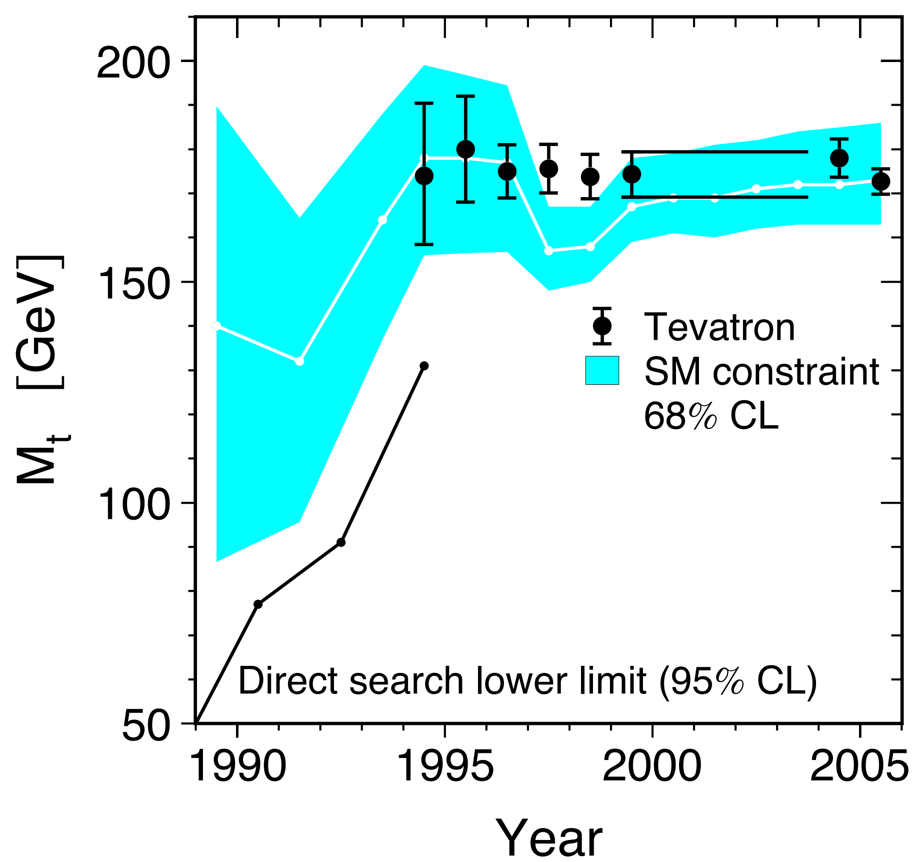
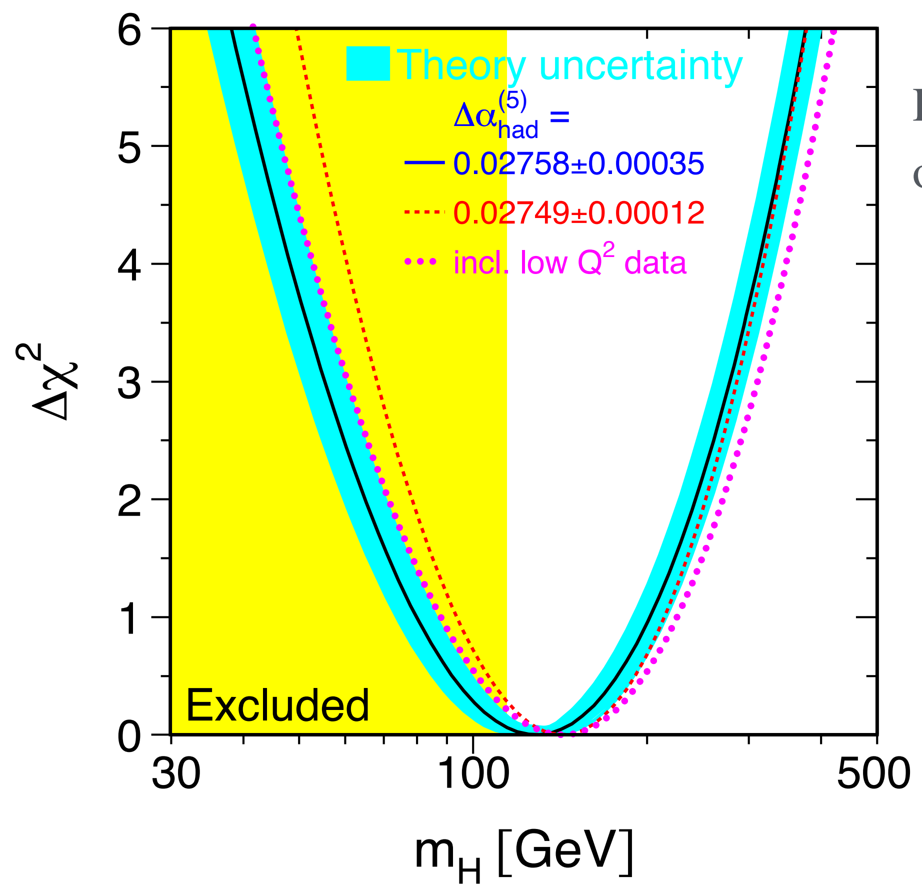
NLO EW

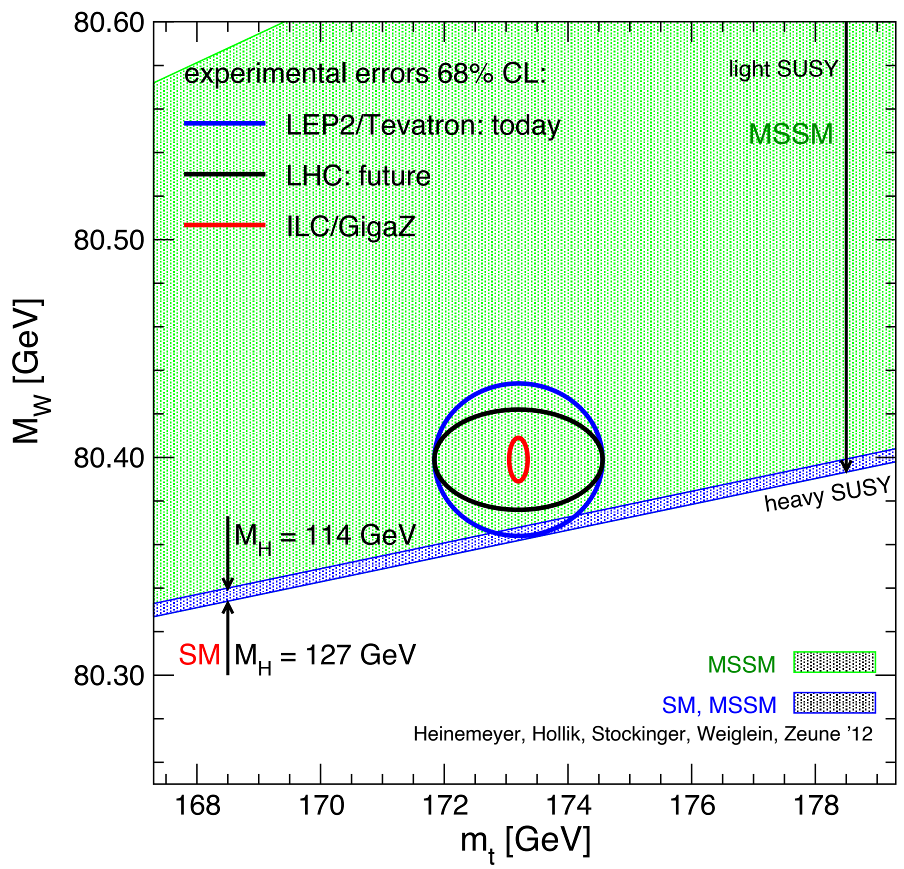
one additional layer of complexity

Relevance of EW Precision Observables





Precision Electroweak measurements on the Z resonance hep-ex/0509008



EWPO were crucial in order to constrain the H-boson and top-quark mass.
Today EWPO can be used to check the internal consistency of the SM.
In models where they can be calculated, as in the MSSM, EWPO can be used to constrain the parameter space.

Relevance of EW Precision Observables

$$\hat{m}_W = \frac{\hat{e}\hat{v}}{2\hat{s}} = 79.794 \text{ GeV}$$ PREDICTION

$$m_{W,pole} = (5) = 80.385 \pm 0.015$$ EXPERIMENT

4σ DEVIATION

SM at tree-level is falsified by experiments. Loops corrections necessary! Imagine if I try to explain this difference with BSM! What a disaster....

$$\alpha = \frac{e^2}{4\pi}, \quad G_F = \frac{1}{\sqrt{2} v^2}$$

$\hat{s}$ from $m_{Z,pole} = \hat{m}_Z$

$$\hat{s}^2(1-\hat{s}^2) = \frac{\pi \hat{\alpha}(M_Z)}{\sqrt{2}\hat{G}_F \hat{m}_Z^2}$$

Relevance of EW Precision Observables

(Un)fortunately we cannot calculate here in these lectures the actual loops

$$m^2_{W,\text{pole}} \sim \hat{c}^2 \hat{m}_Z^2 \left[1 + \frac{3\alpha}{16\pi \hat{s}^2 (\hat{c}^2 - \hat{s}^2)} \frac{m_t^2}{\hat{m}_Z^2}\right] - \frac{5\hat{\alpha}}{24\pi} \frac{\hat{c}^2 \hat{m}_Z^2}{\hat{c}^2 - \hat{s}^2} \log\left(\frac{m_H^2}{\hat{c}^2 \hat{m}_Z^2}\right)$$

dominant effects

QUADRATIC IN m_t

logaritmic in m_H

[GeV]	TREE	1-loop (t,b)	1-loop (t,b,H)
m_W	79.794	80.368	80.333

NLO EW

what about cross sections at the LHC?

In general small % effects on total cross sections, ***BUT*** ..

Process	Syntax	Cross section (in pb) LO	NLO	Correction (in %)
$pp \to e^+\nu_e$	`p p > e+ ve QCD=0 QED=2 [QED]`	$5.2498 \pm 0.0005 \cdot 10^{3}$	$5.2113 \pm 0.0006 \cdot 10^{3}$	-0.73 ± 0.01
$pp \to e^+\nu_e j$	`p p > e+ ve j QCD=1 QED=2 [QED]`	$9.1468 \pm 0.0012 \cdot 10^{2}$	$9.0449 \pm 0.0014 \cdot 10^{2}$	-1.11 ± 0.02
$pp \to e^+\nu_e jj$	`p p > e+ ve j j QCD=2 QED=2 [QED]`	$3.1562 \pm 0.0003 \cdot 10^{2}$	$3.0985 \pm 0.0005 \cdot 10^{2}$	-1.83 ± 0.02
$pp \to e^+e^-$	`p p > e+ e- QCD=0 QED=2 [QED]`	$7.5367 \pm 0.0008 \cdot 10^{2}$	$7.4997 \pm 0.0010 \cdot 10^{2}$	-0.49 ± 0.02
$pp \to e^+e^- j$	`p p > e+ e- j QCD=1 QED=2 [QED]`	$1.5059 \pm 0.0001 \cdot 10^{2}$	$1.4909 \pm 0.0002 \cdot 10^{2}$	-1.00 ± 0.02
$pp \to e^+e^- jj$	`p p > e+ e- j j QCD=2 QED=2 [QED]`	$5.1424 \pm 0.0004 \cdot 10^{1}$	$5.0410 \pm 0.0007 \cdot 10^{1}$	-1.97 ± 0.02
$pp \to e^+e^-\mu^+\mu^-$	`p p > e+ e- mu+ mu- QCD=0 QED=4 [QED]`	$1.2750 \pm 0.0000 \cdot 10^{-2}$	$1.2083 \pm 0.0001 \cdot 10^{-2}$	-5.23 ± 0.01
$pp \to e^+\nu_e\mu^-\bar{\nu}_\mu$	`p p > e+ ve mu- vm~ QCD=0 QED=4 [QED]`	$5.1144 \pm 0.0007 \cdot 10^{-1}$	$5.3019 \pm 0.0009 \cdot 10^{-1}$	$+3.67 \pm 0.02$
$pp \to He^+\nu_e$	`p p > h e+ ve QCD=0 QED=3 [QED]`	$6.7643 \pm 0.0001 \cdot 10^{-2}$	$6.4914 \pm 0.0012 \cdot 10^{-2}$	-4.03 ± 0.02
$pp \to He^+e^-$	`p p > h e+ e- QCD=0 QED=3 [QED]`	$1.4554 \pm 0.0001 \cdot 10^{-2}$	$1.3700 \pm 0.0002 \cdot 10^{-2}$	-5.87 ± 0.02
$pp \to Hjj$	`p p > h j j QCD=0 QED=3 [QED]`	$2.8268 \pm 0.0002 \cdot 10^{0}$	$2.7075 \pm 0.0003 \cdot 10^{0}$	-4.22 ± 0.01
$pp \to W^+W^-W^+$	`p p > w+ w- w+ QCD=0 QED=3 [QED]`	$8.2874 \pm 0.0004 \cdot 10^{-2}$	$8.8017 \pm 0.0012 \cdot 10^{-2}$	$+6.21 \pm 0.02$
$pp \to ZZW^+$	`p p > z z w+ QCD=0 QED=3 [QED]`	$1.9874 \pm 0.0001 \cdot 10^{-2}$	$2.0189 \pm 0.0003 \cdot 10^{-2}$	$+1.58 \pm 0.02$
$pp \to ZZZ$	`p p > z z z QCD=0 QED=3 [QED]`	$1.0761 \pm 0.0001 \cdot 10^{-2}$	$0.9741 \pm 0.0001 \cdot 10^{-2}$	-9.47 ± 0.02
$pp \to HZZ$	`p p > h z z QCD=0 QED=3 [QED]`	$2.1005 \pm 0.0003 \cdot 10^{-3}$	$1.9155 \pm 0.0003 \cdot 10^{-3}$	-8.81 ± 0.02
$pp \to HZW^+$	`p p > h z w+ QCD=0 QED=3 [QED]`	$2.4408 \pm 0.0000 \cdot 10^{-3}$	$2.4809 \pm 0.0005 \cdot 10^{-3}$	$+1.64 \pm 0.02$
$pp \to HHW^+$	`p p > h h w+ QCD=0 QED=3 [QED]`	$2.7827 \pm 0.0001 \cdot 10^{-4}$	$2.4259 \pm 0.0027 \cdot 10^{-4}$	-12.82 ± 0.10
$pp \to HHZ$	`p p > h h z QCD=0 QED=3 [QED]`	$2.6914 \pm 0.0003 \cdot 10^{-4}$	$2.3926 \pm 0.0003 \cdot 10^{-4}$	-11.10 ± 0.02
$pp \to t\bar{t}W^+$	`p p > t t~ w+ QCD=2 QED=1 [QED]`	$2.4119 \pm 0.0003 \cdot 10^{-1}$	$2.3025 \pm 0.0003 \cdot 10^{-1}$	-4.54 ± 0.02
$pp \to t\bar{t}Z$	`p p > t t~ z QCD=2 QED=1 [QED]`	$5.0456 \pm 0.0006 \cdot 10^{-1}$	$5.0033 \pm 0.0007 \cdot 10^{-1}$	-0.84 ± 0.02
$pp \to t\bar{t}H$	`p p > t t~ h QCD=2 QED=1 [QED]`	$3.4480 \pm 0.0004 \cdot 10^{-1}$	$3.5102 \pm 0.0005 \cdot 10^{-1}$	$+1.81 \pm 0.02$
$pp \to t\bar{t}j$	`p p > t t j QCD=3 QED=0 [QED]`	$3.0277 \pm 0.0003 \cdot 10^{2}$	$2.9683 \pm 0.0004 \cdot 10^{2}$	-1.96 ± 0.02
$pp \to jjj$	`p p > j j j QCD=3 QED=0 [QED]`	$7.9639 \pm 0.0010 \cdot 10^{6}$	$7.9472 \pm 0.0011 \cdot 10^{6}$	-0.21 ± 0.02
$pp \to tj$	`p p > t j QCD=0 QED=2 [QED]`	$1.0613 \pm 0.0001 \cdot 10^{2}$	$1.0539 \pm 0.0001 \cdot 10^{2}$	-0.70 ± 0.02

couple of weeks on $\mathcal{O}(200)$ CPUs

$$\delta_{\rm EW} = \frac{\Sigma_{\rm NLO_2}}{\Sigma_{\rm LO_1}} = \frac{\rm NLO}{\rm LO} - 1\,.$$

Enhancements: final-state radiation (FSR)

In sufficiently exclusive observable FSR induces corrections $\sim \alpha Q_\ell^2 \log^2\left(p_T^2(\ell)/m_\ell{}^2\right)$.
Photon-fermion recombination reduces the size of this effect.

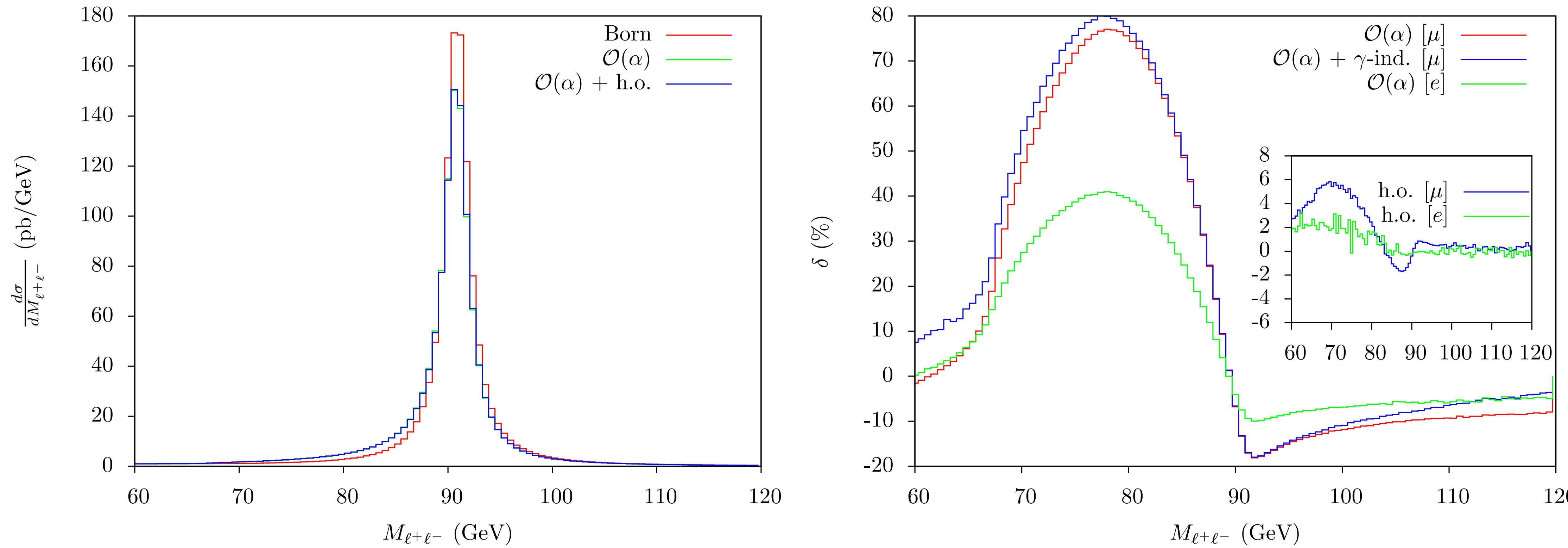
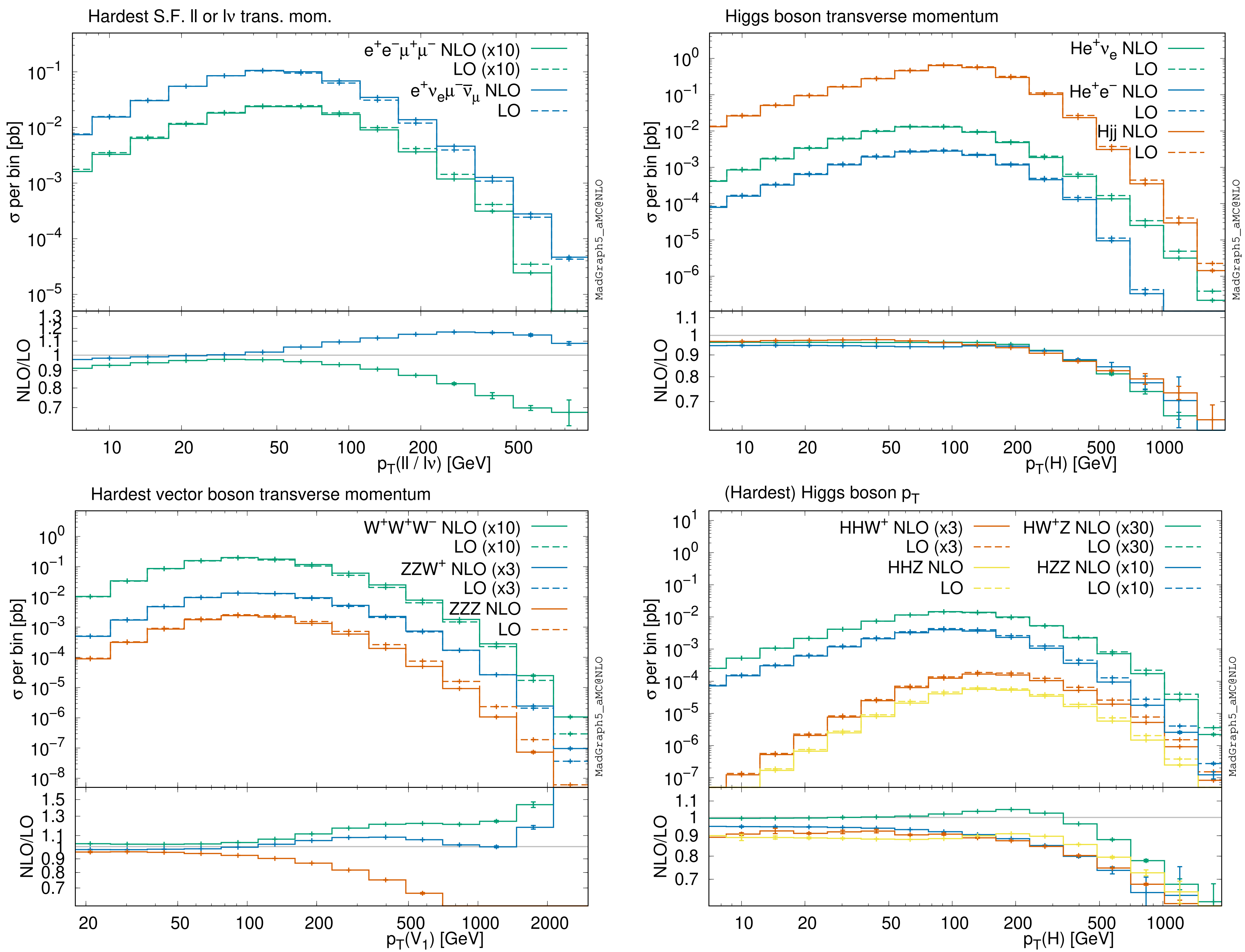


Figure 5: Invariant mass distribution around the Z peak (left) and relative effect of different contributions (right), for bare muons and recombined electrons.

Carloni Calame, Montagna, Nicrosini, Vicini ‘15

Enhancements: Sudakov logarithms

Weak corrections at large scales are not negligible for a general process due to the Sudakov Logarithms $\sim \alpha \log^2\left(Q^2/M_W^2\right)$.



Frederix, Frixione, Hirschi, DP, Shao, Zaro '18

General aspects of one-loop EW amplitudes and their renormalisation

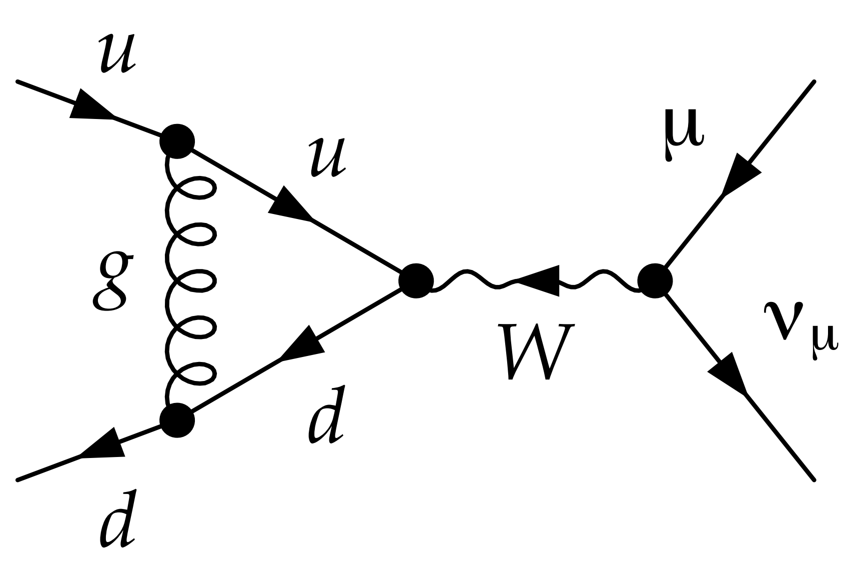
Ansgar Denner, arXiv:0709.1075

I focus in this lecture on the LHC, but in a precision machine such as ILC the content of the following slides is even more important!

Why is NLO EW more complicated than NLO QCD?

$$pp \to \mu^+ \nu_\mu$$

One-Loop QCD diagrams*:

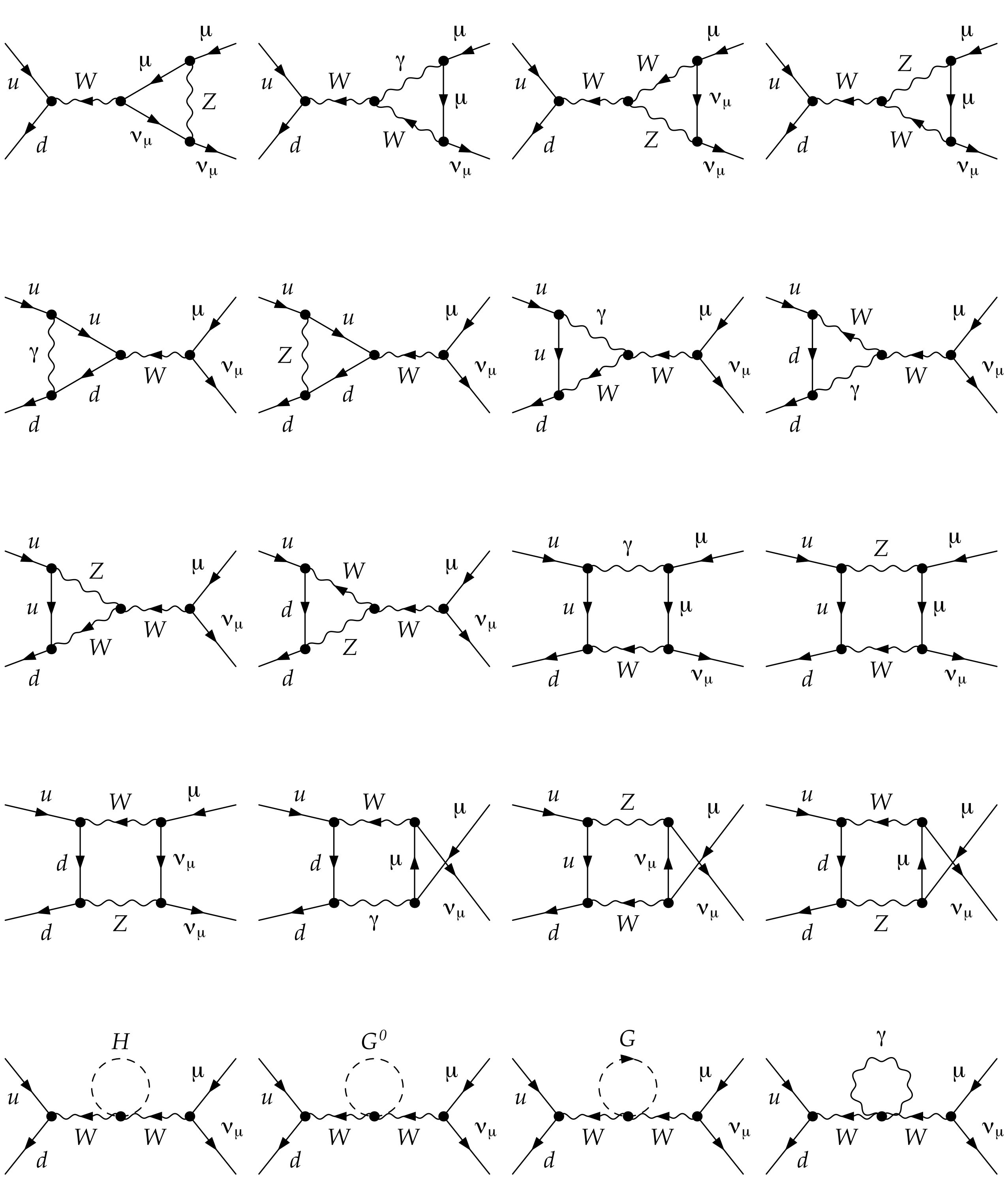


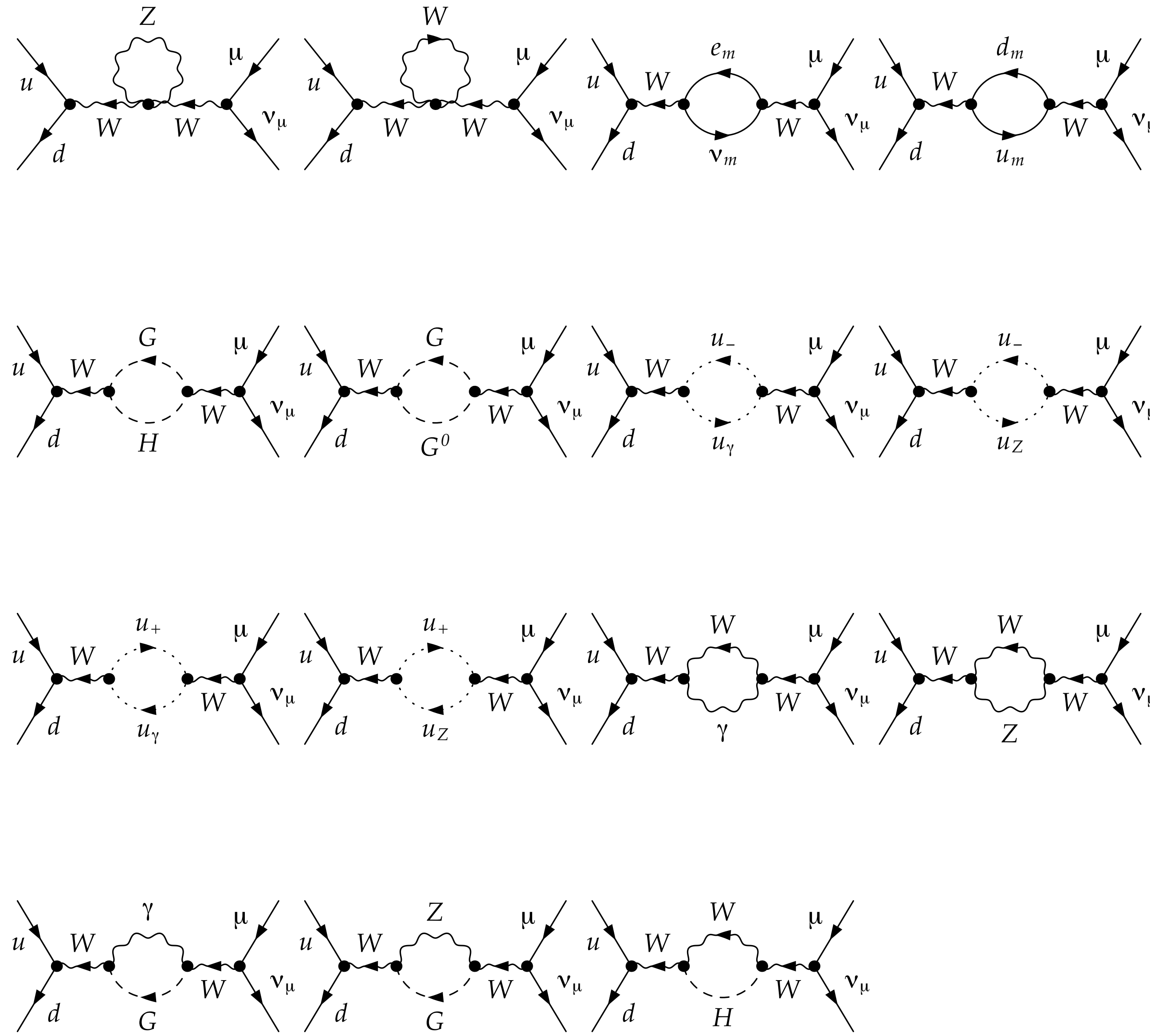
* Remember that in the on-shell scheme for masses and wave-function renormalisation, no loops corrections on the external legs have to be explicitly computed (LSZ).

Why is NLO EW more complicated than NLO QCD?

$pp \rightarrow \mu^{+}\nu_{\mu}$

One-Loop EW diagrams





More diagrams, more particles, more interactions, more scales

More renormalisation parameters, but not more than 13

In the EW sector of the SM there are **3** independent parameters for the **gauge interactions**, the **mass of the Higgs**, and **9 fermion masses** (CKM diagonal in these slides).
In the so called alpha(0)-scheme, with massive particles renormalised on-shell, the **3** independent parameters for the **gauge sector** are**:**

$$\alpha \quad M_W \quad M_Z$$

with α measured in the Thomson scattering, with zero-momentum transferred.
All the other EW parameters are predictions:

$$v\,, \quad G_F\,, \quad \sin(\theta_W)\,, \quad y_t\,, \quad \lambda\,, \quad \rho\,, \quad \text{etc}\ldots$$

On the other hand, one can change input parameters :

$\{\alpha\,,\ M_W\,,\ M_Z\} \rightarrow \{G_\mu\,,\ M_W\,,\ M_Z\}\,, \{\alpha\,,\ G_\mu\,,\ M_W\}$

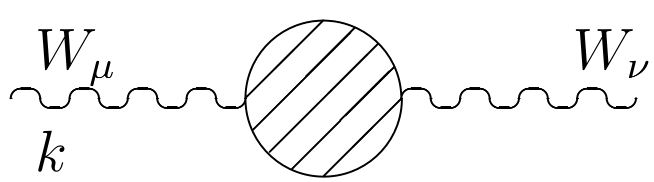
or renormalisation conditions for masses on-shell —> MSbar,

and couplings $\alpha(0) \rightarrow \alpha(M_Z)\,,\ G_\mu\,,\ \alpha^{\overline{MS}}$

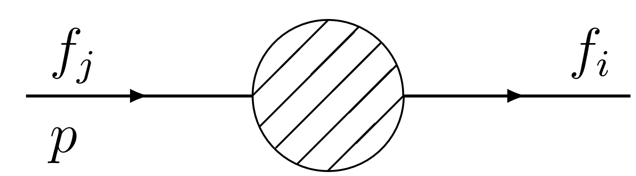
Renormalisation alpha(0)-scheme, on-shell masses

$$
\begin{aligned}
e_0 &= Z_e e = (1+\delta Z_e)e, \\
M_{W,0}^2 &= M_W^2 + \delta M_W^2, \\
M_{Z,0}^2 &= M_Z^2 + \delta M_Z^2, \\
M_{H,0}^2 &= M_H^2 + \delta M_H^2, \\
m_{f,i,0} &= m_{f,i} + \delta m_{f,i},
\end{aligned}
$$

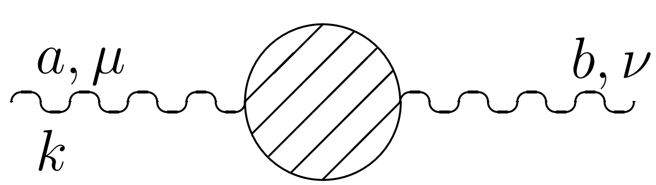
The bare fields(parameters) in the Lagrangian are split into renormalised fields(parameters) and renormalisation constants, which in turn are derived via the following renormalised Green functions:



$= \hat{\Gamma}^{W}_{\mu\nu}(k)$

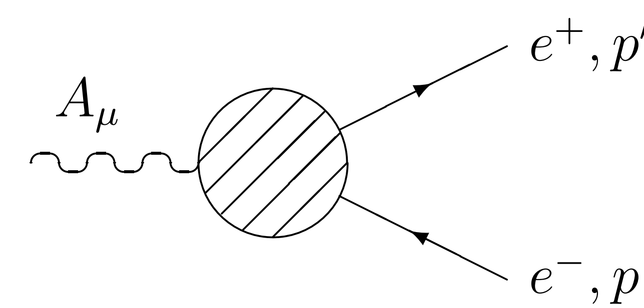


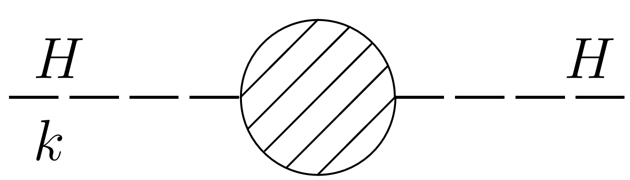
$= \hat{\Gamma}^{f}_{ij}(p)$



$= \hat{\Gamma}^{ab}_{\mu\nu}(k) \qquad a, b = A, Z$

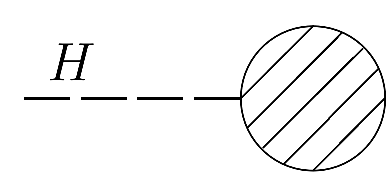
$\hat{\Gamma}^{ee\gamma}_{\mu}(p, p') =$





$= \hat{\Gamma}^{H}(k)$

$\hat{T} =$



Renormalisation alpha(0)-scheme, on-shell masses

Requiring that tadpoles do not shift the minimum of the Higgs potential and that the renormalised masses of the bosons are the on-shell masses:

$$\delta t = -T,$$

$$\delta M_W^2 = \widetilde{\mathrm{Re}}\,\Sigma_T^W(M_W^2), \qquad \delta Z_W = -\mathrm{Re}\left.\frac{\partial \Sigma_T^W(k^2)}{\partial k^2}\right|_{k^2=M_W^2},$$

$$\delta M_Z^2 = \mathrm{Re}\,\Sigma_T^{ZZ}(M_Z^2), \qquad \delta Z_{ZZ} = -\mathrm{Re}\left.\frac{\partial \Sigma_T^{ZZ}(k^2)}{\partial k^2}\right|_{k^2=M_Z^2},$$

$$\delta Z_{AZ} = -2\mathrm{Re}\,\frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2}, \qquad \delta Z_{ZA} = 2\frac{\Sigma_T^{AZ}(0)}{M_Z^2},$$

$$\delta Z_{AA} = -\left.\frac{\partial \Sigma_T^{AA}(k^2)}{\partial k^2}\right|_{k^2=0},$$

$$\delta M_H^2 = \mathrm{Re}\,\Sigma^H(M_H^2), \qquad \delta Z_H = -\mathrm{Re}\left.\frac{\partial \Sigma^H(k^2)}{\partial k^2}\right|_{k^2=M_H^2}.$$

Renormalisation alpha(0)-scheme, on-shell masses

Requiring that also the renormalised masses of the fermion are the on-shell masses:

$$\delta m_{f,i} = \frac{m_{f,i}}{2}\widetilde{\mathrm{Re}}\left(\Sigma_{ii}^{f,L}(m_{f,i}^2) + \Sigma_{ii}^{f,R}(m_{f,i}^2) + 2\Sigma_{ii}^{f,S}({m_{f,i}}^2)\right),$$

$$\delta Z_{ij}^{f,L} = \frac{2}{m_{f,i}^2 - m_{f,j}^2}\widetilde{\mathrm{Re}}\left[m_{f,j}^2\Sigma_{ij}^{f,L}(m_{f,j}^2) + m_{f,i}m_{f,j}\Sigma_{ij}^{f,R}(m_{f,j}^2)\right.$$
$$\left. +(m_{f,i}^2 + m_{f,j}^2)\Sigma_{ij}^{f,S}(m_{f,j}^2)\right], \qquad i \neq j,$$

$$\delta Z_{ij}^{f,R} = \frac{2}{m_{f,i}^2 - m_{f,j}^2}\widetilde{\mathrm{Re}}\left[m_{f,j}^2\Sigma_{ij}^{f,R}(m_{f,j}^2) + m_{f,i}m_{f,j}\Sigma_{ij}^{f,L}(m_{f,j}^2)\right.$$
$$\left. +2m_{f,i}m_{f,j}\Sigma_{ij}^{f,S}(m_{f,j}^2)\right], \qquad i \neq j,$$

$$\delta Z_{ii}^{f,L} = -\widetilde{\mathrm{Re}}\,\Sigma_{ii}^{f,L}(m_{f,i}^2) - m_{f,i}^2\frac{\partial}{\partial p^2}\widetilde{\mathrm{Re}}\left[\Sigma_{ii}^{f,L}(p^2) + \Sigma_{ii}^{f,R}(p^2) + 2\Sigma_{ii}^{f,S}(p^2)\right]\Big|_{p^2=m_{f,i}^2},$$

$$\delta Z_{ii}^{f,R} = -\widetilde{\mathrm{Re}}\,\Sigma_{ii}^{f,R}(m_{f,i}^2) - m_{f,i}^2\frac{\partial}{\partial p^2}\widetilde{\mathrm{Re}}\left[\Sigma_{ii}^{f,L}(p^2) + \Sigma_{ii}^{f,R}(p^2) + 2\Sigma_{ii}^{f,S}(p^2)\right]\Big|_{p^2=m_{f,i}^2}.$$

Renormalisation alpha(0)-scheme, on-shell masses

The alpha(0) condition for the QED interaction among the photon and the electron

$$\bar{u}(p)\Gamma_{\mu}^{ee\gamma}(p,p)u(p)\Big|_{p^2=m_e^2} = ie\bar{u}(p)\gamma_\mu u(p)$$

leads to

$$\delta Z_e = -\frac{1}{2}\delta Z_{AA} - \frac{s_W}{c_W}\frac{1}{2}\delta Z_{ZA} = \frac{1}{2}\frac{\partial\Sigma_T^{AA}(k^2)}{\partial k^2}\Big|_{k^2=0} - \frac{s_W}{c_W}\frac{\Sigma_T^{AZ}(0)}{M_Z^2}$$

All the other counterterms can be obtained as function of those already obtained, for example,

$$\sin^2\theta_W = s_W^2 = 1 - \frac{M_W^2}{M_Z^2}$$

$$\Rightarrow$$

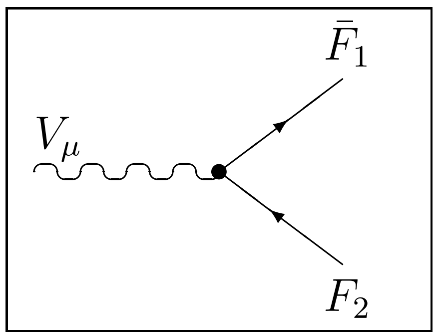
$$\frac{\delta c_W}{c_W} = \frac{1}{2}\left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}\right) = \frac{1}{2}\widetilde{\mathrm{Re}}\left(\frac{\Sigma_T^W(M_W^2)}{M_W^2} - \frac{\Sigma_T^{ZZ}(M_Z^2)}{M_Z^2}\right),$$

$$\frac{\delta s_W}{s_W} = -\frac{c_W^2}{s_W^2}\frac{\delta c_W}{c_W} = -\frac{1}{2}\frac{c_W^2}{s_W^2}\widetilde{\mathrm{Re}}\left(\frac{\Sigma_T^W(M_W^2)}{M_W^2} - \frac{\Sigma_T^{ZZ}(M_Z^2)}{M_Z^2}\right).$$

Renormalisation alpha(0)-scheme, on-shell masses

But also all the possible counterterms, here some examples:

VFF-coupling:



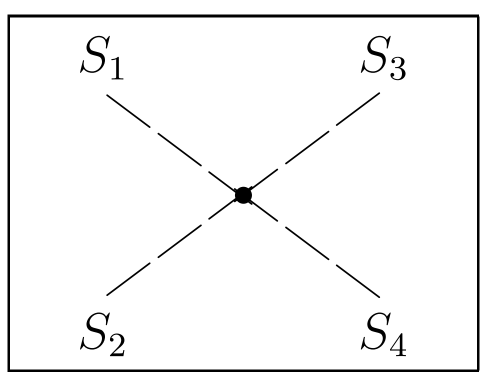
$= ie\gamma_\mu(C^-\omega_- + C^+\omega_+)$

This is the QED vertex, embedded in the SU(2)xU(1) broken symmetry of the SM.

with the actual values of V, $\bar{F}_1$, F_2 and C^+, C^-

$$\gamma \bar{f}_i f_j \quad : \quad \begin{cases} C^+ = -Q_f\Big[\delta_{ij}\Big(1+\delta Z_e + \frac{1}{2}\delta Z_{AA}\Big) + \frac{1}{2}(\delta Z^{f,R}_{ij} + \delta Z^{f,R\dagger}_{ij})\Big] + \delta_{ij} g_f^+ \frac{1}{2}\delta Z_{ZA}, \\ C^- = -Q_f\Big[\delta_{ij}\Big(1+\delta Z_e + \frac{1}{2}\delta Z_{AA}\Big) + \frac{1}{2}(\delta Z^{f,L}_{ij} + \delta Z^{f,L\dagger}_{ij})\Big] + \delta_{ij} g_f^- \frac{1}{2}\delta Z_{ZA}, \end{cases}$$

SSSS-coupling:



$= ie^2 C$

This is the quartic Higgs self coupling. Do you see λ from the Higgs potential?

with the actual values of S_1, S_2, S_3, S_4 and C

$$HHHH \quad : \quad C = -\frac{3}{4s^2}\frac{M_H^2}{M_W^2}\Big[1 + 2\delta Z_e - 2\frac{\delta s}{s} + \frac{\delta M_H^2}{M_H^2} + \frac{e}{2s}\frac{\delta t}{M_W M_H^2} - \frac{\delta M_W^2}{M_W^2} + 2\delta Z_H\Big],$$

Renormalisation scheme

Should I use alpha(0) for any calculation?

NO!

Only vertices involving a final-state on-shell photon are in a physical configuration similar to the Thomson scattering. Otherwise, for large scales,

$$\frac{\Sigma_T^{AA}(k^2)}{k^2} - \frac{\Sigma_T^{AA}(k^2)}{k^2}\bigg|_{k^2=0} \sim \sum_{m_f} \frac{\alpha}{3\pi} Q_f^2 K_f^2 \left[-\frac{5}{3} + \log(Q^2/m_f^2) + \mathcal{O}(m_f^2/Q^2) \right] \qquad \text{for } Q >> 2m_f$$

In other words, this scheme is "Infrared-sensitive" and induces large corrections due to the running of alpha from the scale of the mass of the electron (0.5 MeV) to the typical ~ 0.1-1 TeV scale at the LHC.

Unless final-state on-shell photons are considered, other input parameters and renormalisation conditions are preferable. Indeed, in these cases the alpha(0) scheme artificially enhances loop corrections.

alpha(Mz)

Starting from the alpha(0) scheme

$$\delta Z_e = -\frac{1}{2}\delta Z_{AA} - \frac{s_W}{c_W}\frac{1}{2}\delta Z_{ZA} \qquad \delta Z_{AA} = -\frac{\partial \Sigma_T^{AA}(k^2)}{\partial k^2}\Big|_{k^2=0}$$

we can define the alpha(Q) scheme

$$\delta Z_e|_{\alpha(Q^2)} \equiv \delta Z_e|_{\alpha(0)} - \frac{1}{2}\Delta\alpha(Q^2) \qquad \Delta\alpha(Q^2) \equiv \Pi^{AA}_{f\neq t}(0) - \Re\{\Pi^{AA}_{f\neq t}(Q^2)\}$$

This is not alpha measured at the Q scale, it is alpha(0) + the difference of the QED contributions of the vacuum polarisation with 5 active flavours evaluated at the scale Q and me. In practice, it takes care of QED running.

$$\Delta\alpha(Q^2) = \Delta\alpha_{\text{leptons}}(Q^2, m_f) + \Delta\alpha^{(5)}_{\text{quarks}}(Q^2, m_f) \qquad \Delta\alpha_{\text{leptons}}(Q^2, m_f) \equiv \sum_{f=e,\mu,\tau} \Delta\alpha_{\text{fermion}}(Q^2, m_f)$$

For $Q > 2m_f$ we obtain

$$\Delta\alpha_{\text{fermion}}(Q^2, m_f) = \tfrac{\alpha}{3\pi} Q_f^2 K_f^2 \Big[-\tfrac{8}{3} + \beta^2 - \tfrac{1}{2}\beta(3-\beta^2)\log\left(\tfrac{1-\beta}{1+\beta}\right)\Big] = \tfrac{\alpha}{3\pi} Q_f^2 K_f^2 \Big[-\tfrac{5}{3} + \log(Q^2/m_f^2) + \mathcal{O}(m_f^2/Q^2)\Big]$$

with $\beta = \sqrt{1 - \frac{4m_f^2}{Q^2}}$

alpha(M_Z)

On the other hand, while the leptonic contribution can be perturbatively calculated, the hadronic contribution must be extracted by measurements.

Still the best prediction for alpha at Q=M_Z can be obtained from the measurements of alpha(0) and the measurements of the hadronic contribution.

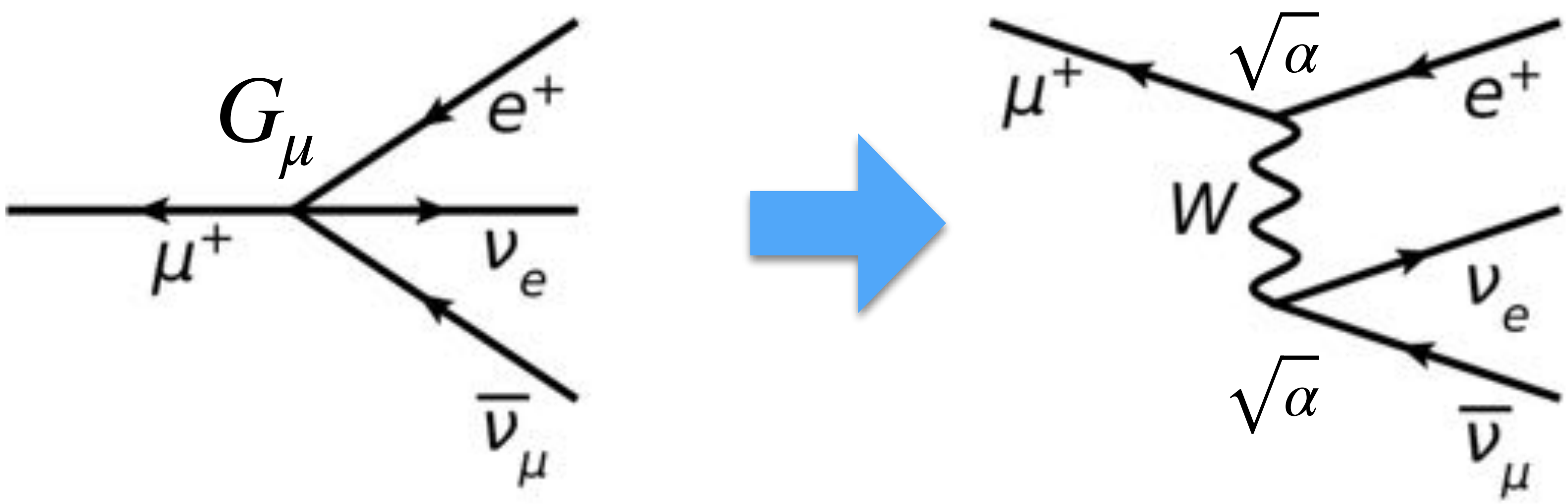
The alpha(M_Z) scheme is not IR sensitive and has an MSbar-like structure for the counterterm of the electric coupling.
In other words, alpha in the MSbar scheme can be obtained from alpha(Q) + a finite term.

$$\alpha_{\overline{\rm MS}}(Q^2) = \alpha(Q^2)(1 + \bar{\Delta}\alpha(Q^2)) + \mathcal{O}(\alpha^3 + \alpha\alpha_s)$$

$$\bar{\Delta}\alpha(Q^2) \equiv \frac{\alpha}{\pi}\left(\frac{100}{27} - \frac{4}{9}\log(\frac{m_t^2}{Q^2})\theta(Q^2 - m_t^2) + \frac{7}{4}\left(\log\frac{m_W^2}{Q^2}\right) - \frac{1}{6}\right) \qquad Q > m_W$$

G_μ-scheme

We can also extract the EW interactions from muon decay.



The muon-lifetime τ_μ can be written in the Fermi Model plus QED as

$$\frac{1}{\tau_\mu} = \frac{G_\mu^2 m_\mu^5}{192\pi^3}\left(1 - 8\frac{m_e^2}{m_\mu^2}\right)(1 + F(\alpha)) \tag{5.1}$$

where $F(\alpha)$ is a function that incorporates all the possible QED corrections.

One can measure τ_μ and determine G_μ. Then, one can calculate τ_μ in the SM and thus re-express G_μ via the SM parameters. One gets

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}s_W^2 m_W^2}(1 + \Delta r(\alpha(0), m_W, m_Z, m_H, m_t, m_f)) \tag{5.2}$$

Starting from one loop, all SM masses are entering

G_μ-scheme

$$\Delta r = \Pi^{AA}(0) + \frac{\Sigma_T^{WW}(0) - \Re(\Sigma_T^{WW}(m_W^2))}{m_W^2} - \frac{c_W^2}{s_W^2}\Re\left(\frac{\Sigma_T^{ZZ}(m_Z^2)}{m_Z^2} - \frac{\Sigma_T^{WW}(m_W^2)}{m_W^2}\right)$$
$$+ 2\frac{c_W}{s_W}\frac{\Sigma_T^{AZ}(0)}{m_Z^2} + \frac{\alpha}{4\pi s_W^2}\left(6 + \frac{7-4s_W^2}{2s_W^2}\log\left(\frac{m_W^2}{m_Z^2}\right)\right)$$

Let's see what are the dominant contributions. A doublet of light fermions give a contribution

$$\Delta r_{\text{light fermions}} = \frac{\alpha}{3\pi}\sum_{f=\pm}\left[Q_f^2\left(\log\left(\frac{m_Z^2}{m_f^2}\right) - \frac{5}{3}\right) + \ldots\right]$$

while a doublet with an heavy up-type and light down-type

$$\Delta r_{\text{heavy/light doublet}} = -\frac{\alpha}{4\pi}\frac{c_W^2}{4s_W^2}\frac{m_+^2}{m_W^2} + \ldots$$

G_μ-scheme

$$\Delta r = \Pi^{AA}(0) + \frac{\Sigma_T^{WW}(0) - \Re(\Sigma_T^{WW}(m_W^2))}{m_W^2} - \frac{c_W^2}{s_W^2}\Re\left(\frac{\Sigma_T^{ZZ}(m_Z^2)}{m_Z^2} - \frac{\Sigma_T^{WW}(m_W^2)}{m_W^2}\right)$$
$$+ 2\frac{c_W}{s_W}\frac{\Sigma_T^{AZ}(0)}{m_Z^2} + \frac{\alpha}{4\pi s_W^2}\left(6 + \frac{7-4s_W^2}{2s_W^2}\log\left(\frac{m_W^2}{m_Z^2}\right)\right)$$

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$$\Delta r_{\text{light fermions}} =$$

the contribution of the light fermions to $\Delta\alpha(m_Z^2)$

while a doublet with an heavy up-type and light down-type

$$\Delta r_{\text{heavy/light doublet}} = -\frac{c_W^2}{s_W^2}\Delta\rho$$

ρ parameter defined via charged and neutral neutrino currents

G_μ-scheme

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ρ parameter defined via charged and neutral neutrino currents

The crucial point is that we can write

$$\boxed{\Delta r = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho + \Delta r_{\text{rem}}}$$

with Δr_{rem} including all subdominant corrections

$$\Delta\alpha \sim 0.06, \quad \Delta\rho \sim 0.01, \quad \Delta r_{\text{rem}} \sim 0.01 \quad \rightarrow \quad \Delta r \sim 0.03$$

G_μ-scheme

The correct prescription for linking G_μ to SM parameters resumming higher-orders $\Delta\rho$ and $\Delta\alpha$ effects is

$\bar{s}_W^2 \equiv s_W^2 + c_W^2 \Delta\rho$

$$G_\mu = \frac{\pi\alpha(0)}{\sqrt{2}\bar{s}_W^2 m_W^2} \frac{(1+\Delta r_{\rm rem})}{1-\Delta\alpha} = \frac{\pi\alpha(m_Z^2)}{\sqrt{2}\bar{s}_W^2 m_W^2}(1+\Delta r_{\rm rem})$$

In conclusion the renormalization condition is defined as

$$\delta Z_e|_{G_\mu} \equiv \delta Z_e|_{\alpha(0)} - \frac{1}{2}\Delta r = \delta Z_e|_{\alpha(m_Z^2)} - \frac{1}{2}\left(-\frac{c_W^2}{s_W^2}\Delta\rho + \Delta r_{\rm rem}\right)$$

Every time we have a W interaction with a quark the coupling is $g = e/s_W$, so

$$\delta g = \frac{e}{s_W}\delta Z_e - \frac{e}{s_W}\frac{\delta s_W}{s_w} = g(\delta Z_e - \frac{\delta s_W}{s_w}) = g(\delta Z_e + \frac{c_W}{s_W^2}\delta c_W) \sim g(\delta Z_e - \frac{1}{2}\frac{c_W^2}{s_W^2}\Delta\rho)$$

the finite effect induced by the δs_W is canceled by the renormalization condition for the charge

Another way to see this: every $g^2 = \alpha/s_W^2$ term in the cross-section receives corrections to s_W^2 from

$$s_W^2 + \delta s_W^2 \sim s_W^2 + c_W^2 \delta\rho$$

which are then canceled by $\delta Z_e|_{G_\mu}$.

Schemes recap

$\{\alpha(0)\,,\ \ M_W\,,\ \ M_Z\} \to \alpha(0)$ scheme

$$\delta Z_e = -\frac{1}{2}\delta Z_{AA} - \frac{s_W}{c_W}\frac{1}{2}\delta Z_{ZA}$$

$\{\alpha(M_Z)\,,\ \ M_W\,,\ \ M_Z\} \to \alpha(M_Z)$ scheme

$$\delta Z_e|_{\alpha(Q^2)} \equiv \delta Z_e|_{\alpha(0)} - \frac{1}{2}\Delta\alpha(Q^2)$$

$\{G_\mu\,,\ \ M_W\,,\ \ M_Z\} \to G_\mu$ scheme

$$\delta Z_e|_{G_\mu} \equiv \delta Z_e|_{\alpha(0)} - \frac{1}{2}\Delta r =$$

$$\delta Z_e|_{\alpha(m_Z^2)} - \frac{1}{2}\left(-\frac{c_W^2}{s_W^2}\Delta\rho + \Delta r_{\rm rem}\right)$$

$\alpha(0) \sim 1/137 \qquad \alpha(M_Z) \sim 1/128 \qquad \alpha|_{G_\mu} \sim 1/132$

As a rule of thumb, for a generic process at the LHC, the Gmu scheme is superior and has to be preferred. However, if a photon is present in the Born final-state, alpha(0) and the corresponding renormalisation should be used for the associated QED vertex.

Do you remember that in QCD:

NLO $\mathcal{O}(\alpha_s^3)$ →

$q\bar{q} \to t\bar{t}$: $2\Re(\mathcal{M}_{\text{tree}}\mathcal{M}^*_{1-\text{loop}})$

$gg \to t\bar{t}$: $2\Re(\mathcal{M}_{\text{tree}}\mathcal{M}^*_{1-\text{loop}})$

} Virtual

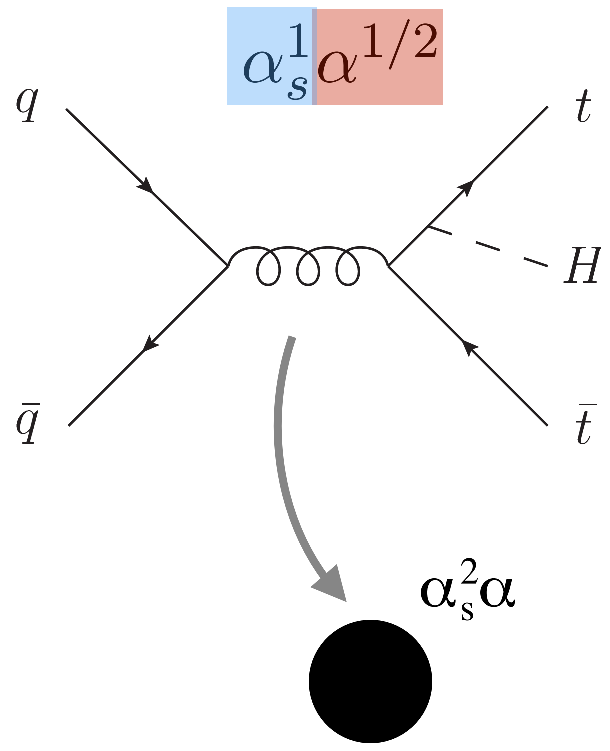
it can be more complex:

NLO EW and Complete-NLO

Structure of NLO EW-QCD corrections

$t\bar{t}H$

as example



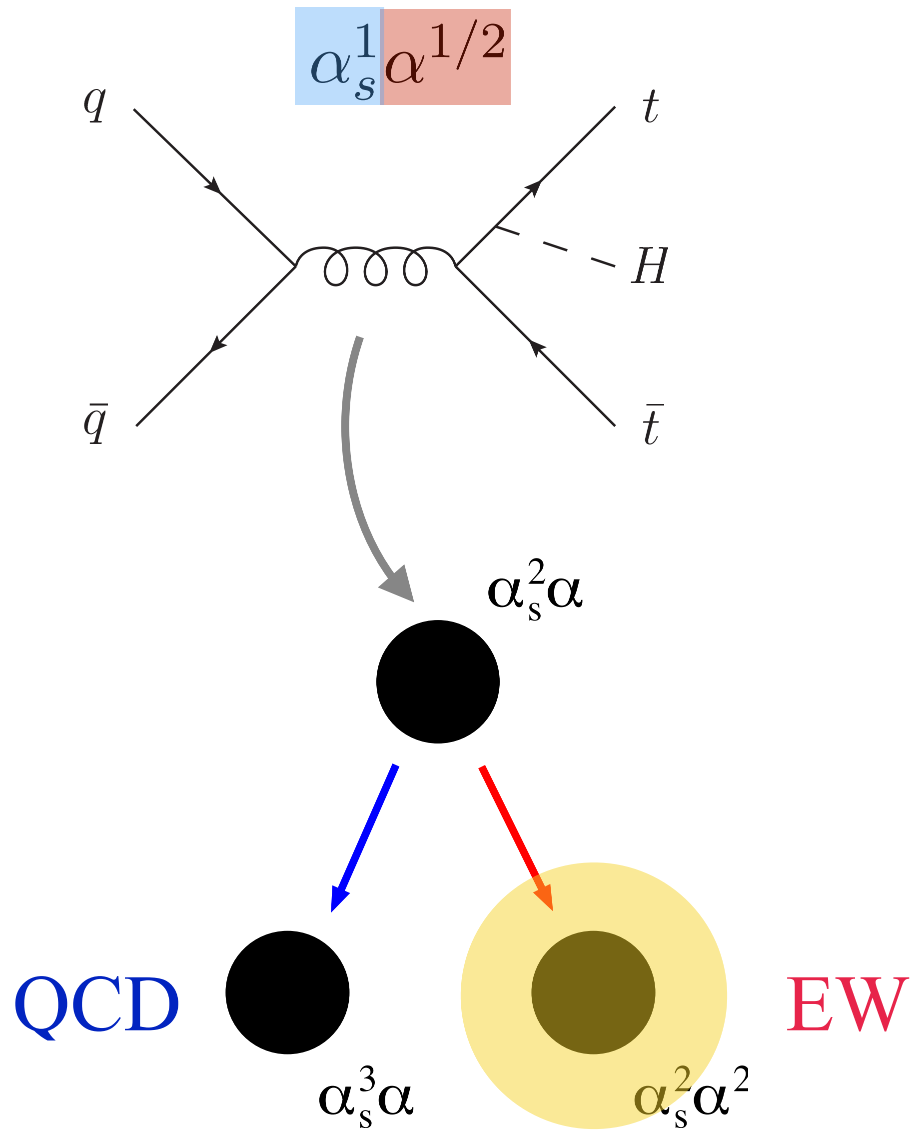
LO

Structure of NLO EW-QCD corrections

$t\bar{t}H$
as example

LO

NLO



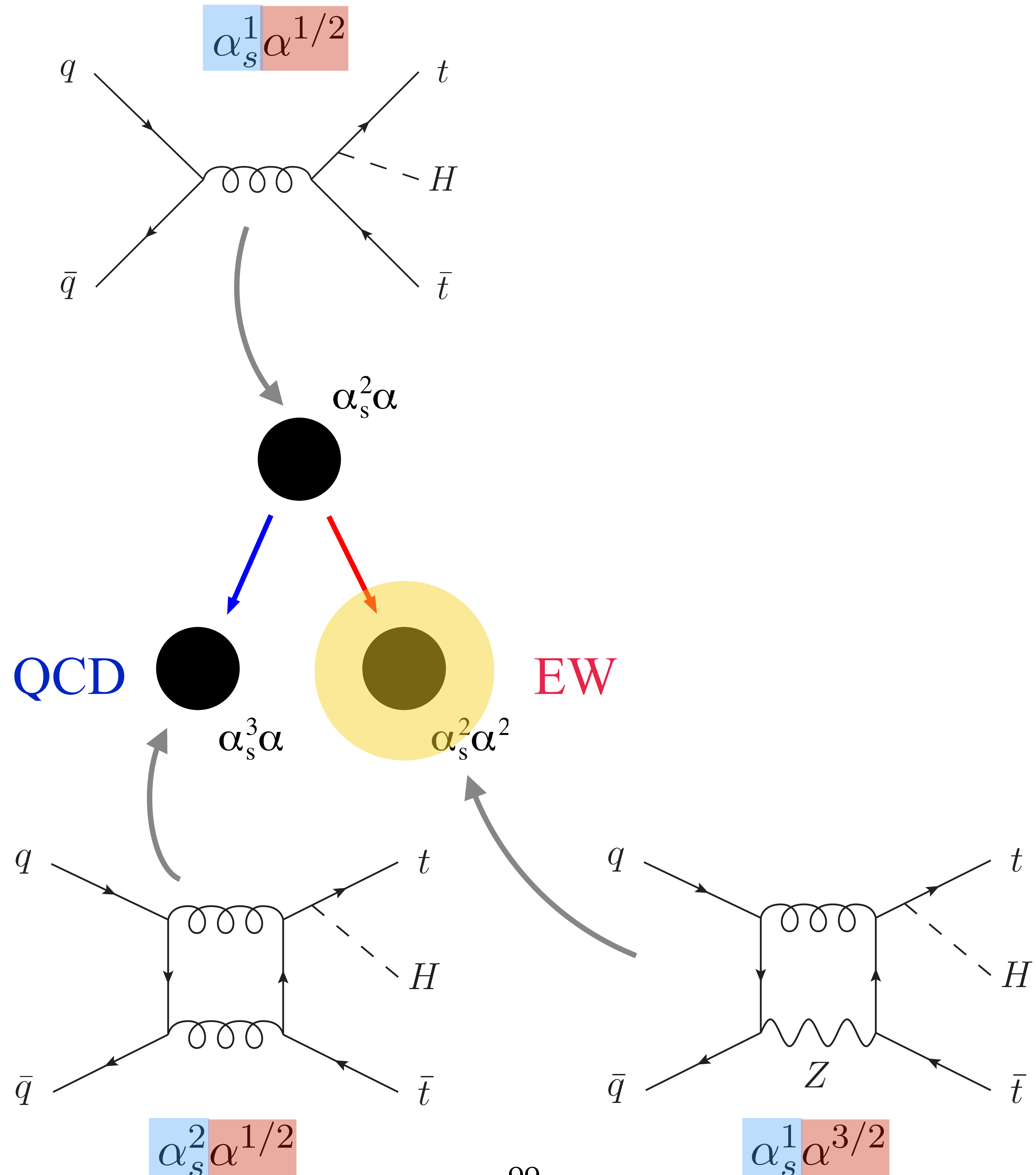
Structure of NLO EW-QCD corrections

$t\bar{t}H$

as example

LO

NLO



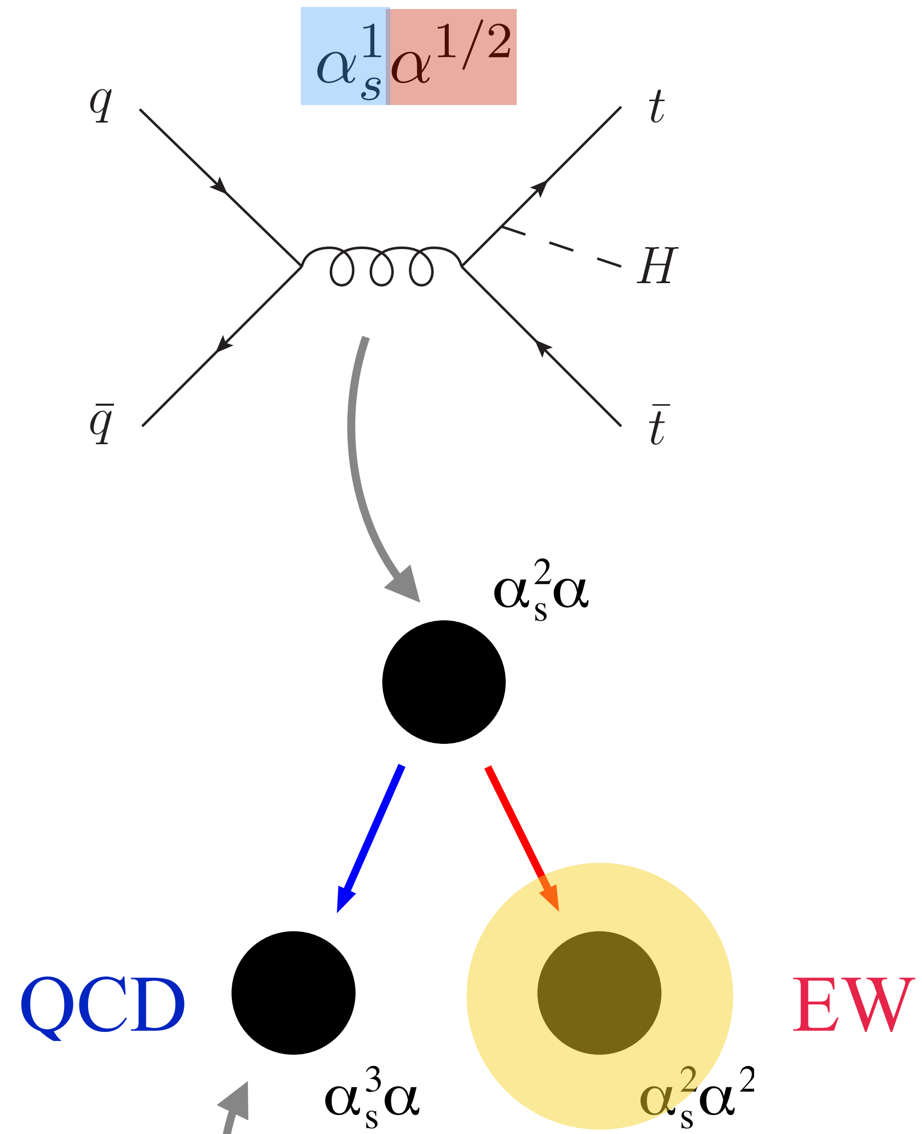
Structure of NLO EW-QCD corrections

$t\bar{t}H$

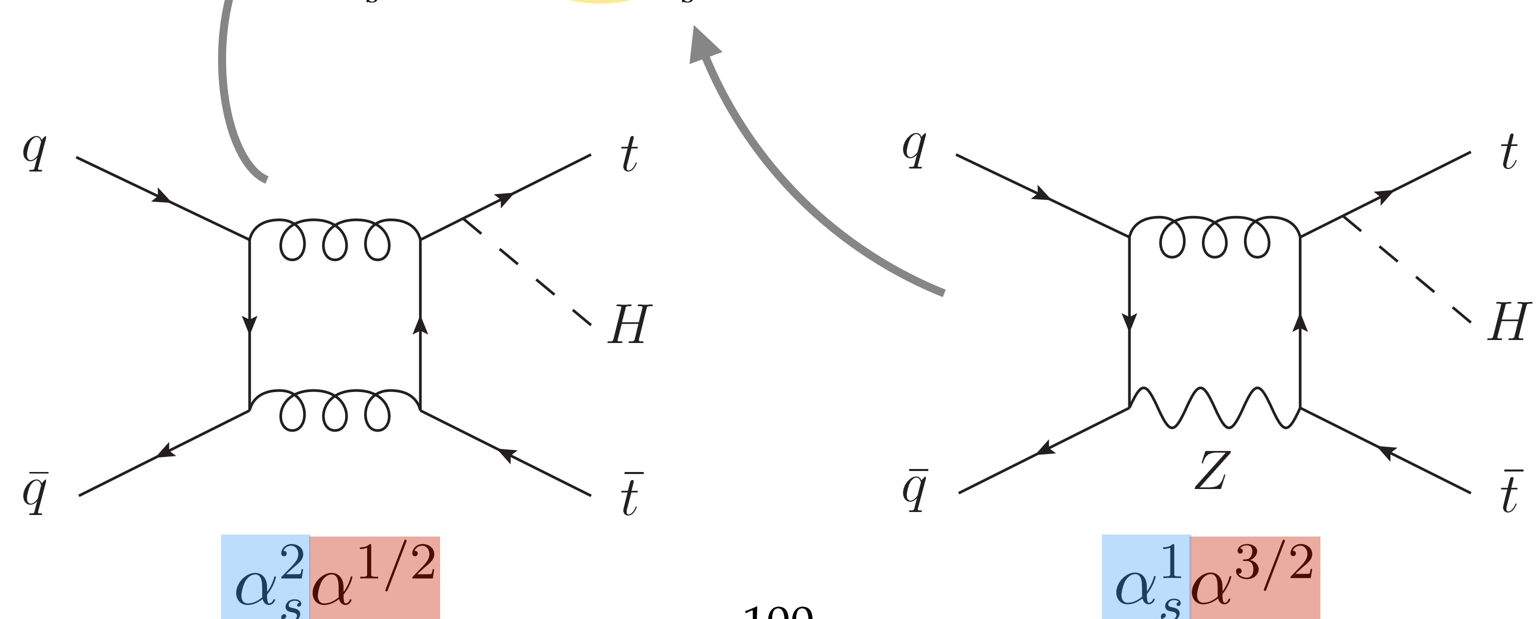
as example

LO

NLO



Is it the only way for obtaining $\mathcal{O}(\alpha)$ corrections?



Structure of NLO EW-QCD corrections

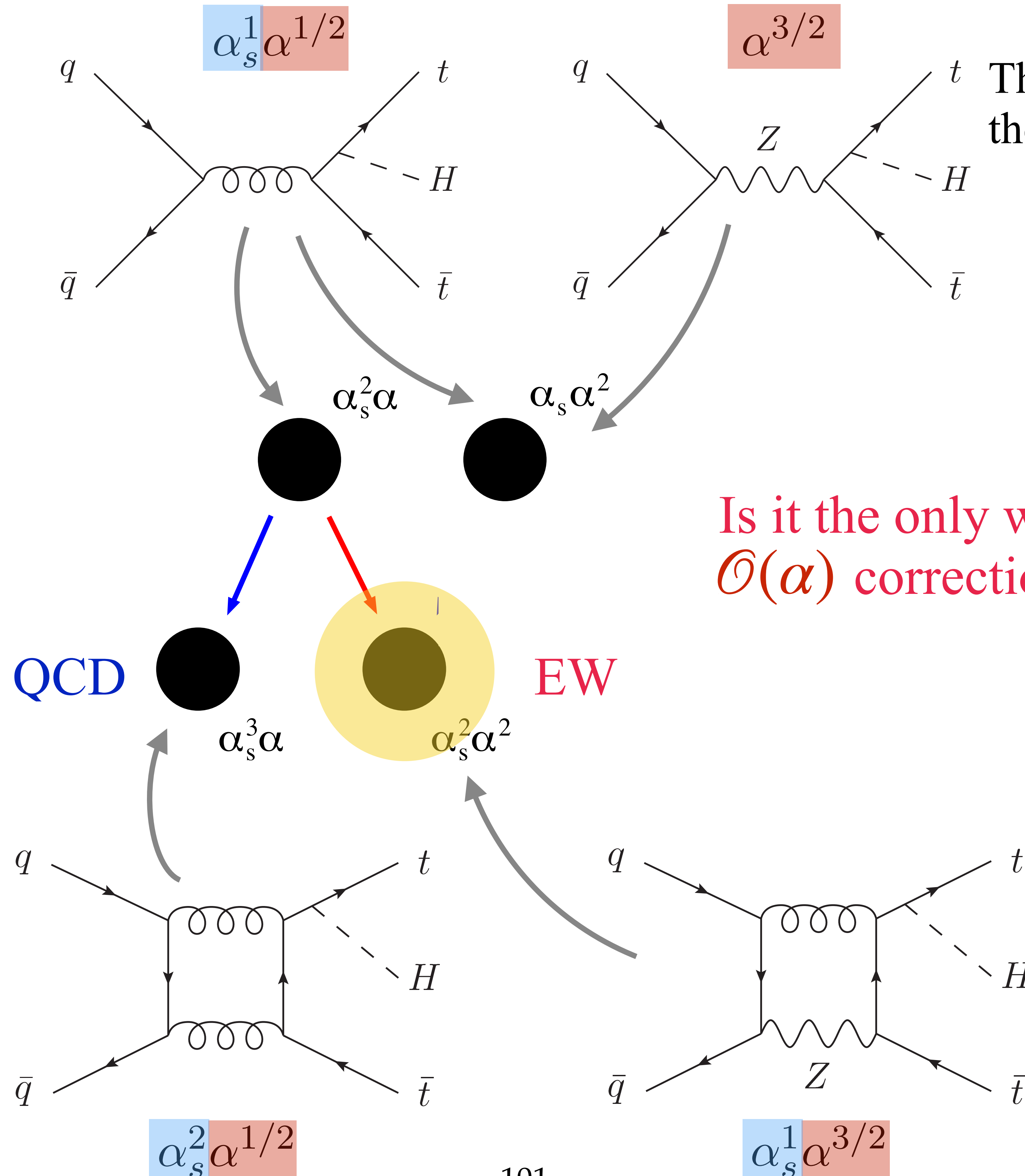
$t\bar{t}H$

as example

The interference of these 2 diagrams is zero,

LO

NLO



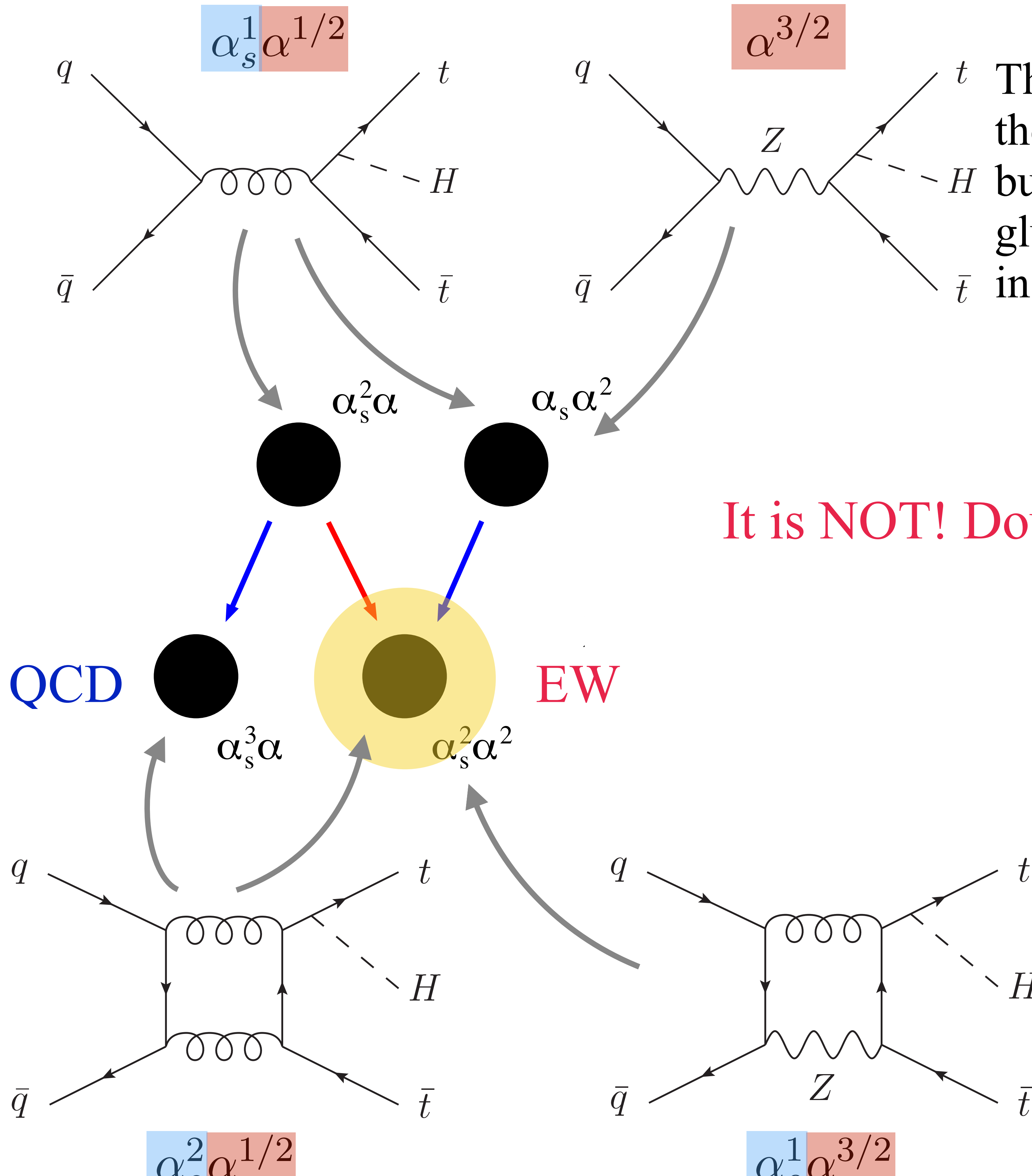
Is it the only way for obtaining $\mathcal{O}(\alpha)$ corrections?

Structure of NLO EW-QCD corrections

$t\bar{t}H$
as example

LO

NLO



The interference of these 2 diagrams is zero, but if I add another gluon between q and t in the first, it is not!

It is NOT! Double structure!

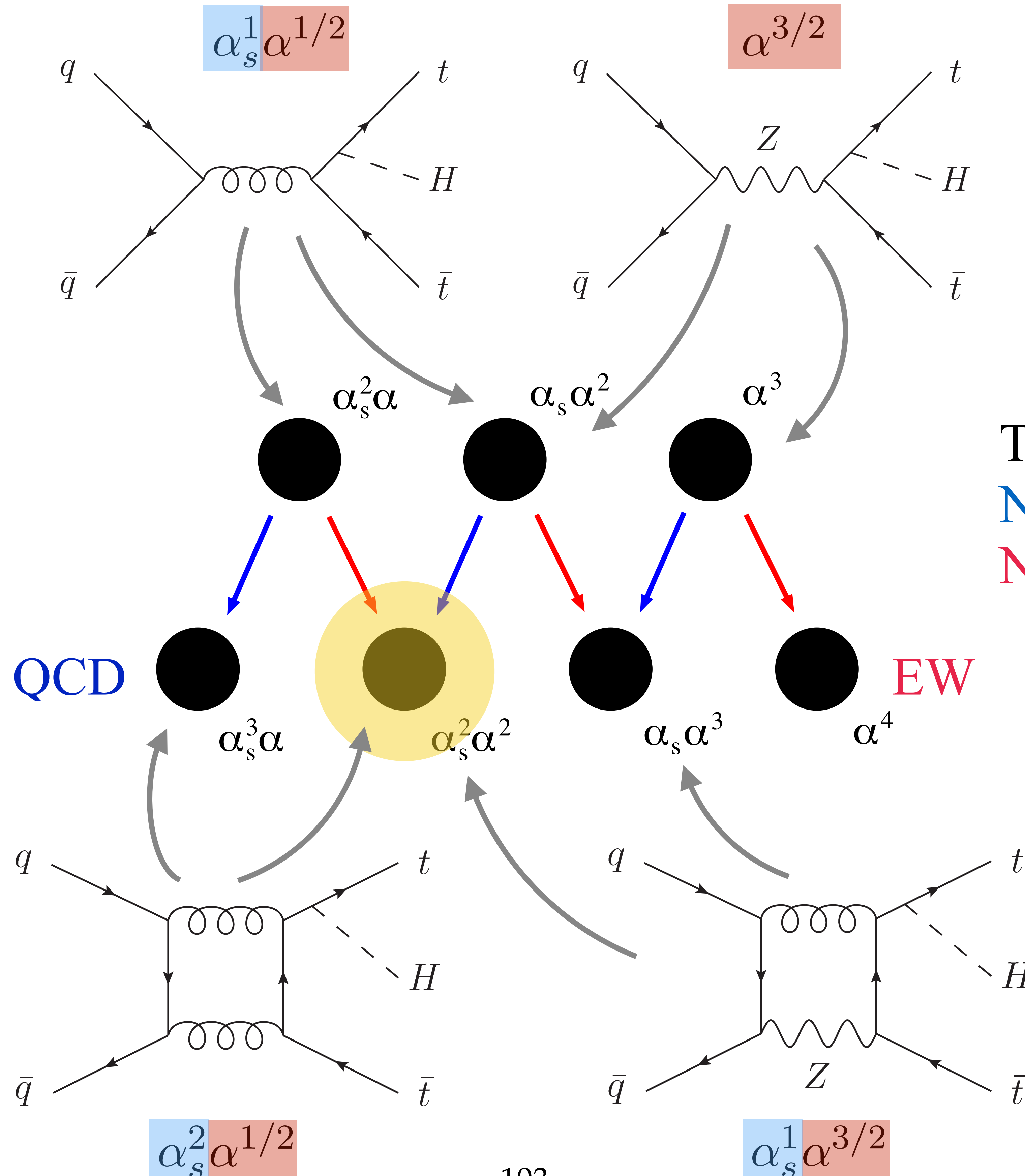
Structure of NLO EW-QCD corrections

$t\bar{t}H$

as example

LO

NLO



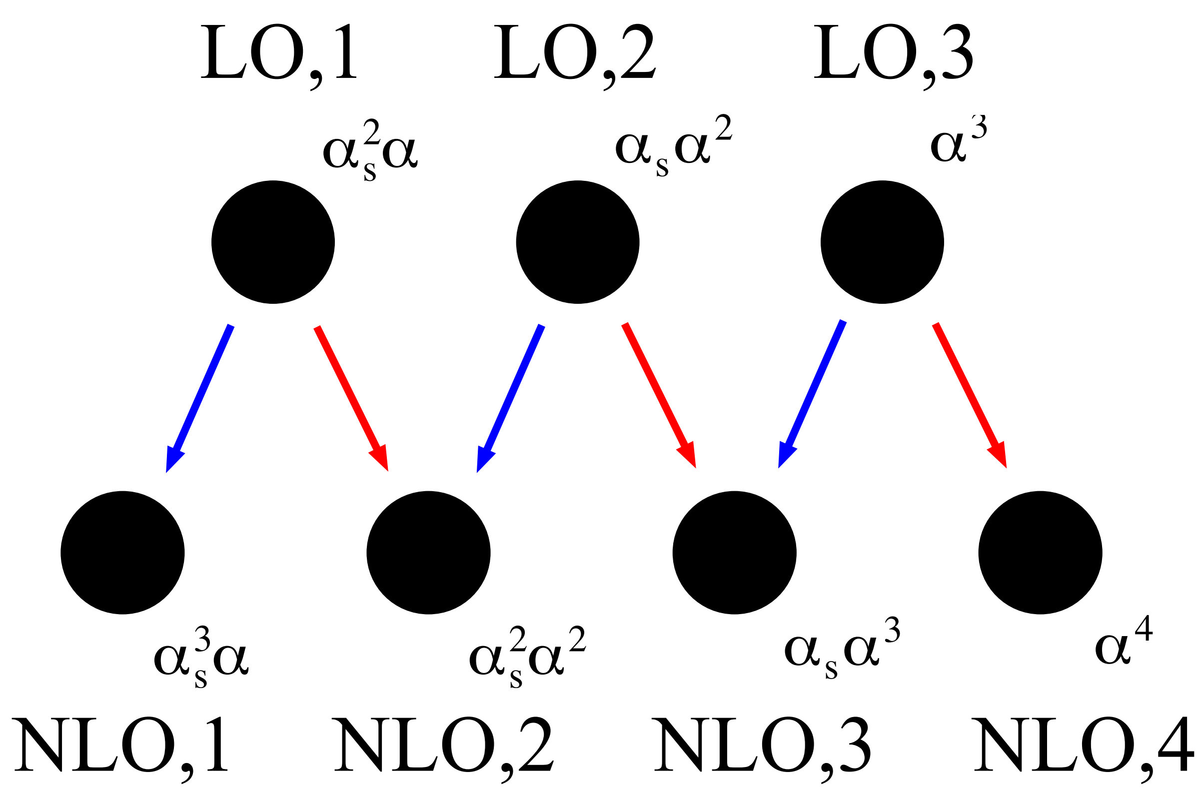
There is not just NLO QCD and NLO EW.

Structure of NLO EW-QCD corrections

$t\bar{t}H$

as example

We can denote the complete set of LO,i and NLO,i as **“Complete NLO”**.



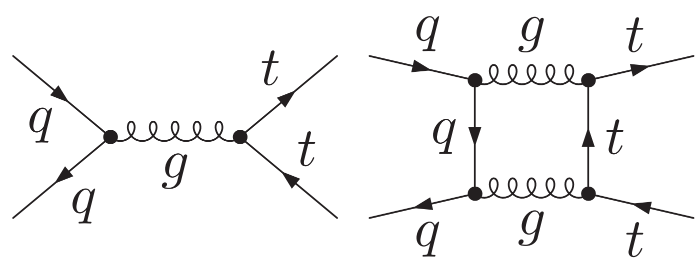
NLO,1 = NLO QCD
NLO,2 = NLO EW

In general, NLO,3 and NLO,4 sizes are negligible, but there are exceptions.

Complications for IR divergencies (let's forget about H)

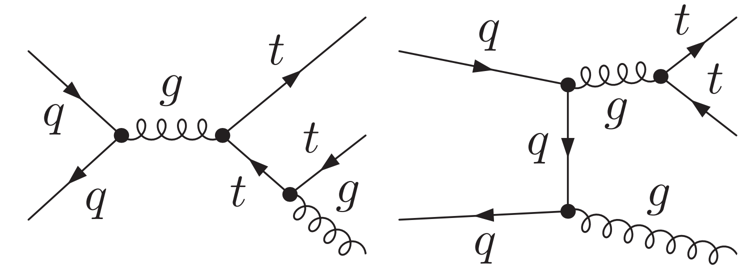
$$q\bar{q} \to t\bar{t}$$

$$\overline{|\mathcal{M}^{t\bar{t}}|}^2_{\mathcal{O}(\alpha_s^3)}$$



$$q\bar{q} \to t\bar{t}g$$

$$\overline{|\mathcal{M}^{t\bar{t}g}|}^2_{\mathcal{O}(\alpha_s^3)}$$



At NLO QCD, IR divergencies in the loops are canceled by corresponding gluon real emissions. At NLO EW we have also emissions of photons, but also the double structure we have shown before. So what happens?

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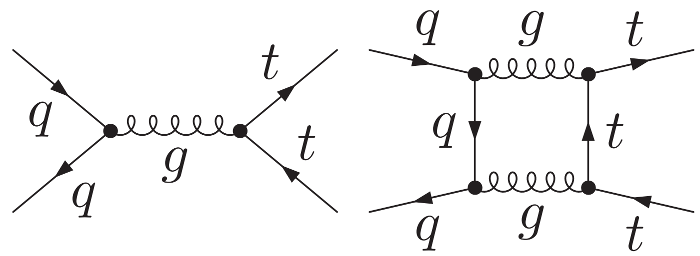

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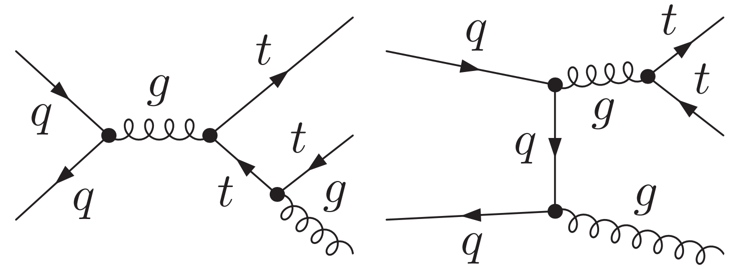


At NLO QCD, IR divergencies in the loops are canceled by corresponding gluon real emissions. At NLO EW we have also emissions of photons, but also the double structure we have shown before. So what happens?

SHORT DIGRESSION

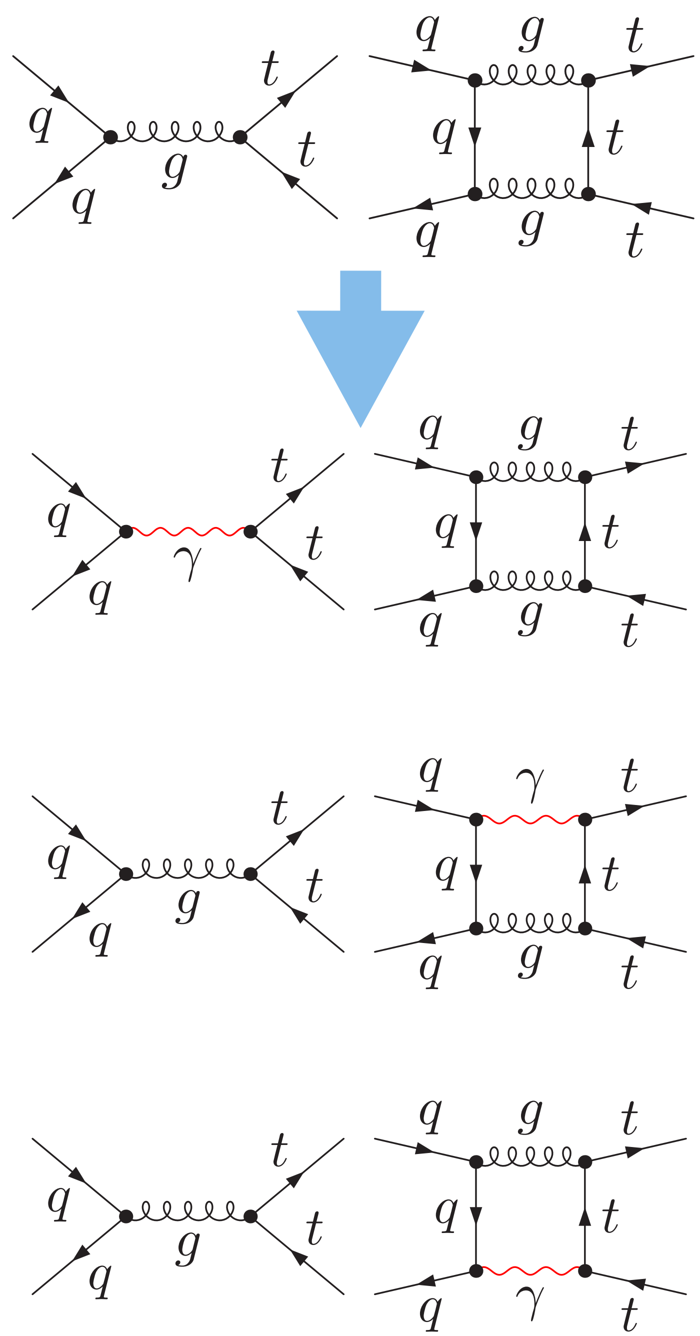
This discussions concern the QED part of the NLO EW corrections. NLO EW can be divided into a **QED** part (photonic emission or loop corrections from fermions) and a **purely Weak** part (all the rest, including vacuum polarisations). However this separation is **not** in general **gauge-invariant**!

Complications for IR divergencies (let's forget about H)

$$q\bar{q} \to t\bar{t}$$

$\overline{|\mathcal{M}^{t\bar{t}}|}^2_{\mathcal{O}(\alpha_s^3)}$

$\overline{|\mathcal{M}^{t\bar{t}}|}^2_{\mathcal{O}(\alpha_s^2\alpha)}$

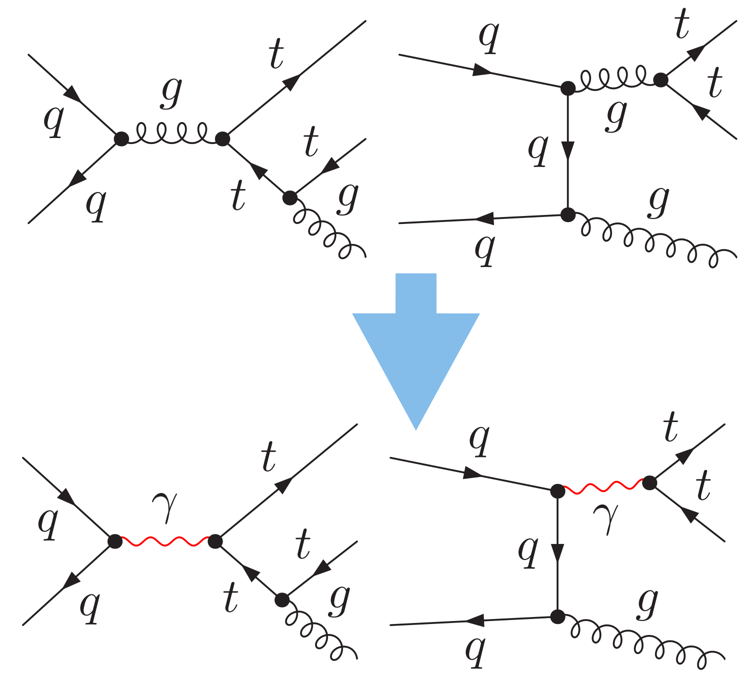


Photons and gluons at the same time leads to more diagrams and more IR divergencies!

$$q\bar{q} \to t\bar{t}g$$

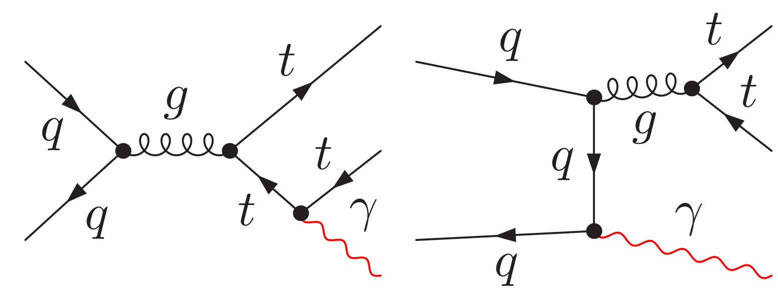
$\overline{|\mathcal{M}^{t\bar{t}g}|}^2_{\mathcal{O}(\alpha_s^3)}$

$\overline{|\mathcal{M}^{t\bar{t}g}|}^2_{\mathcal{O}(\alpha_s^2\alpha)}$



$$q\bar{q} \to t\bar{t}\gamma$$

$\overline{|\mathcal{M}^{t\bar{t}\gamma}|}^2_{\mathcal{O}(\alpha_s^2\alpha)}$

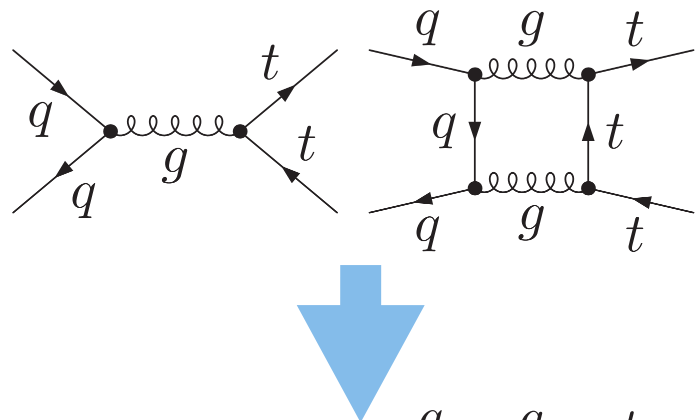


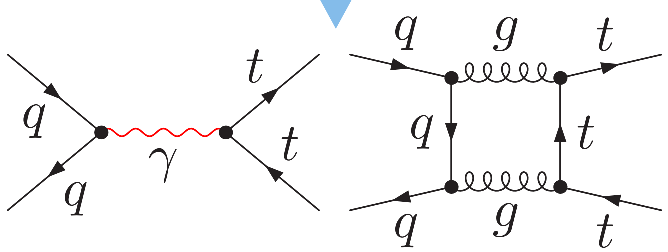
Radiation of photons and gluons have both to be considered for obtaining IR finiteness at NLO EW.

Complications for IR divergencies (let's forget about H)

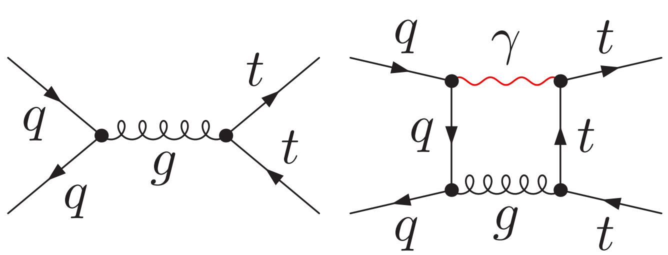
$$q\bar{q} \to t\bar{t}$$

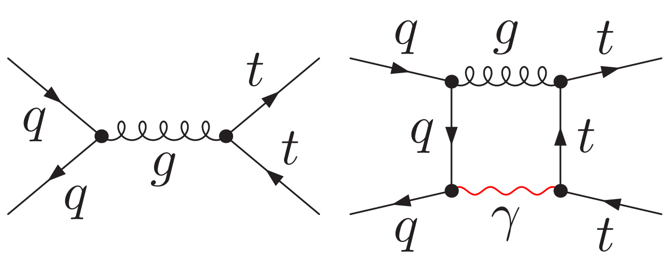
$\overline{|\mathcal{M}^{t\bar{t}}|^2}_{\mathcal{O}(\alpha_s^3)}$





$\overline{|\mathcal{M}^{t\bar{t}}|^2}_{\mathcal{O}(\alpha_s^2\alpha)}$





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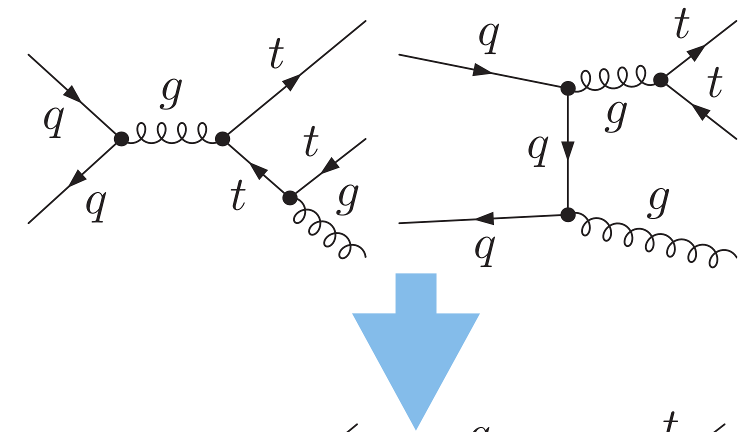
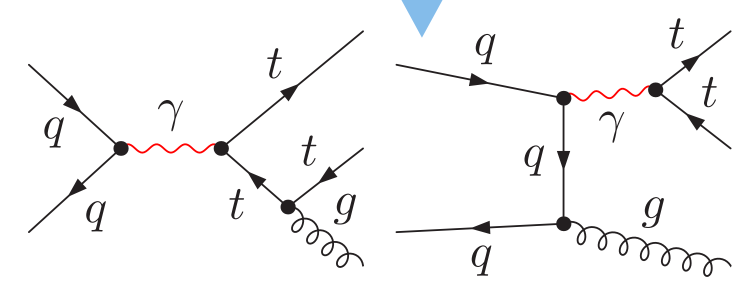


$\overline{|\mathcal{M}^{t\bar{t}g}|^2}_{\mathcal{O}(\alpha_s^2\alpha)}$



$$q\bar{q} \to t\bar{t}\gamma$$

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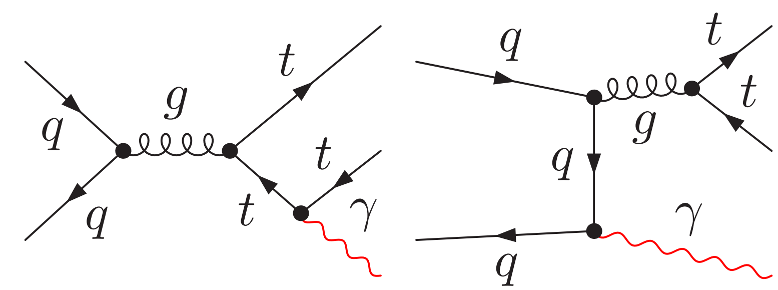


The situation is simpler for collinear radiation … unless jets and photons in the Born final state are considered.
Jet and photon definitions at NLO EW deserve too many slides for today ..

Automation of EW corrections in MadGraph5_aMC@NLO

Automation of NLO corrections in Madgraph5_aMC@NLO

What do we mean with automation of EW corrections?

The possibility of calculating QCD and EW corrections for SM processes (matched to shower effects) with a process-independent approach.

```
generate process [QCD]
output process_QCD
```

→

```
generate process [QCD EW]
output process_QCD_EW
```

again:
you need to know what's going on in order to understand the results

Recently, also the case of e^+e^- collisions has become available!

Bertone, Cacciari, Frixione, Stagnitto, Zaro, Zhao '22

Results: NLO EW

just type:

```
set complex mass scheme true
import model loop_qcd_qed_sm_Gmu
generate process [QED]
output process_NLO_EW_corrections
```

And then wait for the results ………….

CMS is necessary every time a resonance is present in the calculation (Z, W, top, …)

Results: NLO EW

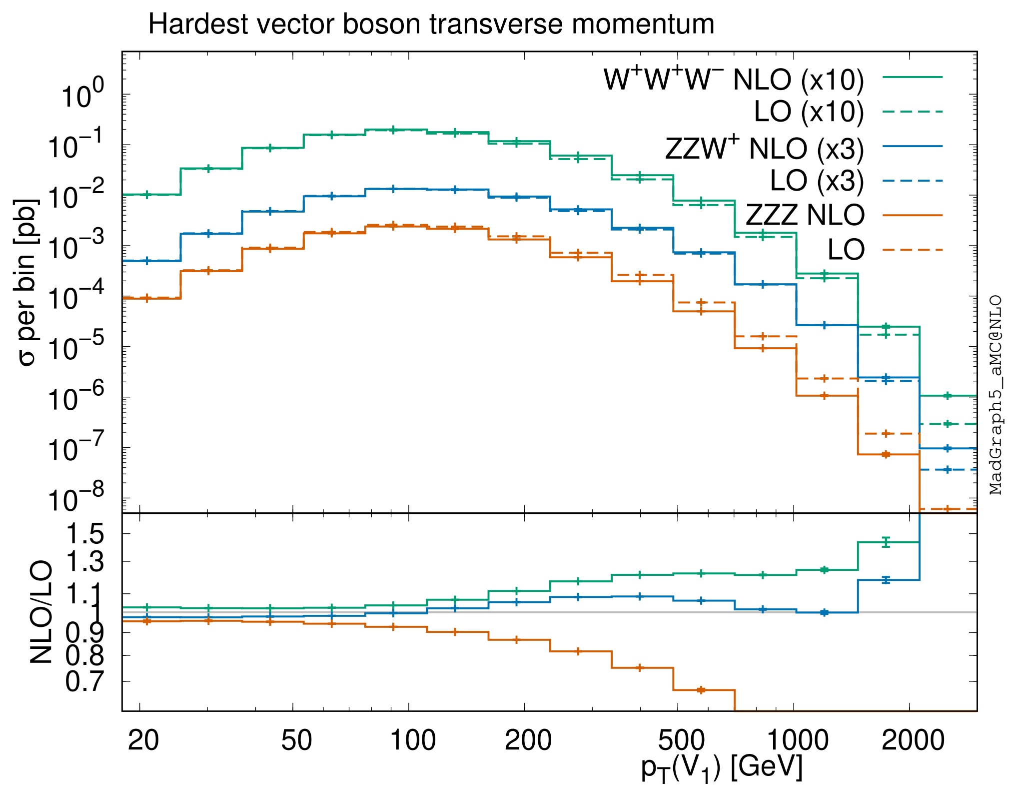
Process	Syntax	Cross section (in pb) LO	NLO	Correction (in %)
$pp \to e^+\nu_e$	`p p > e+ ve QCD=0 QED=2 [QED]`	$5.2498 \pm 0.0005 \cdot 10^3$	$5.2113 \pm 0.0006 \cdot 10^3$	-0.73 ± 0.01
$pp \to e^+\nu_e j$	`p p > e+ ve j QCD=1 QED=2 [QED]`	$9.1468 \pm 0.0012 \cdot 10^2$	$9.0449 \pm 0.0014 \cdot 10^2$	-1.11 ± 0.02
$pp \to e^+\nu_e jj$	`p p > e+ ve j j QCD=2 QED=2 [QED]`	$3.1562 \pm 0.0003 \cdot 10^2$	$3.0985 \pm 0.0005 \cdot 10^2$	-1.83 ± 0.02
$pp \to e^+e^-$	`p p > e+ e- QCD=0 QED=2 [QED]`	$7.5367 \pm 0.0008 \cdot 10^2$	$7.4997 \pm 0.0010 \cdot 10^2$	-0.49 ± 0.02
$pp \to e^+e^- j$	`p p > e+ e- j QCD=1 QED=2 [QED]`	$1.5059 \pm 0.0001 \cdot 10^2$	$1.4909 \pm 0.0002 \cdot 10^2$	-1.00 ± 0.02
$pp \to e^+e^- jj$	`p p > e+ e- j j QCD=2 QED=2 [QED]`	$5.1424 \pm 0.0004 \cdot 10^1$	$5.0410 \pm 0.0007 \cdot 10^1$	-1.97 ± 0.02
$pp \to e^+e^-\mu^+\mu^-$	`p p > e+ e- mu+ mu- QCD=0 QED=4 [QED]`	$1.2750 \pm 0.0000 \cdot 10^{-2}$	$1.2083 \pm 0.0001 \cdot 10^{-2}$	-5.23 ± 0.01
$pp \to e^+\nu_e\mu^-\bar{\nu}_\mu$	`p p > e+ ve mu- vm~ QCD=0 QED=4 [QED]`	$5.1144 \pm 0.0007 \cdot 10^{-1}$	$5.3019 \pm 0.0009 \cdot 10^{-1}$	$+3.67 \pm 0.02$
$pp \to He^+\nu_e$	`p p > h e+ ve QCD=0 QED=3 [QED]`	$6.7643 \pm 0.0001 \cdot 10^{-2}$	$6.4914 \pm 0.0012 \cdot 10^{-2}$	-4.03 ± 0.02
$pp \to He^+e^-$	`p p > h e+ e- QCD=0 QED=3 [QED]`	$1.4554 \pm 0.0001 \cdot 10^{-2}$	$1.3700 \pm 0.0002 \cdot 10^{-2}$	-5.87 ± 0.02
$pp \to Hjj$	`p p > h j j QCD=0 QED=3 [QED]`	$2.8268 \pm 0.0002 \cdot 10^0$	$2.7075 \pm 0.0003 \cdot 10^0$	-4.22 ± 0.01
$pp \to W^+W^-W^+$	`p p > w+ w- w+ QCD=0 QED=3 [QED]`	$8.2874 \pm 0.0004 \cdot 10^{-2}$	$8.8017 \pm 0.0012 \cdot 10^{-2}$	$+6.21 \pm 0.02$
$pp \to ZZW^+$	`p p > z z w+ QCD=0 QED=3 [QED]`	$1.9874 \pm 0.0001 \cdot 10^{-2}$	$2.0189 \pm 0.0003 \cdot 10^{-2}$	$+1.58 \pm 0.02$
$pp \to ZZZ$	`p p > z z z QCD=0 QED=3 [QED]`	$1.0761 \pm 0.0001 \cdot 10^{-2}$	$0.9741 \pm 0.0001 \cdot 10^{-2}$	-9.47 ± 0.02
$pp \to HZZ$	`p p > h z z QCD=0 QED=3 [QED]`	$2.1005 \pm 0.0003 \cdot 10^{-3}$	$1.9155 \pm 0.0003 \cdot 10^{-3}$	-8.81 ± 0.02
$pp \to HZW^+$	`p p > h z w+ QCD=0 QED=3 [QED]`	$2.4408 \pm 0.0000 \cdot 10^{-3}$	$2.4809 \pm 0.0005 \cdot 10^{-3}$	$+1.64 \pm 0.02$
$pp \to HHW^+$	`p p > h h w+ QCD=0 QED=3 [QED]`	$2.7827 \pm 0.0001 \cdot 10^{-4}$	$2.4259 \pm 0.0027 \cdot 10^{-4}$	-12.82 ± 0.10
$pp \to HHZ$	`p p > h h z QCD=0 QED=3 [QED]`	$2.6914 \pm 0.0003 \cdot 10^{-4}$	$2.3926 \pm 0.0003 \cdot 10^{-4}$	-11.10 ± 0.02
$pp \to t\bar{t}W^+$	`p p > t t~ w+ QCD=2 QED=1 [QED]`	$2.4119 \pm 0.0003 \cdot 10^{-1}$	$2.3025 \pm 0.0003 \cdot 10^{-1}$	-4.54 ± 0.02
$pp \to t\bar{t}Z$	`p p > t t~ z QCD=2 QED=1 [QED]`	$5.0456 \pm 0.0006 \cdot 10^{-1}$	$5.0033 \pm 0.0007 \cdot 10^{-1}$	-0.84 ± 0.02
$pp \to t\bar{t}H$	`p p > t t~ h QCD=2 QED=1 [QED]`	$3.4480 \pm 0.0004 \cdot 10^{-1}$	$3.5102 \pm 0.0005 \cdot 10^{-1}$	$+1.81 \pm 0.02$
$pp \to t\bar{t}j$	`p p > t t j QCD=3 QED=0 [QED]`	$3.0277 \pm 0.0003 \cdot 10^2$	$2.9683 \pm 0.0004 \cdot 10^2$	-1.96 ± 0.02
$pp \to jjj$	`p p > j j j QCD=3 QED=0 [QED]`	$7.9639 \pm 0.0010 \cdot 10^6$	$7.9472 \pm 0.0011 \cdot 10^6$	-0.21 ± 0.02
$pp \to tj$	`p p > t j QCD=0 QED=2 [QED]`	$1.0613 \pm 0.0001 \cdot 10^2$	$1.0539 \pm 0.0001 \cdot 10^2$	-0.70 ± 0.02

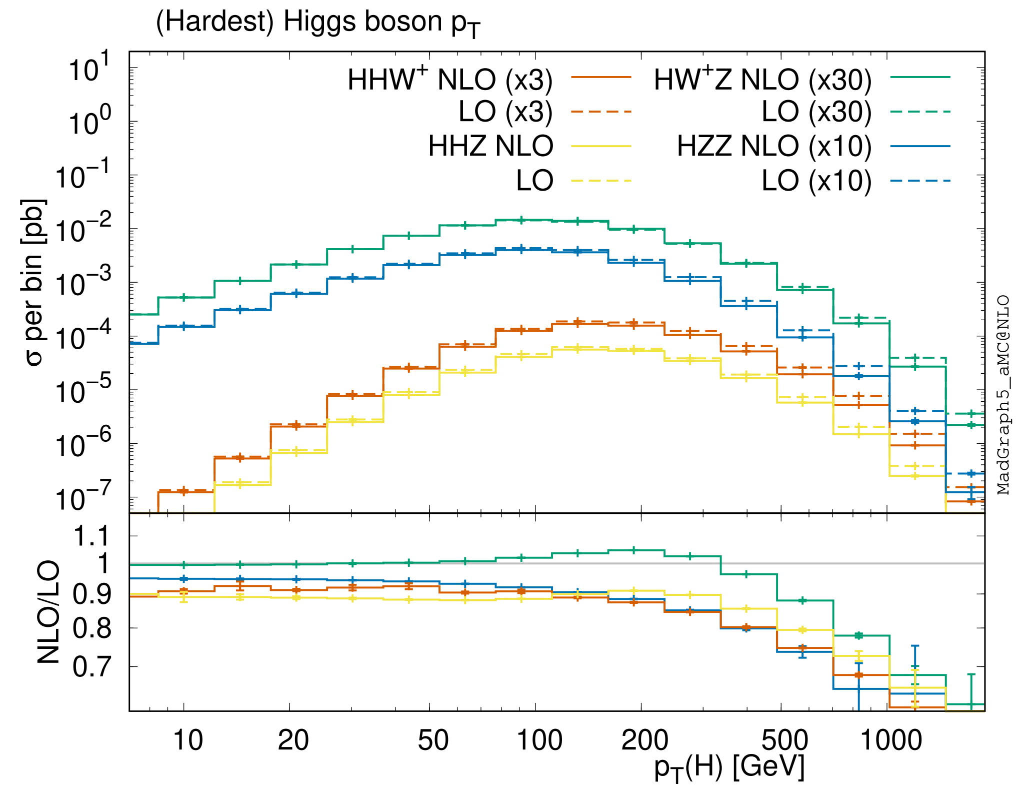
Calculated for the 1st time

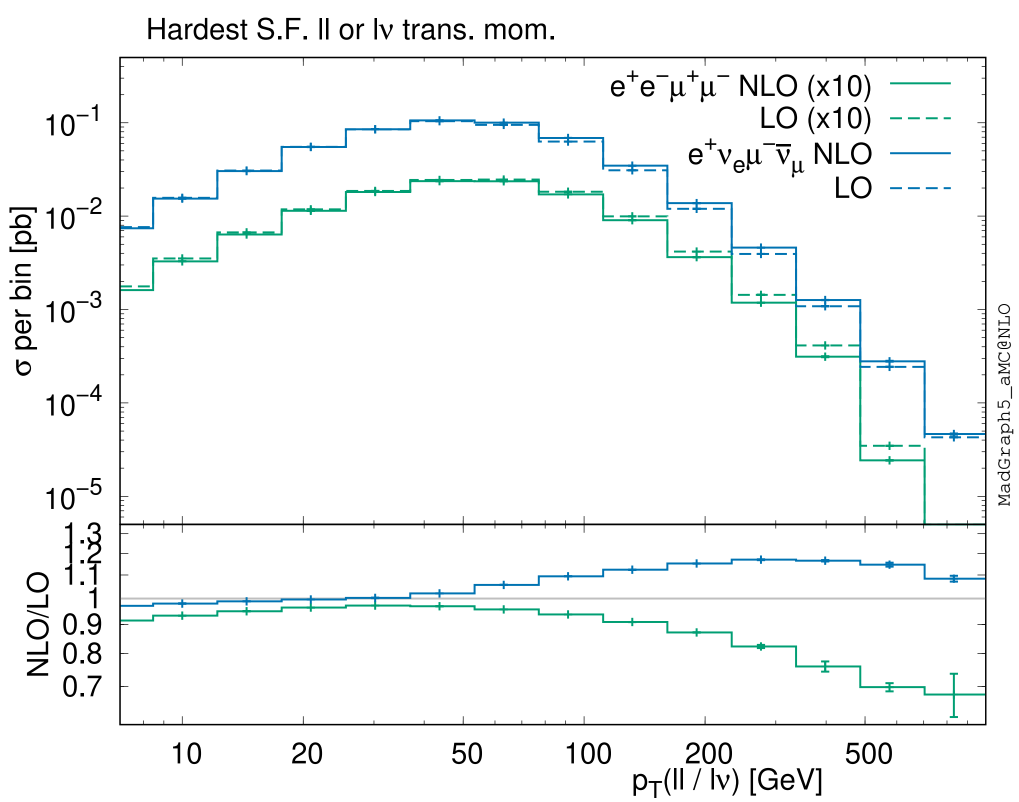
couple of weeks on $\mathcal{O}(200)$ CPUs

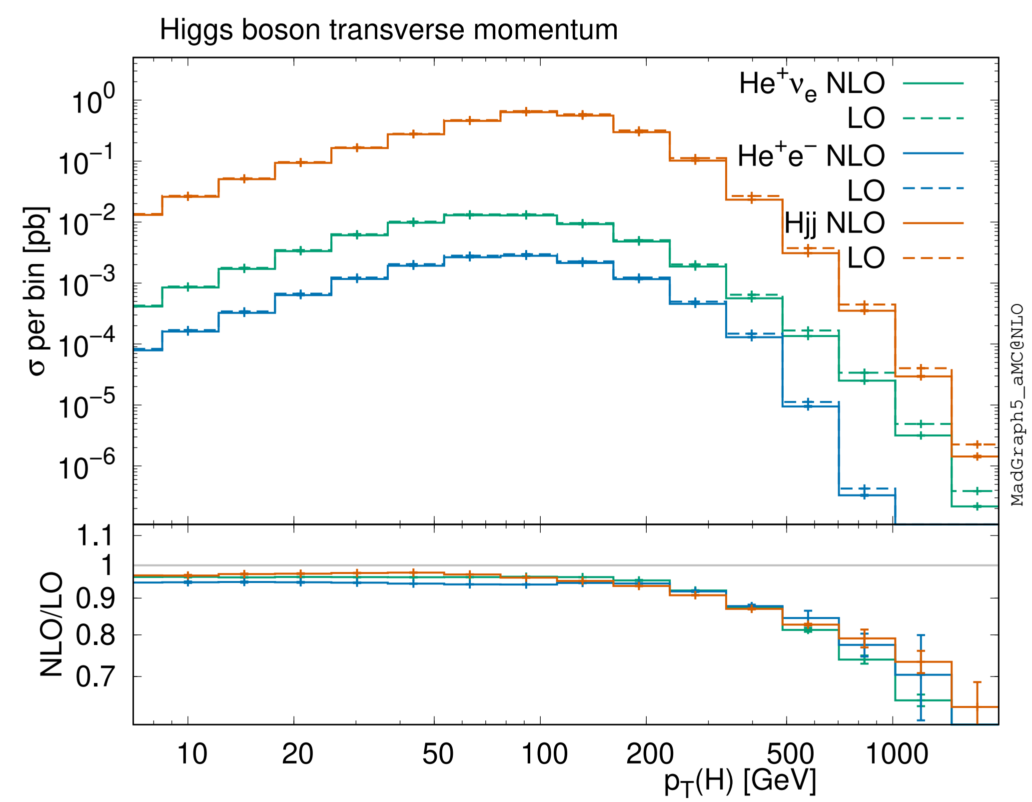
$$\delta_{\mathrm{EW}} = \frac{\Sigma_{\mathrm{NLO}_2}}{\Sigma_{\mathrm{LO}_1}} = \frac{\mathrm{NLO}}{\mathrm{LO}} - 1\,.$$

Results: NLO EW









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Results: Complete NLO

just type:

```
set complex mass scheme true
import model loop_qcd_qed_sm_Gmu
generate process QCD=99 QED=99 [QCD QED]
output process_NLO_EW_corrections
```

And then wait for the results …………..

Results: Complete NLO

NEW

	$pp \to t\bar{t}$	$pp \to t\bar{t}Z$	$pp \to t\bar{t}W^+$	$pp \to t\bar{t}H$	$pp \to t\bar{t}j$
LO_1	$4.3803 \pm 0.0005 \cdot 10^2$ pb	$5.0463 \pm 0.0003 \cdot 10^{-1}$ pb	$2.4116 \pm 0.0001 \cdot 10^{-1}$ pb	$3.4483 \pm 0.0003 \cdot 10^{-1}$ pb	$3.0278 \pm 0.0003 \cdot 10^2$ pb
LO_2	$+0.405 \pm 0.001$ %	-0.691 ± 0.001 %	$+0.000 \pm 0.000$ %	$+0.406 \pm 0.001$ %	$+0.525 \pm 0.001$ %
LO_3	$+0.630 \pm 0.001$ %	$+2.259 \pm 0.001$ %	$+0.962 \pm 0.000$ %	$+0.702 \pm 0.001$ %	$+1.208 \pm 0.001$ %
LO_4					$+0.006 \pm 0.000$ %
NLO_1	$+46.164 \pm 0.022$ %	$+44.809 \pm 0.028$ %	$+49.504 \pm 0.015$ %	$+28.847 \pm 0.020$ %	$+26.571 \pm 0.063$ %
NLO_2	-1.075 ± 0.003 %	-0.846 ± 0.004 %	-4.541 ± 0.003 %	$+1.794 \pm 0.005$ %	-1.971 ± 0.022 %
NLO_3	$+0.552 \pm 0.002$ %	$+0.845 \pm 0.003$ %	$+12.242 \pm 0.014$ %	$+0.483 \pm 0.008$ %	$+0.292 \pm 0.007$ %
NLO_4	$+0.005 \pm 0.000$ %	-0.082 ± 0.000 %	$+0.017 \pm 0.003$ %	$+0.044 \pm 0.000$ %	$+0.009 \pm 0.000$ %
NLO_5					$+0.005 \pm 0.000$ %

$$\frac{\Sigma_{\mathrm{LO}_i}}{\Sigma_{\mathrm{LO}_1}}\,, \qquad i = 2, 3, 4\,,$$

$$\frac{\Sigma_{\mathrm{NLO}_i}}{\Sigma_{\mathrm{LO}_1}}\,, \qquad i = 1, \ldots 5\,;$$

NLO3 in ttW is ~12%:

A thorough phenomenological study is necessary!

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$t\bar{t}W^{\pm}$

R. Frederix, D.P., M. Zaro

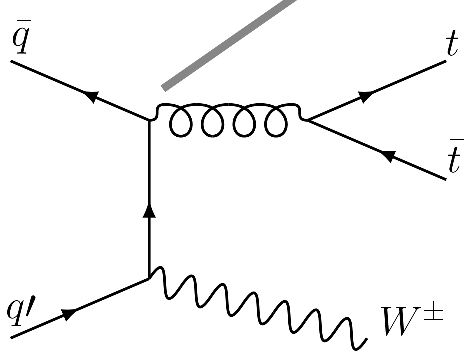
JHEP 1802 (2018) 031 (arXiv:1711.02116)

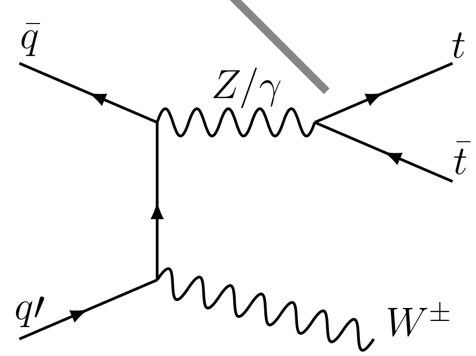
Complete-NLO

Frederix, DP, Zaro ’17

$$\Sigma_{\rm LO}^{t\bar{t}W^\pm}(\alpha_s,\alpha) = \alpha_s^2\alpha\Sigma_{3,0}^{t\bar{t}W^\pm} + \alpha_s\alpha\Sigma_{3,1}^{t\bar{t}W^\pm} + \alpha^2\Sigma_{3,2}^{t\bar{t}W^\pm}$$
$$\equiv \Sigma_{\rm LO_1} + \cancel{\Sigma_{\rm LO_2}} + \Sigma_{\rm LO_3}\,,$$

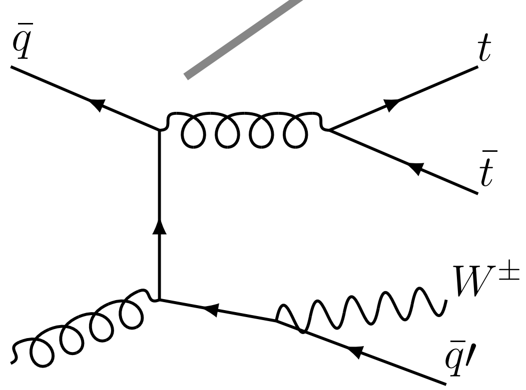
Only initial states without gluons are present.

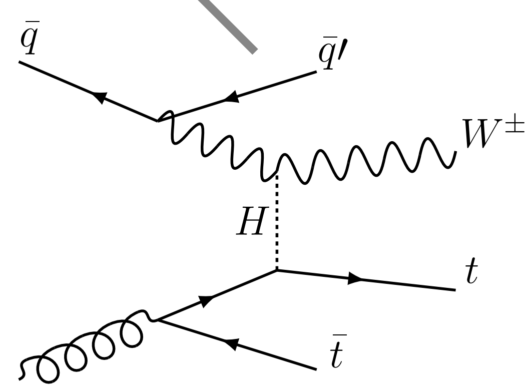




$\Sigma_{\rm LO_1} \longrightarrow {\rm LO_{QCD}}$

$$\Sigma_{\rm NLO}^{t\bar{t}W^\pm}(\alpha_s,\alpha) = \alpha_s^3\alpha\Sigma_{4,0}^{t\bar{t}W^\pm} + \alpha_s^2\alpha^2\Sigma_{4,1}^{t\bar{t}W^\pm} + \alpha_s\alpha^3\Sigma_{4,2}^{t\bar{t}W^\pm} + \alpha^4\Sigma_{4,3}^{t\bar{t}W^\pm}$$
$$\equiv \Sigma_{\rm NLO_1} + \Sigma_{\rm NLO_2} + \Sigma_{\rm NLO_3} + \Sigma_{\rm NLO_4}\,,$$





$\Sigma_{\rm NLO_1} \longrightarrow {\rm NLO_{QCD}}$

$\Sigma_{\rm NLO_2} \longrightarrow {\rm NLO_{EW}}$

MadGraph5_aMC@NLO

Cross sections: order by order

$$\delta_{(\mathrm{N})\mathrm{LO}_i}(\mu) = \frac{\Sigma_{(\mathrm{N})\mathrm{LO}_i}(\mu)}{\Sigma_{\mathrm{LO}_{\mathrm{QCD}}}(\mu)}$$

Numbers in parentheses refer to the case of a jet veto $p_T(j) > 100$ GeV and $|y(j)| < 2.5$ applied

13 TeV

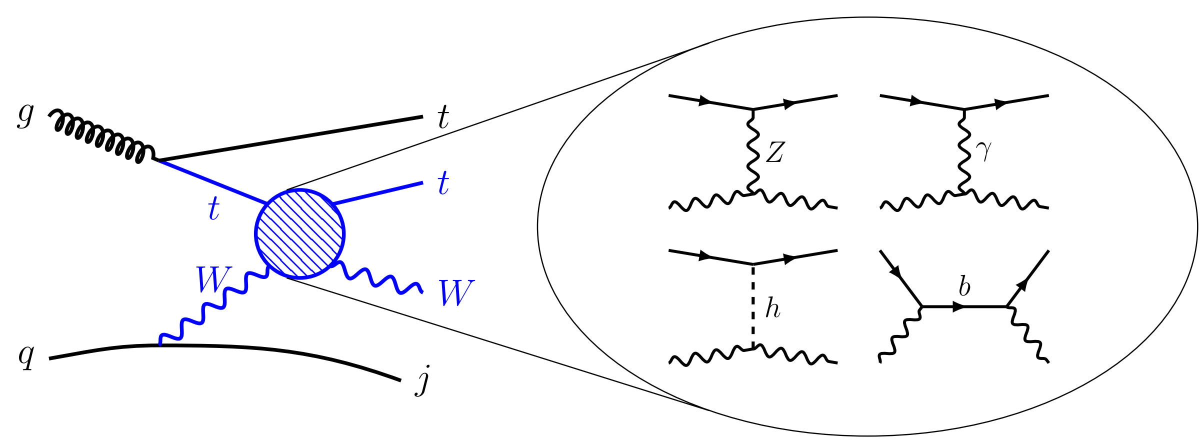
$\delta[\%]$	$\mu = H_T/4$	$\mu = H_T/2$	$\mu = H_T$	Naive estimate
LO_2	-	-	-	10
LO_3	0.8	0.9	1.1	1
NLO_1	34.8 (7.0)	50.0 (25.7)	63.4 (42.0)	10
NLO_2	−4.4 (−4.8)	−4.2 (−4.6)	−4.0 (−4.4)	1
NLO_3	11.9 (8.9)	12.2 (9.1)	12.5 (9.3)	0.1
NLO_4	0.02 (−0.02)	0.04 (−0.02)	0.05 (−0.01)	0.01

100 TeV

$\delta[\%]$	$\mu = H_T/4$	$\mu = H_T/2$	$\mu = H_T$
LO_2	-	-	-
LO_3	0.9	1.1	1.3
NLO_1	159.5 (69.8)	149.5 (71.1)	142.7 (73.4)
NLO_2	−5.8 (−6.4)	−5.6 (−6.2)	−5.4 (−6.1)
NLO_3	67.5 (55.6)	68.8 (56.6)	70.0 (57.6)
NLO_4	0.2 (0.1)	0.2 (0.2)	0.3 (0.2)

NLO3 is large and it is not suppressed by the jet veto (numbers in parentheses) as much as NLO QCD corrections.

NLO QCD corrections depend on the scale, while NLO EW and NLO3 do not.



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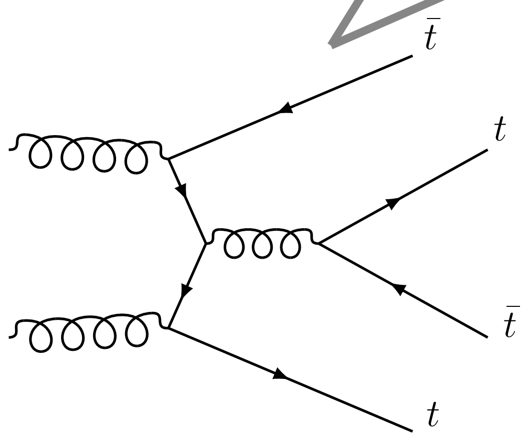
$t\bar{t}t\bar{t}$

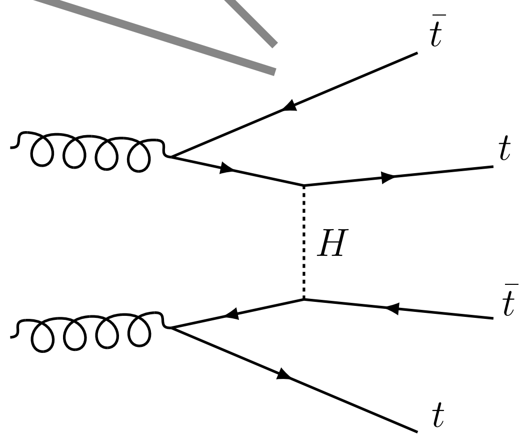
R. Frederix, D.P., M. Zaro
JHEP 1802 (2018) 031 (arXiv:1711.02116)

Complete-NLO

Frederix, DP, Zaro '17

$$\Sigma^{t\bar{t}t\bar{t}}_{\rm LO}(\alpha_s,\alpha) = \alpha_s^4\Sigma^{t\bar{t}t\bar{t}}_{4,0} + \alpha_s^3\alpha\Sigma^{t\bar{t}t\bar{t}}_{4,1} + \alpha_s^2\alpha^2\Sigma^{t\bar{t}t\bar{t}}_{4,2} + \alpha_s^3\alpha\Sigma^{t\bar{t}t\bar{t}}_{4,3} + \alpha^4\Sigma^{t\bar{t}t\bar{t}}_{4,4}$$
$$\equiv \Sigma_{\rm LO_1} + \Sigma_{\rm LO_2} + \Sigma_{\rm LO_3} + \Sigma_{\rm LO_4} + \Sigma_{\rm LO_5}\,.$$



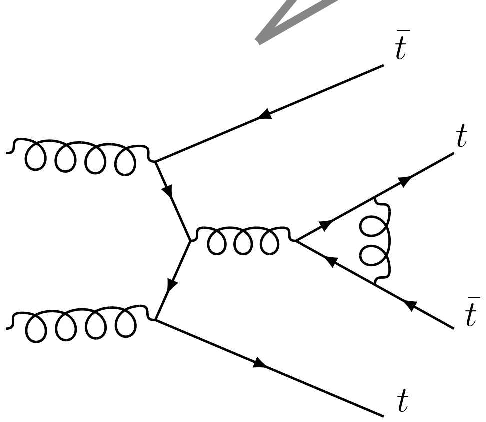


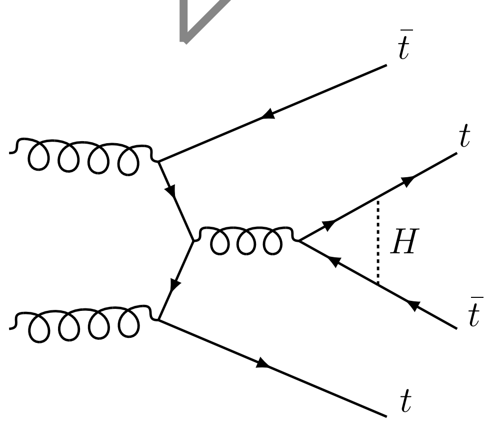
The gg initial state amounts to ~90% of LO cross section at 13 TeV and almost all the cross section at 100 TeV.

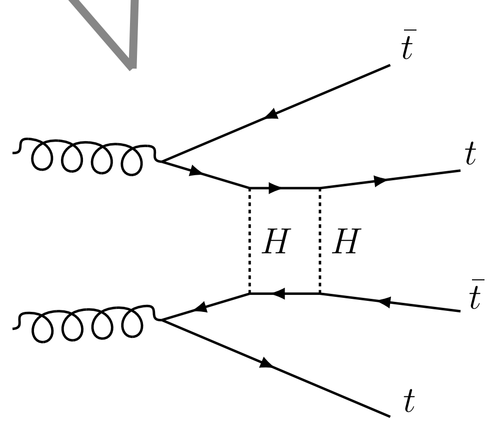
There is no gg contribution at LO4 and LO5.

120

$$\Sigma^{t\bar{t}t\bar{t}}_{\rm NLO}(\alpha_s,\alpha) = \alpha_s^5\Sigma^{t\bar{t}t\bar{t}}_{5,0} + \alpha_s^4\alpha^1\Sigma^{t\bar{t}t\bar{t}}_{5,1} + \alpha_s^3\alpha^2\Sigma^{t\bar{t}t\bar{t}}_{5,2} + \alpha_s^2\alpha^3\Sigma^{t\bar{t}t\bar{t}}_{5,3} + \alpha_s^1\alpha^4\Sigma^{t\bar{t}t\bar{t}}_{5,4} + \alpha^5\Sigma^{t\bar{t}t\bar{t}}_{5,5}$$
$$\equiv \Sigma_{\rm NLO_1} + \Sigma_{\rm NLO_2} + \Sigma_{\rm NLO_3} + \Sigma_{\rm NLO_4} + \Sigma_{\rm NLO_5} + \Sigma_{\rm NLO_6}\,.$$







There is no gg contribution at NLO5 and NLO6.

MadGraph5_aMC@NLO

Cross sections

13 TeV | Naive estimate | **100 TeV**

13 TeV:

$\delta[\%]$	$\mu = H_T/8$	$\mu = H_T/4$	$\mu = H_T/2$	Naive estimate
LO_2	-26.0	-28.3	-30.5	10
LO_3	32.6	39.0	45.9	1
LO_4	0.2	0.3	0.4	0.1
LO_5	0.02	0.03	0.05	0.01
NLO_1	14.0	62.7	103.5	10
NLO_2	8.6	-3.3	-15.1	1
NLO_3	-10.3	1.8	16.1	0.1
NLO_4	2.3	2.8	3.6	0.01
NLO_5	0.12	0.16	0.19	0.001
NLO_6	< 0.01	< 0.01	< 0.01	0.0001
$\mathrm{NLO}_2 + \mathrm{NLO}_3$	-1.7	-1.6	0.9	

100 TeV:

$\delta[\%]$	$\mu = H_T/8$	$\mu = H_T/4$	$\mu = H_T/2$
LO_2	-18.7	-20.7	-22.8
LO_3	26.3	31.8	37.8
LO_4	0.05	0.07	0.09
LO_5	0.03	0.05	0.08
NLO_1	33.9	68.2	98.0
NLO_2	-0.3	-5.7	-11.6
NLO_3	-3.9	1.7	8.9
NLO_4	0.7	0.9	1.2
NLO_5	0.12	0.14	0.16
NLO_6	< 0.01	< 0.01	< 0.01
$\mathrm{NLO}_2 + \mathrm{NLO}_3$	-4.2	-4.0	2.7

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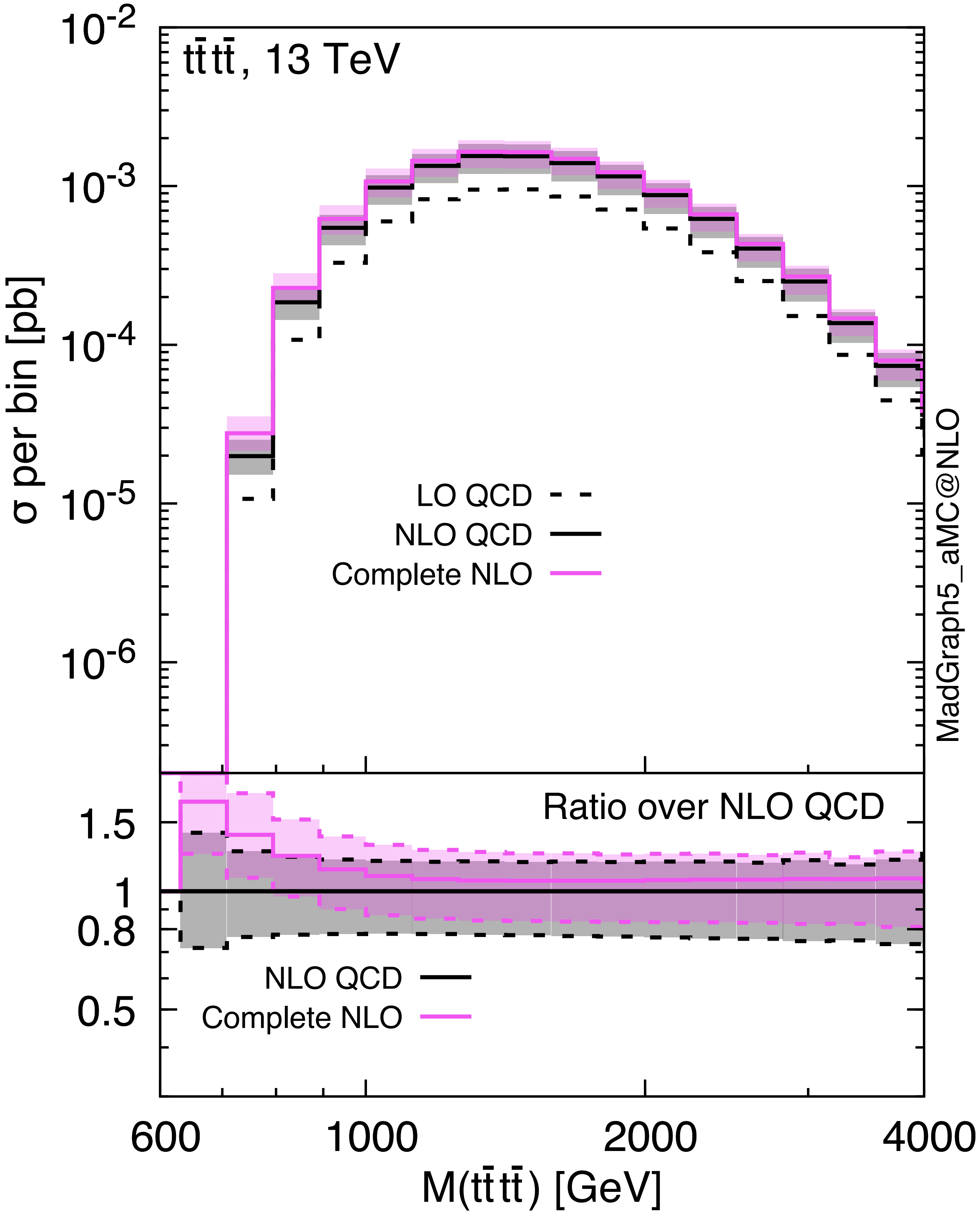
LO_2 and LO_3 are large and have also large cancellations.

NLO_2 and NLO_3 are mainly given by 'QCD corrections' on top of them, so they are large and strongly depend on the scale choice, at variance with standard EW corrections.

Accidentally, relatively to LO_1, NLO_2+NLO_3 scale dependence almost disappears.

What happens if BSM enters into the game? Anomalous y_t ?

Distributions



13 TeV

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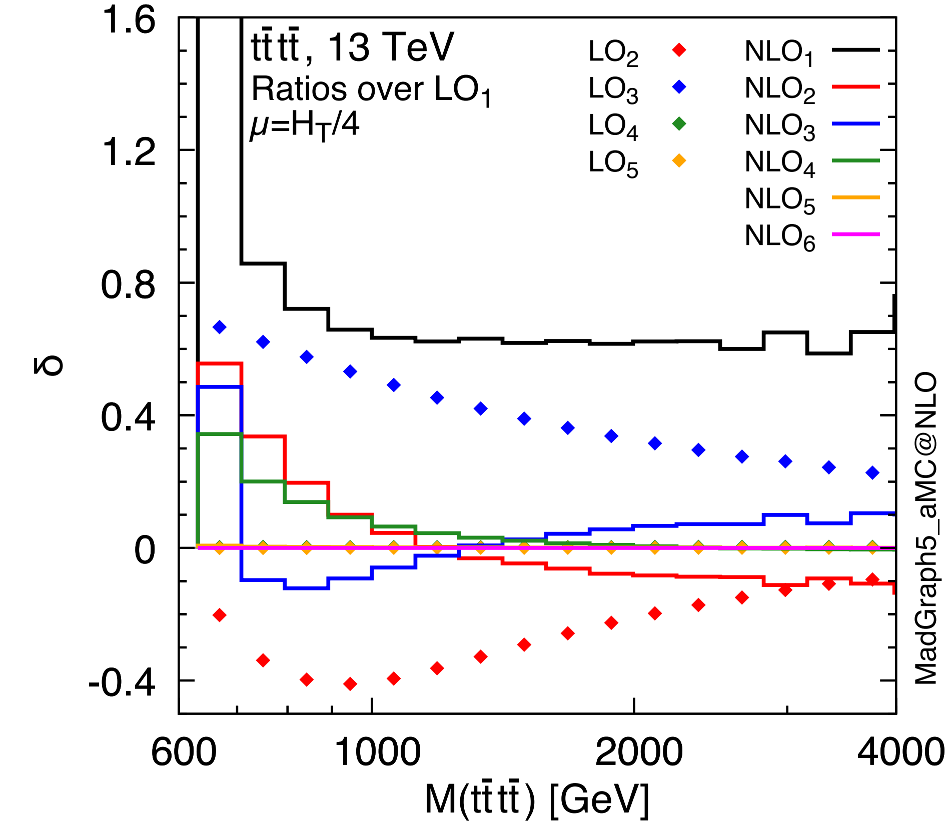
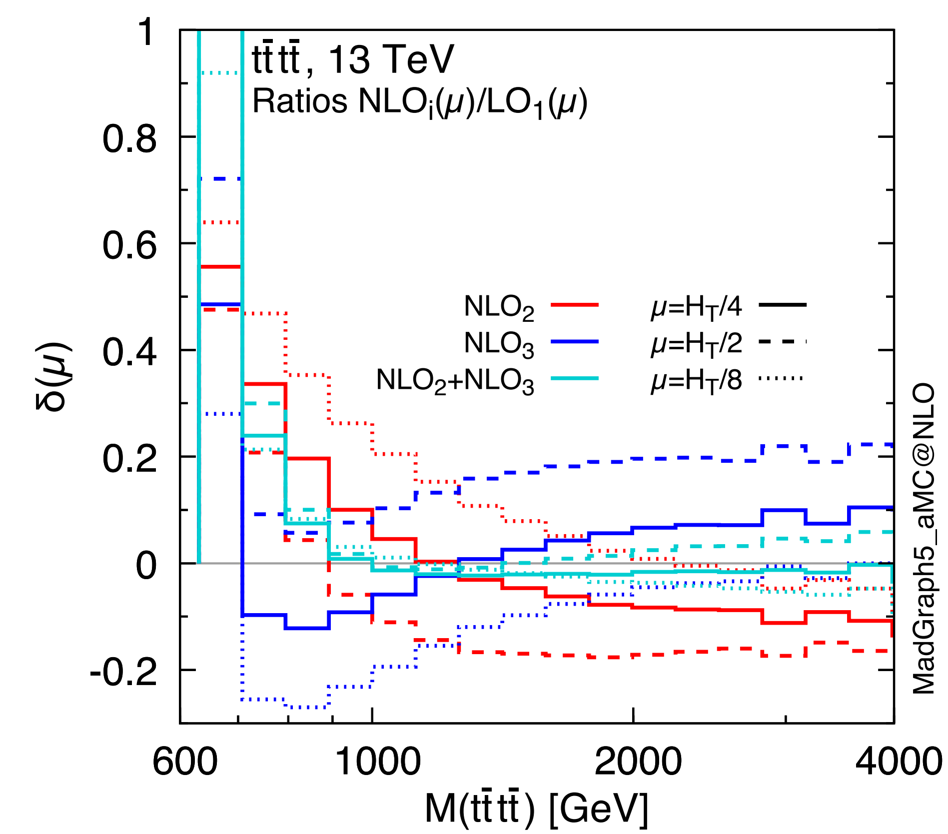




Large cancellations among (N)LO2 and (N)LO3 are present also at the differential level.

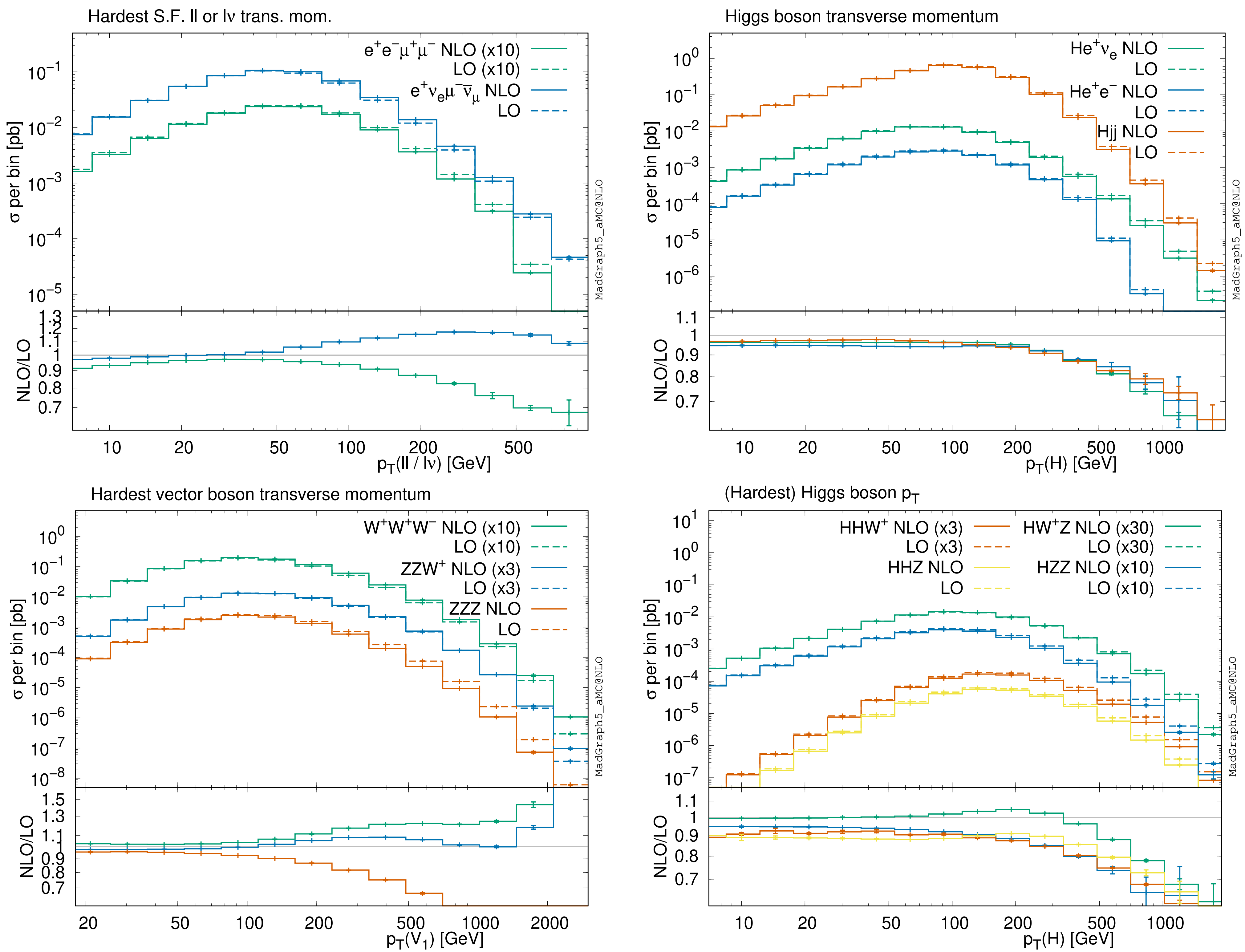
At the threshold also NLO4 is large.

NLO EW

Sudakov logarithms

Differential distributions: Sudakov enhancements

Sudakov enhancements are NOT exceptions and involve at NLO corrections of order $-\alpha \log^k(s/m_W^2)$ with $k = 1{,}2$.



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What are EW Sudakov logarithms?

QCD: virtual and real terms are separately IR divergent ($1/\epsilon$ poles). In physical cross sections the contributions are combined and poles cancel.

QED: same story, but I can also regularise IR divergencies via a photon-mass λ. So $1/\epsilon$ poles $\to \log(Q^2/\lambda^2)$, where Q is a generic scale.

EW: with weak interactions $\lambda \to m_W, m_Z$ and W and Z radiation are typically not taken into account, which is anyway IR-safe.

Therefore, at high energies EW loops induce corrections of order

$$-\alpha^k \log^n(s/m_W^2)$$

where k is the number of loops and $n \leq 2k$. These logs are physical. Even including the real radiation of W and Z, there is not the full cancellation of this kind of logarithms.

Why automate Sudakov in Madgraph5_aMC@NLO?

NLO EW corrections already fully include one-loop EW Sudakov logarithms $(n = 1,\ k = 1{,}2)$, why automate them?

- They can be calculated **analytically via tree-level** amplitudes only. They are a very good approximation of NLO EW at high energy and they can be computed much **faster**. No cancellations among virtual and real, so very stable results.

- When **NLO EW** becomes **large and negative**, **Sudakov logarithms** have to be **resummed**. Having in one tool separately the exact NLO EW and its Sudakov component will allow **matching of NLO EW and EW LL resummed.**

- They **depend only on** properties of the **external particles**: masses, momenta, ***helicities***, charges, SU(2) components, hypercharges. The **generalisation to** the **BSM** case is therefore much **easier** than the NLO EW case.

one-loop EW virtual corrections $\mathcal{O}(\alpha)$

=

α **[Sudakov Logs $\mathcal{O}(-\log^k(s/m_W^2), k=1,2)$ +**

constant term $\mathcal{O}(1)$ +

mass-suppressed terms $\mathcal{O}(m_W^2/s)$]

At high energies, mass suppressed terms vanish so NLO logarithmically grows but (NLO-Sudakov)/LO ~ $\mathcal{O}(1\%)$

Our revisitation and automation: Amplitude level

We have **revisited** and **automated** in aMG5 the **Denner&Pozzorini algorithm** for the evaluation of one-loop EW Sudakov corrections to amplitudes (*Denner, Pozzorini '01*). In particular we have introduced the **following novelties**.

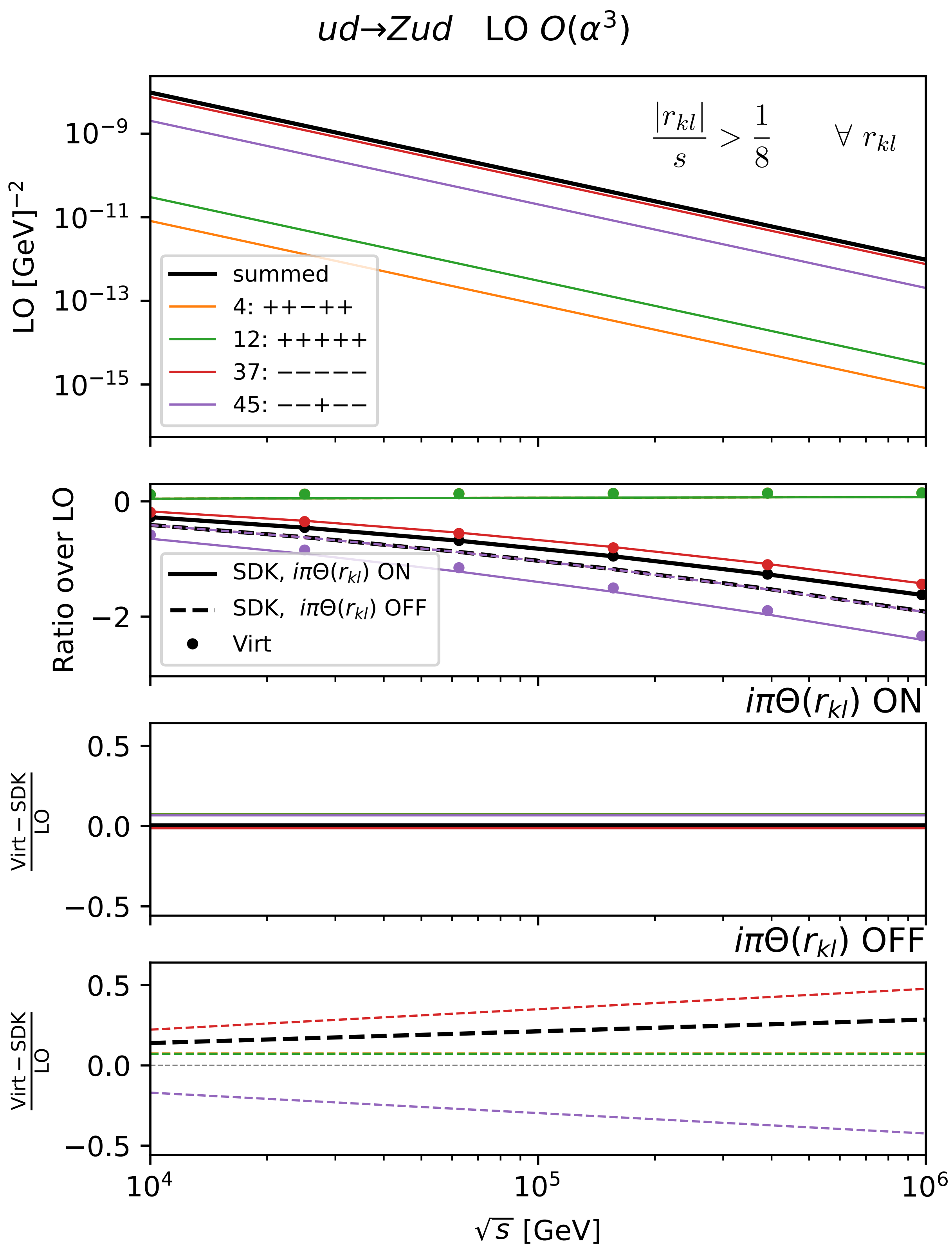
- **IR QED** divergencies are dealt with **via D**imensional **R**egularisation, with strictly massless photons and light fermions.

- **Additional logarithms** that involve ratios between invariants, and therefore **angular** dependences, are taken into account.

- We correctly take into account an **imaginary term** that was **previously omitted** in the literature. Relevant for $2 \to n$ processes with $n > 2$

- Moving to the level of interferences of tree and one-loop amplitudes, we take into account NLO EW contributions originating from **QCD loops on top of subleading LO terms**.

Example $(2 \to 3)$: Imaginary term and scan in s



Denner&Pozzorini algorithm works only with non mass-suppressed LO processes: we select only helicity configurations > 10^(-3) of the dominant one.

Dots: NLO EW (MadLoop). **Lines =** Sudakov.
Dashed: standard approach, $i\pi\Theta(r_{kl})$ omitted
Solid: our formulation, $i\pi\Theta(r_{kl})$ included

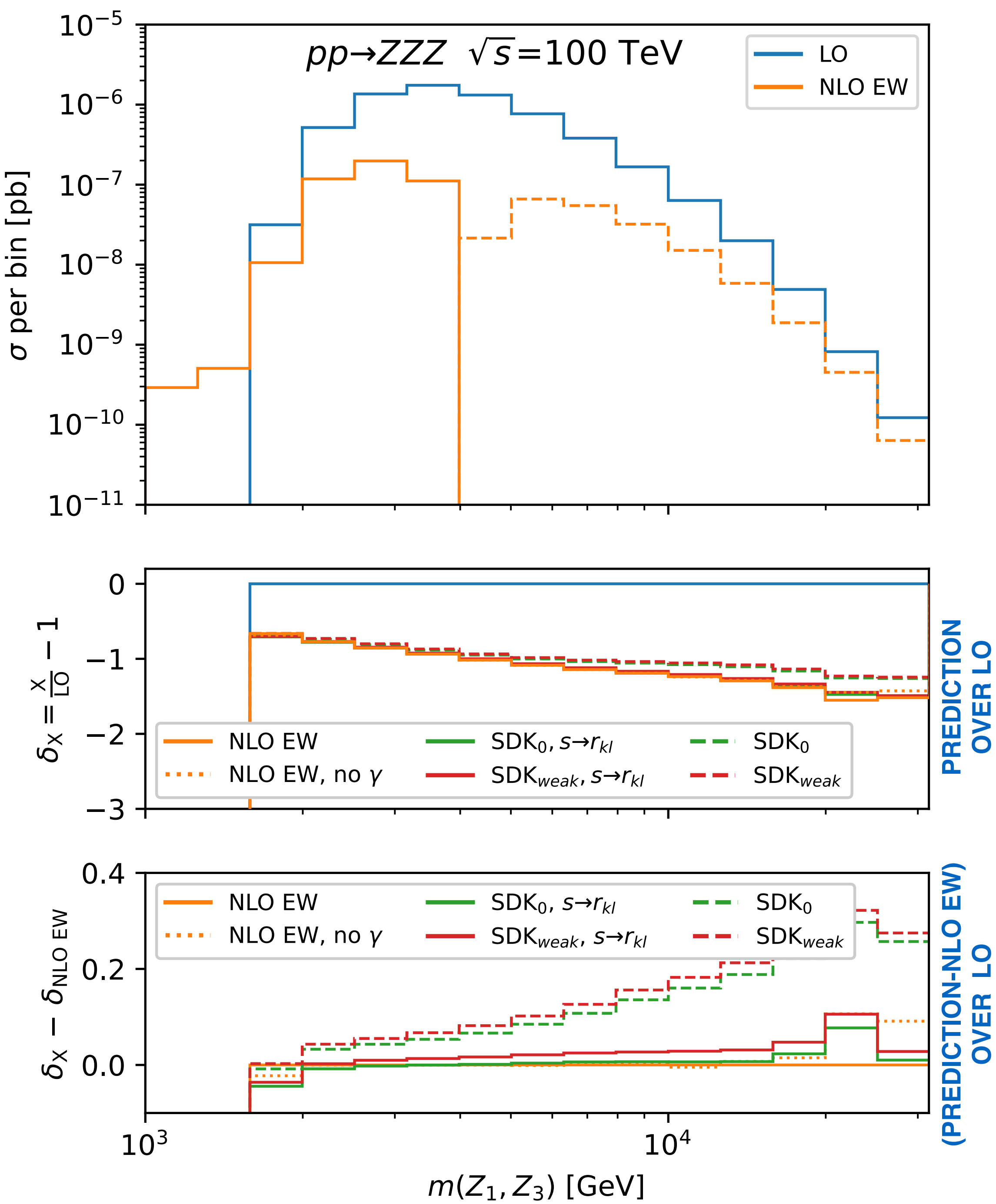
Dots-Solid/LO: horizontal, the correct Log dependence is captured.

Dots-Dashed/LO: not horizontal, the correct Log dependence is lost.

SOME EXAMPLES AT 100 TeV

ZZZ production at 100 TeV

$$p_T(Z_i) > 1\ \text{TeV}, \quad |\eta(Z_i)| < 2.5, \quad m(Z_i, Z_j) > 1\ \text{TeV}, \quad \Delta R(Z_i, Z_j) > 0.5.$$



Orange: NLO EW, (**dotted**: NLO EW no γ PDF)
Green = SDK_0, **Red** = $\mathrm{SDK}_{\mathrm{weak}}$
Dashed: standard approach for amplitudes.
Solid: our formulation (more angular information)

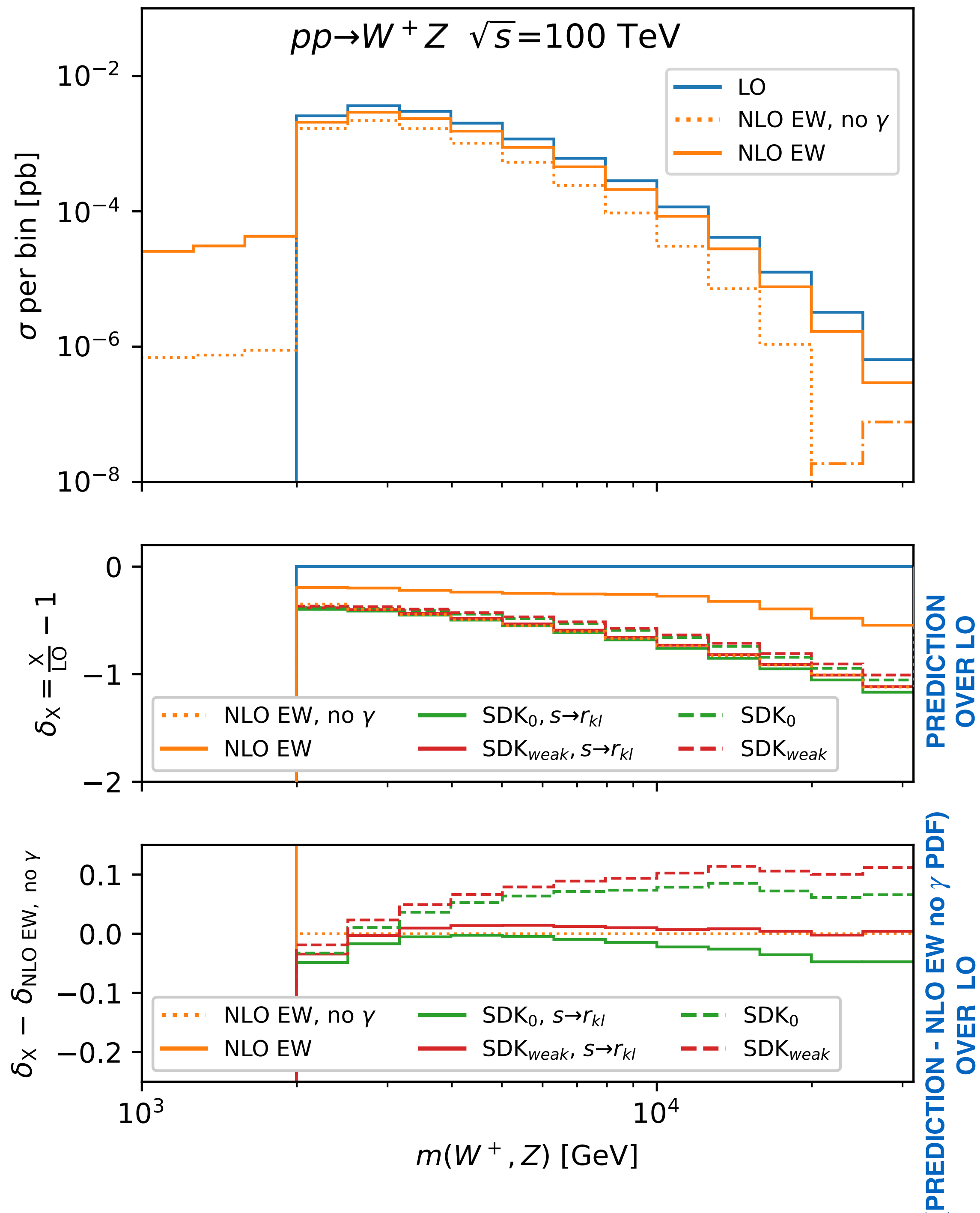
Reference Prediction: Red-solid line

$\mathrm{SDK}_{\mathrm{weak}}$ and SDK_0 approaches return very similar results (neutral final state).

Only the solid lines, having more angular information, correctly capture NLO EW.

WZ production at 100 TeV

$$p_T(V_i) > 1\ \text{TeV}\,, \qquad |\eta(V_i)| < 2.5\,, \qquad m(W^+, Z) > 1\ \text{TeV}\,, \qquad \Delta R(W^+, Z) > 0.5\,.$$



Orange: NLO EW, (**dotted**: NLO EW no γ PDF)

Green = SDK_0, **Red** = $\mathrm{SDK}_{\mathrm{weak}}$

Dashed: standard approach for amplitudes.

Solid: our formulation (more angular information)

Reference Prediction: Red-solid line

The Sudakov approximation cannot approximate large logs from the opening of new channels. The fair comparison is with NLO EW no γ PDF.

Only the $\mathrm{SDK}_{\mathrm{weak}}$ approach correctly captures the NLO EW prediction.

Only the solid lines, having more angular information, correctly capture NLO EW.

CONCLUSIONS

- NLO QCD, NLO EW and Complete NLO predictions are essential in order to provide precise and reliable theoretical predictions for the LHC.

- A lot of technology underlies the commands that let you calculate NLO with MadGraph5_aMC@NLO. Try to learn the basics idea if you want to critically understand the numerical outputs.

- There is more than NLO (e.g. NNLO, NNNLO and other techniques besides fixed order). They are already relevant and they will be even more in the future.