

Parton Showers and their  
**matching/merging**  
with NLO computations

Rikkert Frederix  
Lund University



# Yesterday's summary

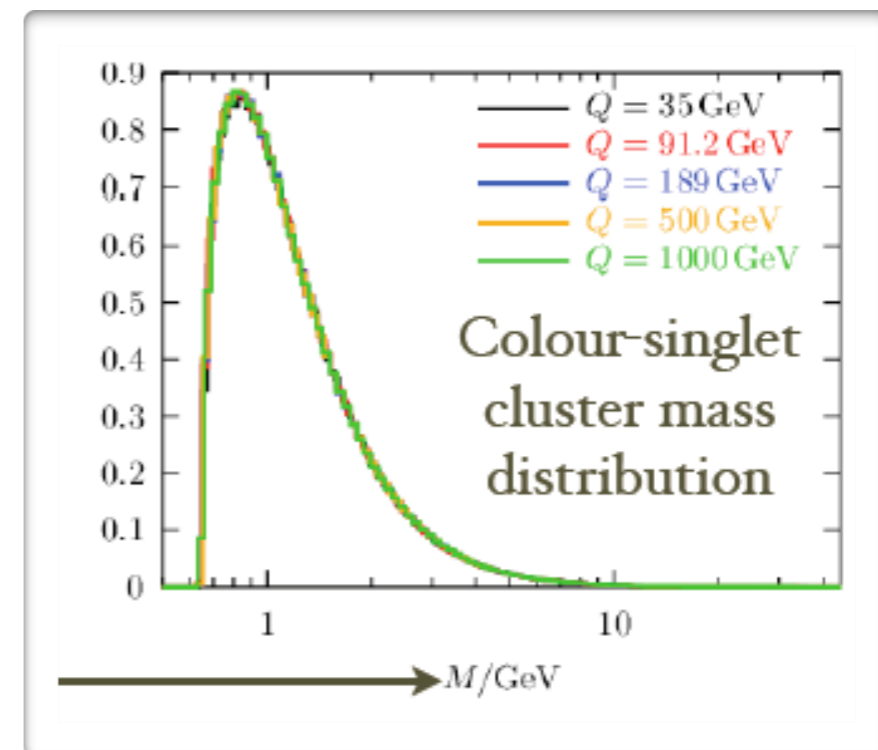
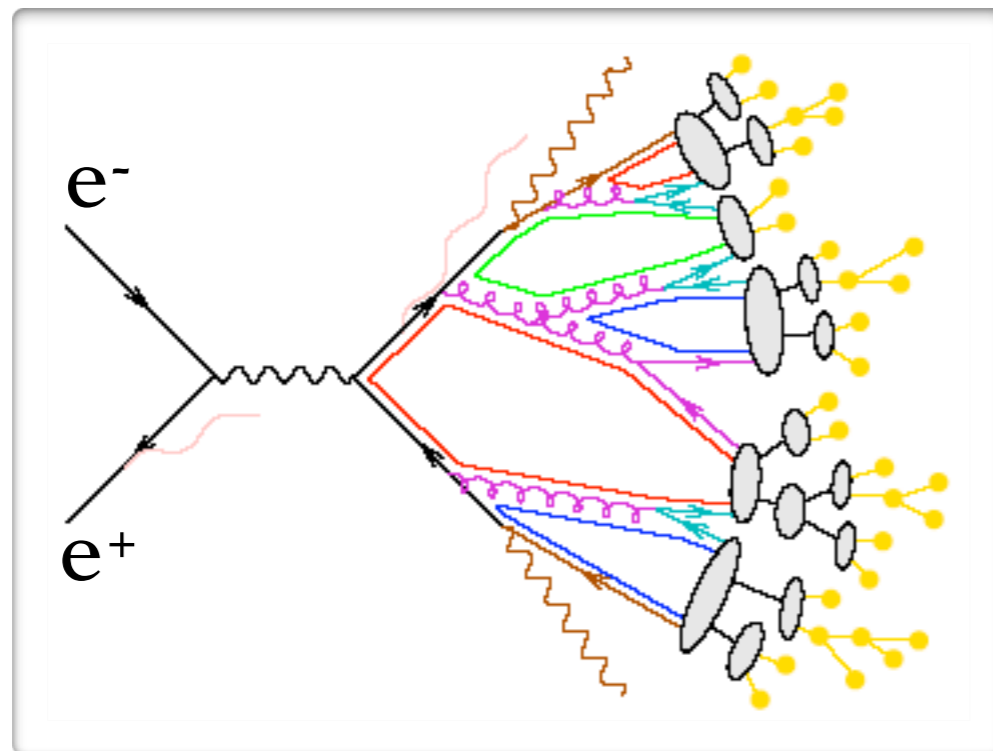
- The parton shower dresses partons with radiation. This makes the inclusive parton-level predictions (i.e. inclusive over extra radiation) completely exclusive
  - In the **soft and collinear limits** the partons showers are exact, but in practice they are used outside this limit as well
  - Partons showers are **universal** (i.e. independent from the process)
- There is a cut-off in the shower (below which we don't trust perturbative QCD) at which a hadronization model takes over
  - Hadronization models are universal and independent from the energy of the collision

# Hadronisation

- The shower stops if all partons are characterised by a scale at the IR cut-off:  $Q_0 \sim 1 \text{ GeV}$
- Physically, we observe hadrons, not (coloured) partons
- We need a non-perturbative model in passing from partons to colourless hadrons
- There are two models, based on physical and phenomenological considerations

# Cluster model

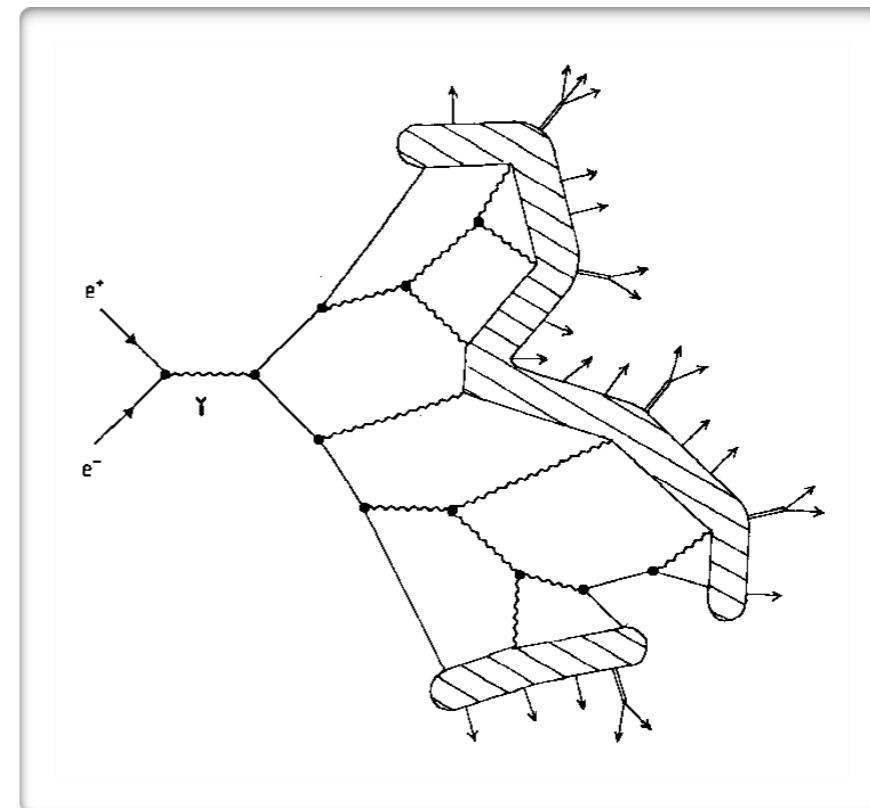
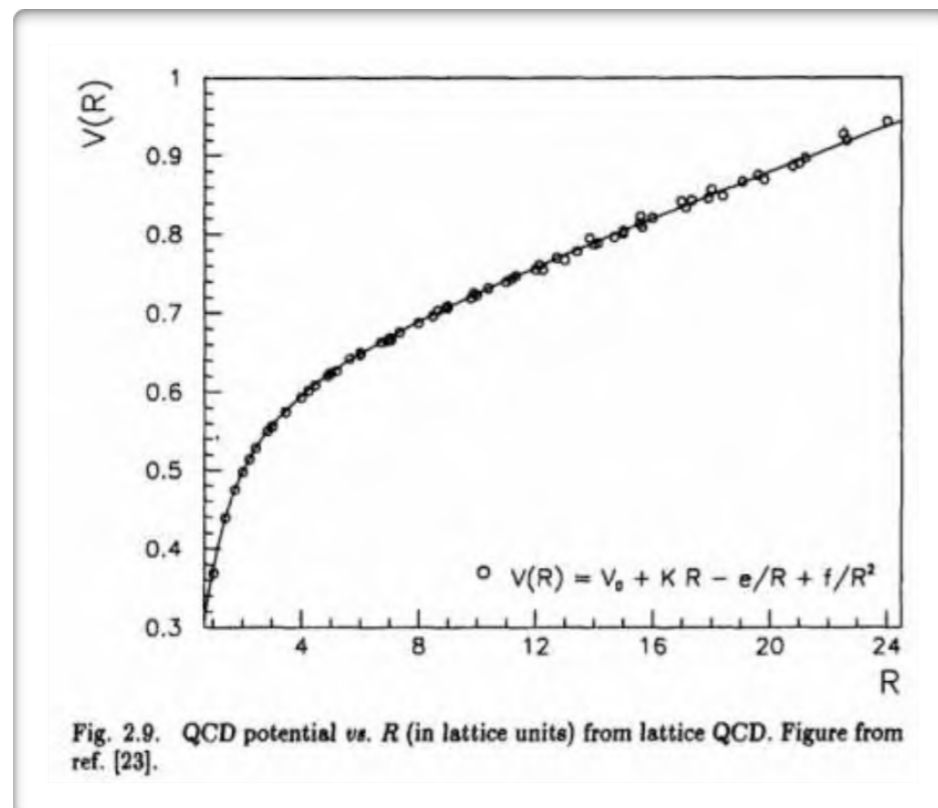
The structure of the perturbative evolution including angular ordering, leads naturally to the clustering in phase-space of colour-singlet parton pairs (pre-confinement). Long-range correlations are strongly suppressed. Hadronisation will only act locally, on low-mass colour singlet clusters.





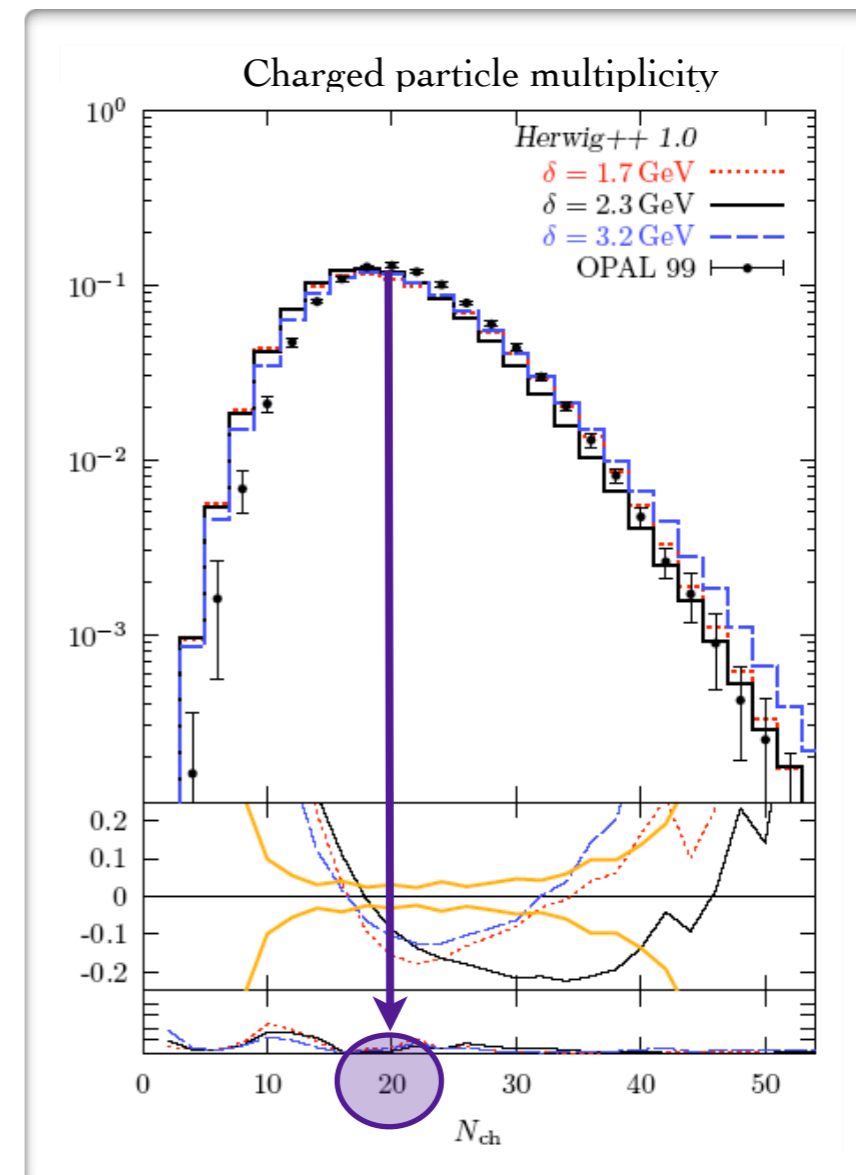
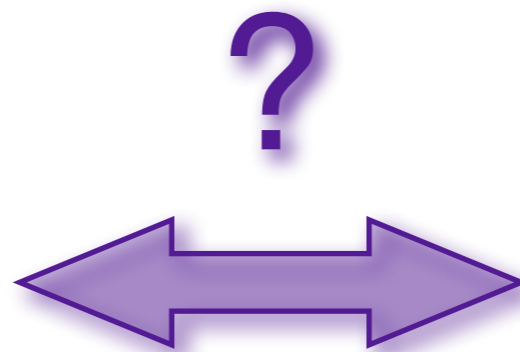
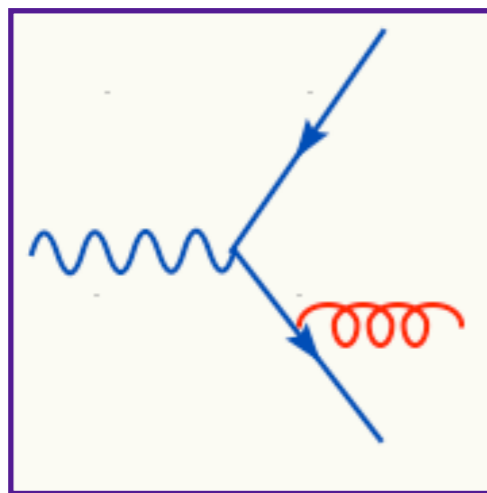
# String model

From lattice QCD one sees that the colour confinement potential of a quark-antiquark grows linearly with their distance:  $V(r) \sim kr$ , with  $k \sim 0.2$  GeV, This is modelled with a string with uniform tension (energy per unit length)  $k$  that gets stretched between the  $qq\bar{}$  pair.



When quark-antiquarks are too far apart, it becomes energetically more favourable to break the string by creating a new  $qq\bar{}$  pair in the middle.

# Exclusive observable



A parton shower program associates one of the possible histories (and pre-histories in case of pp collisions) of an hard event in an explicit and fully detailed way, such that the sum of the probabilities of all possible histories is unity.

# Parton Shower MC event generators

A parton shower program associates one of the possible histories (and pre-histories in case of pp) of an hard event in an explicit and fully detailed way, such that the sum of the probabilities of all possible histories is unity.

- General-purpose tools
- Always the first experimental choice
- Complete exclusive description of the events: hard scattering, showering & hadronisation (and underlying event)
- Reliable and well-tuned tools
- Significant and intense progress in the development of new showering algorithms with the final aim to go at NLO in QCD

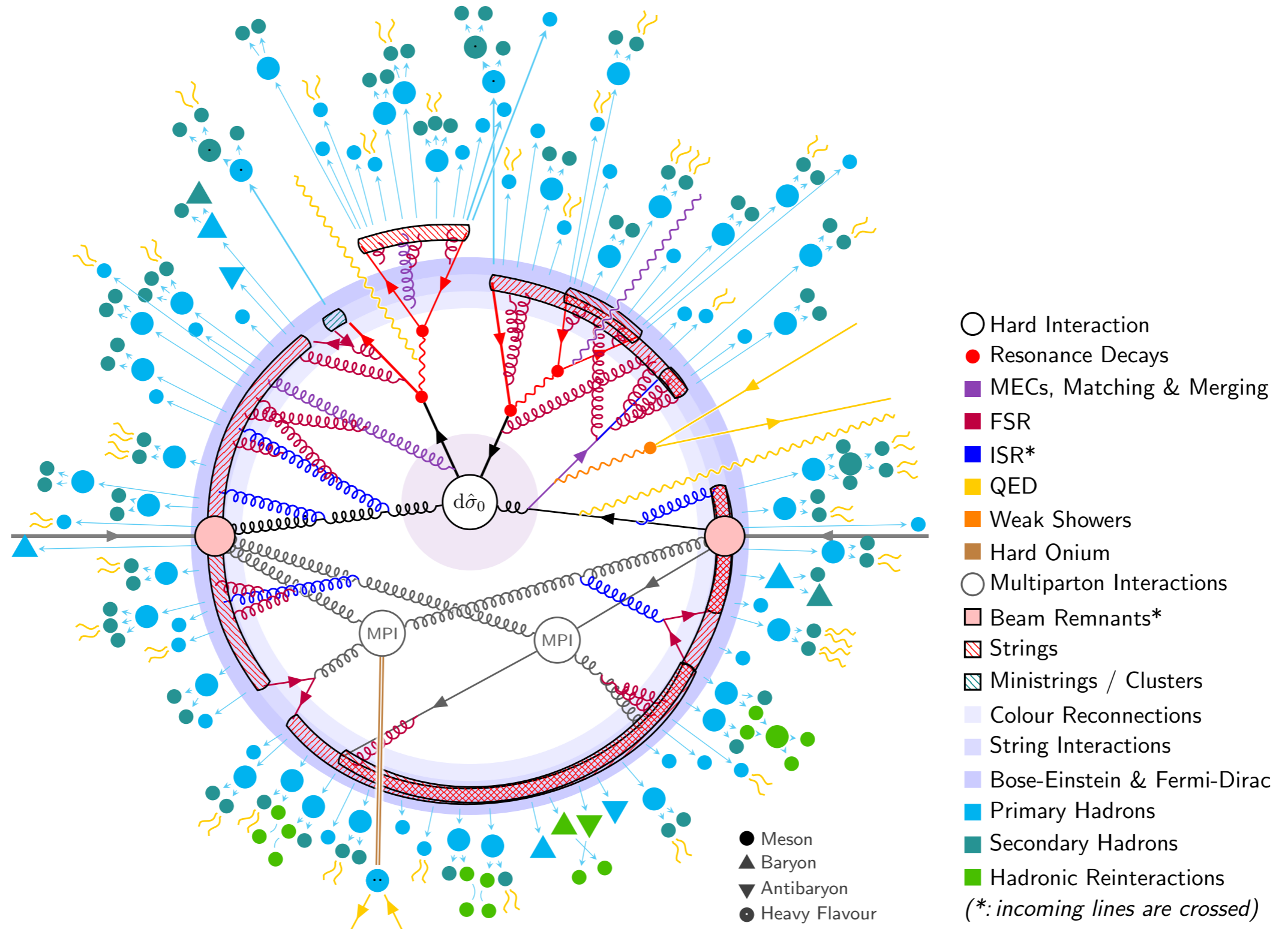
# Pythia, Sherpa, Herwig, ...

- Significant differences between shower implementations (choice of evolution variable and kernel, momentum mappings, phase-space boundaries, massive quarks, photon emissions, etc.)
- All are tuned to data, and describe it reasonably well (typically better than expected from their formal accuracy)
- Some are (formally) more correct than others
  - However, not easy to assess accuracy for a general observable
  - Assessment (and improvement!) of formal accuracy is an active field of research

# Improving parton showers in the hard region

- Parton shower MC programs are only correct in the soft-collinear region. Hard radiation cannot be described correctly
- There are two ways to improve a Parton Shower Monte Carlo event generator with matrix elements:
  - ME+PS merging: include matrix elements with more final state partons to describe hard, well-separated radiation better
  - NLO+PS matching: include full NLO corrections to the matrix elements to reduce theoretical uncertainties in the matrix elements. The real-emission matrix elements will describe the hard radiation

# An LHC collision, factorised



# Improving Parton Showers

## LO merging



# Matrix elements vs. Parton showers

ME



1. Fixed order calculation
2. Computationally expensive
3. Limited number of particles
4. Valid when partons are **hard and well separated**
5. Quantum interference correct
6. Needed for multi-jet description

Shower MC



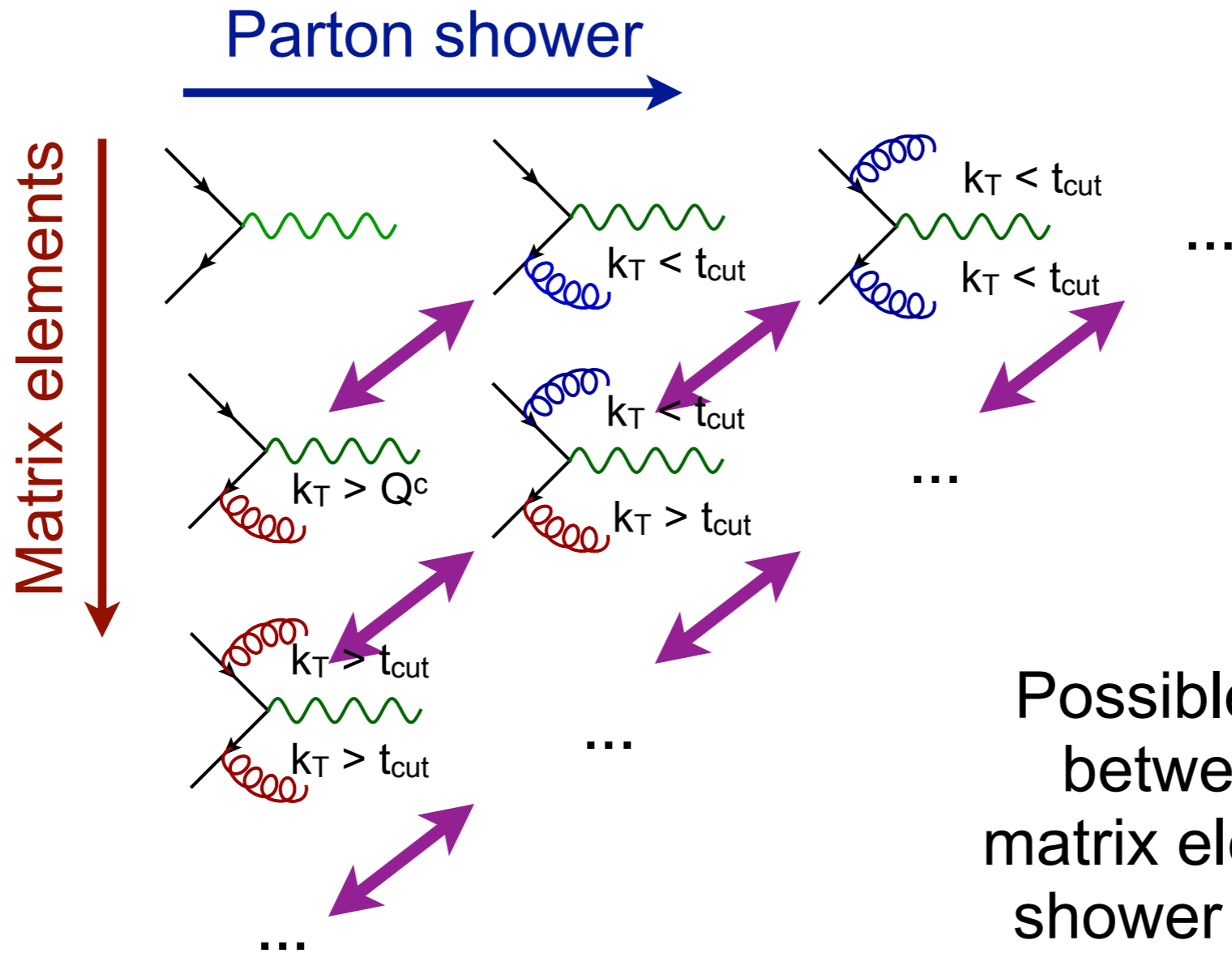
1. Resums logs to all orders
2. Computationally cheap
3. No limit on particle multiplicity
4. Valid when partons are **collinear and/or soft**
5. Partial interference through angular ordering
6. Needed for hadronisation

**Approaches are complementary: merge them!**

**Avoid double counting, ensure smooth distributions**



# Possible double counting

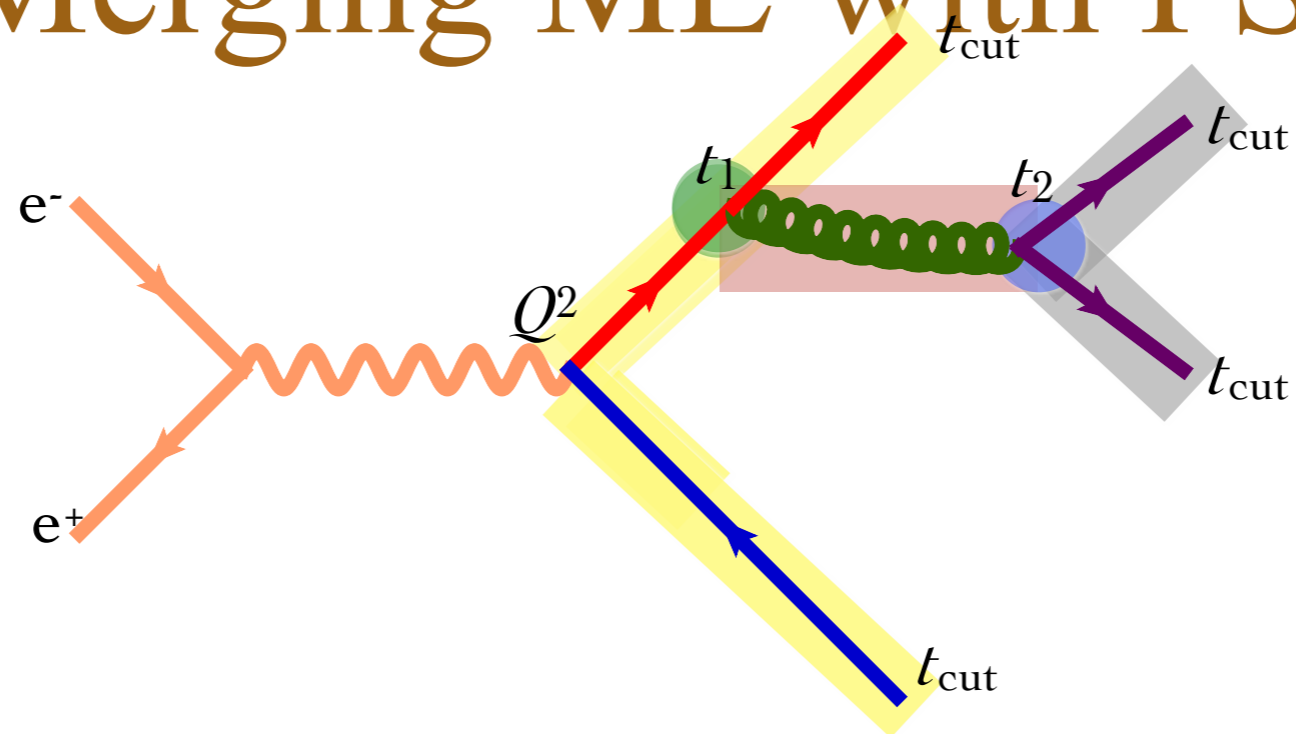


Possible double counting between partons from matrix elements and parton shower easily avoided by applying a cut in phase space

# Merging ME with PS

- So double counting no problem, but what about getting smooth distributions that are independent of the precise value of  $t_{\text{cut}}$ ?
- Below cutoff, distribution is given by PS
  - need to make ME look like PS near cutoff
- Let's take another look at the PS!

# Merging ME with PS



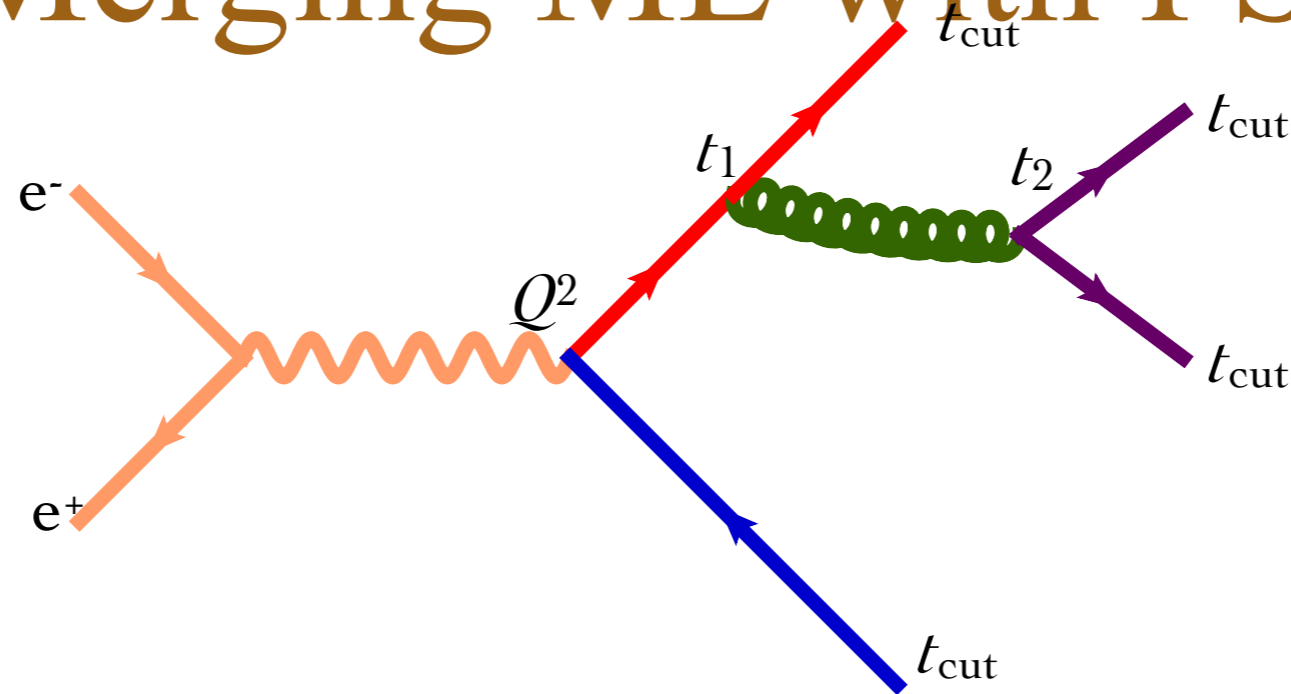
- How does the PS generate the configuration above (i.e. starting from  $e^+e^- \rightarrow qq\bar{q}$  events)?
- Probability for the splitting at  $t_1$  is given by

$$(\Delta_q(Q^2, t_1))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

and for the whole tree

$$(\Delta_q(Q^2, t_{\text{cut}}))^2 \Delta_g(t_1, t_2) (\Delta_q(t_2, t_{\text{cut}}))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$

# Merging ME with PS

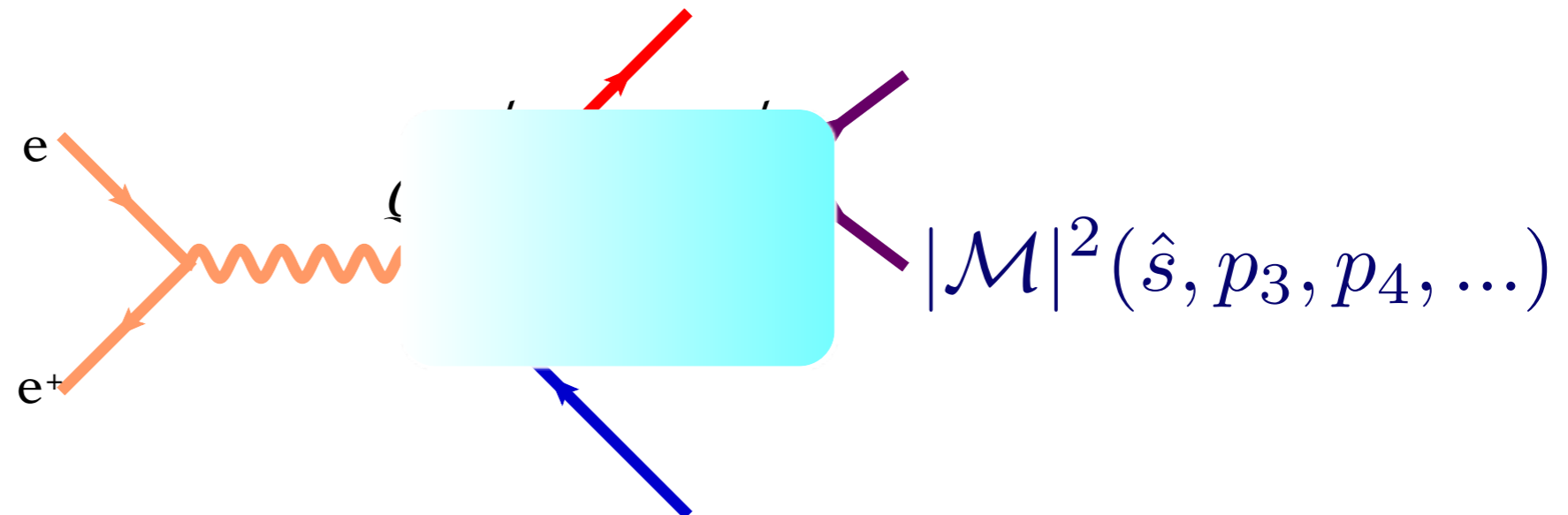


$$(\Delta_q(Q^2, t_{\text{cut}}))^2 \Delta_g(t_1, t_2) (\Delta_q(t_2, t_{\text{cut}}))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$

Leading Logarithmic approximation of the matrix element  
BUT with  $\alpha_s$  evaluated at the scale of each splitting

Sudakov suppression due to disallowing additional radiation  
above the scale  $t_{\text{cut}}$

# Merging ME with PS



To get an equivalent treatment of the corresponding matrix element, do as follows:

1. Cluster the event using some clustering algorithm  
- this gives us a corresponding “parton shower history”
2. Reweight  $\alpha_s$  in each clustering vertex with the clustering scale

$$|\mathcal{M}|^2 \rightarrow |\mathcal{M}|^2 \frac{\alpha_s(t_1)}{\alpha_s(Q^2)} \frac{\alpha_s(t_2)}{\alpha_s(Q^2)}$$

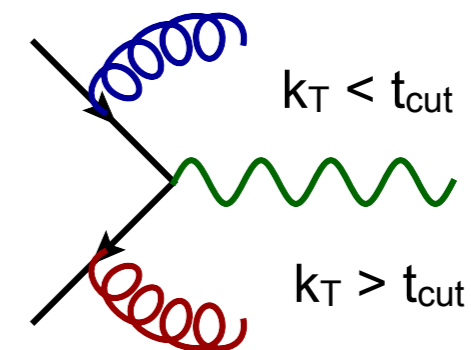
3. Use some algorithm to apply the equivalent Sudakov suppression

$$(\Delta_q(Q^2, t_{\text{cut}}))^2 \Delta_g(t_1, t_2) (\Delta_q(t_2, t_{\text{cut}}))^2$$

# Merging ME with PS

CKKW (2004) and MLM (2004)

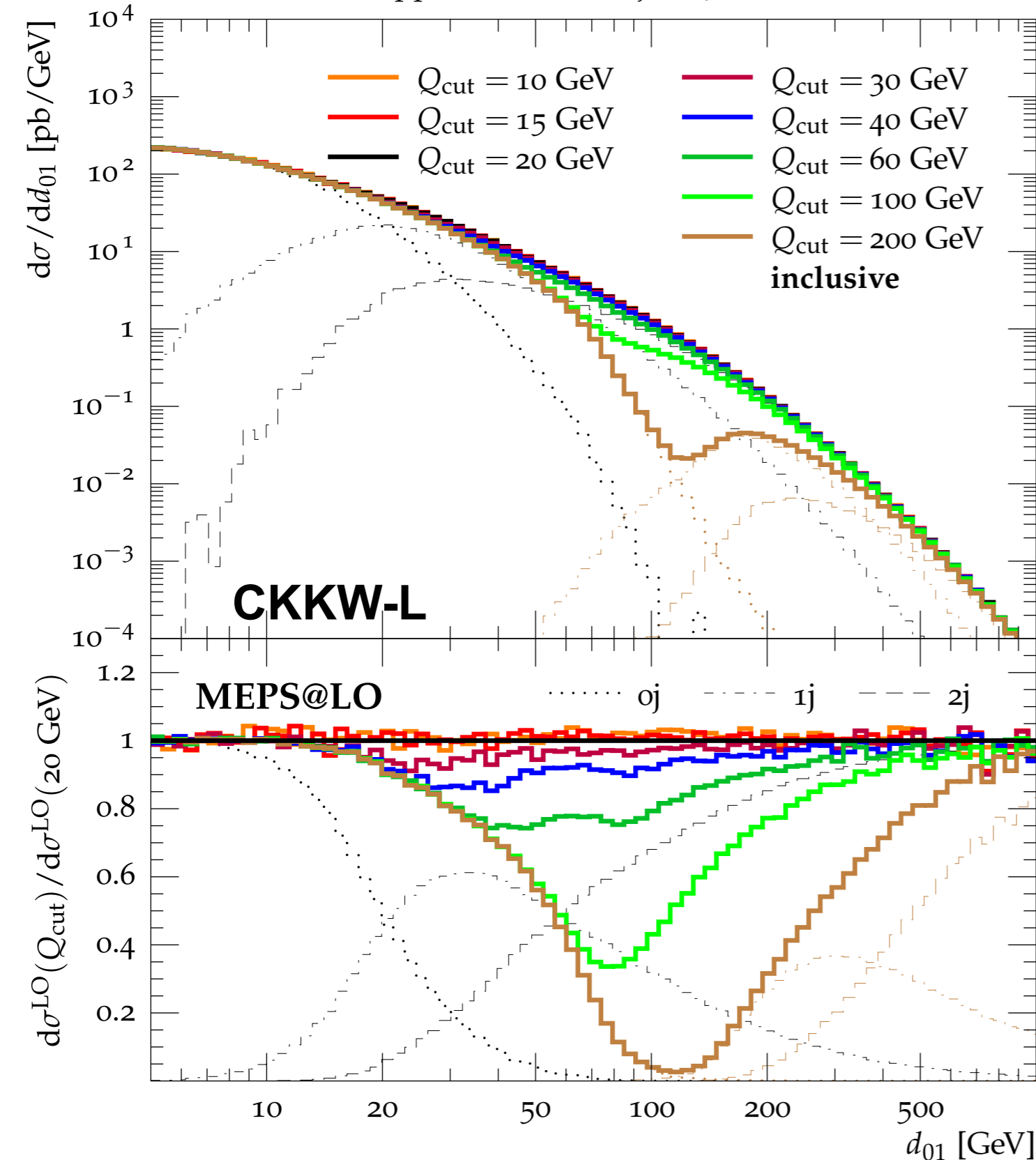
- In summary:
- Double counting no problem: we simply throw events away when the matrix-element partons are too soft, or when the parton shower generates too hard radiation
- Applying the matrix-element cut is easy: during phase-space integration, we only generate events with partons above the matching scale
- For the cut on the shower, there are two methods. Throwing events away after showering is not very efficient, although it is working (“**MLM method**”)
- Instead we can also multiply the Born matrix elements by suitable product of Sudakov factors (i.e. the no-emission probabilities)  $\Delta(Q_{\max}, t_{\text{cut}})$  and start the shower at the scale  $t_{\text{cut}}$  (“**CKKW method**”).
- For a given multiplicity we have



$$\sigma_{n,\text{excl}}^{\text{LO}} = B_n \Theta(k_{T,n} - t_{\text{cut}}) \Delta_n(Q, t_{\text{cut}})$$

# Merging results

$pp \rightarrow \ell^- \bar{\nu} + 0,1,2j @ 13 \text{ TeV}$



- W+jets production: diff. jet rate for  $0 \rightarrow 1$  transition ( $\sim p_T$  of hardest jet)
- Small dependence on the merging scale for small values,  $\sim 10\%$ 
  - When taken too large, the parton shower cannot fill the region all the way up to the merging scale anymore, leading to large deficits

[Kallweit, Lindert, Maierhöfer, Pozzorini, Schönherr 2016]

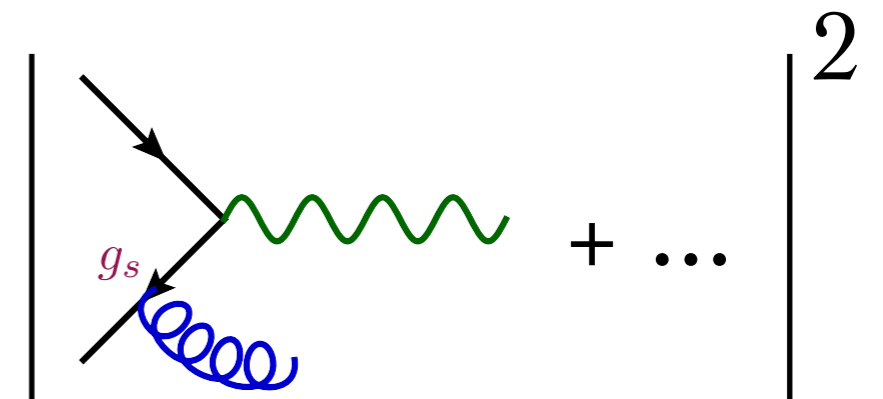
# ME+PS (at LO)

- Merging matrix elements of various multiplicities with parton showers improves the predictive power of the parton shower outside the collinear/soft regions
  - These merged samples give good description of the data (except for the total normalisation)
- There is a dependence on the parameters responsible for the cut in phase-space (i.e. the matching scale)
- By letting the matrix elements mimic what the parton shower does in the collinear/soft regions (PDF/ $\alpha_s$  reweighting and including the Sudakov suppression) the dependence is greatly reduced
- In practice, one should check explicitly that this is the case by plotting differential jet-rate plots for a couple of values for the matching scale



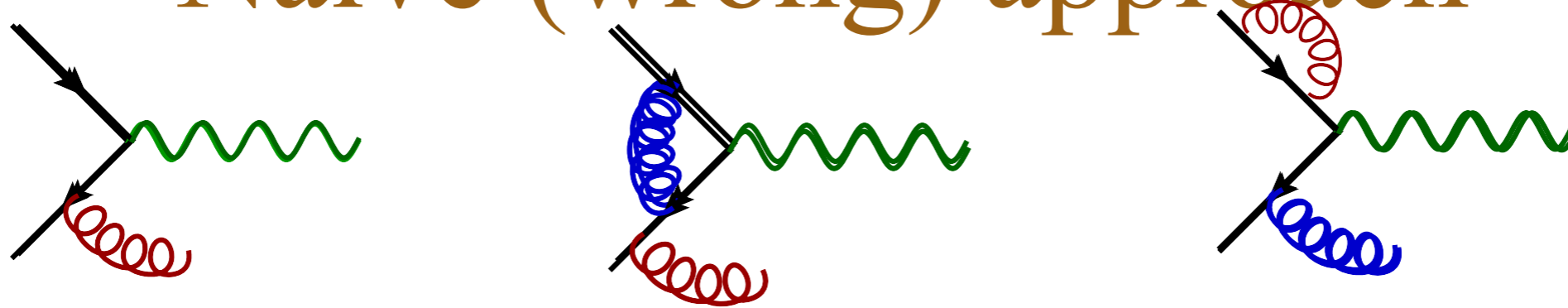
# Improving Parton Showers: NLO matching

# At NLO



- We have to integrate the real emission over the **complete** phase-space of the one particle that can go soft or collinear to obtain the infra-red poles that will cancel against the virtual corrections
- Hence, we cannot introduce a "cut" that says that:
  - **hard** radiation needs to be described by the matrix elements
  - and **soft** radiation by the parton shower
- We have to invent a new procedure to match NLO matrix elements with parton showers

# Naive (wrong) approach



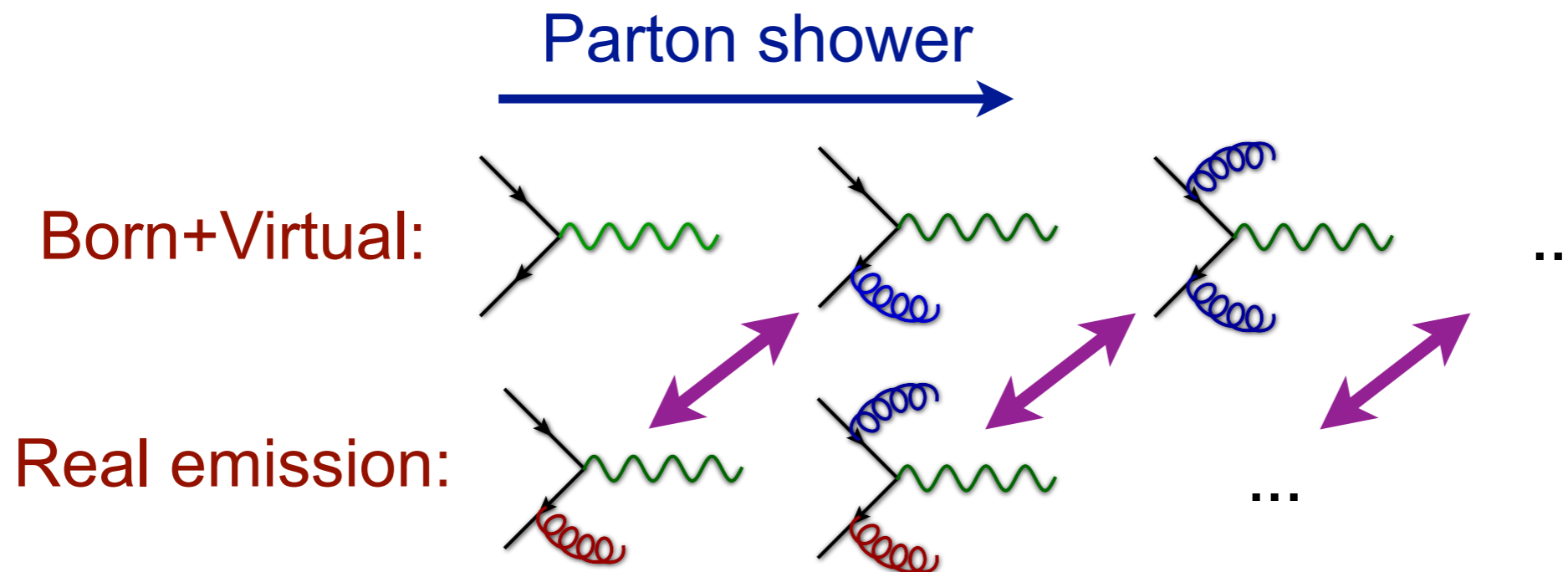
- In a fixed order calculation we have contributions with  $m$  final state particles and with  $m+1$  final state particles

$$\sigma^{\text{NLO}} \sim \int d^4\Phi_m B(\Phi_m) + \int d^4\Phi_m \int_{\text{loop}} d^d l V(\Phi_m) + \int d^d\Phi_{m+1} R(\Phi_{m+1})$$

- We could try to shower them independently
- Let  $I_{\text{MC}}^{(k)}(O)$  be the parton shower spectrum for an observable  $O$ , showering from a  $k$ -body initial condition
- We can then try to shower the  $m$  and  $m+1$  final states independently

$$\frac{d\sigma_{\text{NLOwPS}}}{dO} = \left[ d\Phi_m (B + \int_{\text{loop}} V) \right] I_{\text{MC}}^{(m)}(O) + \left[ d\Phi_{m+1} R \right] I_{\text{MC}}^{(m+1)}(O)$$

# NLO+PS: sources of double counting



- There is double counting between the real emission matrix elements and the parton shower: the extra radiation can come from the matrix elements or the parton shower
- There is also an overlap between the virtual corrections and the Sudakov suppression in the zero-emission probability
- We have to integrate the real emission over the **complete** phase-space of the one particle that can go soft or collinear to obtain the infra-red poles that will cancel against the virtual corrections
- We can NOT use the same merging procedure as used at LO (MLM or CKKW): requiring that all partons should produce separate jets is not infrared safe

# Double counting in virtual/Sudakov

- The Sudakov factor  $\Delta$  (which is responsible for the resummation of all the radiation in the shower) is the no-emission probability
- It's defined to be  $\Delta = 1 - P$ , where  $P$  is the probability for a branching to occur
- By using this conservation of probability in this way,  $\Delta$  contains contributions from the virtual corrections implicitly
- Because at NLO the virtual corrections are already included via explicit matrix elements,  $\Delta$  is double counting with the virtual corrections
- In fact, because the shower is unitary, what we are double counting in the real emission corrections is exactly equal to what we are double counting in the virtual corrections (but with opposite sign)!

# Avoiding double counting

- There are two widely methods to circumvent this double counting
  - MC@NLO (Frixione & Webber)
  - POWHEG (Nason)
  - KRKNLO (Cracow group), Vincia (Skands et al.), Geneva (Alioli et al.), ...

# MC@NLO procedure

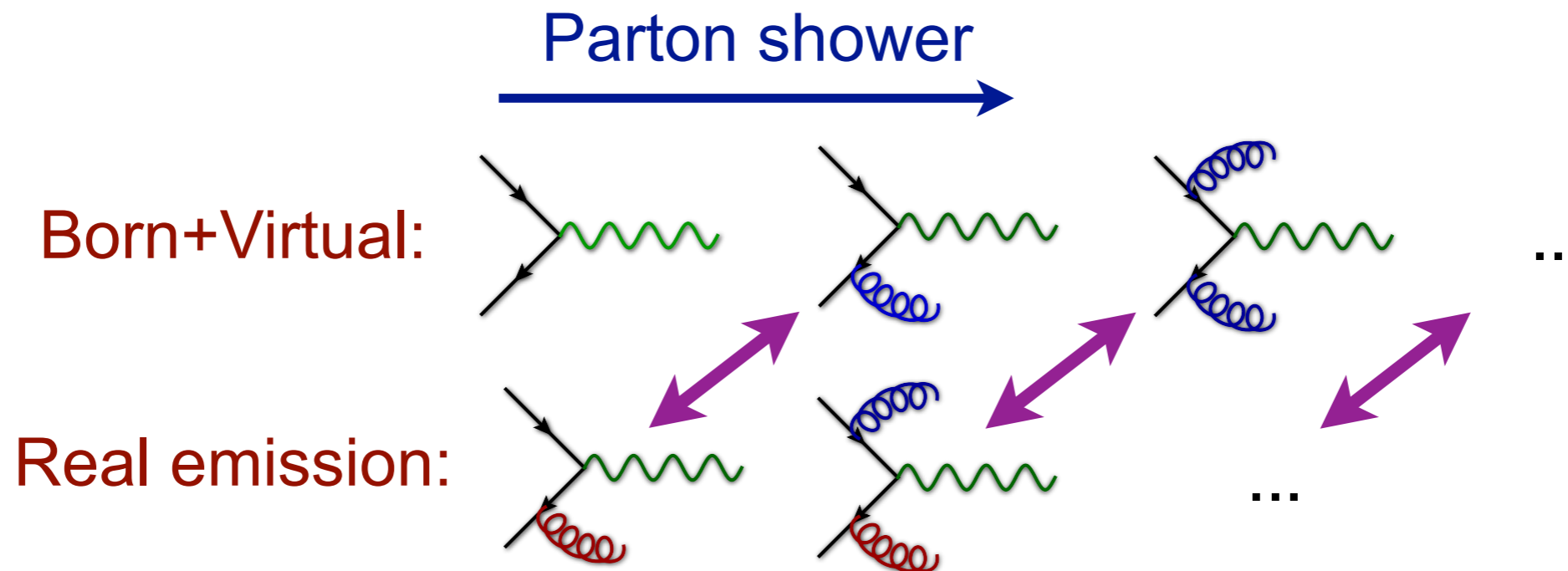
Frixione & Webber (2002)

- To remove the double counting, we can add and subtract the same term to the  $m$  and  $m+1$  body configurations

$$\frac{d\sigma_{\text{MC@NLO}}}{dO} = \left[ d\Phi_m (B + \int_{\text{loop}} V + \int d\Phi_1 MC) \right] I_{\text{MC}}^{(m)}(O) \\ + \left[ d\Phi_{m+1} (R - MC) \right] I_{\text{MC}}^{(m+1)}(O)$$

Where the  $MC$  are defined to be the contribution of the parton shower to get from the  $m$  body Born final state to the  $m+1$  body real emission final state

# MC@NLO procedure



- Double counting is explicitly removed by including the “shower subtraction terms”

$$\frac{d\sigma_{\text{NLO wPS}}}{dO} = \left[ d\Phi_m (B + \int_{\text{loop}} V + \int d\Phi_1 MC) \right] I_{\text{MC}}^{(m)}(O) + \left[ d\Phi_{m+1} (R - MC) \right] I_{\text{MC}}^{(m+1)}(O)$$



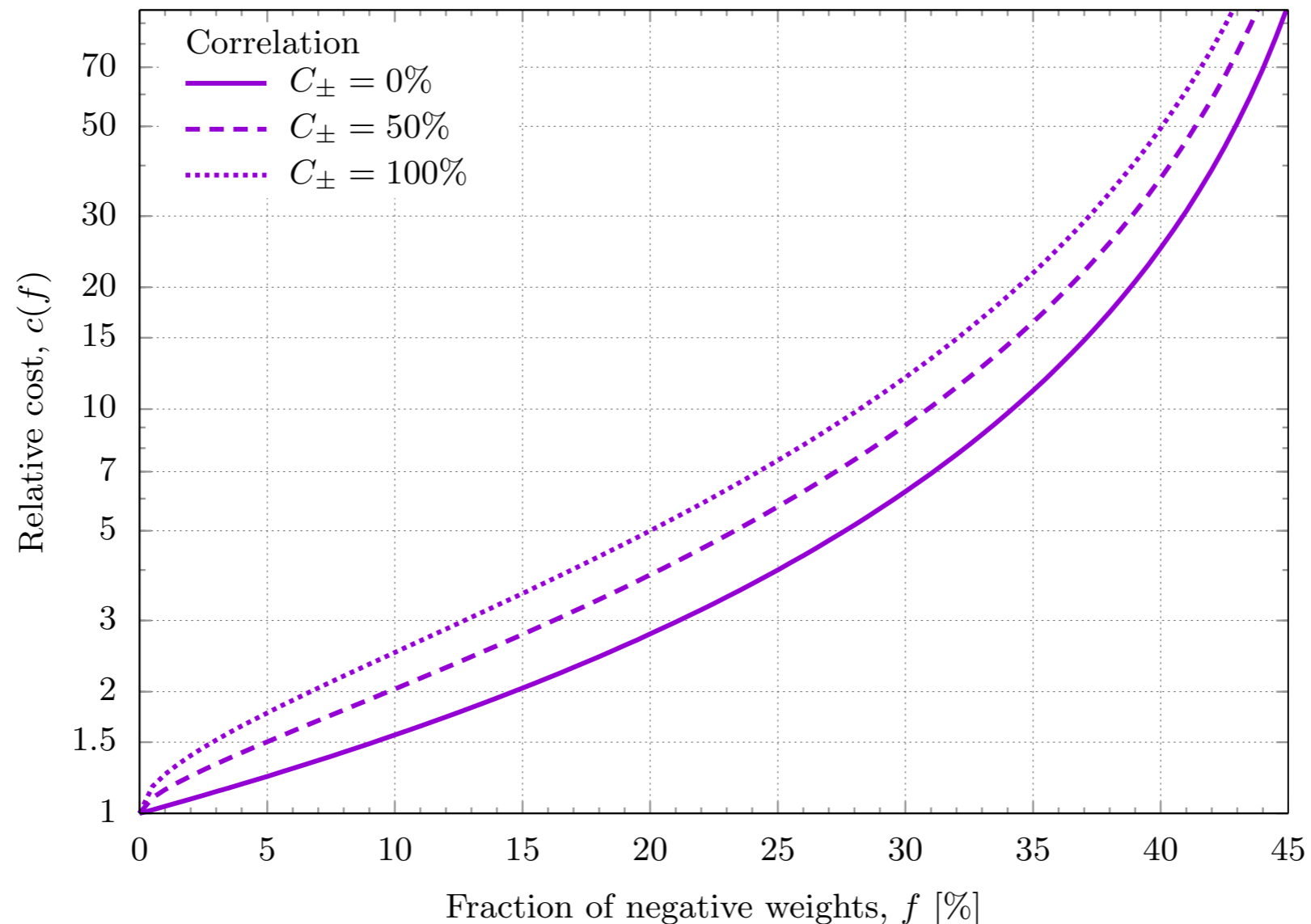
# Negative weights

$$\frac{d\sigma_{\text{NLOwPS}}}{dO} = \left[ d\Phi_m(B + \int_{\text{loop}} V + \int d\Phi_1 MC) \right] I_{\text{MC}}^{(m)}(O) \\ + \left[ d\Phi_{m+1}(R - MC) \right] I_{\text{MC}}^{(m+1)}(O)$$

- We generate events for the two terms between the square brackets (S- and H-events) separately
- There is no guarantee that these contributions are separately positive (even though predictions for infra-red safe observables should always be positive!)
- Therefore, when we do event unweighting we can only unweight the events **up to a sign**. These signs should be taken into account when doing a physics analysis (i.e. making plots etc.)
- The events are only physical when they are showered

# Cost of negative weights

- The computational costs to generate events negative weights can be enormous: for physical observables they cancel against positive weight events, but the overall sample still carries the statistical uncertainty of the full sample:



# NLO+PS

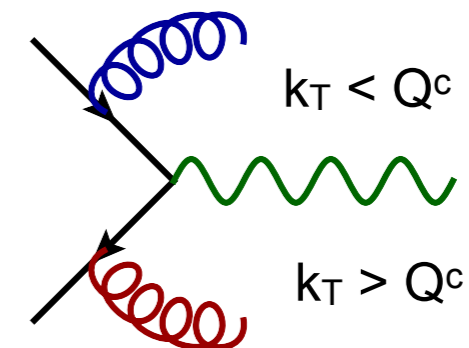
- Advantages:
  - Total cross section and differential distributions related to the hard process are NLO accurate
  - Reduced renormalisation and factorisation scale uncertainties
  - Shower to include multiple emissions; there are models for hadronisation and underlying event
    - Fully exclusive description of the event
- Disadvantages
  - Other observables (e.g., "multi-jet") are only LO accurate, or only generated by the shower

Merging ME+PS at NLO accuracy

# Merging LO ME with PS

CKKW (2004) and MLM (2004)

- In summary:
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- For a given multiplicity we have



$$\sigma_{n,\text{excl}}^{\text{LO}} = B_n \Theta(k_{T,n} - Q^c) \Delta_n(Q_{\max}, Q^c)$$

# Merging at NLO

- To make a LO prediction exclusive in the number of jets, we need to multiply it by a Sudakov damping factor; this is CKKW method:

$$\sigma_{n,\text{excl}}^{\text{LO}} = B_n \Theta(k_{T,n} - Q^c) \Delta_n(Q_{\text{max}}, Q^c)$$

This makes the prediction exclusive at leading logarithmic accuracy

- Similarly we can make an NLO prediction exclusive at leading logarithm

$$\sigma_{n,\text{excl, LL}}^{\text{NLO}} = \left\{ B_n + V_n + \int d\Phi_1 R_{n+1} \right\} \Theta(k_{T,n} - Q^c) \Delta_n(Q_{\text{max}}, Q^c)$$

- We can improve here and use the real-emission matrix elements instead of just the Sudakov:

$$\sigma_{n,\text{excl, LL}}^{\text{NLO}} = \left\{ B_n + V_n + \int_0^{Q^c} d\Phi_1 R_{n+1} - B_n \Delta_n^{(1)}(Q_{\text{max}}, Q^c) \right\} \Theta(k_{T,n} - Q^c) \Delta_n(Q_{\text{max}}, Q^c)$$

# Exclusive MC@NLO: FxFx merging and MEPS@NLO

- Converting the NLO exclusive predictions in the number of jets to the MC@NLO event generation is straight-forward:

$$\text{S-events: } \left\{ B_n + V_n + \int_0^{Q^c} d\Phi_1 \text{MC} - B_n \Delta_n^{(1)}(Q_{\text{max}}, Q^c) \right\}$$

$$\Theta(k_{T,n}^B - Q^c) \Delta_n(Q_{\text{max}}^B, Q^c)$$

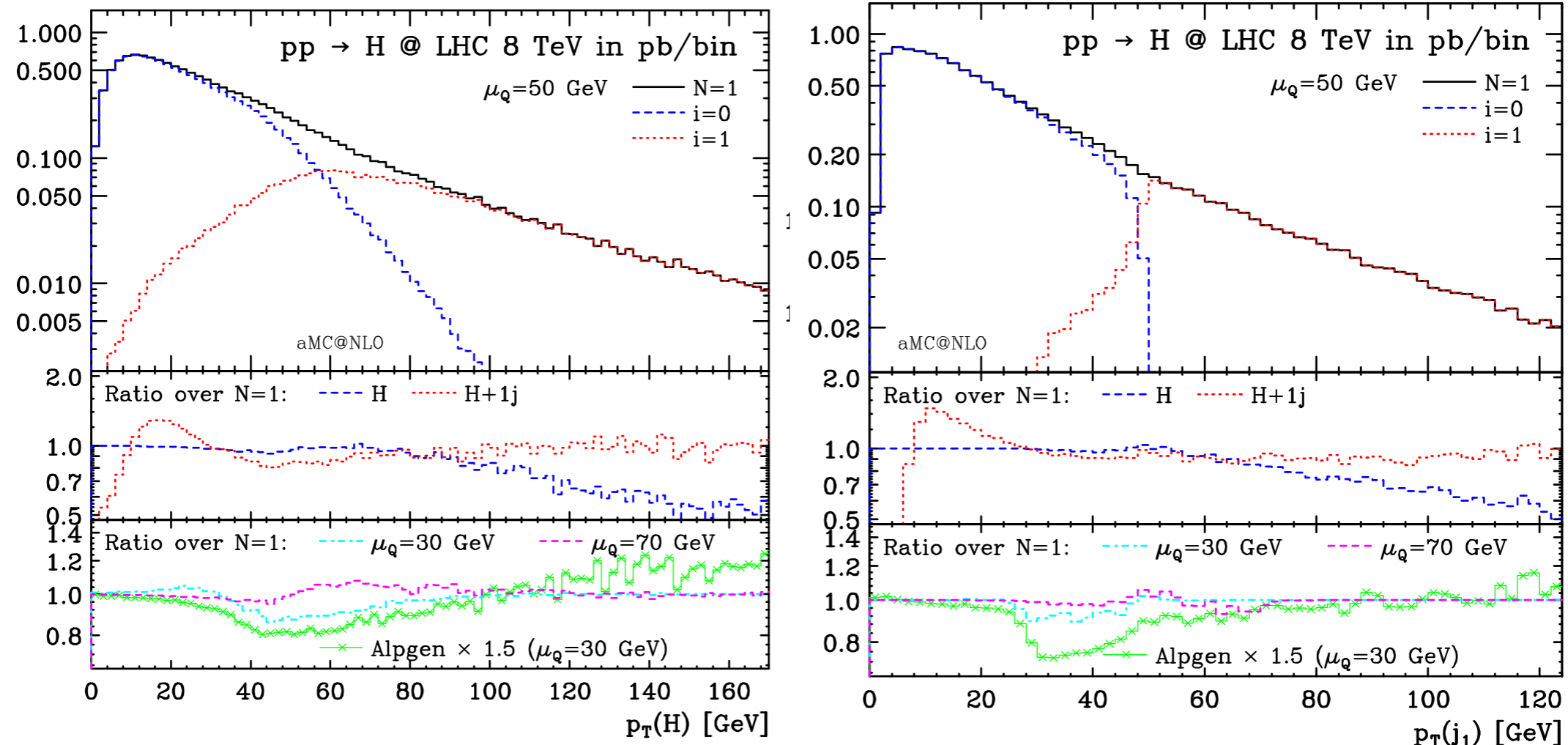
$$\text{H-events: } \left\{ R_{n+1} \Theta(k_{T,n}^R - Q^c) - \text{MC} \Theta(k_{T,n}^B - Q^c) \right\}$$

$$\Theta(Q^c - k_{T,n+1}^R) \Delta_n(Q_{\text{max}}^R, Q^c)$$

- But the evil is in the details...

# FxFx merging: Higgs boson production

[RF and Frixione]

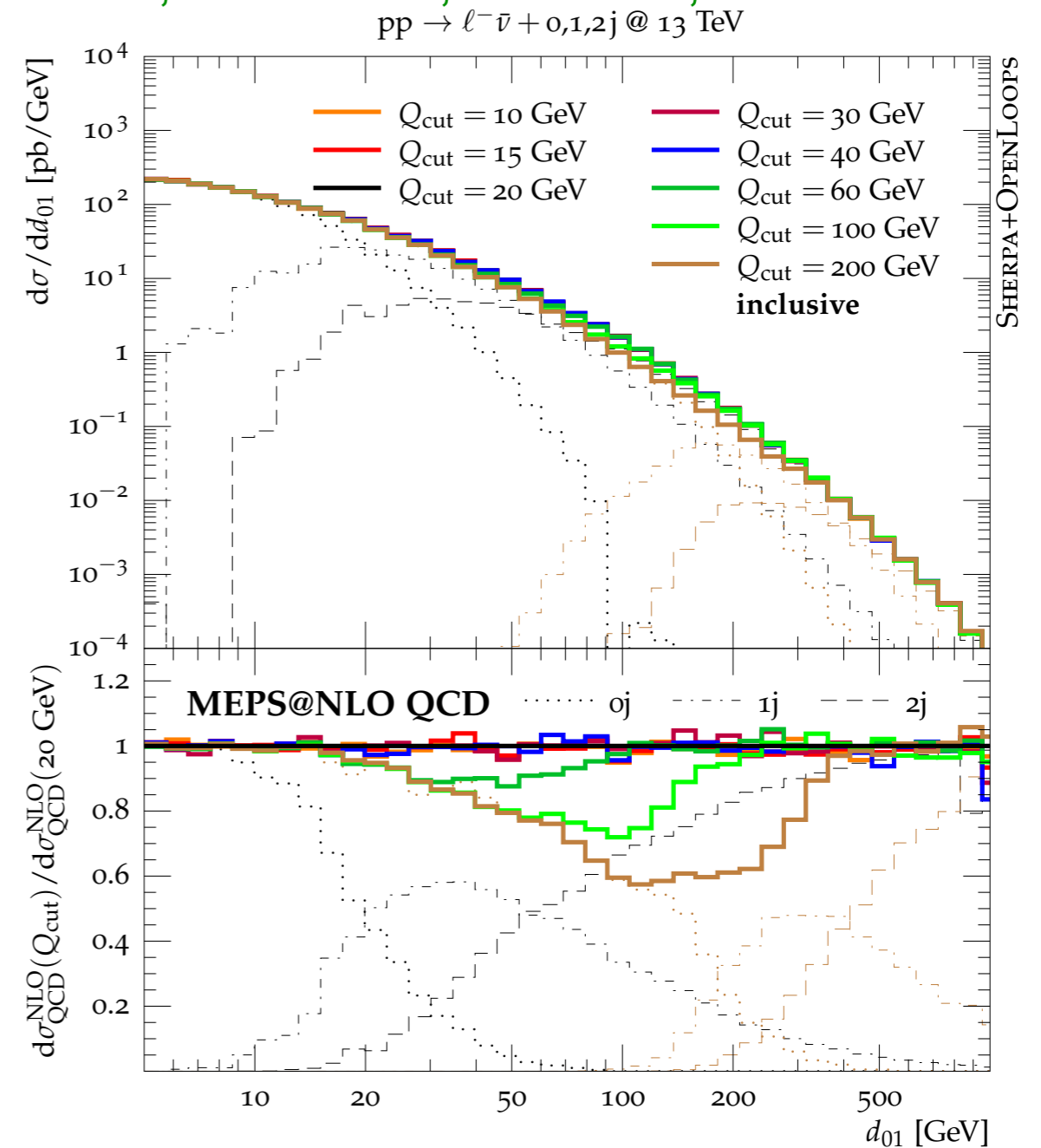
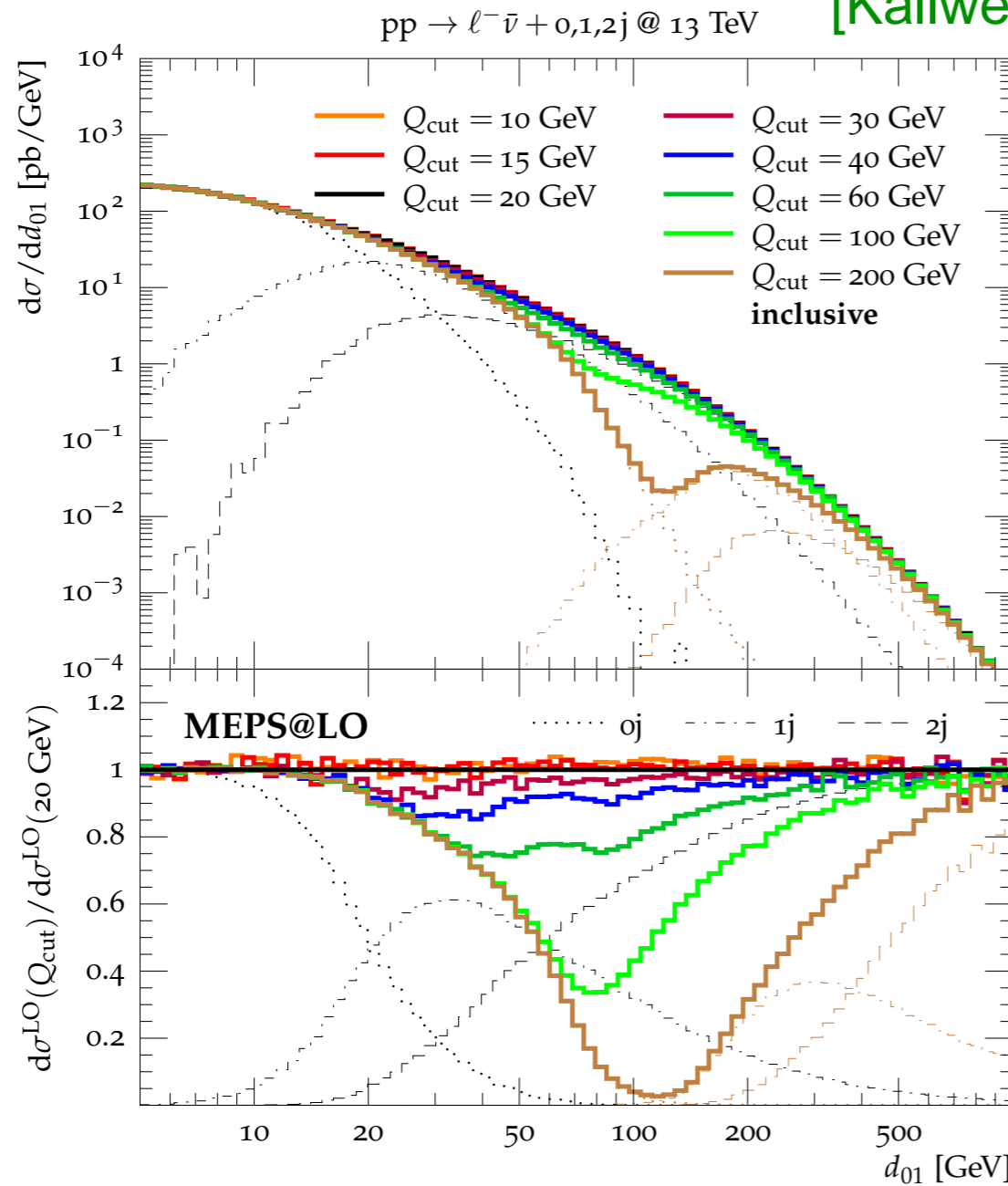


- Transverse momentum of the Higgs and of the 1st jet.
- Agreement with H+0j at MC@NLO and H+1j at MC@NLO in their respective regions of phase-space; Smooth matching in between; Small dependence on matching scale



# Merging scale dependence

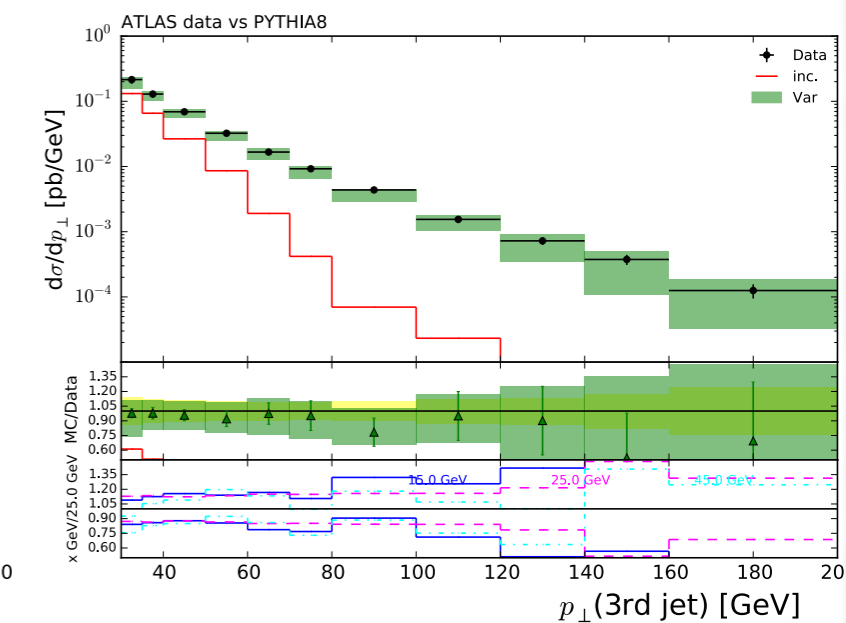
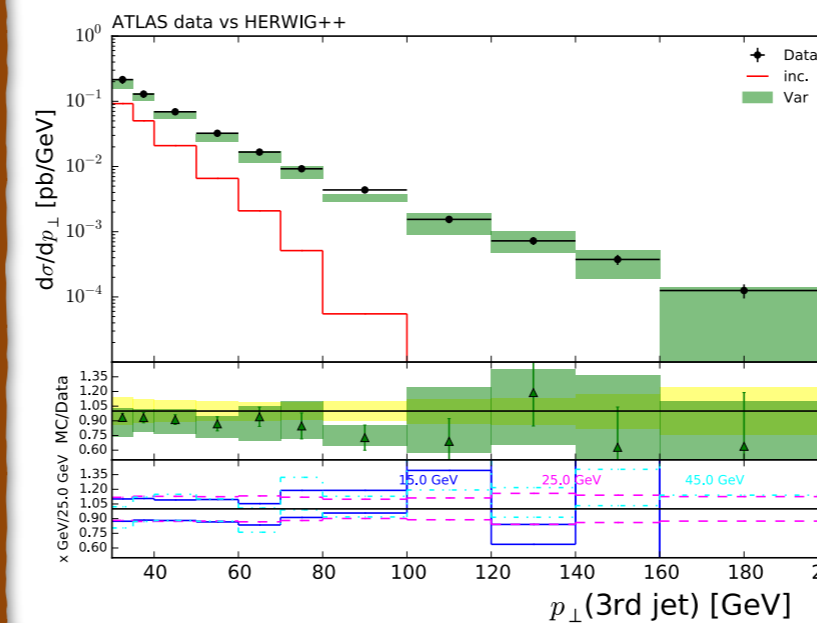
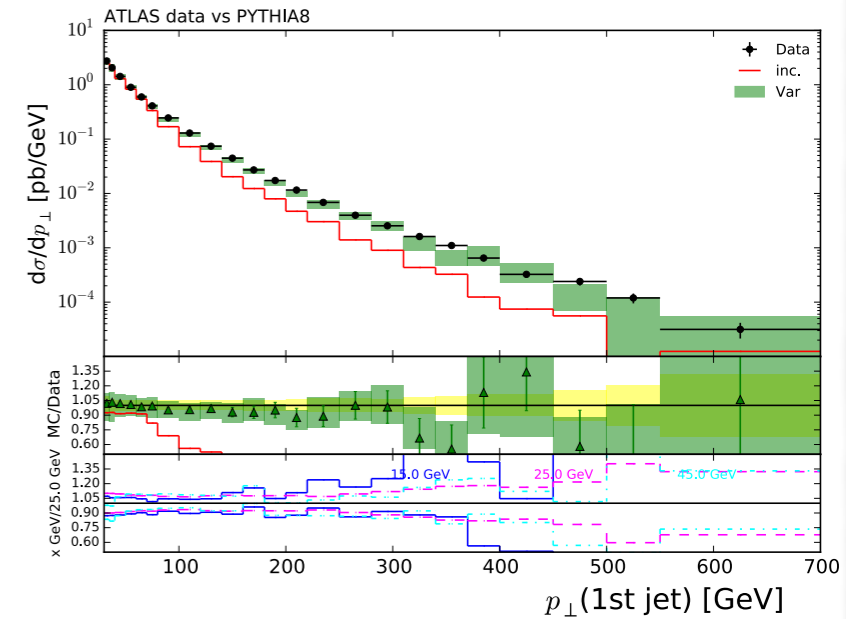
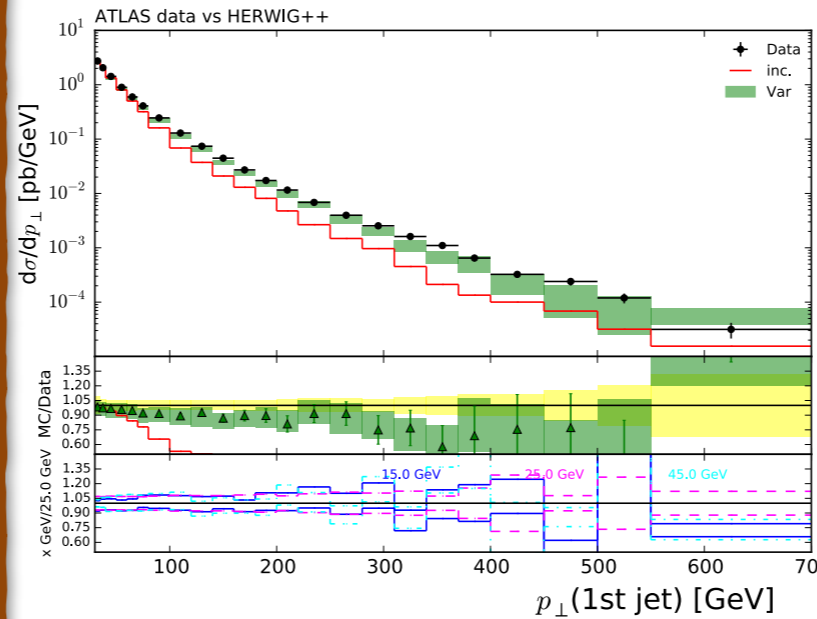
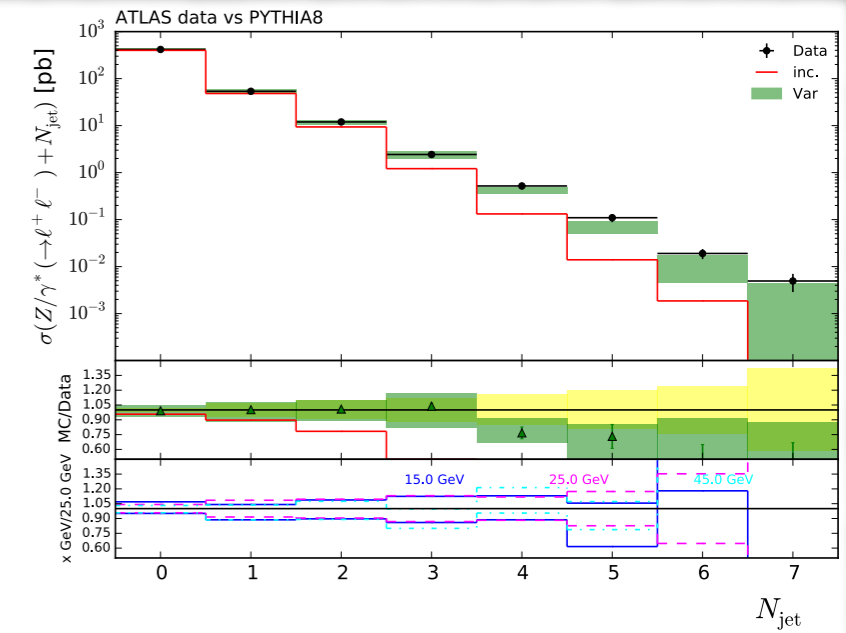
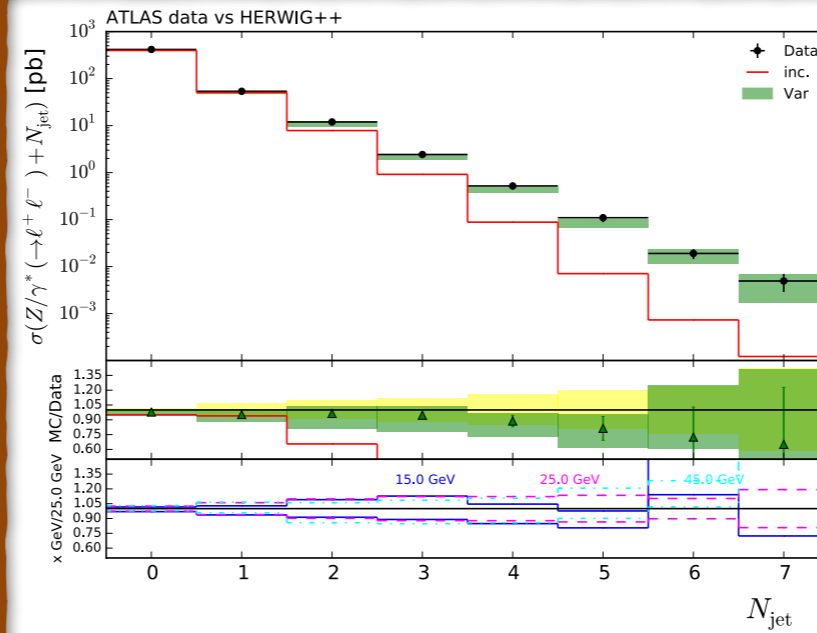
[Kallweit, Lindert, Maierhöfer, Pozzorini, Schönherr 2016]



- Besides having the benefits from higher-accurate matrix elements, there is also a **smaller merging scale dependence at NLO**

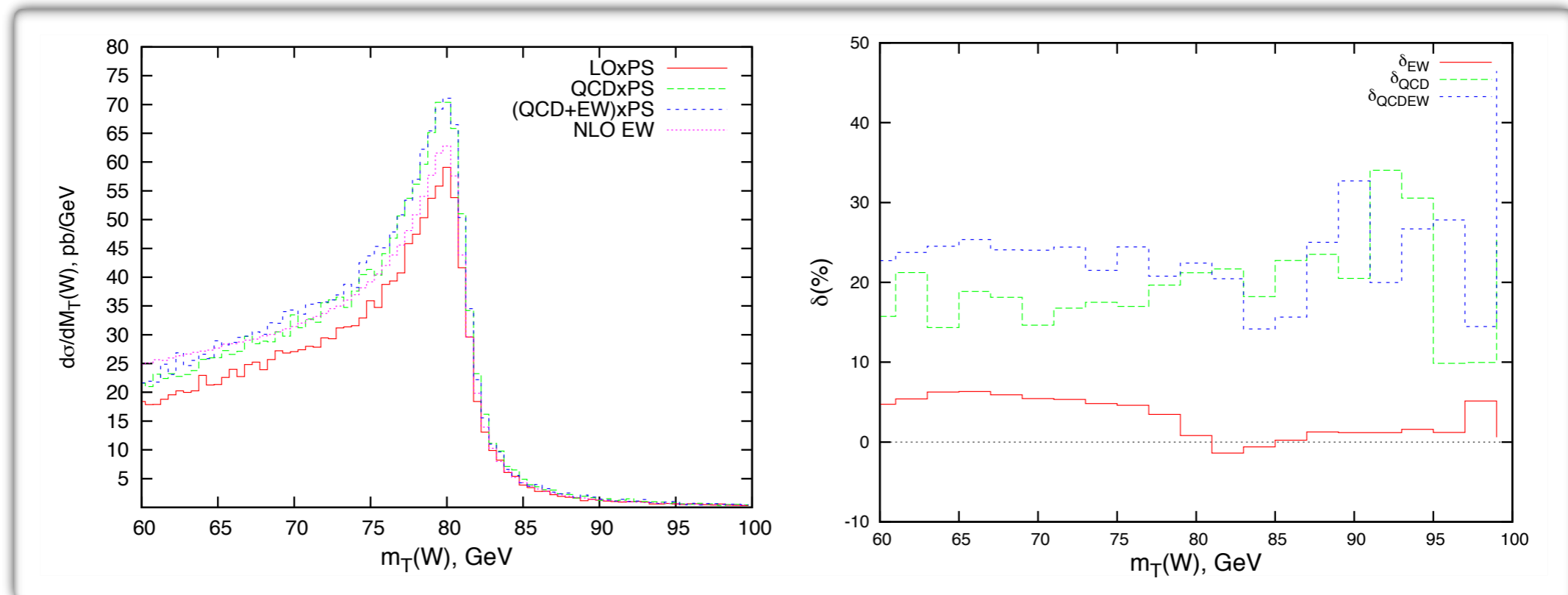
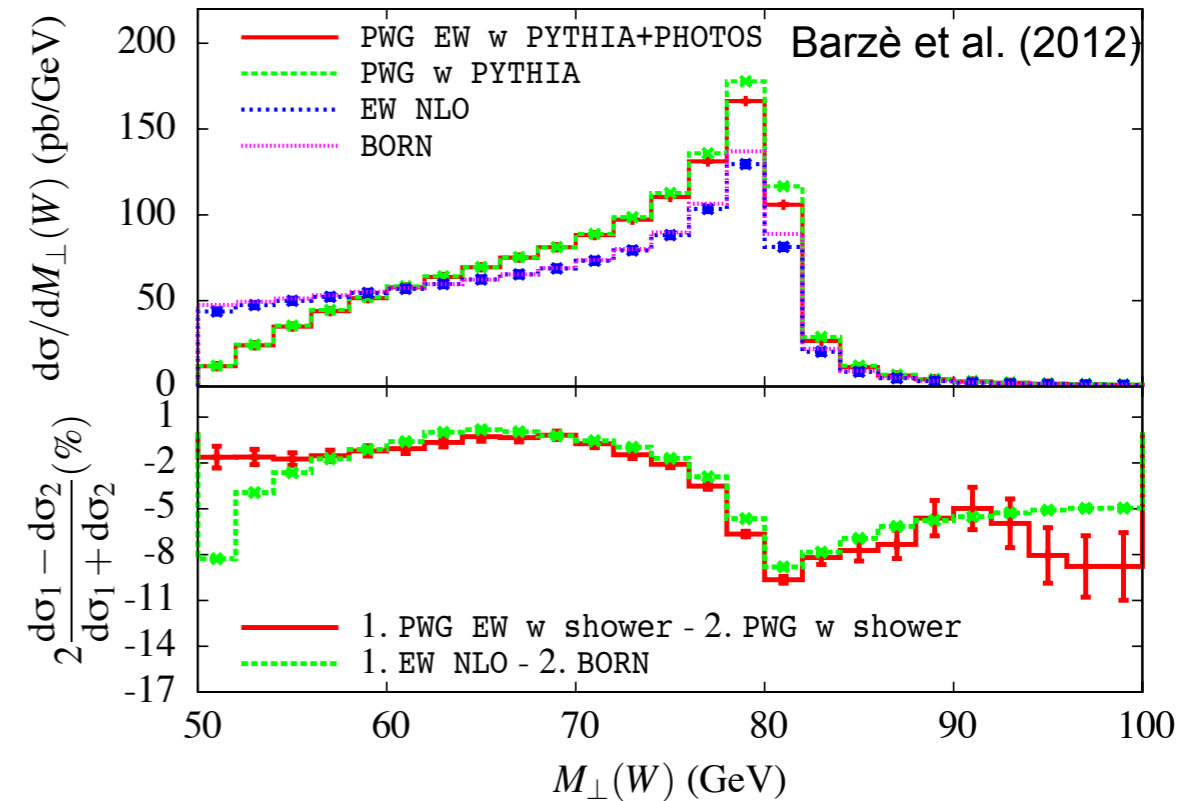
# FxFx

- Comparison to data
- Z+jets
- Exclusive jet multiplicity and hardest and 3rd hardest jet pT spectra
- Uncertainty band contains ren. & fac. scale, PDF & merging scale dependence
- Rather good agreement between data and theory



# NLO+PS matching including EW corrections

- In POWHEG, two independent implementations of QCD+EW corrections to W-boson production exist [Bernaciak & Wackerath (2012); Barzè et al. (2012)]
- MG5\_aMC and Sherpa working towards automation. Some first results with Sherpa+OpenLoops have been presented, although they include only EW corrections of virtual origin [Kallweit et al. (2015)]



# Conclusions

- In the last couple of years the accuracy of event generation has greatly improved, and full automation has been achieved at NLO accuracy
- A lot of freedom in tuning has been replaced by accurate theory descriptions:
  - More predictive power
  - Better control on uncertainties
  - Greater trust in the measurements
- Recent developments for which I have had no time
  - NLO EW corrections and the parton shower
  - Combining NNLO in QCD and the parton shower: MINNLO
  - MC@NLO-Delta: reducing negative weights in MC@NLO