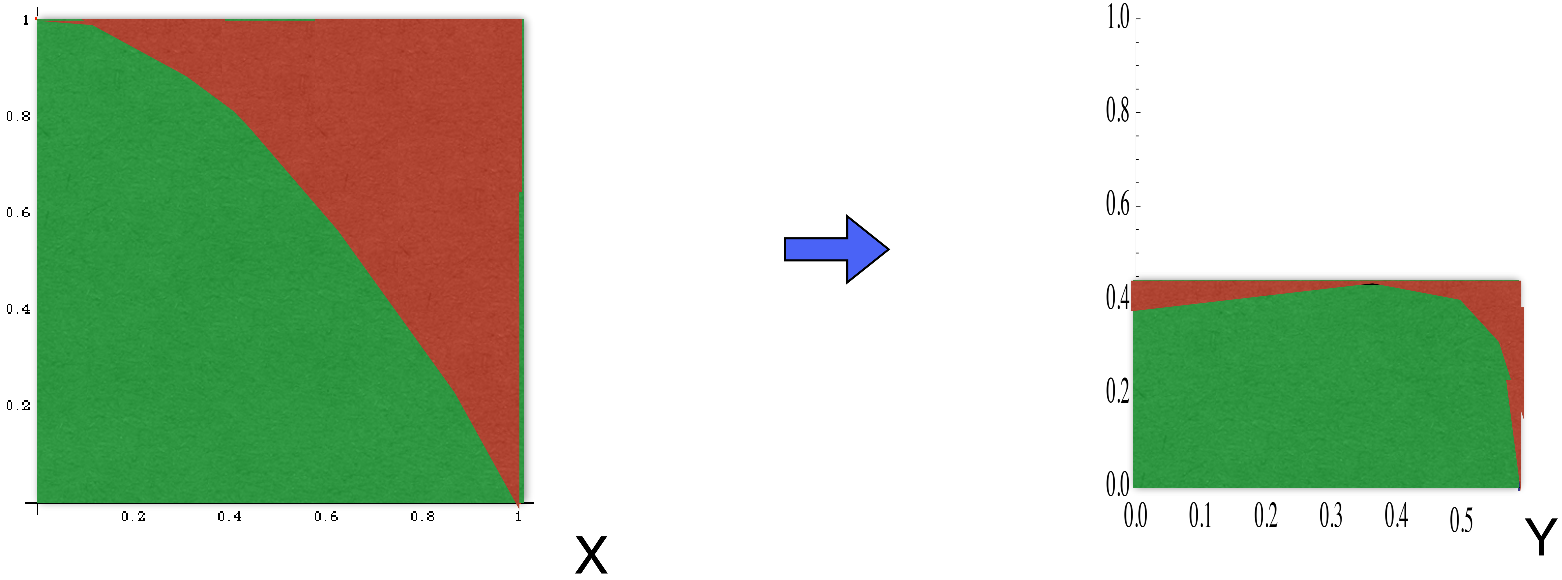


1. pick x_i
2. calculate $f(x_i)$
3. pick $y \in [0, max(f)]$
4. Compare:
   if $y < f(x_i)$ accept event,

   else reject it.

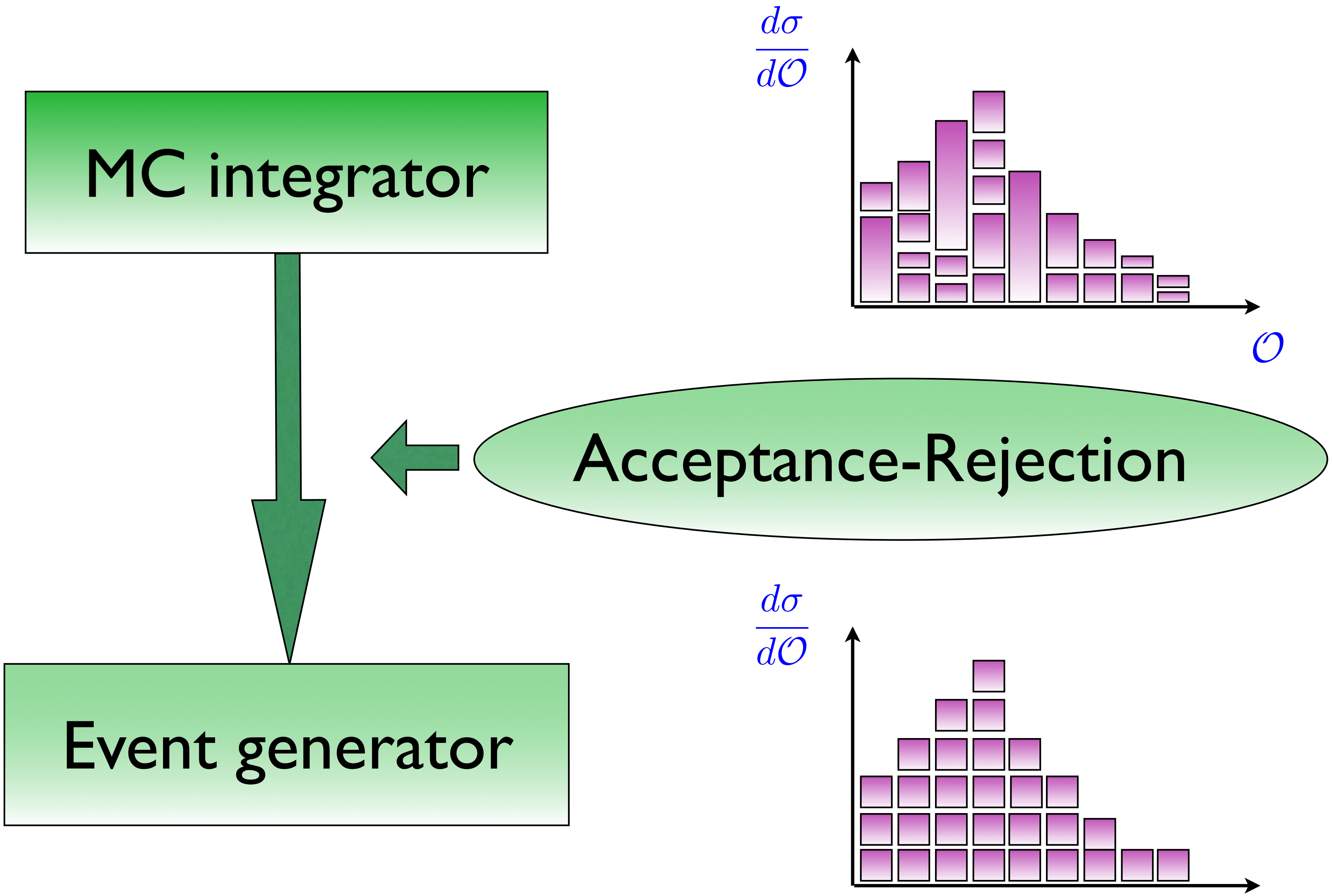
Event generation

$$\int f(x)dx = \int dy \frac{f(y)}{p(y)} = \frac{1}{N}\sum_{i=1}^{N} \frac{f(y_i)}{p(y_i)} = \frac{1}{N}\sum_{i=1}^{N} \frac{f(y_i)}{p(y_i)\ w_{thres}} w_{thres}$$



- Having smaller variance (flatter function) also allows to have $\frac{w}{w_{thres}}$ or $\frac{w}{max(w)}$ closer to one and therefore better unweighting efficiency (i.e. faster code)

Event generation



☞ **This is possible only if f(x)<∞ AND has definite sign!**

Monte-Carlo Summary

Bad Point

- Slow Convergence (especially in low number of Dimension)
- Need to know the function
    - Impact on cut

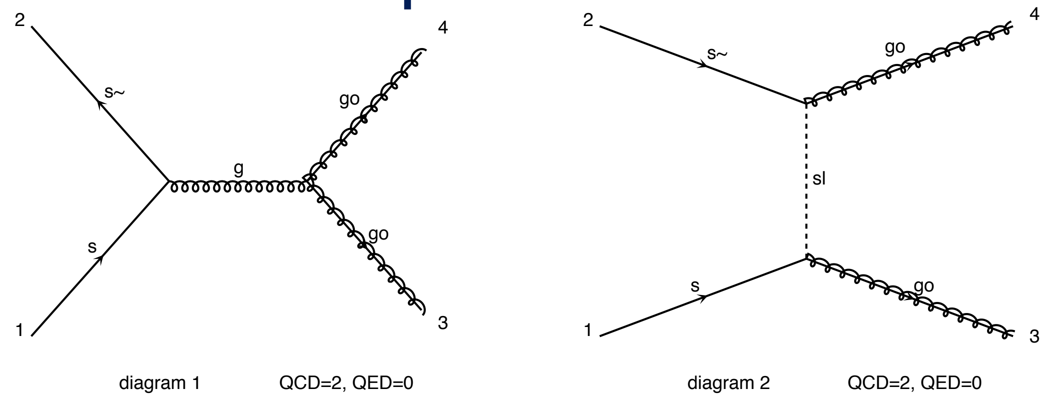
Good Point

- Complex area of Integration
- Easy error estimate
- quick estimation of the integral
- Possibility to have unweighted events

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



Easy enough

- Evaluate the matrix-element

$|\mathcal{M}|^2$ ➡Need Feynman Rules!

Hard

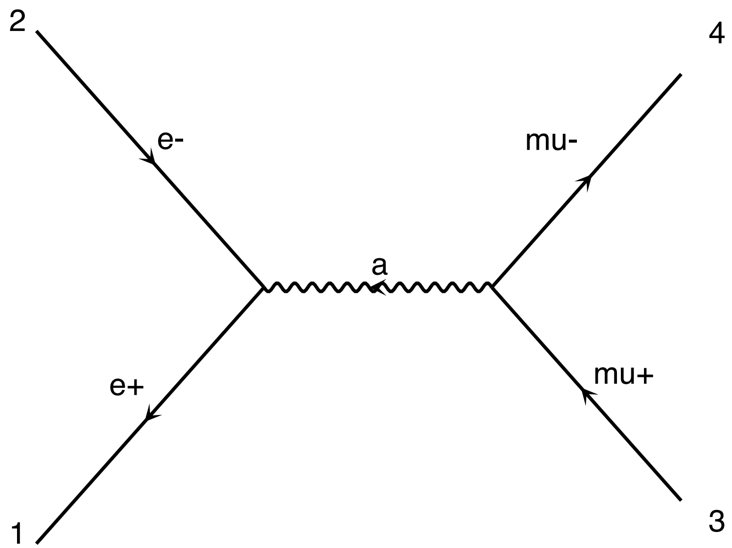
Now

- Phase-Space Integration

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

Very Hard (in general)

Matrix Element



$$\mathcal{M} = e^2 (\bar{u}\gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\gamma^\nu u)$$

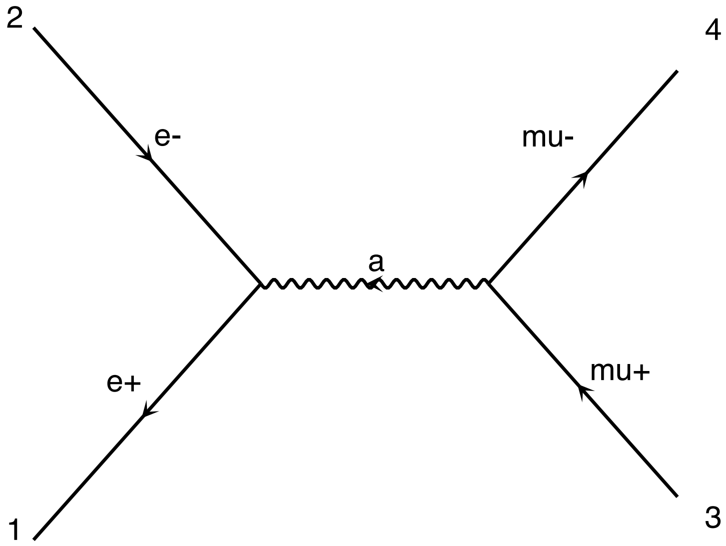
$$\frac{1}{4} \sum_{pol} |\mathcal{M}|^2 = \frac{1}{4} \sum_{pol} \mathcal{M}^* \mathcal{M}$$

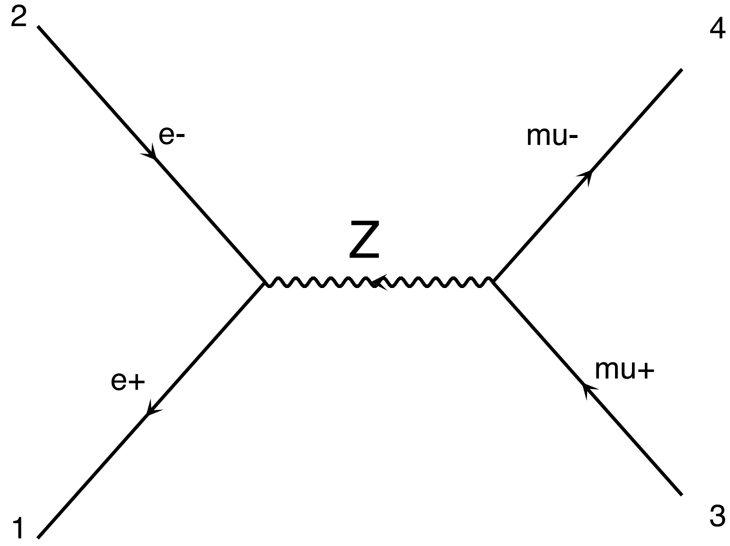
$$\sum_{pol} \bar{u}u = \not{p} + m$$

$$\rightarrow \frac{e^4}{4q^4} Tr[\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu] Tr[\not{p}_3 \gamma_\mu \not{p}_4 \gamma_\nu]$$

$$\rightarrow \frac{8e^4}{q^4} [(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3)]$$

Very Efficient !!!





Need to compute $|M_a|^2$ $|M_z|^2$ $2Re(M_a^* M_z)$

So for M Feynman diagram we need to compute M^2 different term

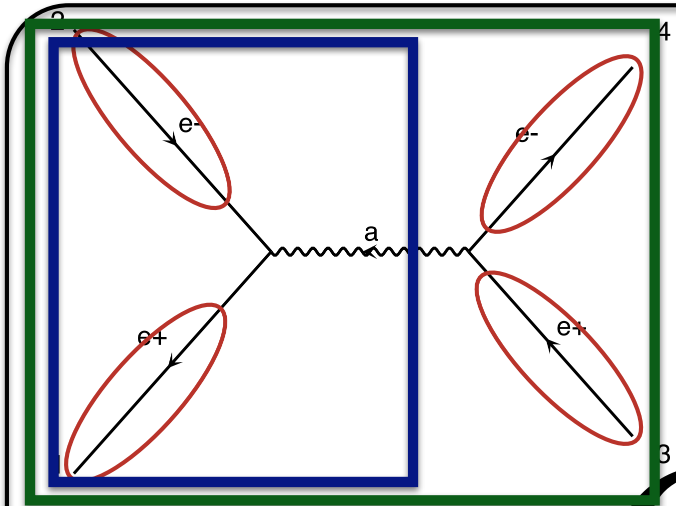
The number of diagram scales factorially with the number of particle

In practise possible up to 2>4

Helicity Amplitude

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡Loop on Helicity and average the results



$$\mathcal{M} = ((\bar{u} e \gamma^\mu v)\frac{g_{\mu\nu}}{q^2})(\bar{v} e \gamma^\nu u))$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

Finally evaluate amplitude (c-number)

Lines present in the code.

$$\bar{v}_1 = fct(\vec{p}_1, m_1)$$
$$u_2 = fct(\vec{p}_2, m_2)$$
$$v_3 = fct(\vec{p}_3, m_3)$$
$$\bar{u}_4 = fct(\vec{p}_4, m_4)$$
$$W_a = fct(\bar{v}_1, u_2, e, m_a, \Gamma_a) = e\bar{v}_1\gamma^\mu u_2 \frac{g_{\mu\nu}}{q^2 - m_a^2 + i m_a \Gamma_a}$$
$$\mathcal{M} = fct(\bar{v}_3, u_4, W^a_\nu, e) = e\bar{v}_3\gamma_\nu \bar{u}_4 W^a_\nu$$

$$u(p) = \begin{pmatrix} \omega_{-\lambda}(p)\chi_\lambda(\vec{p}) \\ \omega_\lambda(p)\chi_\lambda(\vec{p}) \end{pmatrix}$$

$$\omega_\pm(p) \equiv \sqrt{E \pm |\vec{p}|}.$$

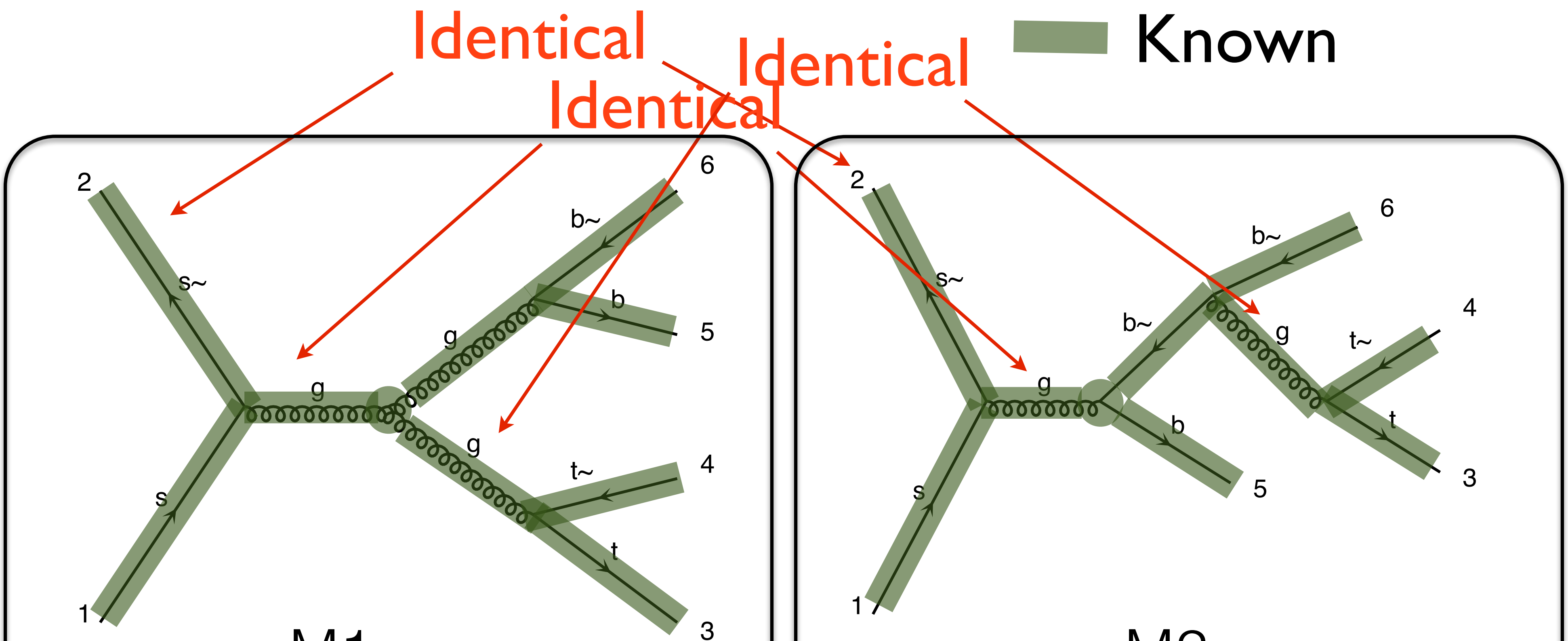
$$\chi_+(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}}\begin{pmatrix} |\vec{p}|+p_z \\ p_x + ip_y \end{pmatrix},$$

$$\chi_-(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}}\begin{pmatrix} -p_x + ip_y \\ |\vec{p}|+p_z \end{pmatrix}.$$

Comparison

	M diag	N particle
Analytical	M^2	$(N!)^2$
Helicity	M	$(N!)\, 2^N$

Real case



M1

Number of routines: [illegible]0

2(N+1)

M2

Number of routines: [illegible]0

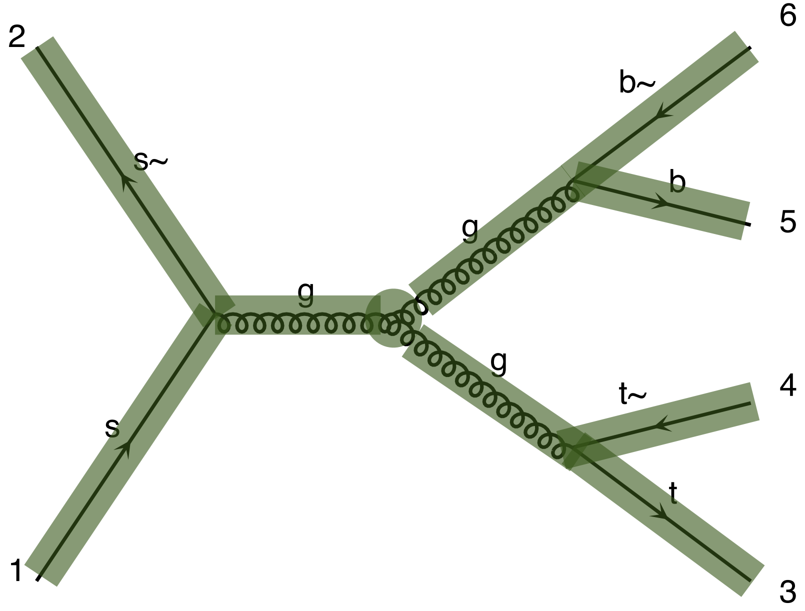
2(N+1)

Number of routines for both: [illegible]

$$|M|^2 = |M_1 + M_2|^2$$

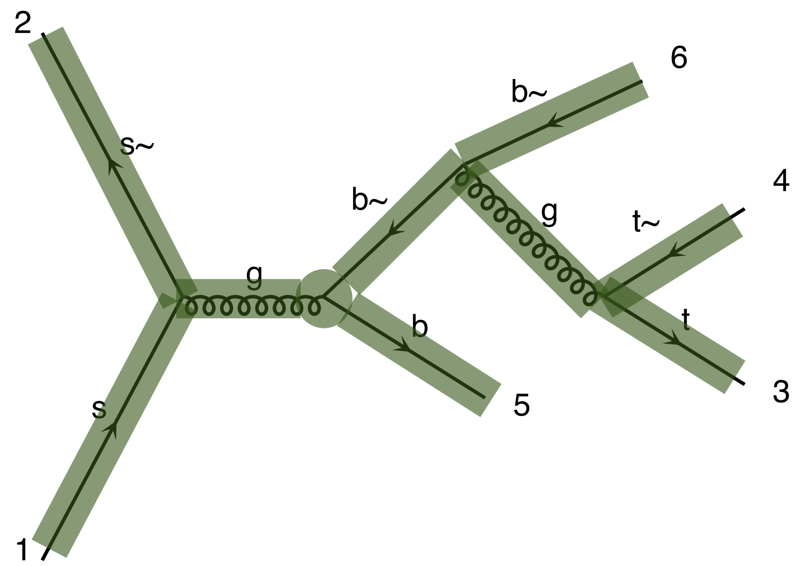
Real case

Known



Number of routines: 10

2(N+1)



Number of routines: 10

2(N+1)

Number of routines for both: 12

N!*2(N+1) ⟶ N!

Comparison

	M diag	N particle
Analytical	M^2	$(N!)^2$
Helicity	M	$(N!)\,2^N$
Recycling	M	$(N-1)!\,2^{(N-1)}$

Color handling

- Can we de the same for colour
  - ➡ Fixed color for final state
  - ➡ Loop over them
- No coherent sum for colour
  - ➡ $gg \to (n-2)g$ scales like 8^{2n}

N	Fixed colour	$((n-1)!)^2$
4	16777216	36
5	1073741824	576
6	68719476736	14400
7	4398046511104	518400
8	281474976710656	25401600

All gluon solution

- Decompose the QCD amplitude on an basis

➡ $$\mathcal{M}(ng) = \sum_{P(2,\ldots,n)} \mathrm{Tr}(t^1 \ldots t^n)\, M(1,\ldots,n) \equiv \sum_{\sigma} F_\sigma M_\sigma$$

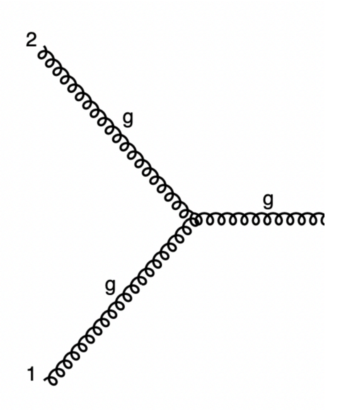
➡ Where the sum is over the permutation of index with $(n-1)!$ term

- Amplitude square is then

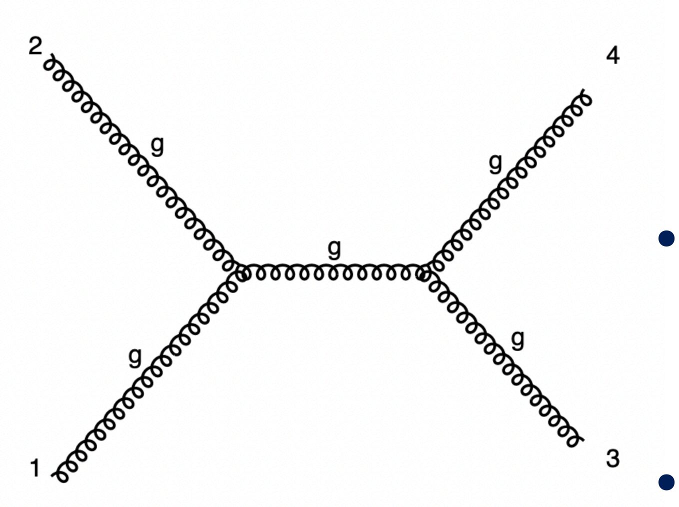
$$|\mathcal{M}(1,\ldots,n)|^2 = \sum_{\sigma,\sigma'} M_\sigma \underbrace{F_\sigma F^*_{\sigma'}}_{C_{\sigma\sigma'}} M^*_{\sigma'},$$

- $C_{\sigma\sigma'}$ is called the colour matrix

Example



$$f^{c_1c_2c_3}\,(P_1^{\mu_3}\eta_{\mu_1\mu_2} - P_2^{\mu_3}\eta_{\mu_1\mu_2} - P_1^{\mu_2}\eta_{\mu_1\mu_3} + P_3^{\mu_2}\eta_{\mu_1\mu_3} + P_2^{\mu_1}\eta_{\mu_2\mu_3} - P_3^{\mu_1}\eta_{\mu_2\mu_3})$$



$$\mathcal{M} = f^{c_1c_2\lambda} f_{\lambda}^{\ c_3c_4} M$$

- M is the amplitude stripped of colour
  - ➡ Computed like in QED
- We can use colour algebra to compute which term are contributing

$$f^{c_1c_2\lambda} f^{\lambda c_3c_4} \equiv_{c_\sigma} \sum_{\sigma} c_\sigma \mathrm{Tr}(t_1^\sigma t_2^\sigma t_3^\sigma t_4^\sigma)$$

Can we do better? YES

- Recursion relation (used in Sherpa) [WIP]
- New in MG5aMC: Helicity Recycling 2102.00773
- 5 Dimensional helicity wave function 2203.10440
- Not full color computation [2210.07267]

	M diag	N particle
Analytical	M^2	$(N!)^2$
Helicity	M	$(N!)\,2^N$
Recycling	M	$(N-1)!\,2^{(N-1)}$
Hel Recycling	M	$\approx (N-1)!2^{N/2}$

• 2102.00773 [hep-ph]

Can we go faster? YES

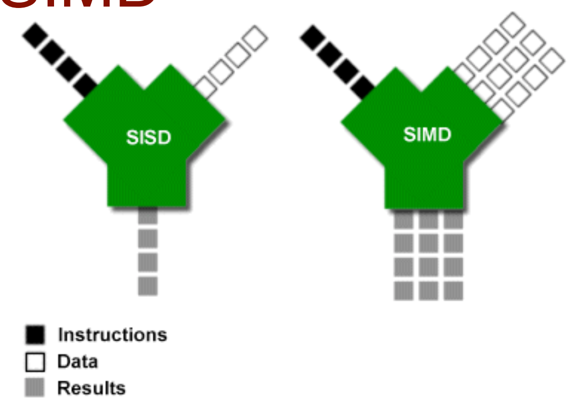
GPU

- ➡ Was first done a while ago (cuda)

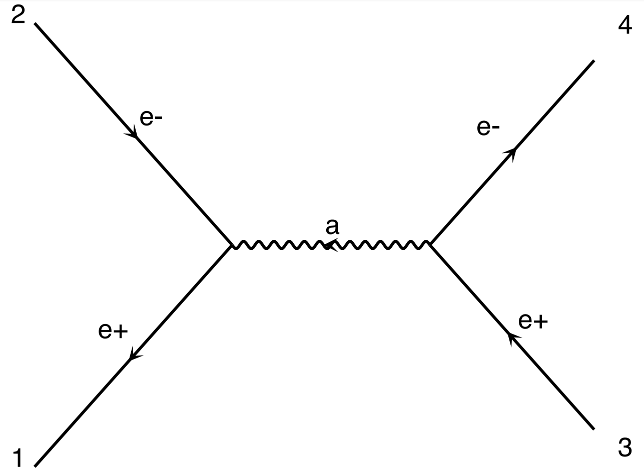
  arXiv:0908.4403,arXiv:1305.0708v2

- ➡ New recent focus in this direction
  - ➡Not only cuda:
  - ➡Kokkos, syCL, tensorflow
- ➡ Good performance but not yet integrated with the phase-space

SIMD

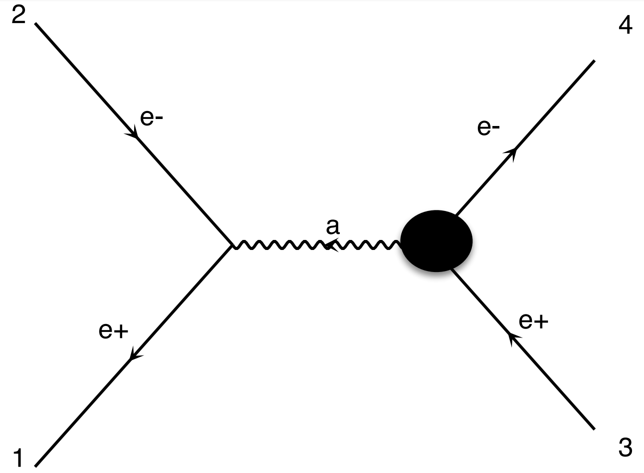


- ➡ Modern CPU can act as a baby GPU
- ➡ They can perform N identical operation as fast as one
- ➡ Close to be released



$$\mathcal{M} = \bar{u}\gamma^{\mu} v P_{\mu\nu} \bar{u}\gamma^{\nu} v$$

$$\bar{u}_1 = fct(\vec{p}_1, m)$$
$$v_2 = fct(\vec{p}_2, m)$$
$$\bar{u}_3 = fct(\vec{p}_3, m)$$
$$v_4 = fct(\vec{p}_4, m)$$
$$W_a = fct(\bar{u}_1, v_2, M_a, \Gamma_a)$$
$$\mathcal{M} = fct(\bar{u}_3, v_4, W_a)$$



$$\mathcal{M} = \bar{u}\gamma^{\mu} v P_{\mu\nu} \bar{u}\Gamma^{\mu} v$$

$$\bar{u}_1 = fct(\vec{p}_1, m)$$
$$v_2 = fct(\vec{p}_2, m)$$
$$\bar{u}_3 = fct(\vec{p}_3, m)$$
$$v_4 = fct(\vec{p}_4, m)$$
$$W_a = fct(\bar{u}_1, v_2, M_a, \Gamma_a)$$
$$\mathcal{M} = fct(\bar{u}_3, v_4, W_a)$$

BSM and HELAS

- **Original HELicity Amplitude Subroutine library** [Murayama, Watanabe, Hagiwara]
- **One routine by Lorentz structure**
  - ➡**MSSM** [cho, al] hep-ph/0601063 (2006)
  - ➡**HEFT** [Frederix] (2007)
  - ➡**Spin 2** [Hagiwara, al] 0805.2554 (2008)
  - ➡**Spin 3/2** [Mawatari, al] 1101.1289 (2011)

Chiral Perturbation

BNV Model

SLIH

Effective Field Theory

NMSSM

Chromo-magnetic operator

Full HEFT

Black Holes

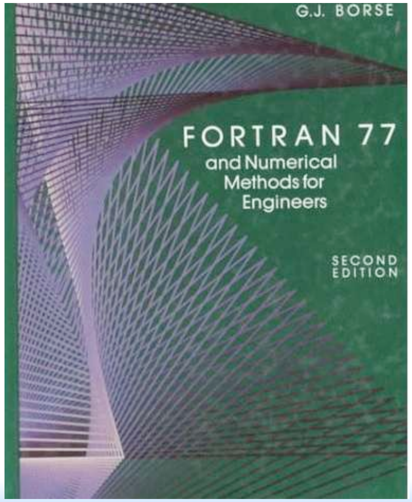
ALOHA

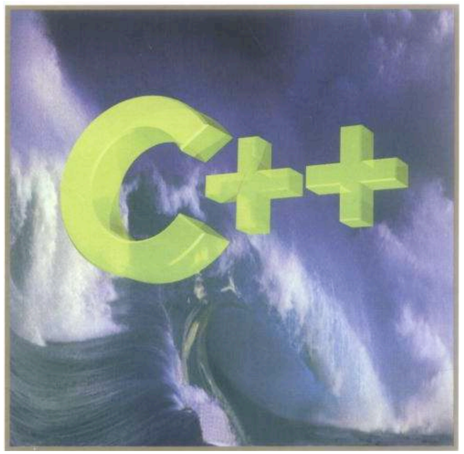
ALOHA
~~Google~~ translate

From: UFO To: Helicity Translate

Basically, any new operator can be handle by MG5/Pythia8 out of the box!

Type text or a website address or translate a document.



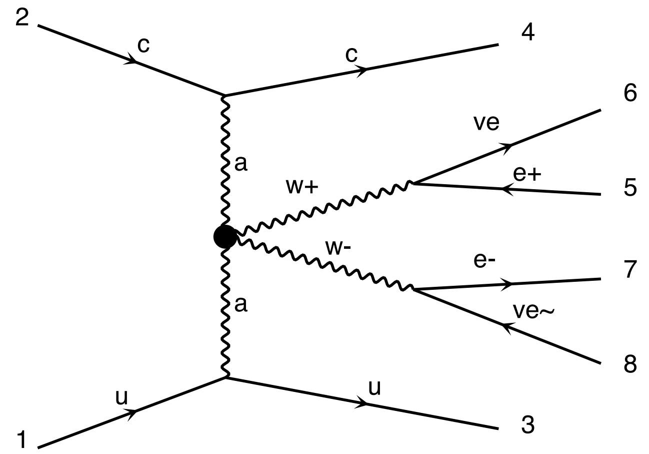


To Remember

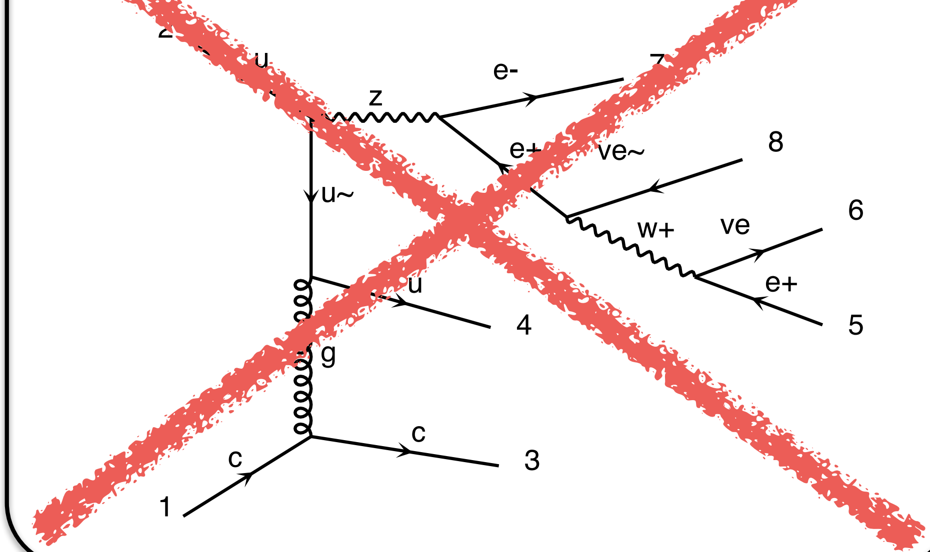
- Numerical computation faster than analytical computation
- We are able to compute matrix-elment
  - ➡ for large number of final state
  - ➡ for any BSM theory
  - ➡ actually also for loop

Decay

Resonant Diagram



Non Resonant Diagram



Problem

- Process complicated to have the full process

➡Including off-shell contribution

Solution

- Only keep on-shell contribution

Narrow-Width Approx.

Theory

$$\int dq^2 \left| \frac{1}{q^2 - M^2 + iM\Gamma} \right|^2 \approx \frac{\pi}{M\Gamma}\ \delta(q^2 - M^2)$$

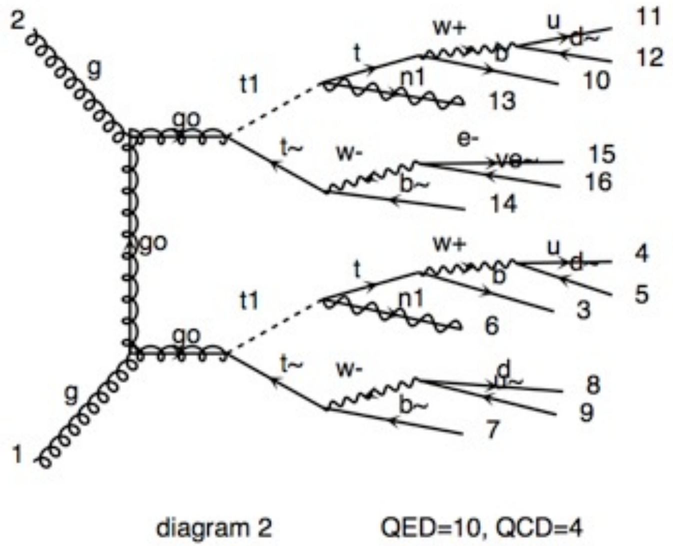
$$\sigma_{full} = \sigma_{prod} * (BR + \mathcal{O}(\frac{\Gamma}{M}))$$

Comment

- This is an Approximation!
- This force the particle to be on-shell!
  - Recover by re-introducing the Breit-wigner up-to a cut-off

Decay chain

- p p > t t~ w+, (t > w+ b, w+ > l+ vl), \
  (t~ > w- b~, w- > j j), \
  w+ > l+ vl



very long
decay chains possible to simulate
directly in MadGraph!

- This syntax has an invariant mass cut associated to t/t~/W
- Other syntax/tools exists for NWA (like MadSpin)

To Remember

- We do assume factorisation into different scale
- Perturbative theory
  - ➡ LO good for shape
  - ➡ Higher order good for cross-section
- We are able to compute matrix-element
  - ➡ for any BSM theory
  - ➡ Also for loop
  - ➡ Not fast enough (we need your help)
- Loop computation need dedicated model

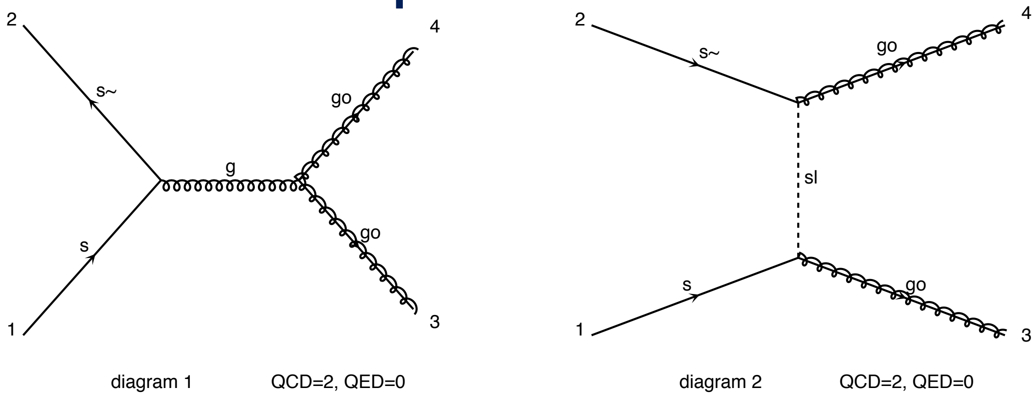
What to remember

- Analytical computation can be slower than numerical method
- Any BSM model are supported (at LO)
- Phase Space integration are slow
  - need knowledge of the function
  - cuts can be problematic
- Event generation are from free.
- All this are automated in MG5_aMC@NLO
- Important to know the physical hypothesis

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



- Evaluate the matrix-element

$|\mathcal{M}|^2$ ➡Need Feynman Rules!

- Phase-Space Integration

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

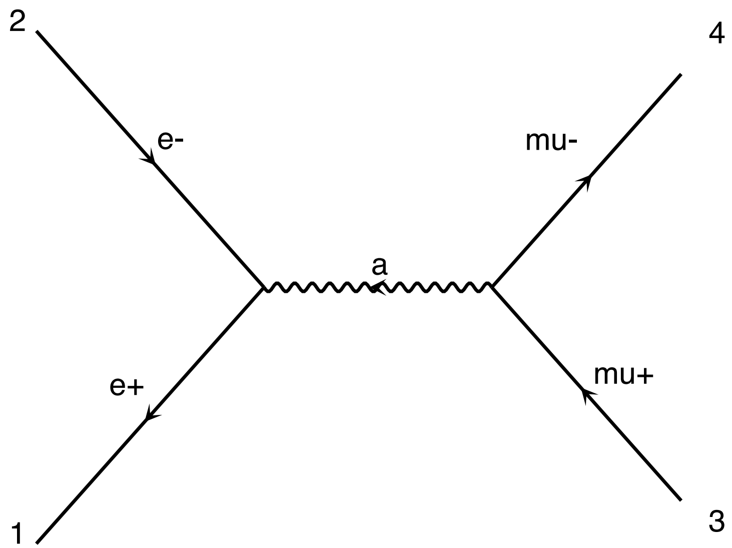
Easy enough

Hard

Tommorow

Very Hard
(in general)

Matrix Element



$$\mathcal{M} = e^2 (\bar{u}\gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v}\gamma^\nu u)$$

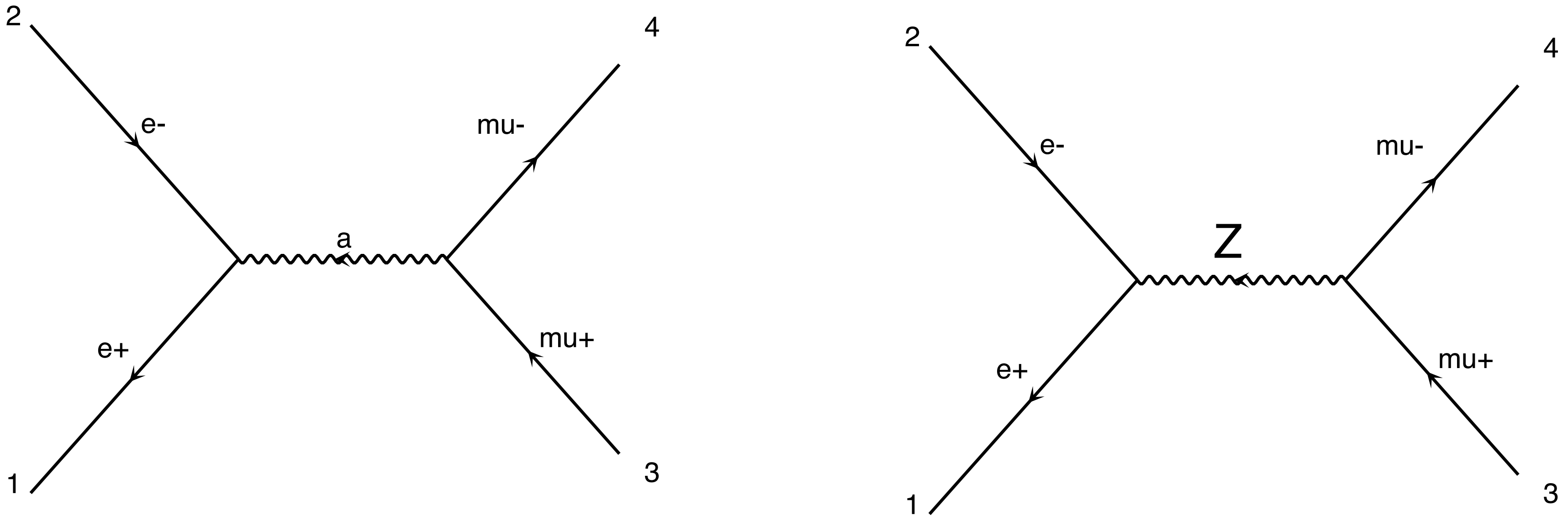
$$\frac{1}{4}\sum_{pol} |\mathcal{M}|^2 = \frac{1}{4}\sum_{pol} \mathcal{M}^*\mathcal{M}$$

$$\sum_{pol} u\bar{u} = \not{p} + m$$

$$\rightarrow \frac{e^4}{4q^4} Tr[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu] Tr[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu]$$

$$\rightarrow \frac{8e^4}{q^4} [(p_1.p_3)(p_2.p_4) + (p_1.p_4)(p_2.p_3)]$$

Very Efficient !!!



Need to compute $|M_a|^2 \quad |M_z|^2 \quad 2Re(M_a^* M_z)$

So for M Feynman diagram we need to compute M^2 different term

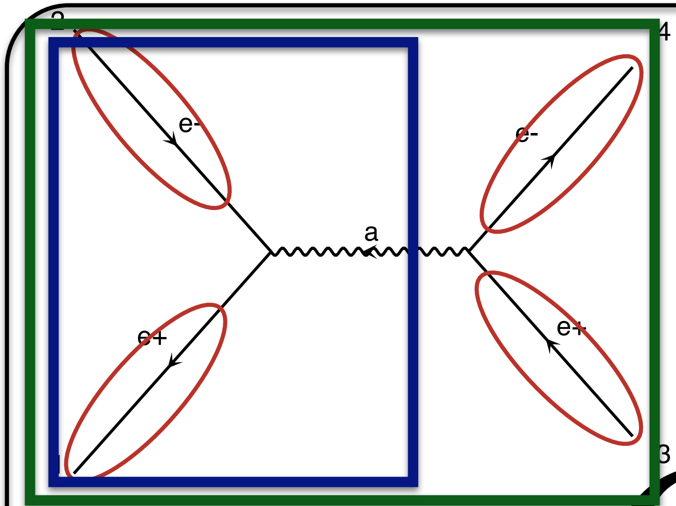
The number of diagram scales factorially with the number of particle

In practise possible up to 2>4

Helicity Amplitude

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> $|\mathcal{M}|$^2
  - ➡Loop on Helicity and average the results



$$\mathcal{M} = ((\bar{u}e\gamma^\mu v)\frac{g_{\mu\nu}}{q^2})(\bar{v}e\gamma^\nu u))$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

Finally evaluate amplitude (c-number)

Lines present in the code.

$$\bar{v}_1 = fct(\vec{p}_1, m_1)$$
$$u_2 = fct(\vec{p}_2, m_2)$$
$$v_3 = fct(\vec{p}_3, m_3)$$
$$\bar{u}_4 = fct(\vec{p}_4, m_4)$$
$$W_a = fct(\bar{v}_1, u_2, e, m_a, \Gamma_a) = e\bar{v}_1\gamma^\mu u_2 \frac{\eta_{\mu\nu}}{q^2 - m_a^2 + i m_a \Gamma_a}$$
$$\mathcal{M} = fct(\bar{v}_3, u_4, W_\nu^a, e) = e\bar{v}_3\gamma_\nu \bar{u}_4 W_\nu^a$$

$$u(p) = \begin{pmatrix} \omega_{-\lambda}(p)\chi_\lambda(\vec{p}) \\ \omega_\lambda(p)\chi_\lambda(\vec{p}) \end{pmatrix}$$

$$\omega_\pm(p) \equiv \sqrt{E \pm |\vec{p}|}.$$

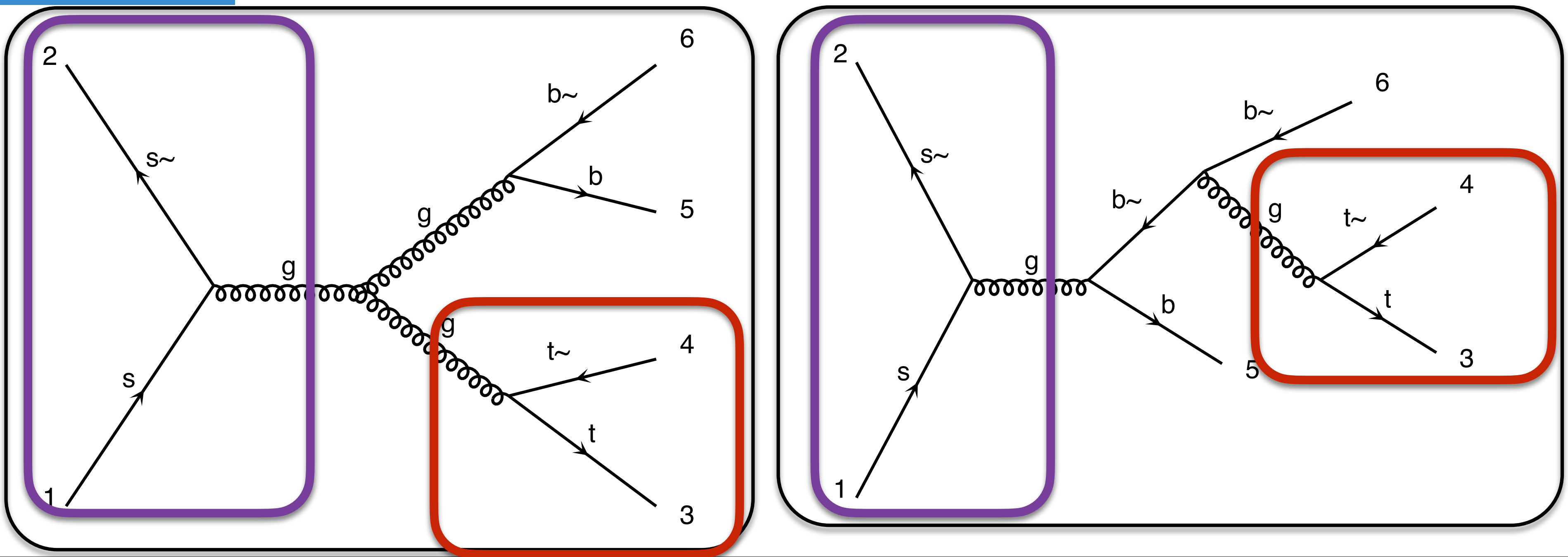
$$\chi_+(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}}\begin{pmatrix} |\vec{p}|+p_z \\ p_x + ip_y \end{pmatrix},$$
$$\chi_-(\vec{p}) = \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}|+p_z)}}\begin{pmatrix} -p_x + ip_y \\ |\vec{p}|+p_z \end{pmatrix}.$$

Comparison

	M diag	N particle
Analytical	M^2	$(N!)^2$
Helicity	M	$(N!)\,2^N$

Comparison

	M diag	N particle
Analytical	M^2	$(N!)^2$
Helicity	M	$(N!)\,2^N$
Recycling	M	$(N-1)!\,2^{(N-1)}$



Can we do better? YES

- Recursion relation (used in Sherpa) [WIP]
- New in MG5aMC: Helicity Recycling 2102.00773
- 5 Dimensional helicity wave function 2203.10440
- Not full color computation [WIP]

	M diag	N particle
Analytical	M^2	$(N!)^2$
Helicity	M	$(N!)\,2^N$
Recycling	M	$(N-1)!\,2^{(N-1)}$
Hel Recycling	M	$\approx (N-1)!2^{N/2}$

- 2102.00773 [hep-ph]

Can we go faster? YES

GPU

➡ Was first done a while ago (cuda)

arXiv:0908.4403,arXiv:1305.0708v2

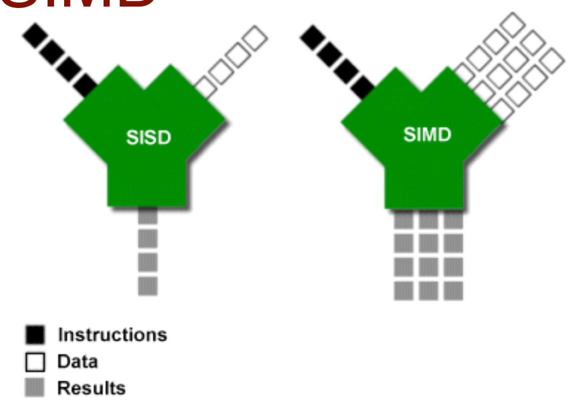
➡ New recent focus in this direction

➡Not only cuda:

➡Kokkos, syCL, tensorflow

➡ Good performance but not yet integrated with the phase-space

SIMD



➡Modern CPU can act as a baby GPU

➡They can perform N identical operation as fast as one

To Remember

- Numerical computation faster than analytical computation
- We are able to compute matrix-element
  - ➡ for large number of final state
  - ➡ for any BSM theory
- Computing the matrix-element is slow
  - ➡ We are still looking for new idea
    - ❑ Physics idea
    - ❑ Better hardware implementation

Intuition for matching and merging

Parton shower

Goal

- We want to an **explicit** description of the SOFT radiation that are **ALREADY** included **implicitly in the LO events** (via the scale)

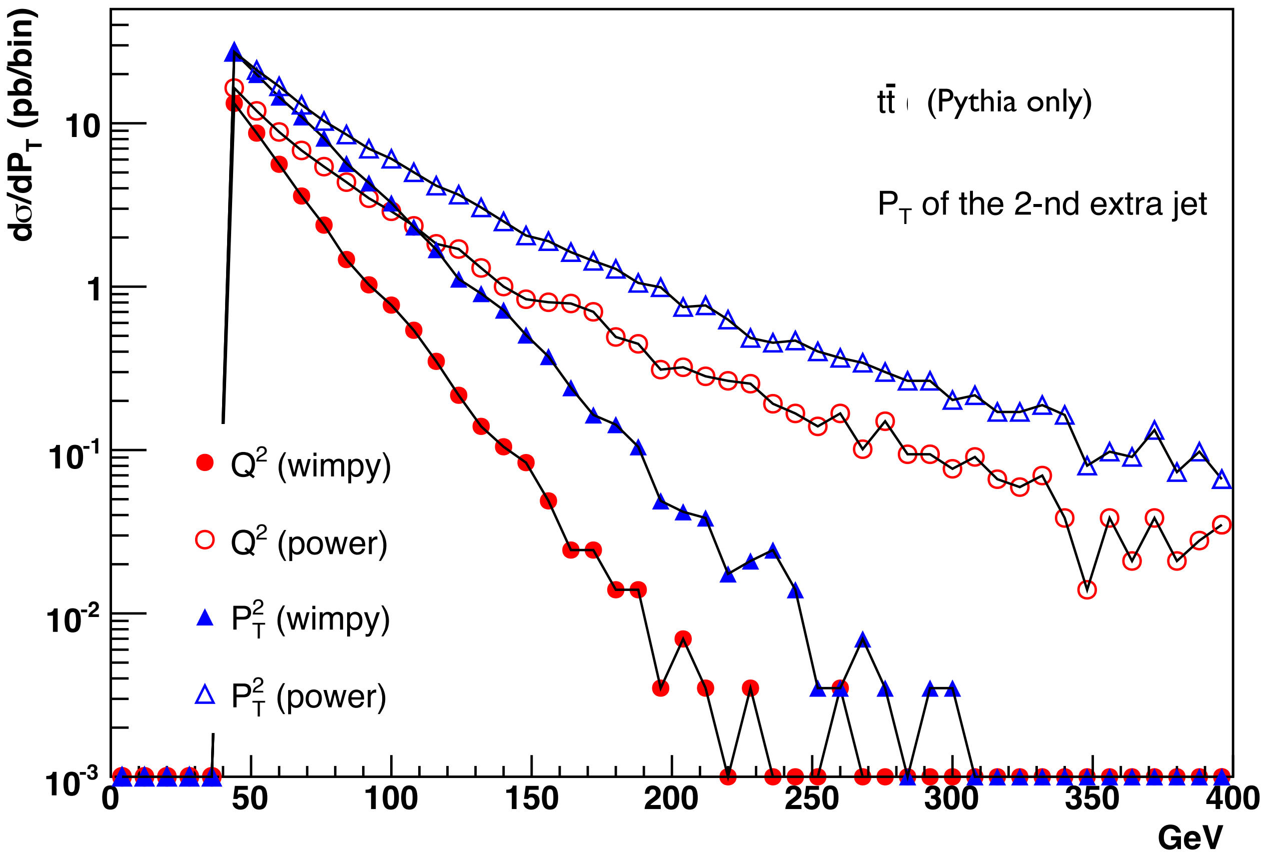
Important

- Parton-Shower is **not ADDING radiation**
  - Such radiations are already included within the event-generator

- This effect should be **unitary:** the inclusive cross section shouldn't change when extra radiation is added

PS alone vs matched samples

In the soft-collinear approximation of Parton Shower MCs, parameters are used to tune the result ⇒ Large variation in results (small prediction power)



Matrix Elements vs. Parton Showers

ME

1. Fixed order calculation
2. Computationally expensive
3. Limited number of particles
4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description

Shower MC

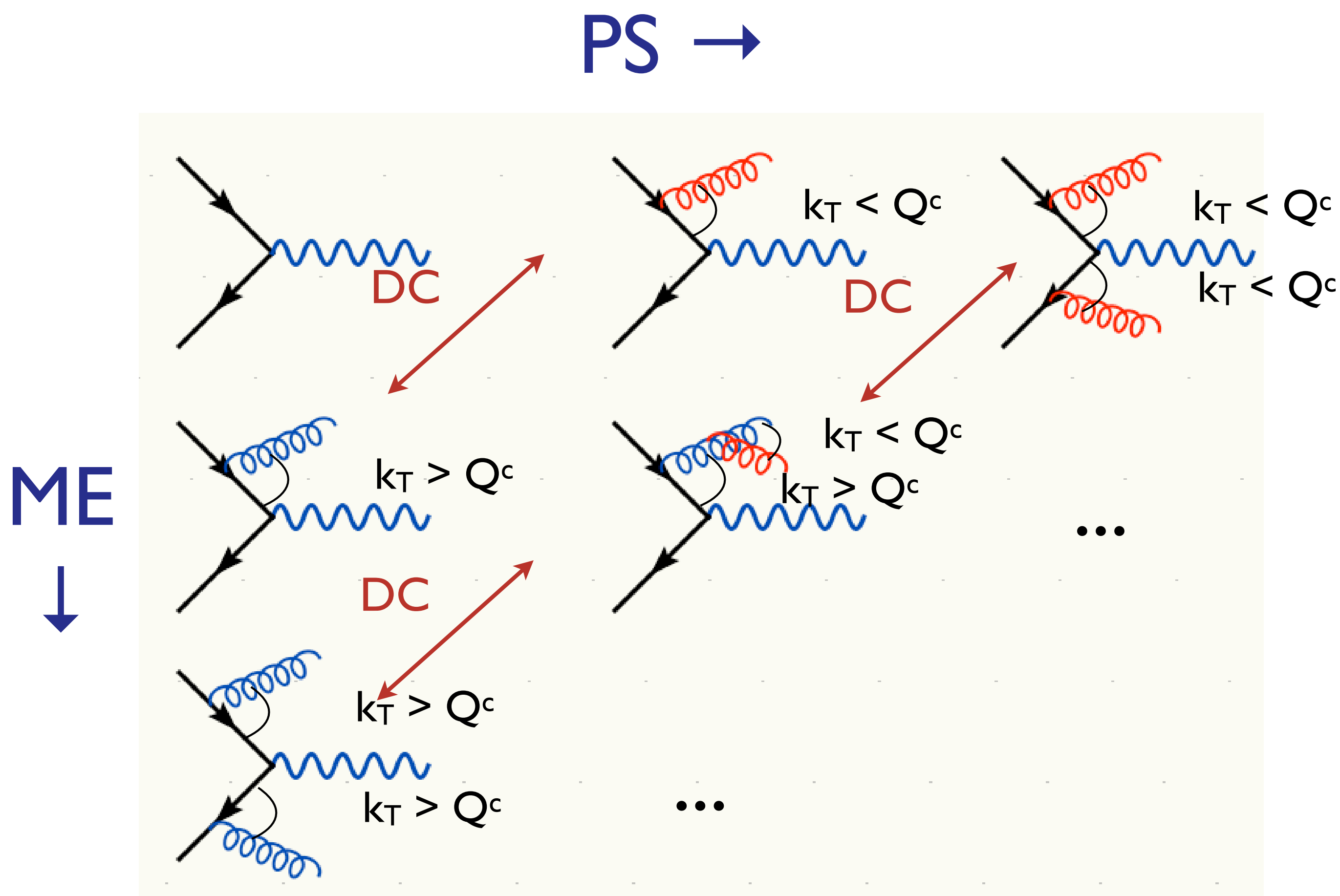
1. Resums logs to all orders
2. Computationally cheap
3. No limit on particle multiplicity
4. Valid when partons are collinear and/or soft
5. Partial interference through angular ordering
6. Needed for hadronization

Approaches are complementary: merge them!

Difficulty: avoid double counting, ensure smooth distributions

Merging ME with PS

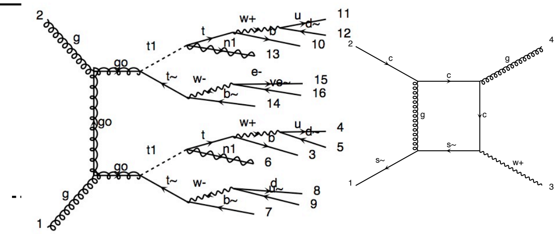
[Mangano]
[Catani, Krauss, Kuhn, Webber]
[Lönnblad]

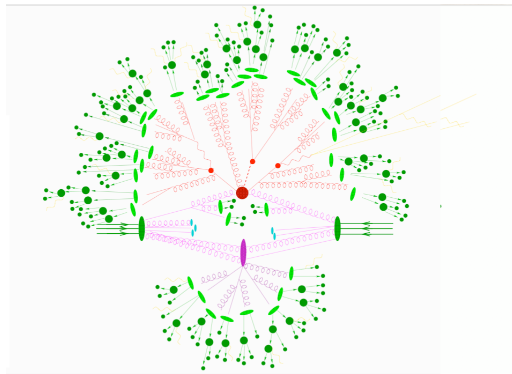


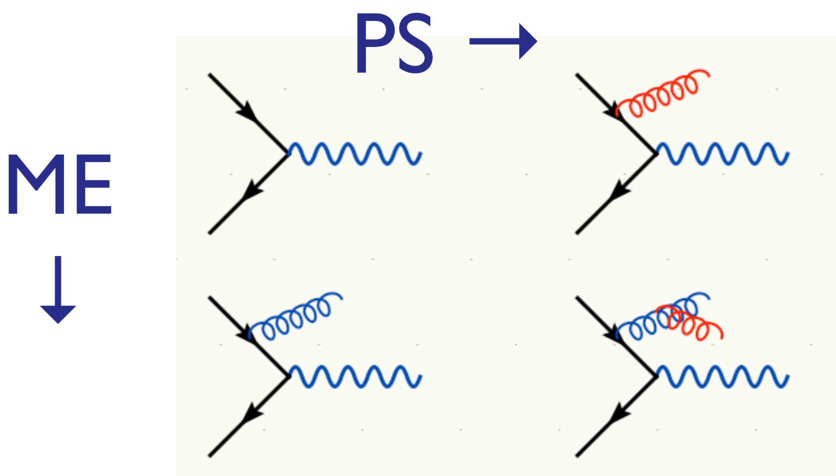
Double counting between ME and PS easily avoided using phase space cut between the two: PS below cutoff, ME above cutoff.

Type of generation

	Tree (B)SM	NLO (QCD) (SM)	NLO (QCD) (BSM)	NLO (EW) (SM)	Loop Induced (B)SM
Fix Order	✓	✓	✓	✓	✓





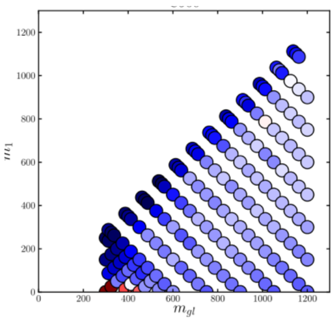


LO Feature

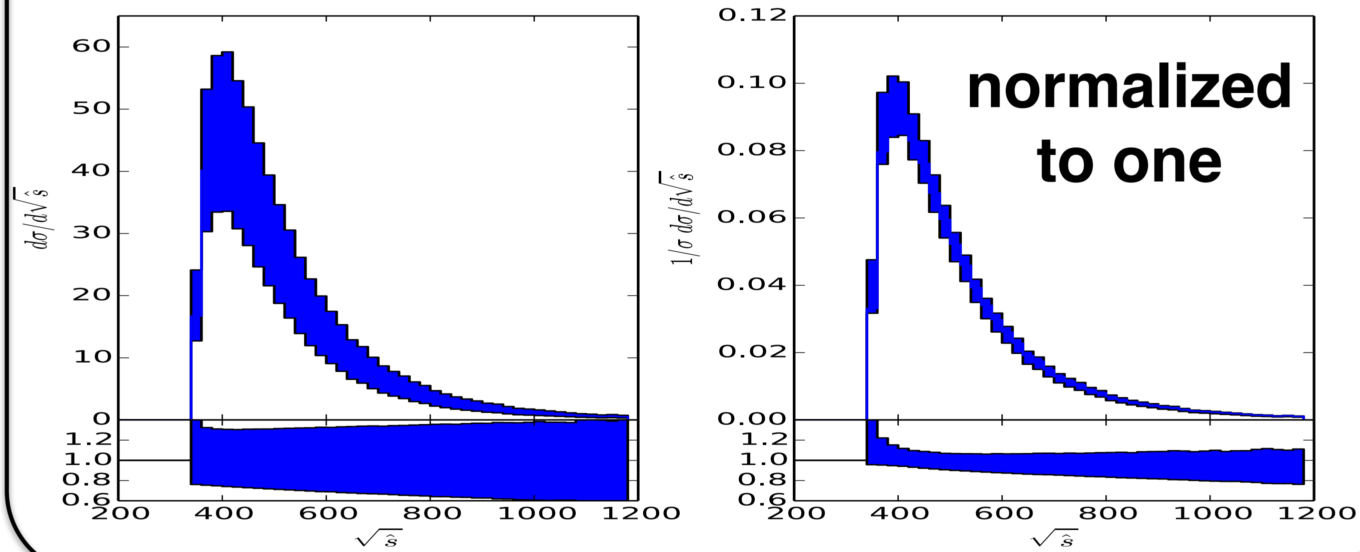
Auto-Width

$$\Gamma = ?$$

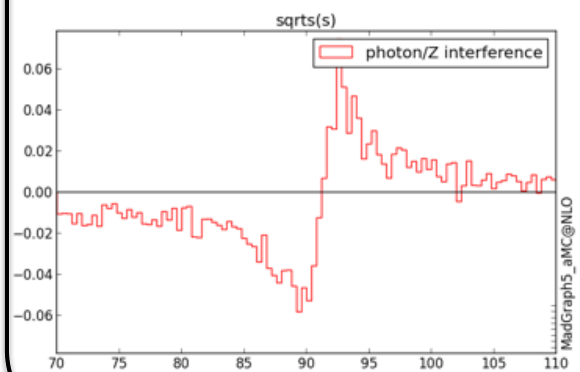
Parameter scan



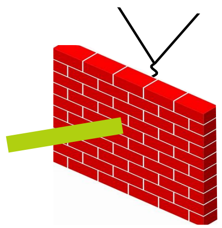
Systematics



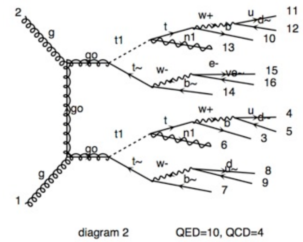
Interference



Plugin



Narrow-width



Interface

MadAnalysis 5

BSM re-weighting

$$|M_{new}|^2 / |M_{old}|^2$$