NLO EW

one additional layer of complexity

Relevance of EW Precision Observables

Precision Electroweak measurements on the Z resonance hep-ex/0509008

EWPO were crucial in order to constrain the Hboson and top-quark mass.

Today EWPO can be used to check the internal consistency of the SM.

In models where they can be calculated, as in the MSSM, EWPO can be used to constrain the parameter space.

Relevance of EW Precision Observables

$$
\hat{m}_{uv} = \frac{\partial \hat{v}}{\partial \hat{s}} = 79.794 \text{ GeV} \text{ Peterson}
$$
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$$
\hat{m}_{uv,pole} = (6) = 80.385 \pm 0.015 \text{ expansion}
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\text{SM} \text{ at } \text{tree-level} \text{ is folification}
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Relevance of EW Precision Observables

(Un) fortunately we connot colater fere in these Certures the actual loops lectures The actual loops
 $m_{w, \text{pole}}^2$ τ $\epsilon^2 m_z^2$ $\left[1 + \frac{3}{46\pi} \frac{3}{5} \frac{2}{(e^2 - 3)^2} \right]$ $\frac{m_e^2}{m_z^2}$ clominant
 $e^{\frac{3\pi}{24\pi}} - \frac{5\hat{\alpha}}{25\pi} \frac{\hat{c}^2 \hat{m}_2^2}{\hat{c}^2 - \hat{s}^2}$ Cof $\left(\frac{\hat{m}_H^2}{\hat{c}^2 \hat{m}_2^2}\right)$ $\frac{[G_{R}U]}{W|_{W}}$ These 1-eagle (t,b) 1-eagle (t,b,H)
 $W|_{W}$ 79.794 80.368 80.333

NLO EW

what about cross sections at the LHC?

In general small % effects on total cross sections, *BUT*.. ρ ilicus oli total cross scutions, DUI ...

$$
\text{couple of weeks on } \mathcal{O}(200) \text{ CPUs} \qquad \qquad \text{75} \qquad \qquad \delta_{\text{EW}} = \frac{\Sigma_{\text{NLO}_2}}{\Sigma_{\text{LO}_1}} = \frac{\text{NLO}}{\text{LO}} - 1 \, .
$$

Enhancements: final-state radiation (FSR)

In sufficiently exclusive observable FSR induces corrections $\sim \alpha Q_e^2 \log^2 (p_T^2(\ell)/m_e^2)$. Photon-fermion recombination reduces the size of this effect.

Figure 5: Invariant mass distribution around the Z peak (left) and relative effect of different contributions (right), for bare muons and recombined electrons.

Carloni Calame, Montagna, Nicrosini, Vicini '15 76

Enhancements: Sudakov logarithms

Weak corrections at large scales are not negligible for a general process due to the Sudakov Logarithms $\sim \alpha \log^2 \left(Q^2 / M_W^2 \right)$.

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hi,
'18 *Frederix, Frixione, Hirschi, DP, Shao, Zaro '18*

General aspects of one-loop EW amplitudes and their renormalisation

Ansgar Denner, [arXiv:0709.1075](http://arxiv.org/abs/arXiv:0709.1075)

I focus in this lecture on the LHC, but in a precision machine such as ILC the content of the following slides is even more important!

Why is NLO EW more complicated than NLO QCD? *u d* A µ i^µ

 $pp \rightarrow \mu^+ \nu_\mu$ One-Loop QCD diagrams*:

* Remember that in the on-shell scheme for masses and wave-function renormalisation, no loops corrections on the external legs have to be explicitly computed (LSZ).

Why is NLO EW more complicated than NLO QCD?

More renormalisation parameters, but not more than 13

In the EW sector of the SM there are **3** independent parameters for the **gauge interactions**, the **mass of the Higgs**, and **9 fermion masses** (CKM diagonal in these slides).

In the so called alpha(0)-scheme, with massive particles renormalised on-shell, the **3** independent parameters for the **gauge sector** are**:**

α MW MZ

with α measured in the Thomson scattering, with zero-momentum transferred. All the other EW parameters are predictions:

$$
v
$$
, G_F , $sin(\theta_W)$, y_t , λ , ρ , etc...

On the other hand, one can change input parameters :

$$
\{\alpha, M_W, M_Z\} \rightarrow \{G_\mu, M_W, M_Z\}, \{\alpha, G_\mu, M_W\}
$$

or renormalisation conditions for masses on-shell \longrightarrow MSbar,

and couplings $\alpha(0) \rightarrow \alpha(M_Z)$, G_μ , $\alpha^{\overline{MS}}$ 81

Renormalisation alpha(0)-scheme, on-shell masses (we denote bare quantities by an index 0) $\mathcal{L}_{\mathcal{A}}$ and $\mathcal{L}_{\mathcal{A}}$ and $\mathcal{L}_{\mathcal{A}}$ and $\mathcal{L}_{\mathcal{A}}$ and $\mathcal{L}_{\mathcal{A}}$ and $\mathcal{L}_{\mathcal{A}}$ Renormalisation alpha(0)-scheme, on-shell ma $\mathbf{H} \rightarrow \mathbf{V}$ tion al (Λ) sah (V)⁻ (U)-.
. $\overline{111}$ π ation alpha (0) -coheme on-chell macces Next wipha (v) benefine, on dien finaded T_{ref} $\left(\bigcap_{i=1}^n A_i\right)$ and $\left(\bigcap_{i=1}^n A_i\right)$ As a construction of the third condition of the condition of the construction of the calculations. particles in the Thomson limit. This means that all corrections that all corrections that all corrections that \mathbf{r} ona(U)-scheme, on-shell

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e_0 = Z_e e = (1 + \delta Z_e)e,
$$
 The bare fields (parameters
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M_{W,0}^2 = M_W^2 + \delta M_W^2,
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 Lagrangian are split into ren
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M_{Z,0}^2 = M_Z^2 + \delta M_Z^2,
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 constants, which in turn are deriv
\nfollowing renormalised Green fu
\n
$$
m_{f,i,0} = m_{f,i} + \delta m_{f,i},
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 q independent parameters we choose the physical parameters we choose the physical parameters specified in α

 $M_{Z,0}^2 = M_Z^2 + \delta M_Z^2$, constants, which in turn are derived $W_{W,0} = W_{W} + \sigma W_{W}$, $\tilde{\text{fields}}(\text{parameters})$ and renorm he bare 81 !!! u.
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ugran \overline{C} The care nere parameters $\frac{1}{5}$ $\frac{1}{5}$ 110100[/]h $\frac{L}{\text{fit}}$ \prod \mathbf{E} \mathbf{a}
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fers :rs) a ndus (parameter)

are split into re

eters) and reno ${\rm \, rangesed from the original context.}$ ❍ Green f \overline{A} n are $\overline{ }$ <u>LU</u> , accessor
een functi \mathbb{R}^n ! !
! The bare fields(parameters) in the ! <u>l</u> ! Lagrangian are split into renormalised ! ! ! fields(parameters) and renormalisation ! $\overline{}$ constants, which in turn are derived via the .
1 \lbrack $\overline{}$ following renormalised Green functions:

$$
W_{\mu} \left(\bigotimes_{k} W_{\nu} \right) = \hat{\Gamma}^{W}_{\mu\nu}(k)
$$

$$
\bigotimes_k^{a,\mu} \bigotimes \bigotimes \bigotimes \bigotimes \bigotimes^b \bigotimes = \hat{\Gamma}^{ab}_{\mu\nu}(k) \qquad a,b=A,Z
$$

$$
\frac{H}{k} \leftarrow \text{---} \text{---} \frac{H}{k} = \hat{\Gamma}^H(k)
$$

Renormalisation alpha(0)-scheme, on-shell masses

Requiring that tadpoles do not shift the minimum of the Higgs potential and that the renormalised masses of the bosons are the on-shell masses:

> $\delta t = -T,$ $\delta M_W^2 = \widetilde{\text{Re}} \, \Sigma_T^W(M_W^2), \qquad \qquad \delta Z_W \ = \ - \text{Re} \, \frac{\partial \Sigma_T^W(k^2)}{\partial k^2}$ ∂k^2 $\begin{array}{c} \hline \end{array}$ $\begin{array}{c} \hline \end{array}$ $\begin{array}{c} \hline \end{array}$ $\begin{array}{c} \hline \end{array}$ $k^2 = M_W^2$ W , $\delta M_Z^2 = \text{Re}\,\Sigma_T^{ZZ}(M_Z^2), \qquad \delta Z_{ZZ} = -\text{Re}\,\frac{\partial \Sigma_T^{ZZ}(k^2)}{\partial k^2}$ ∂k^2 $\begin{array}{c} \hline \end{array}$ $\begin{array}{c} \end{array}$ $\begin{array}{c} \hline \end{array}$ $\begin{array}{c} \hline \end{array}$ $|k^2 = M_Z^2$, $\delta Z_{AZ} = -2\mathrm{Re}\, \frac{\Sigma^{AZ}_T(M_Z^2)}{M_Z^2}$ M_Z^2 $, \quad \delta Z_{ZA}=2$ $\Sigma^{AZ}_T(0)$ M_Z^2 , $\delta Z_{AA} = \, - \frac{\partial \Sigma^{AA}_T(k^2)}{\partial k^2}$ ∂k^2 $\overline{}$ $\begin{array}{c} \hline \end{array}$ $\begin{array}{c} \hline \end{array}$ $\begin{array}{c} \hline \end{array}$ $k^2=0$, $\sum H / \sqrt{2}$.

$$
\delta M_H^2 = \text{Re}\,\Sigma^H(M_H^2), \qquad \delta Z_H = -\text{Re}\,\frac{\partial \Sigma^H(k^2)}{\partial k^2}\bigg|_{k^2 = M_H^2}
$$

Renormalisation alpha(0)-scheme, on-shell masses " k2=0 Renormalisation alpha(0)-scheme " $\frac{1}{2}$

 \mathcal{C} \downarrow 1 fermi masses: Requiring that also the renormalised masses of the fermion are the on-shell

$$
\delta m_{f,i} = \frac{m_{f,i}}{2} \widetilde{\text{Re}} \left(\Sigma_{ii}^{f,L}(m_{f,i}^2) + \Sigma_{ii}^{f,R}(m_{f,i}^2) + 2\Sigma_{ii}^{f,S}(m_{f,i}^2) \right),
$$

a
22 a 22 a 22 a 22

$$
\delta Z_{ij}^{f,L} = \frac{2}{m_{f,i}^2 - m_{f,j}^2} \widetilde{\text{Re}} \left[m_{f,j}^2 \Sigma_{ij}^{f,L}(m_{f,j}^2) + m_{f,i} m_{f,j} \Sigma_{ij}^{f,R}(m_{f,j}^2) + (m_{f,i}^2 + m_{f,j}^2) \Sigma_{ij}^{f,S}(m_{f,j}^2) \right], \qquad i \neq j,
$$

$$
\delta Z_{ij}^{f,R} = \frac{2}{m_{f,i}^2 - m_{f,j}^2} \widetilde{\text{Re}} \left[m_{f,j}^2 \Sigma_{ij}^{f,R}(m_{f,j}^2) + m_{f,i} m_{f,j} \Sigma_{ij}^{f,L}(m_{f,j}^2) + 2m_{f,i} m_{f,j} \Sigma_{ij}^{f,S}(m_{f,j}^2) \right], \qquad i \neq j,
$$

$$
\delta Z_{ii}^{f,L} = -\widetilde{\text{Re}} \,\Sigma_{ii}^{f,L}(m_{f,i}^2) - m_{f,i}^2 \frac{\partial}{\partial p^2} \widetilde{\text{Re}} \left[\Sigma_{ii}^{f,L}(p^2) + \Sigma_{ii}^{f,R}(p^2) + 2\Sigma_{ii}^{f,S}(p^2) \right] \Big|_{p^2 = m_{f,i}^2},
$$

$$
\delta Z_{ii}^{f,R} = -\widetilde{\text{Re}} \,\Sigma_{ii}^{f,R}(m_{f,i}^2) - m_{f,i}^2 \frac{\partial}{\partial p^2} \widetilde{\text{Re}} \left[\Sigma_{ii}^{f,L}(p^2) + \Sigma_{ii}^{f,R}(p^2) + 2\Sigma_{ii}^{f,S}(p^2) \right] \Big|_{p^2 = m_{f,i}^2}.
$$

Renormalisation alpha(0)-scheme, on-shell masses \cdot \cdot \cdot ! **Example 19 Example 19 Except 10 Exercise 10** 1 Rendinansation alpha(v)-seneme, on-shem δZAA) + Λ^f ^V (0) + Λ^f ^S(0) + v^f 1 \Box alpha (0) -sche 2 c^W 2 $\overline{}$ on-shel 2 ∂k² \lfloor L
! ll masses c^W M²

2

 \sim (3.32)

The alpha (0) condition for the QED interaction among the photon and the electron the field renormalization the corrections in the external legislation of the exter The alpha (0) condition for the QED interaction among the pho electron is a ward in the set of a Ward in the due to a Ward in the due to a Ward in the due to a Ward in the derived from the due to a Ward in the derived from the due to a Ward in the derived from the due to a Ward in th ine aipi \overline{y} contracts for the \overline{y} morne INTUON TOT THE QED INTERACTION among the photon and the

$$
\bar{u}(p)\Gamma_\mu^{ee\gamma}(p,p)u(p)\Big|_{p^2=m_e^2}=ie\bar{u}(p)\gamma_\mu u(p)
$$

✚✙

 $0 < \frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

❍❍❨❍

2

leads to f_{max} $\sum_{i=1}^{n}$ leads to

explicitly in (A.15). This yields in fact two conditions, namely

ii +

0 = [−]Q^f (δZ^e ⁺ ^δZf,V

$$
\delta Z_e = -\frac{1}{2} \delta Z_{AA} - \frac{s_W}{c_W} \frac{1}{2} \delta Z_{ZA} = \frac{1}{2} \frac{\partial \Sigma_T^{AA}(k^2)}{\partial k^2} \bigg|_{k^2=0} - \frac{s_W}{c_W} \frac{\Sigma_T^{AZ}(0)}{M_Z^2}
$$

!!

2

All the other counterterms can be obtained as function of the THE GREATING COMMUNICATION CONSEQUENCE OF THE CHOICE OF THE SPECIFIC CHOICE OF THE CONSEQUENTLY THE CON oblanicu, ioi cxampic, $\frac{11}{4}$ obtained, for example, Country, it dimitted, All the other counterterms can be obtained as function of those already $B - 3$

$$
\sin^2 \theta_W = s_W^2 = 1 - \frac{M_W^2}{M_Z^2} \qquad \frac{\delta c_W}{c_W} = \frac{1}{2} \left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} \right) = \frac{1}{2} \widetilde{\text{Re}} \left(\frac{\Sigma_T^W (M_W^2)}{M_W^2} - \frac{\Sigma_T^{ZZ}(M_Z^2)}{M_Z^2} \right),
$$
\n
$$
\frac{\delta s_W}{s_W} = -\frac{c_W^2}{s_W^2} \frac{\delta c_W}{c_W} = -\frac{1}{2} \frac{c_W^2}{s_W^2} \widetilde{\text{Re}} \left(\frac{\Sigma_T^W (M_W^2)}{M_W^2} - \frac{\Sigma_T^{ZZ}(M_Z^2)}{M_Z^2} \right).
$$

Renormalisation alpha(0)-scheme, on-shell masses

But also all the possible counterterms, here some examples:

VFF-coupling:

embedded in the $SU(2)xU(1)$ This is the QED vertex, broken symmetry of the SM.

with the actual values of V, \bar{F}_1 , F_2 and C^+ , C^-

$$
\gamma \bar{f}_i f_j \qquad : \quad \begin{cases} C^+ = -Q_f \Big[\delta_{ij} \Big(1 + \delta Z_e + \frac{1}{2} \delta Z_{AA} \Big) + \frac{1}{2} (\delta Z_{ij}^{f,R} + \delta Z_{ij}^{f,R\dagger}) \Big] + \delta_{ij} g_f^+ \frac{1}{2} \delta Z_{ZA}, \\ C^- = -Q_f \Big[\delta_{ij} \Big(1 + \delta Z_e + \frac{1}{2} \delta Z_{AA} \Big) + \frac{1}{2} (\delta Z_{ij}^{f,L} + \delta Z_{ij}^{f,L\dagger}) \Big] + \delta_{ij} g_f^- \frac{1}{2} \delta Z_{ZA}, \end{cases}
$$

SSSS-coupling: $\ddot{\ }$

 $\overline{2}$ 2^2 This is the quartic Higgs self coupling. Do you see λ from the Higgs potential?

with the actual values of S_1 , S_2 , S_3 , S_4 and C S_4 and C_5

$$
HHHH \quad : \quad C = -\frac{3}{4s^2} \frac{M_H^2}{M_W^2} \Big[1 + 2\delta Z_e - 2\frac{\delta s}{s} + \frac{\delta M_H^2}{M_H^2} + \frac{e}{2s} \frac{\delta t}{M_W M_H^2} - \frac{\delta M_W^2}{M_W^2} + 2\delta Z_H \Big],
$$

Renormalisation scheme

Should I use alpha(0) for any calculation? NO!

Only vertices involving a final-state on-shell photon are in a physical configuration similar to the Thomson scattering. Otherwise, for large scales,

$$
\frac{\Sigma_T^{AA}(k^2)}{k^2} - \frac{\Sigma_T^{AA}(k^2)}{k^2} \bigg|_{k^2=0} \sim \sum_{m_f} \frac{\alpha}{3\pi} Q_f^2 K_f^2 \bigg[-\frac{5}{3} + \log(Q^2/m_f^2) + \mathcal{O}(m_f^2/Q^2) \bigg] \qquad \text{for } Q > 2m_f
$$

In other words, this scheme is "Infrared-sensitive" and induces large corrections due to the running of alpha from the scale of the mass of the electron (0.5 MeV) to the typical \sim 0.1-1 TeV scale at the LHC.

Unless final-state on-shell photons are considered, other input parameters and renormalisation conditions are preferable. Indeed, in these cases the alpha(0) scheme artificially enhances loop corrections.

z alpha(Mz) 2 λ *pha* $\overline{\mathbf{r}}$ 2 \mathcal{I}_7 @*k*² *^Z*) scheme (and generalization to ↵(*Q*²) scheme) common way is using the ↵(*Q*2) scheme, where typically *Q* = *m^Z* and has been exploited in the precision studies at $\mathbf{alpha}(\mathbf{Mz})$ is not the straightforward general in the precision studies at the *Z* peak at LEP.) for *p*² = *m*² *^e*, *p*0² = *m*²

*^k*2=0 (1.2)

)² = *Q*2.

alization of the condition of eq. 1.1, i.e., ˆ*ee*

The scheme ↵(*Q*2) is defined as

Starting from the alpha(0) scheme As we said, one has to get rid of the *m^f* dependence, but not via the ↵MS-scheme. The Starting from the alpha(0) scheme The first important point to remember is that ↵(*Q*2) is NOT the straightforward gener-Starting from the alpha(0) scheme alization of the condition of eq. 1.1, i.e., ˆ*ee*

 $\delta Z_e = -\frac{1}{2}$ 2 $\delta Z_{AA} - \frac{s_W}{c_W}$ *c^W* 1 2 δZ_{ZA} $\delta Z_{AA} = -\frac{\partial \Sigma_T^{AA}(k^2)}{\partial k^2}\Big|_{k^2=0}$ ∂k^2 $\overline{}$ $\overline{}$ $k^2=0$ $\frac{1}{27}$ $\frac{1}{27}$ $\sigma \Omega_e = 2^{\sigma \Omega_{AA}}$ $c_W 2^{\sigma \Omega_{ZA}}$ $\sigma \Omega_{AA} = 0$ ∂k^2 $|_{k^2=0}$ $\frac{\partial S_W}{\partial \partial \overline{Z}} \frac{1}{2} \delta Z_{ZA} \qquad \qquad \delta Z_{AA} = - \frac{\partial \Sigma_T^{AA}(k^2)}{\partial k^2}$ *Ze|*↵(*Q*2) ⌘ *Ze|*↵(0) ¹ $\left|\frac{T^{(k-1)}(k)}{2!2}\right|$

 $h^{(n)}$ and define the elpha (Ω) sepand we can acting the argum(χ) seneme alization of the condition of eq. 1.1, i.e., ˆ*ee ^µ* (*p, p*⁰ *^e*, *p*0² = *m*²) for *p*² = *m*² we can define the alpha(Q) scheme where I is the translation relation to the translation \mathcal{L}

$$
\delta Z_e|_{\alpha(Q^2)} \equiv \delta Z_e|_{\alpha(0)} - \frac{1}{2} \Delta \alpha(Q^2) \qquad \Delta \alpha(Q^2) \equiv \Pi_{f \neq t}^{AA}(0) - \Re{\{\Pi_{f \neq t}^{AA}(Q^2)\}}
$$

I find is not alpha measured at the Q scale, it is alpha
 GED contributions of the vacuum polarisation *f*62. For the contributions of the vacuum polarization with 3
evaluated at the scale Q and me. In practice, it takes care of Q $\mathbf t$ top quark. We remind that the definition of vacuum polarization of vacuum polarization is $\mathbf t$ $f(t)$ is the contribution to the vacuum polarization in the vacuum polarization in $f(t)$ alpha measured at the Q scale, it is alpha(0) + the difference **b** of QED running. This is not alpha measured at the Q scale, it is alpha (0) + the difference of the additional term in the renormalization conditions condition the respective term in the condition of the stream in the shift of the stream in the condition of the stream in the stream in the condition of the stream in the c eq. (3.8). Given the RGE structure, we can even define ↵(*Q*2) = ↵(0) aunt polaitisau evaluated at the scale Q and me. In practice, it takes care of QED running. QED contributions of the vacuum polarisation with 5 active flavours $f(x) = f(x)$ terms. Coming back to the real world, since α is the real world, since α is the since α ⇧*AA*(*k*2) = ⌃*AA* k top contributes to ↵(*Q*2) as α at the beare χ a ference of the

$$
\Delta \alpha(Q^2) = \Delta \alpha_{\text{leptons}}(Q^2, m_f) + \Delta \alpha_{\text{quarks}}^{(5)}(Q^2, m_f) \qquad \Delta \alpha_{\text{leptons}}(Q^2, m_f) \equiv \sum_{f=e,\mu,\tau} \Delta \alpha_{\text{fermion}}(Q^2, m_f)
$$

For $Q > 2m$ we obtain σ ² $\frac{d}{dx}$ 2*m*^{*f*} we obtain For $Q > 2m_f$ we obtain

For
$$
Q > 2m_f
$$
 we obtain
\n
$$
\Delta \alpha_{\text{fermion}}(Q^2, m_f) = \frac{\alpha}{3\pi} Q_f^2 K_f^2 \left[-\frac{8}{3} + \beta^2 - \frac{1}{2}\beta(3-\beta^2) \log \left(\frac{1-\beta}{1+\beta} \right) \right] =
$$
\n
$$
\frac{\alpha}{3\pi} Q_f^2 K_f^2 \left[-\frac{5}{3} + \log(Q^2/m_f^2) + \mathcal{O}(m_f^2/Q^2) \right]
$$
\nwith $\beta = \sqrt{1 - \frac{4m_f^2}{Q^2}}$

alpha(Mz) $\overline{}$ 1 p $\overline{\ }$ ⌘ $\mathfrak{u}(\mathbf{M}_Z)$ + log(*m*² o

On the other hand, while the leptonic contribution can be perturbatively calculated, the hadronic contribution must be extracted by measurements. *^Q*²)

⇣

Still the best prediction for alpha at Q=Mz can be obtained from the measurements of alpha(0) and the measurements of the hadronic contribution. ב
1 *e*¹/_a_l_/**d**</sup> at Q=M_z can be obtai $\overline{\mathbf{1}}$ \overline{U} c obtoined ed from the dronic *c* contribution.

The alpha(Mz) scheme is not IR sensitive and has an MSbar-like structure for the counterterm of the electric coupling. *alpha(Mz)* **scheme is not II** *k*
k sen \mathbf{n} log *^m*² *Q*² **1**
1 $\frac{1}{2}$ alpha(Mz) scheme is not IR sensitive and has an MSbar-lik UP COUNTENTING THE ELECTIC COUPLING. $\ddot{}$ IR sensitive and has θ electric coupling. $\frac{1!}{2!}$ \mathbf{r} succes

In other words, alpha in the MSbar scheme can be obtained from alpha (Q) + a finite term. in ouier words, alpha in α innic term. t e MSl the MSbar scheme can be obtained from alpha (Q) +

$$
\alpha_{\overline{\rm MS}}(Q^2) = \alpha(Q^2)(1 + \bar{\Delta}\alpha(Q^2)) + \mathcal{O}(\alpha^3 + \alpha\alpha_s)
$$

$$
\bar{\Delta}\alpha(Q^2) \equiv \frac{\alpha}{\pi} \left(\frac{100}{27} - \frac{4}{9} \log(\frac{m_t^2}{Q^2}) \theta(Q^2 - m_t^2) + \frac{7}{4} \left(\log \frac{m_W^2}{Q^2} \right) - \frac{1}{6} \right) \qquad Q > m_W
$$

Gµ-scheme

We can also extract the EW interactions from muon decay. of *W*-induced corrections to *Z^e* we just considered the QED-like interactions with photons.

5 *Gµ*-scheme

The muon-lifetime τ , can be written in the Fermi Model plus OF μ interaction. Traction diagram for ordinary model diagram for ordinary muon decay indicating the contraction of μ The muon-lifetime τ_{μ} can be written in the Fermi Model plus QED as

$$
\frac{1}{\tau_{\mu}} = \frac{G_{\mu}^2 m_{\mu}^5}{192\pi^3} \left(1 - 8\frac{m_e^2}{m_{\mu}^2}\right) \left(1 + F(\alpha)\right) \tag{5.1}
$$

 $F(s)$ is a function that incorporates all the pessible $\triangle F$ com. where $I(\alpha)$ is a ranchon that meet porates an the possible will conwhere $F(\alpha)$ is a function that incorporates all the possible QED corrections.

One can measure τ_μ and determine G_μ . Then, one can calculate τ_μ in the SM and thus re-express G_μ via the SM parameters. One gets

$$
G_{\mu} = \frac{\pi \alpha}{\sqrt{2s_W^2 m_W^2}} (1 + \Delta r(\alpha(0), m_W, m_Z, m_H, m_t, m_f))
$$
(5.2)

Starting from one loop, all SM masses are entering *G^F* . The roots of *G^F* are Fermi's theory—based on an analogy between the

G_μ-scheme it, the already present in *F*(μ) must not be double-counted. The contributions already present in *F*(α) must not be double-counted. The contributions already present in α

$$
\Delta r = \Pi^{AA}(0) + \frac{\Sigma_T^{WW}(0) - \Re(\Sigma_T^{WW}(m_W^2))}{m_W^2} - \frac{c_W^2}{s_W^2} \Re\left(\frac{\Sigma_T^{ZZ}(m_Z^2)}{m_Z^2} - \frac{\Sigma_T^{WW}(m_W^2)}{m_W^2}\right) + 2\frac{c_W}{s_W} \frac{\Sigma_T^{AZ}(0)}{m_Z^2} + \frac{\alpha}{4\pi s_W^2} \left(6 + \frac{7 - 4s_W^2}{2s_W^2} \log\left(\frac{m_W^2}{m_Z^2}\right)\right)
$$

Let's see what are the dominant contributions. A doublet of light fermions give a contribution

$$
\Delta r_{\text{light fermions}} = \frac{\alpha}{3\pi} \sum_{f=\pm} \left[Q_f^2 \left(\log \left(\frac{m_Z^2}{m_f^2} \right) - \frac{5}{3} \right) + \ldots \right]
$$

while a doublet with an heavy up-type and light down-type

$$
\Delta r_{\rm heavy/light\ doublet} = -\frac{\alpha}{4\pi} \frac{c_W^2}{4s_W^2} \frac{m_+^2}{m_W^2} + \dots
$$

G_{μ} -scheme Δ see what are the dominant contributions. A doublet of Δ doublet of light fermions give a contributions give a contri-" Ξ ! *r* α ^{*r*} + *...* (5.5) *s^W* $\sum_{i=1}^{\infty}$ dominant contributions. A dominant contributions give a contributions give a contributions give a contri-

+ 2

$$
\Delta r = \Pi^{AA}(0) + \frac{\Sigma_T^{WW}(0) - \Re(\Sigma_T^{WW}(m_W^2))}{m_W^2} - \frac{c_W^2}{s_W^2} \Re\left(\frac{\Sigma_T^{ZZ}(m_Z^2)}{m_Z^2} - \frac{\Sigma_T^{WW}(m_W^2)}{m_W^2}\right) + 2\frac{c_W}{s_W} \frac{\Sigma_T^{AZ}(0)}{m_Z^2} + \frac{\alpha}{4\pi s_W^2} \left(6 + \frac{7 - 4s_W^2}{2s_W^2} \log\left(\frac{m_W^2}{m_Z^2}\right)\right)
$$

Let's see what are the dominant contributions. A doublet of light fermions give a contribution < " Ξ ! ! \cdot \uparrow dou $\frac{1}{2}$ et o see what are the dominant contributions. A doublet of light fermions give a contri-
n *Z m*² *Z m*² *W m*² *W* $\overline{\operatorname{li}}$ $\frac{1}{2}$ $\frac{1}{2}$ *cw* (5.6) One can see that eq. (5.4) includes exactly the contribution of the light fermions to ↵(*m*² which we call here simply α includes α and α and α and α and α and α and α

f=*±*

$$
\Delta r_{\rm light\ fermions} =
$$

$$
\Delta r_{\rm light\ fermions} = \boxed{\rm \qquad the\ contribution\ of\ the\ light\ fermions\ to\ }\Delta\alpha(m_Z^2)}
$$

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2*s*²

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while a doublet with an heavy up-type and light down-type which we call here simply a simply a simply and more. The term includes <u>a</u>nd more. The term in eq. (5.5) and more. The term i $\sum_{i=1}^n \sum_{i=1}^n \sum_{j=1}^n \sum_{j$ while a doublet with an heavy up-type and light down-type W *n*¹*z*ne *m*²

$$
\Delta r_{\rm heavy/light\ doublet} = \left| \begin{array}{cc} -\frac{c_W^2}{s_W^2} \Delta \rho & \rho \text{ parameter defined via} \\ \frac{c_W^2}{s_W^2} \Delta \rho & \text{charged and neutral neutrino currents} \end{array} \right|
$$

*m*²

*m*²

G_{μ} -scheme Δ see what are the dominant contributions. A doublet of Δ doublet of light fermions give a contributions give a contri-" Ξ ! *r* α ^{*r*} + *...* (5.5) *s^W* $\sum_{i=1}^{\infty}$ dominant contributions. A dominant contributions give a contributions give a contributions give a contriwhile a doublet with a doublet with an \mathbf{U}_{μ} -SCHEIN

+ 2

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*m*² *f*

$$
\Delta r = \Pi^{AA}(0) + \frac{\Sigma_T^{WW}(0) - \Re(\Sigma_T^{WW}(m_W^2))}{m_W^2} - \frac{c_W^2}{s_W^2} \Re\left(\frac{\Sigma_T^{ZZ}(m_Z^2)}{m_Z^2} - \frac{\Sigma_T^{WW}(m_W^2)}{m_W^2}\right) + 2\frac{c_W}{s_W} \frac{\Sigma_T^{AZ}(0)}{m_Z^2} + \frac{\alpha}{4\pi s_W^2} \left(6 + \frac{7 - 4s_W^2}{2s_W^2} \log\left(\frac{m_W^2}{m_Z^2}\right)\right)
$$

Let's see what are the dominant contributions. A doublet of light fermions give a contribution < " Ξ ! ! \cdot \uparrow dou et o see what are the dominant contributions. A doublet of light fermions give a contri-
n *Z m*² *Z m*² *W m*² *W* $\overline{\operatorname{li}}$ $\frac{1}{2}$ $\frac{1}{2}$ *cw* (5.6) One can see that eq. (5.4) includes exactly the contribution of the light fermions to ↵(*m*² which we call here simply α includes α and α and α and α and α and α and α are the dominant contributions. A do *m*² *W Z m*² *Z n*
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2011 *W m*² *W* light fermio = 2 *cw*

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The old times and neutral neut

$$
\Delta r_{\rm light\ fermions} =
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$$
\Delta r_{\rm light\ fermions} = \boxed{\rm \qquad the\ contribution\ of\ the\ light\ fermions\ to\ }\Delta\alpha(m_Z^2)}
$$

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*m*²

while a doublet with an heavy up-type and light down-type which we call here simply a simply a simply and more. The term includes <u>a</u>nd more. The term in eq. (5.5) and more. The term i $\sum_{i=1}^n \sum_{i=1}^n \sum_{j=1}^n \sum_{j$ while a doublet with an heavy up-type and light down-type W *n*¹*z*ne while a doublet with an heavy up-type and light down-type

3⇡

f=*±*

$$
\Delta r_{\text{heavy/light doublet}} = \frac{c_W^2}{s_W^2} \Delta \rho
$$
 p parameter defined via charged and neutral neutrino currents
The crucial point is that we can write

*m*²

*m*²

The crucial point is that we can write The crucial p The crucial point is that we can write

$$
\Delta r = \Delta \alpha - \frac{c_W^2}{s_W^2} \Delta \rho + \Delta r_{\rm rem}
$$

 \overline{a} $\frac{1}{\sqrt{2\pi}}$ (*m*₂) and subdominant corrections WILII $\Delta r_{\rm r}$ $\begin{tabular}{c} \multicolumn{2}{c}{\textbf{subdominant cor}}\\ \end{tabular}$ *W* with Δr_{rem} including all subdominant corrections ctions *remains*

so it is a correction in the seen as a correction to the seen as a correction to the μ parameter as a correction to the μ $0.06, \quad \Delta \rho \sim$ $0.01, \quad \Delta r_{\text{rem}}$ 0.01 $\rightarrow \Delta r \sim 0.03$ $\Delta\alpha$ \sim 0.00, $\Delta\rho$ \sim 0.01, Δr *r* \overline{a} \overline{b} \overline{c} \overline{c} \overline{c} $\Delta \alpha \sim 0.06, \quad \Delta \rho \sim 0.01, \quad \Delta r_{\rm rem} \sim 0.01 \quad \rightarrow \quad \Delta r \sim 0.03$

*m*² *Z*

$G_μ$ -scheme

The correct prescription for linking G_{μ} to SM parameters resumming higher-orders $\Delta \rho$ and $\Delta \alpha$ effects is $\bar{s}_W^2 \equiv s_W^2 + c_W^2 \Delta \rho$ $\frac{M}{2}$

$$
G_{\mu} = \frac{\pi \alpha(0)}{\sqrt{2}\bar{s}_{W}^{2}m_{W}^{2}} \frac{(1 + \Delta r_{\text{rem}})}{1 - \Delta \alpha} = \frac{\pi \alpha(m_{Z}^{2})}{\sqrt{2}\bar{s}_{W}^{2}m_{W}^{2}} (1 + \Delta r_{\text{rem}})
$$

³I guess it would not depend on *m^f* if another IR-safe definition of ↵ would be used In conclusion the renormalization condition is defined as \mathbf{I} conclusion is defined as a condition is defined as \mathbf{I} as a condition is defined as \mathbf{I} as a condition is defined as a condi nondition is defined as

^c^W , as can be inferred comparing

g = *e/s^W* , so we can see that

In conclusion the renormalization condition is defined as
\n
$$
\delta Z_e|_{G_\mu} \equiv \delta Z_e|_{\alpha(0)} - \frac{1}{2}\Delta r = \delta Z_e|_{\alpha(m_Z^2)} - \frac{1}{2}\left(-\frac{c_W^2}{s_W^2}\Delta \rho + \Delta r_{\text{rem}}\right)
$$

Every time we have a W interaction with a quark the coupling is $q = e/s_W$ so \mathcal{L} the renormalization for the charge. We use \mathcal{L} Every time we have a *W* interaction with a quark the coupling is $g = e/s_W$, The ⇢ term is the leading contribution to 2*c^W* **c**
We inferred comparing compar Every time we have a *W* interaction with a quark the coupling is $g = e/s_W$, so Every time we have a *W* interaction with a quark the coupling is \mathbb{R} is the coupling is \mathbb{R} time we have a *W* interaction with a quark the coupling is $g = e/s$

–8–

eqs. (5.3) and (5.6). Every time we have a *W* interaction with a quark the coupling is

s^W

. (5.3) and (5.5). Every time we have a *maximum with a we have a with a with a with a with a quark the coupling i*
.

that is if *Z^e* is in the *Gµ*-scheme (eq. (5.11)) the finite effect induced by the *s^W* coun-

Z^e ^e

s^W

sw

⁼ *^g*(*Z^e s^W*

eqs. (5.3) and (5.6). Every time we have a *W* interaction with a quark the coupling is

$$
\delta g = \frac{e}{s_W} \delta Z_e - \frac{e}{s_W} \frac{\delta s_W}{s_w} = g(\delta Z_e - \frac{\delta s_W}{s_w}) = g(\delta Z_e + \frac{c_W}{s_W^2} \delta c_W) \sim g(\delta Z_e - \frac{1}{2} \frac{c_W^2}{s_W^2} \Delta \rho)
$$

*n*dition for the charge *W W W W* dition for the *charge* that induced by the *La*ge is especied by the rememe (institution condition for the spen t_{max} is called by the respective by the renormalization condition for the charge. the finite effect induced by the δs_W is canceled by the renormalization condition for the charge. t_{t} or the renormalization for the renormalization for the renormalization for the relationships of the relati *che* innue enect induced by the σs_W is canceled by the step.

n receives corrections that is if *Z^e* is in the *Gµ*-scheme (eq. (5.11)) the finite effect induced by the *s^W* coun-Another way to see this: every $g^{\omega}=\alpha/s_W^{\omega}$ term in the cross-section receives cor: $\int \ln \frac{s_W^2}{s_W^2}$ from \overline{c} \overline{c} \overline{c} that is in the $\frac{1}{2}$ is induced by the finite effect induced by the finite effect induced by the s $\frac{1}{2}$ Another way to see this. Every $g - \alpha/s_W$ term in the cross-section receives corrections
to s_{xx}^2 from ² *cw*⇢ in the steps. terterm is canceled by the renormalization for the renormalization for the relation \mathcal{L} Another way to see this: every $g^2 = \alpha/s_W^2$ term. δ Another way to see this: every $a^2 - \alpha/s^2$, term in the cross-section rece \overline{c} $\frac{1}{\sqrt{N}}$ the correction receives correction $\frac{2}{\sqrt{N}}$ $A \times g = \alpha / s_W$ term in the cross-section receives corrections Another way to see this: every $g^2 = \alpha/s_W^2$ term in the cross-section receives corrections

$$
s_W^2 + \delta s_W^2 \sim s_W^2 + c_W^2 \delta \rho
$$

which a $\frac{d}{dx}$ $\frac{d}{dx}$. *^W* ⇠ *^s*² which are then canceled by $\delta Z_e|_{G_\mu}$. which are then canceled by $\delta Z_e|_{G_\mu}$.

to *s*²

Schemes recap $\frac{1}{\sqrt{2}}$ schemes recap

S(0) → Qf \sim

e Gmu scheme is $WUVU$, it a photon is present in the DOM ding renormalisation should be used for the renormalisation should be used for top contributes to ↵(*Q*2) as ↵fermion(*Q*2*, m^f*) = ↵ $\overline{\mathcal{O}}$ wever, if a photon is present in the Born *^q*² (*B*0(*q*2; *m, m*) *^B*0(0; *m, m*))]*}* ⁺ *^O*(✏) (3.4) *s*^{*s*}*ss***^{***s***}** *s***^{***s***}***s*</sup> *s*^{*s*}*s***^{***s***}</sup>** *s<i>s***^{***s***}</sup>** *s<i>s***^{***s***}</sup>** *s***^{***s***}** *s***^{***s*} *Z^e ^e* has *e*, for a generic process at the LHC, the C *s^W z*
z e prefei ϵ *W s a z* process at u
Iowever, if a *sw g*₂ b₁ d₂^{*d*} + *g*^{α} + *g*<sup> $\$ *s*2 final-state, alpha(0) and the corresponding renormalisation should be used for As a rule of mumo, for a generic process at the LHC, the GMU scheme is
superior and has to be preferred. However, if a photon is present in the Born the associated QED vertex. As a rule of thumb, for a generic process at the LHC, the Gmu scheme is

 \vdots corresponding content G_{μ}

 μ

Do you remember that in QCD:

it can be more complex: NLO EW and Complete-NLO

Structure of NLO EW-QCD corrections Γ Sudduct of NLO EW-QU \mathbb{P}^{∞} of NII \cap EWI \cap CD corrections The orient to the same of the corrections

Structure of NLO EW-QCD corrections Γ \mathbb{P}^{∞} of NII \cap EWI \cap CD corrections The orient to the same of the corrections

Structure of NLO EW-QCD corrections Γ \mathbb{P}^{∞} of NII \cap EWI \cap CD corrections The orient to the same of the corrections

Structure of NLO EW-QCD corrections Γ \mathbb{P}^{∞} of NII \cap EWI \cap CD corrections The orient to the same of the corrections

Structure of NLO EW-QCD corrections Γ \mathbb{P}^{∞} of NII \cap EWI \cap CD corrections The OTIME DE M-CON CONSCITONS

Structure of NLO EW-QCD corrections Γ \mathbb{P}^{∞} of NII \cap EWI \cap CD corrections The OTIME DE M-CON CONSCITONS

Structure of NLO EW-QCD corrections

 $t\bar{t}H$ We can denote the complete set of LO,i and NLO,i as "Complete NLO". **"Complete NLO"**.

as example

 $NLO,1 = NLO QCD$ $NLO,2 = NLO EW$

 $A = NLO QCD$ In general, NLO,3 and NLO,4 sizes are negligible, In general, NLO,3 and NLO,4 sizes are negligible, but there are exceptions.

that are the control of α $\mathcal{L} = \frac{1}{2} \sum_{i=1}^{n} \frac{1}{i}$ ivergencies (let 's forget about H) \mathbf{y} in the \mathcal{L} case and the substitution of a gluon with a photon. The substitution of a gluon with a photon. = $\frac{1}{t}$ $\frac{q}{t}$ $\frac{t}{\sqrt{t}}$ interference of Fig. 2 and \sim 4 and \sim 4 \sim \sim ¯ $=$ $\sqrt{1}$ \sim 2 \mathbf{r} $\sim \frac{q}{\sqrt{q}}$ q $\begin{array}{cc} g & t \\ 0 & 0 & 1 \end{array}$ "
" $\overline{}$ $q\bigcup_{\alpha}q$ $\frac{1}{1-\lambda}$ $\begin{array}{cc} \hline \end{array}$ a, $\begin{array}{cc} \hline \end{array}$ d f_g t \overline{q} t g Figure 2: Real emissions of gluon: photon in the propagator \mathcal{P}_c t g \overline{q} t t Complications for IR divergencies (let's forget about H) $\frac{1}{\sqrt{2}}$ $q\bar{q} \rightarrow t\bar{t}g$ $t \nearrow$ and the three gluon propagator three gluon propagator three gluon propagator that appears in the O(a $q\bar{q} \rightarrow tt$ \bar{t} \overline{a} \overline{a} $t \rightarrow \frac{q}{\sqrt{q}} \frac{g}{\sqrt{q}} \frac{t}{\sqrt{q}}$ $\frac{1}{t}$ $\overline{|\mathcal{M}^{t\bar{t}}|}^2_{\mathcal{O}(\alpha^3)}$ of \overline{q} of t the $\overline{|\mathcal{M}^{t\bar{t}g}|}^2_{\mathcal{O}(\alpha^2)}$ section

 $\overline{|\mathcal{M}^{t\bar{t}}|}$

 $\mathcal{O}(\alpha_s^3)$

 $\binom{3}{s}$

 \sqrt{q} s-channel q and $\frac{q}{q}$ and $\frac{q}{f}$ to the $\frac{q}{q}$

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 $\sum_{\alpha,\beta} g_{\alpha\beta}$ or $\sum_{\beta} g_{\beta\alpha}$

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At NLO QCD, IR divergencies in the loops are canceled by corresponding (Fig. 2). Following the must consider the color structure argument we color structure also emissions of photons of η the double structure we have shown before. So what happens? μ iuon real chinssions. At INEO E W We have also chinssions of photons, but also y conceponding
Shotong but also $\frac{A}{\sigma}$ At NLO QCD, IR divergencies in the l IV
11011 Freah emissions. At NLO EW we have a $\frac{S}{1}$ \int_{0}^{π} At NLO QCD, IR divergencies in the loops are canceled by corresponding gluon real emissions. At NLO EW we have also emissions of photons, but also

 \mathcal{A}

 $\overline{\left|{\cal M}^{t \bar t g}\right|^2_\ell}$

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 \overline{q} don't depend on the external momenta and \overline{q}

 $\frac{1}{4}$

 ${\cal O}(\alpha_s^3)$

 \overline{q}

 $\stackrel{\cdot}{q}$

that are the control of α $\mathcal{L} = \frac{1}{2} \sum_{i=1}^{n} \frac{1}{i}$ ivergencies (let 's forget about H) \mathbf{y} interference of Fig. 2 and \sim 4 and \sim 4 \sim \sim ¯ Figure 2: Real emissions of gluon: photon in the propagator \mathcal{P}_c Complications for IR divergencies (let's forget about H)

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At NLO QCD, IR divergencies in the loops are canceled by corresponding (Fig. 2). Following the must consider the color structure argument we color structure also emissions of photons of η the double structure we have shown before. So what happens? μ iuon real chinssions. At INEO E W We have also chinssions of photons, but also y conceponding
Shotong but also $\frac{A}{\sigma}$ l IV
11011 Freah emissions. At NLO EW we have a $\frac{S}{1}$ \int_{0}^{π} At NLO QCD, IR divergencies in the loops are canceled by corresponding gluon real emissions. At NLO EW we have also emissions of photons, but also

\overline{S} $\overline{9}$ **PRT DIGRESSION** $\boldsymbol{\delta} H \boldsymbol{U} K I \boldsymbol{U} I \boldsymbol{U} K L \boldsymbol{\delta} \boldsymbol{\delta} I \boldsymbol{U} I \boldsymbol{V}$ *SHORT DIGRESSION*

This discussions concern the OED part of the NI Ω EW corrections NI Ω This discussions concern the QED part of the NLO EW corrections. NLO EW $\sum_{i=1}^{n}$ and $\sum_{i=1}^{n}$ part of the NLO EW concernois, NLO can be divided mio a **QED** part (photome emission of can be divided mo a QED part (photome emission or loop corrections
fermions) and a **purely Weak** part (all the rest, including vacuum polarisation ithinons) and a purery wean part (an the rest, incruding vacuu can be divided into a **QED** part (photonic emission or loop corrections from corrections of parameters is not in general gauge in the theore. fermions) and a **purely Weak** part (all the rest, including vacuum polarisations). However this separation is **not** in general **gauge-invariant**!

$\ddot{}$ t $\overline{\mathbf{r}}$ $\overline{}$ noiog (1 t t q ا
⊢ا t that are the control of α $\mathcal{L} = \frac{1}{2} \sum_{i=1}^{n} \frac{1}{i}$ ivergencies (let 's forget about H) $\frac{1}{2}$ interference of Fig. 2 and \sim 4 and \sim 4 \sim \sim ¯ QCD) and QED (Ftt QED) cases, and obtain the ratio of them. es (let's forget about H) q γ t ibout Figure 2: Real emissions of gluon: photon in the propagator \mathcal{P}_c Complications for IR divergencies (let's forget about H) $\frac{1}{\sqrt{2}}$

 $\frac{1}{2}$

 $\overline{\tilde{\mathcal{M}}^{t\bar{t}}g}$

 $\frac{1}{4}$

 $\frac{1}{1-\lambda}$ \mathcal{A}

 $\overline{\left|{\cal M}^{t \bar t g}\right|^2_\ell}$

 $\frac{1}{2}\log$

 $\mathcal{O}(\alpha_s^3)$

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 $|\mathcal{M}^{t\bar{t}\gamma}|$

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 $\mathcal{O}(\alpha_s^2)$

 $\begin{array}{c} \text{hotons and gluons at the same time} \\ \text{obtainir} \end{array}$ encies! diagrams and more in ر
diagram diagrams a divergencies! d leads to more diagrams and more IR obtaining IR finiteness and more IR Photons and gluons at the same time have both to be considered. divergencies! divergences, only infrared singularities can arise. After regularities can arise term α mass term α mass term α

 \mathbb{R} argument we can identify the considered for structure and the considered for \mathbb{R}^n $\chi^t = \frac{q}{r}$ to the results of \mathbf{r} and \mathbf{r} the results of \mathbf{r} \mathbf{r} obtai \mathbf{r} nng ny finiteness at NL $\frac{1}{2}$ EQUATION OF PROTONS and gluons. aining IR finiteness at \mathbf{I} obtaining IR finiteness at NLO EW. \mathbf{r} $\frac{1}{1}$ t $\overline{\mathbf{C}}$ \mathbf{y} $\left\langle \frac{q}{q}\right\rangle$ $\left\langle \frac{q}{q}\right\rangle$ $\left\langle \frac{q}{q}\right\rangle$ Radiation of photons and gluons These diagram are shown in Fig. 3, also a diagram with the trigluon vertex can be designed vertex can be de our to be considered to W . have both to be considered for

 $\frac{q}{q}$ in $\frac{q}{q}$ in the charges of incoming in the charges of incoming

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 $q\bar{q} \rightarrow t\bar{t}$

 $\sum_{i=1}^{n} a_i$

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 $t \searrow \gamma$

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 $\sum_{t=1}^{n}$

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16 · 9

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 $q\bar{q} \rightarrow t\bar{t}g$

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 $t \sim q \sqrt{\gamma}$

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 & 0 & 0 & 0 & 0\n\end{array}$

' '

 $\frac{g}{\left|\mathcal{M}^{t\bar{t}g}\right|^2}$ are two different color structures and the result depends on d2 $\frac{g}{q}$

 $\frac{t}{\sqrt{t}}$

t

t

 $\frac{1}{2}$

T r!

 $\begin{array}{cc} \begin{array}{ccc} \begin{array}{ccc} \hline \end{array} & q & \end{array} & \begin{array}{ccc} g & \bullet & \end{array} \\ \hline \begin{array}{ccc} a & \end{array} & \begin{array}{ccc} g & \bullet & \end{array} & \end{array}$

 \sim subprocess can be evaluated through the results obtained for \sim

 \overline{g}

Figure 3: Real emissions of gluon

 $q\bar{q} \rightarrow ttg$

and combining the result of $\frac{q}{t}$

g crossed, so it's easy to understand how asymmetric term can arise, but its contribution to AF B

substituting with a photon propagator of the three gluon propagator three gluon propagator that appears in the t

 $\sum_{\alpha,\beta} g_{\alpha\beta}$ or $\sum_{\beta} g_{\beta\alpha}$

 \overline{q} transformation

 $\int_{t}^{t} e^{-g}$

 $t \nearrow$ and the three gluon propagator three gluon propagator three gluon propagator that appears in the O(a

the presence of SU(3) generators in the vertexes, so summing over color in the final state and

 $\bar t\gamma$

 $\frac{q}{\sqrt{1-q}}$

 \overline{g}

 \checkmark

 $\frac{q}{q}$

t

 $\frac{1}{2}$

 $qq \rightarrow \iota \iota \gamma$

1

 $\bigwedge^{\infty} \gamma$

t

 $\begin{array}{cc} & t\searrow g\ & \delta_{\delta_{\lambda}}\searrow q\ \end{array}$

q g

 $\frac{1}{\sqrt{2}}$

 $\frac{1}{t}$

t

 $\frac{t}{t}$

t

 \jmath

 $\begin{array}{c} \n\text{F} \text{F} \n\end{array}$

t

 $q\bar{q}$ –

 $t \swarrow$

g

 $\frac{1}{q}$ and $\frac{1}{q}$ emissions of g

 $\frac{q}{t}$

t

 \ddotsc

 \overline{q}

 $\stackrel{\cdot }{q}$

 \overline{q}

=

 \mathscr{L}_{a}

 e^{2} or α_s^3 of α_s^3 or α_s^4

 $\stackrel{\cdot}{q}$

 \overline{q}

$\ddot{}$ t $\overline{\mathbf{r}}$ $\overline{}$ noiog (1 t q ا
⊢ا that are the control of α $\mathcal{L} = \frac{1}{2} \sum_{i=1}^{n} \frac{1}{i}$ ivergencies (let 's forget about H) $\frac{1}{2}$ interference of Fig. 2 and \sim 4 and \sim 4 \sim \sim ¯ QCD) and QED (Ftt QED) cases, and obtain the ratio of them. es (let's forget about H) q γ t ibout Figure 2: Real emissions of gluon: photon in the propagator \mathcal{P}_c Complications for IR divergencies (let's forget about H)

 $\frac{1}{2}$

t

 $\frac{1}{2}$

 $\frac{1}{2}$ botons and gluons at the same time $\frac{1}{2}$ Born find
bods to more discrems and more ID at and encies! diagrams and more in ر
diagram diagrams a divergencies! d leads to more diagrams and more IR Jet and photon definition divergencies!
deserve too many slides for today .. Photons and gluons at the same time Born final state are cons divergences, only infrared singularities can arise. After regularities can arise term α mass term α mass term α

¯

t

q

 $\frac{1}{2}$

T r!

Figure 3: Real emissions of gluon

 $\gamma \rightarrow g$ subset the found that, in the situation is simpler for collinear $\frac{1}{2}$. Following the color structure argument we can identify the color structure and the color st deser $\frac{1}{1}$ $\overline{}$ e too many slid $\overline{\text{10}}$ deserve too many slides for today \overline{q} \overline{q} , \overline{r} adiation \overline{q} and photons in the α and α fullat state ale considér dest let and phot α definitions at $\frac{1}{2}$ (11) tinitions at NLO E
slides for today ل
Lation is simpler for \sim and photon definitions $\overline{\mathcal{L}}$ Jet and photon definitions at NLO EW ∫
, slides for today .. $\overline{\mathcal{L}}$ $\overline{}$ ets and photor npler for collinear $\overline{}$ Γ re too many slides for today.. ation ... unless jets and photons in the μ ation is simpler for collinear Born final state are considered. ρ FW \sqrt{q} , \sqrt{q} , \sqrt{q} , \sqrt{r} adiation … unless jets and photons in the The situation is simpler for collinear

Automation of EW corrections in MadGraph5_aMC@NLO

Automation of NLO corrections in Madgraph5 $aMC@NLO$

What do we mean with automation of EW corrections?

The possibility of calculating QCD and EW corrections for SM processes (matched to shower effects) with a process-independent approach.

again:

you need to know what's going on in order to understand the results

Recently, also the case of e^+e^- *collisions has become* available!

 $\overline{1}$ *Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao '22*

Results: NLO EW

just type:

set complex mass scheme true import model loop_qcd_qed_sm_Gmu generate process [QED] output process_NLO_EW_corrections

And then wait for the results …………..

CMS is necessary every time a resonance is present in the calculation $(Z, W, top, ...)$

Results: NLO EW precision arithmetic, and Ninja is used first. If the internal numerical stability tests are not

$$
\text{couple of weeks on } \mathcal{O}(200) \text{ CPUs} \qquad \qquad \text{112} \qquad \qquad \delta_{\text{EW}} = \frac{\Sigma_{\text{NLO}_2}}{\Sigma_{\text{LO}_1}} = \frac{\text{NLO}}{\text{LO}} - 1 \, .
$$

Results: NLO EW

 \int P i den interfax, interference, interschit, Dit, Shino, and the processes of the pro 18 113 *Frederix, Frixione, Hirschi, DP, Shao, Zaro '18*

Results: Complete NLO

just type:

set complex mass scheme true import model loop_qcd_qed_sm_Gmu generate process QCD=99 QED=99 [QCD QED] output process_NLO_EW_corrections

And then wait for the results …………..

Results: Complete NLO

immediately after the p and j definitions given by $\mathbf{N}\mathbf{E}\mathbf{W}$ NEW

$$
\frac{\Sigma_{\text{LO}_i}}{\Sigma_{\text{LO}_1}}, \qquad i = 2, 3, 4,
$$
\nNLO₃ in ttW is ~12%.

MG5 aMC> define p = p e+ e- mu+ mu- ta+ ta-

NLO3 in ttW is
$$
\sim
$$
12%.

Λ unotega is necessary! A thorough phenomenological study

 $\frac{1}{1}$ Ferons, $\frac{1}{1}$ Ferons, $\frac{1}{1}$ Erons $f\gamma$ *Frederix, Frixione, Hirschi, DP, Shao, Zaro '18*

$t\bar{t}W^{\pm}$

R. Frederix, D.P., M. Zaro
JHEP 1802 (2018) 031 (arXiv:1711.02116) R. Frederix, D.P., M. Zaro JHEP 1802 (2018) 031 (arXiv:1711.02116)

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Cross sections: order by order predictions at LO \sim NLO and LOCD \sim NLO and \sim 100 TeV accuracy. Analogous results at 100 TeV acc displayed in the case of the case of the case in the case in the case in which we apply a jet the case in which we apply a jet of the case in which we apply a jet of the case in which we apply a jet of the case in which we

clear in the discussion below. Further details about the size of the individual (N)LO*ⁱ* terms

 $\frac{1}{2}$ Numbers in parentheses refer to the case of a jet veto $p_T(j) > 100$ GeV and $|y(j)| < 2.5$ applied veto, regelection and the events with the events with the events with the events with $\frac{1}{2}$

 13 TeV at the scale cross section evaluated at the scale $\ddot{ }$ $\ddot{ }$

13 TeV Naive estimate 100 TeV result for LOND the result for LOND since it is a set of LOND since it is a set of LOND since it is a set of LO

 13 TeV Naive estimate 100 TeV

 $NILOOCD$ come the LO 2 corrections. \mathbf{u} NLO3 is large and it is not suppressed by the NLO QCD corrections.

while NLO EW and NLO3 do not. the α NLO QCD corrections depend on the scale,

initiated and have similar kinematic dependence is entirely due to the simulation of the simulation of the similar simulation of the *C*(P ¹¹⁸) suppressed and domestic the *Frederix*, *DP*, *Zaro* $'17$ *Frederix, DP, Zaro '17*

t \overline{t}

R. Frederix, D.P., M. Zaro
JHEP 1802 (2018) 031 (arXiv:1711.02116) R. Frederix, D.P., M. Zaro JHEP 1802 (2018) 031 (arXiv:1711.02116)

Complete-NLO ³*,*⁰ ⁺ ↵*s*↵⌃*ttW*¯ *[±]* \int_{0}^{1} $\sum_{i=1}^{\infty}$

^m+*n,n* quantities. At LO accuracy, we can denote the *ttW*¯ *[±]* and

 $t\bar{t}$ two quantities as $t\bar{t}$ $\alpha_s, \alpha_f = \alpha_s \Delta_{4,0} + \alpha_s \alpha \Delta_{4,1} + \alpha_s \alpha \Delta_{4,2} + \alpha_s \alpha \Delta_{4,3} + \alpha$ $\alpha_s \alpha_{4,0} + \alpha_s \alpha_{4,1} + \alpha_s \alpha_{4,2} + \alpha_s$
 $= \sum_{I \cap \{1,2\}} \sum_{i=1}^{K} \alpha_{i} + \sum_{I \cap \{2,3\}} \sum_{i=1}^{K} \alpha_{i} + \sum_{I \cap \{1,3\}} \sum_{i=1}^{K} \alpha_{i} + \sum_{I \cap$ $\Sigma_{\text{LO}}^{t\bar{t}t\bar{t}}(\alpha_s,\alpha)=\alpha_s^4\Sigma_{4,0}^{t\bar{t}t\bar{t}}+\alpha_s^3\alpha\Sigma_{4,1}^{t\bar{t}t\bar{t}}+\alpha_s^2\alpha^2\Sigma_{4,2}^{t\bar{t}t\bar{t}}+\alpha_s^3\alpha\Sigma_{4,3}^{t\bar{t}t\bar{t}}+\alpha^4\Sigma_{4,4}^{t\bar{t}t\bar{t}}$ 4*,*4

LO(↵*s,* ↵) = ↵⁴ *s*⌃*ttt*¯ *^t* ¯ ⁴*,*⁰ + ↵³ *s*↵⌃*ttt*¯ *^t* ⁴*,*¹ + ↵² $\frac{1}{2}$ $\overbrace{0000}^t$ **.** (2.4) $\overline{0000}$. (2. $\sqrt{000}$ and $\sqrt{000}$ and the NLO corrections and the definitions and the definitions orders can be defined by definitions and the definitions of $\frac{1}{2}$ $\sum t$ *s*↵⌃*ttW*¯ *[±]* ⁴*,*¹ ⁺ ↵*s*↵3⌃*ttW*¯ *[±]* $\equiv \Sigma_{\text{LO}_1} + \Sigma_{\text{LO}_2} + \Sigma_{\text{LO}_3} + \Sigma_{\text{LO}_4} + \Sigma_{\text{LO}_5}$. \blacksquare NLO (↵*s,* ↵) = ↵³ *s*↵⌃*ttW*¯ *[±]* $\frac{1}{2}$ ⁴*,*¹ ⁺ ↵*s*↵3⌃*ttW*¯ *[±]* ⁴*,*² ⁺ ↵4⌃*ttW*¯ *[±]* $\frac{1}{2000}$ $\frac{1}{2000}$ $\frac{1}{2000}$ $\frac{1}{2000}$ $\frac{1}{2000}$ $\frac{1}{2000}$ $\frac{1}{2000}$ 7000 *s*⌃*ttt*¯ *^t* ⁵*,*⁰ + ↵⁴ $\frac{1}{t}$ \sim *s*↵3⌃*ttt*¯ *^t* \overline{a} *√* \mathcal{L} *t t* \bar{t} *t* \bar{t} *t t t* \bar{t} *t t* \bar{t} *H*

different ⌃*ttW*¯ *[±]*

⌃*ttW*¯ *[±]*

¯ *t*

^m+*n*+1*,n* and ⌃*ttt*¯ *^t*

s↵⌃*ttW*¯ *[±]*

¯ observables as ⌃*ttW*¯ *[±]*

LO (↵*s,* ↵) = ↵²

The gg initial state amounts to ~90% of LO cr 4*,*4 \sim 90% of LO cross section at 13 TeV and almost all the cross section at 100 TeV.

Frederix, DP, Zaro '17

⁴*,*² ⁺ ↵4⌃*ttW*¯ *[±]* There is no gg contribution at LO₄
and I O₅ and LO₅.

Cross sections

LO₂ and LO₃ are large and have also large cancellations.

*/*LOQCD ratios at 100 TeV, for different values of *µ* = *µ^r* = *µ^f* . *Frederix, DP, Zaro '17*

NLO₂ and NLO₃ are mainly given by 'QCD corrections' on top of them, so they are large and strongly depend on the scale choice, at variance with production, at variance with most of the other production Accidentally, relatively to LO1, $NLO2+NLO3$ scale depen What happens if BSM enters into the game? Anomalous yt ? *thoice* at variance with standard EW corrections. $\frac{1}{2}$ *the present for any <i>i* α and *i* α *i* α *seed ange also at*, *discussesses* Accidentally, relatively to LO1, NLO2+NLO3 scale dependence almost disappears. and strongly depend on the scale choice, at variance with standard EW corrections.

Distributions

are present also at the differential level. At the threshold also NLO4 is large.

 -0.2

600

0 1000

0 2000

ی
M(tt̃ttt̃) [GeV]

0 4000

కి

T.

NLO EW

Sudakov logarithms

Differential distributions: Sudakov enhancements

Sudakov enhancements are NOT exceptions and involve at NLO corrections of order $-\alpha \log^k(s/m_W^2)$ with $k = 1,2$.

What are EW Sudakov logarithms?

 QCD : virtual and real terms are separately IR divergent ($1/e$ poles). In physical cross sections the contributions are combined and poles cancel.

QED: same story, but I can also regularise IR divergencies via a photon- λ . So $1/\epsilon$ poles $\rightarrow \log(Q^2/\lambda^2)$, where Q is a generic scale.

EW: with weak interactions $\lambda \rightarrow m_W, m_Z$ and W and Z radiation are typically not taken into account, which is anyway IR-safe.

Therefore, at high energies EW loops induce corrections of order

$$
-\alpha^k \log^n(s/m_W^2)
$$

where k is the number of loops and $n \leq 2k$. These logs are physical. Even including the real radiation of W and Z, there is not the full cancellation of this kind of logarithms.

Why automate Sudakov in Madgraph5_aMC@NLO?

NLO EW corrections already fully include one-loop EW Sudakov logarithms $(n = 1, k = 1, 2)$, why automate them?

- They can be calculated **analytically via tree-level** amplitudes only. They are a very good approximation of NLO EW at high energy and they can be computed much **faster**. No cancellations among virtual and real, so very stable results.
- When **NLO EW** becomes **large and negative**, **Sudakov logarithms** have to be **resummed**. Having in one tool separately the exact NLO EW and its Sudakov component will allow **matching of NLO EW and EW LL resummed.**
- They **depend only on** properties of the **external particles**: masses, momenta, *helicities*, charges, SU(2) components, hypercharges. The **generalisation to** the **BSM** case is therefore much **easier** than the NLO EW case.

one-loop EW virtual corrections $O(\alpha)$ **=** α [Sudakov Logs $\mathcal{O}(-\log^k(s/m_W^2), k = 1, 2)$ + constant term $\mathcal{O}(1)$ + ${\sf mass-suppressed \; terms \;} {\mathcal O}(m_W^2/s) {\sf J}$

At high energies, mass suppressed terms vanish so NLO logarithmically grows but $(NLO-Sudakov)/LO \sim 6(1\%)$

Our revisitation and automation: Amplitude level

We have **revisited** and **automated** in aMG5 the **Denner&Pozzorini algorithm** for the evaluation of one-loop EW Sudakov corrections to amplitudes (*Denner, Pozzorini '01*). In particular we have introduced the **following novelties**.

- **IR QED** divergencies are dealt with **via D**imensional **R**egularisation, with strictly massless photons and light fermions.
- **Additional logarithms** that involve ratios between invariants, and therefore **angular** dependences, are taken into account.
- We correctly take into account an **imaginary term** that was **previously omitted** in the literature. Relevant for $2 \rightarrow n$ processes with $n > 2$
- Moving to the level of interferences of tree and one-loop amplitudes, we take into account NLO EW contributions originating from **QCD loops on top of subleading LO terms**.

Example $(2 \rightarrow 3)$: Imaginary term and scan in s in turn parametrises the value of *t*, in the range 10² . ✓ . ⇡*/*2. We have fixed the value $I \cap \Delta$ ($I \cap \Delta$ \rightarrow $I \cap \Delta$). Imaginary tarm t the previous section for the previous section \mathcal{L}

 $\frac{1}{8}$ \forall r_{kl} **Denner&Pozzorini algorithm works only with** dominant one.

Dots-Dashed/LO: not horizontal, the anticipated before, this happens also for the sum over the helicities, which for this particular correct Log dependence is lost.

SOME EXAMPLES AT 100 TeV

ZZZ production at 100 TeV

 $p_T(Z_i) > 1 \text{ TeV}, \qquad |\eta(Z_i)| < 2.5, \qquad m(Z_i, Z_j) > 1 \text{ TeV}, \qquad \Delta R(Z_i, Z_j) > 0.5.$

WZ production at 100 TeV $\mathbf{V} \mathbf{V}$ \mathbf{Z} proudults have been obtained by using the following the following the following cuts is the following cuts for \mathbf{Z}

 $p_T(V_i) > 1 \text{ TeV}, \qquad |\eta(V_i)| < 2.5, \qquad m(W^+, Z) > 1 \text{ TeV}, \qquad \Delta R(W^+, Z) > 0.5.$

Orange: NLO EW, (dotted: NLO EW no γ PDF) **Green =** , **Red =** SDK0 SDKweak in the real-radiation processes or even at the Born. **Dashed**: standard approach for amplitudes. **Solid**: our formulation (more angular information). \blacksquare $\frac{10}{\sqrt{25}}$ by and collinear splittings $\frac{10}{\sqrt{25}}$ collinear splittings appearing ap

Red-solid line

The Sudakov approximation cannot **approximate large logs from the opening of new channels. The fair comparison is with NLO EW no** $γ$ **PDF.**

Only the SDK_{weak} approach correctly captures the NLO EW prediction.

Only the solid lines, having more angular information, correctly capture NLO EW.

CONCLUSIONS

- NLO QCD, NLO EW and Complete NLO predictions are essential in order to provide precise and reliable theoretical predictions for the LHC.
- A lot of technology underlies the commands that let you calculate NLO with MadGraph5_aMC@NLO. Try to learn the basics idea if you want to critically understand the numerical outputs.
- There is more than NLO (e.g. NNLO, NNNLO and other techniques besides fixed order). They are already relevant and they will be even more in the future.