Beyond the Standard Model physics made easy with FEYNRULES

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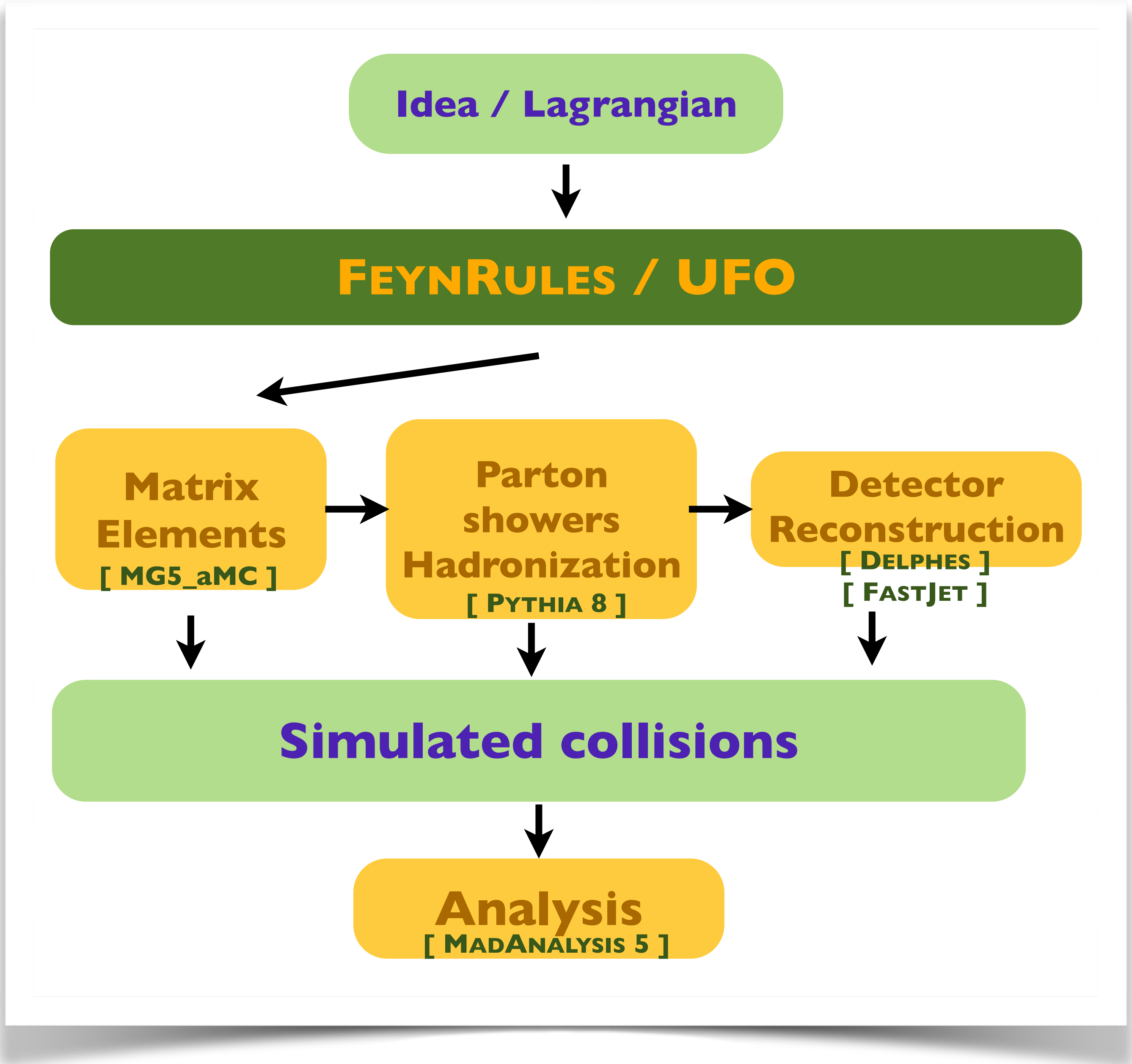
Outline

A comprehensive approach to MC simulations

[Christensen, de Aquino, Degrande, Duhr, BF, Herquet, Maltoni & Schumann (EPJC`11)]

Tools connecting an idea to simulated collisions



- Model building
- Hard scattering
  - ★ Feynman diagram and amplitude generation
  - ★ Monte Carlo integration
  - ★ Event generation
- QCD environment
  - ★ Parton showering
  - ★ Hadronisation
  - ★ Underlying event
- Detector simulation
  - ★ Simulation of the detector response
  - ★ Object reconstruction
- Event analysis
  - ★ Signal/background analysis
  - ★ LHC recasting

FeynRules in a nutshell

What is FeynRules?

- A framework to develop new physics models
- Automatic export to Monte Carlo event generators

  ➡ Facilitate phenomenological investigations of BSM models

  ➡ Facilitate the confrontation of BSM models to data

- Validation of implementations with several Monte Carlo programmes

Main features

- Mathematica package
- Core function: derives Feynman rules from a Lagrangian
- Requirements: locality, Lorentz and gauge invariance
- Supported fields: scalars, (two- and four-component) fermions, vectors (and ghosts), spin-3/2, tensors, superfields

From FeynRules to Monte Carlo tools...

[Christensen, Duhr (CPC '09); Alloul, Christensen, Degrande, Duhr, BF (CPC'14)]

Model

- Particles^
- Gauge symmetries
- Parameters
- Lagrangian

FeynRules

- TeX output
- Feynman rules

Translation interfaces

- CalcHep
- FeynArts
- Whizard*

The UFO

- Aloha
- GoSam
- Herwig ++
- Sherpa
- MadGraph5_aMC@NLO
- Whizard
- MadAnalysis 5

[Degrande, Duhr, BF, Grellscheid, Mattelaer, Reiter (CPC '12)]

Computational tools

- Superspace module [Duhr, BF (CPC '11); BF (IJMPA '12)]
- Mass diagonalisation [Alloul *et al.* (EPJC '13)]
- Decay package [Alwall *et al.* (CPC`16)]
- NLO module [Degrande (CPC`15); Frixione *et al.* (JHEP`19)]

* Whizard interface: Christensen, Duhr, BF, Reuter, Speckner (EPJC '12)

^ Support for spin 3/2: Christensen, de Aquino, Deutschmann, Duhr, BF, Garcia-Cely, Mattelaer, Mawatari, Oexl, Takaesu (EPJC '13)

Outline

Supersymmetric QCD: particle content

Particle content

- Two matter supermultiplets in the fundamental representation of $SU(3)_c$
  - ★ One massive Dirac fermion: a quark
  - ★ Two massive scalar fields: a left-handed and a right-handed squark
- One ($SU(3)_c$) gauge supermultiplet
  - ★ One massive Majorana fermion: a gluino
  - ★ One massless gauge boson: the gluon

(Broken) supersymmetric QCD: the model

The dynamics of the model is embedded in the Lagrangian

$$\begin{aligned}\mathcal{L} = \; & -\frac{1}{4} g_{\mu\nu} g^{\mu\nu} + \frac{i}{2}\bar{\tilde{g}} \not{D} \tilde{g} + D_\mu \tilde{q}_L^\dagger D^\mu \tilde{q}_L + D_\mu \tilde{q}_R^\dagger D^\mu \tilde{q}_R + i\bar{q}\not{D} q \\ & - m^2_{\tilde{q}_L} \tilde{q}_L^\dagger \tilde{q}_L - m^2_{\tilde{q}_L} \tilde{q}_R^\dagger \tilde{q}_R - m_q \bar{q} q - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}} \tilde{g} \\ & - \frac{g_s^2}{2}\Big[- \tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R \Big]\Big[- \tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R \Big] \\ & + \sqrt{2} g_s \Big[- \tilde{q}_L^\dagger T^a \big(\bar{\tilde{g}}^a P_L q\big) + \big(\bar{q} P_L \tilde{g}^a\big) T^a \tilde{q}_R + \text{h.c.} \Big]\end{aligned}$$

R

- Kinetic terms for all fields (first line)
- Mass terms for the squarks, quarks and gluino (second line)
- Supersymmetric gauge interactions for all fields (last two lines)

A FEYNRULES model file

A FEYNRULES model file is an .fr file written in MATHEMATICA syntax

Preamble
- ★ Author information
- ★ Model information
- ★ Index definitions

Declaration of the gauge group
- ★ Abelian or not
- ★ Representation matrices
- ★ Structure constants
- ★ Coupling constant
- ★ Gauge boson or vector superfield

Declaration of fields
- ★ Names, spins, PDG codes
- ★ Indices, quantum numbers
- ★ Masses, widths
- ★ Classes and class members

Declaration of parameters
- ★ External and internal
- ★ Scalar and tensor

A Lagrangian

Preamble: general information

An electronic signature for the model implementation

- Important for traceability, documentation, contact with the authors, *etc*.
- Reference publications
- Webpage information

```
M$ModelName = "SUSYQCD";

M$Information = {
  Authors      -> {"Benjamin Fuks"},
  Date         ->  "19.11.18”,
  Version      ->  "1.0",
  Institutions -> {"LPTHE / Sorbonne U."},
  Emails       -> {"fuks@lpthe.jussieu.fr"}
};
```

Preamble: indices

Index dimensions

- Fundamental $SU(3)_C$ indices for squarks and quarks: *Colour*, dimension 3
- Adjoint $SU(3)_C$ indices for gluons and gluinos: *Gluon*, dimension 8
- Lorentz and spin indices automatically handled

```
IndexRange[Index[Gluon ]] = NoUnfold[Range[8]];
IndexRange[Index[Colour]] = NoUnfold[Range[3]];
```

QCD ≡ special role in event generators ➝ special names

- *Colour, Sextet and Gluon* for colour indices
- *G* for the gluon field
- *T* for the fundamental representation matrices
- *f* and *d* for the structure constants
- G/*gs* and aS for the coupling constants

The style of the indices can be specified

- Fundamental indices starting with *m*
- Adjoint indices starting with *a*

```
IndexStyle[Colour, m];
IndexStyle[Gluon,  a];
```

Gauge groups (I)

Each element the group is declared in the *M$GaugeGroups* list

- One group ≡ one set of MATHEMATICA replacement rules
  - ★ We must only declare SU(3)$_C$ (name: *SU3C*)

```
M$GaugeGroups = {
  SU3C == {
    Abelian           -> False,
    GaugeBoson        -> G,
    CouplingConstant  -> gs,
    StructureConstant -> f,
    Representations   -> { {T,Colour} }
  }
};
```

- One rule ≡ one group property
  - ★ *Abelian*: abelian or non-abelian group
  - ★ *GaugeBoson*: associated gauge boson
  - ★ *CouplingConstant, StructureConstant*: coupling and structure constants
  - ★ *Representations*: list of 2-tuples linking indices to representations

See the manual for more details on gauge groups

Gauge groups (2)

Advantages of a proper gauge group declaration

- Writing Lagrangians more easily
  - ★ Covariant derivatives (*DC[field, Lorentz index]*)
  - ★ Field strength tensors (*FS[field, Lorentz index 1, Lorentz index 2]*)
  - ★ Useful for Lagrangian building in superspace (briefly covered in the last slides)

- Example: gluon and gluino kinetic terms

$$-\frac{1}{4}g_{\mu\nu}g^{\mu\nu} + \frac{i}{2}\bar{\tilde{g}}\not{D}\tilde{g}$$

```
-1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
  I/2 gobar.Ga[mu].DC[go, mu]
```

The gluon field

Each field is an element of the *M$ClassesDescription* list

- A declaration ≡ a set of MATHEMATICA replacement rules
- In our SUSY-QCD model, we first declare the $SU(3)_C$ gauge boson: *G*

```
M$ClassesDescription = {
  V[1] == {
    ClassName       -> G,
    SelfConjugate   -> True,
    Indices         -> {Index[Gluon]},
    Mass            -> 0,
    Width           -> 0,
    PDG             -> 21
  },
  …
```

- One rule ≡ one property of the field
  - ★ Vector field ➙ the label is V[1] (with V, and not F, S, R, T, *etc.*)
  - ★ ClassName: defines a symbol for Lagrangians ➙ *G*
  - ★ Indices: the gluon in the adjoint representation
    - ➙ tagged as gauge boson of $SU(3)_c$ ➙ *Gluon* internally linked to the adjoint
  - ★ Others: vanishing mass and width, PDG code ≡ 21, self-conjugate

See the manual for more options for field declarations

A Majorana gluino field

A second element in the *M$ClassesDescription* list

```
F[1] == {
  ClassName        -> go,
  SelfConjugate    -> True,
  Indices          -> {Index[Gluon]},
  Mass             -> {Mgo,500},
  Width            -> {Wgo,10},
  PDG              -> 1000021
},
```

- Differences with the gluon field declaration
  - ★ Four-component fermionic field ➙ F[1] (starting with an F)
  - ★ Classname: two symbols for the Lagrangian ➙ *go* and *gobar*
  - ★ Mass/width: two symbols and associated numerical values

See the manual for more details on field declarations

Weyl/Majorana gluino fields

Second option: two-component spinors

```
W[1] == {
  ClassName      -> gow,
  Unphysical     -> True,
  Chirality      -> Left,
  SelfConjugate  -> False,
  Indices        -> {Index[Gluon]},
},
```

```
F[1] == {
  ClassName         -> go,
  WeylComponents    -> gow,
  SelfConjugate     -> True,
  Indices           -> {Index[Gluon]},
  Mass              -> {Mgo,500},
  Width             -> {Wgo,10},
  PDG               -> 1000021
},
```

- Extra options available
  - ★ Two-component and four-component fermions ➞ W[1] (with a W) and F[1]
  - ★ Weyl fermions unphysical ➞ linked to four-component fermions (WeylComponents)
  - ★ Several symbols defined ➞ *go* and *gobar; gow* and *gowbar*
  - ★ Chirality to specify

See the manual for more details on field declarations

The (top) quark field

A third element in the *M$ClassesDescription* list

```
F[2] == {
  ClassName         -> q,
  SelfConjugate     -> False,
  Indices           -> {Index[Colour]},
  Mass              -> {Mq, 173},
  Width             -> {Wq, 1.50833649},
  PDG               -> 6
},
```

- Nothing special compared to the other fields
  - ★ Fundamental QCD indices specified

More on quarks: generation indices

Three generations of up-type quarks

```
F[3] == {
  ClassName        -> uq,
  ClassMembers     -> {u, c, t},
  SelfConjugate    -> False,
  Indices          -> {Index[Gen], Index[Colour]},
  FlavorIndex      -> Gen,
  Mass             -> {Mu, {MU,2.55*^-3}, {MC,1.42}, {MT,173}},
  Width            -> {0, 0, {WT, 1.50833649}},
  PDG              -> {2, 4, 6}
},
```

- Slight differences for a bunch of options (+ new options)
  - ★ ClassMembers: all the members of the class
  - ★ A generation index: *Gen* (FlavorIndex defines the flavour index)
  - ★ Mass, Width, PDG: one per class member (+ generic mass symbol)

Squarks

Extra elements in the *M$ClassesDescription* list

```
S[3] == {
  ClassName        -> sq1,
  SelfConjugate    -> False,
  Indices          -> {Index[Colour]},
  Mass             -> {Msq1,300},
  Width            -> {Wsq1,10},
  PDG              -> 1000006
},
S[4] == {
  ClassName        -> sq2,
  SelfConjugate    -> False,
  Indices          -> {Index[Colour]},
  Mass             -> {Msq2,800},
  Width            -> {Wsq2,2},
  PDG              -> 2000006
}
```

- Nothing special
  - ★ Scalar field → S[3]
  - ★ Classname → pairs of symbols: *sq1*, *sq2*, *sq1bar* and sq2*bar*
  - ★ Mass, width, Indices, *etc.* (standard)

Missing: squark mixing

Squark mixing

Mixing implemented via physical and unphysical fields

```
S[1] == {
  ClassName        -> sqL,
  SelfConjugate    -> False,
  Indices          -> {Index[Colour]},
  Unphysical       -> True,
  Definitions      -> {sqL[c_] -> Cos[theta] sq1[c] - Sin[theta] sq2[c]}
},
S[2] == {
  ClassName        -> sqR,
  SelfConjugate    -> False,
  Indices          -> {Index[Colour]},
  Unphysical       -> True,
  Definitions      -> {sqR[c_] -> Sin[theta] sq1[c] + Cos[theta] sq2[c]}
},
```

- Squark fields mix as

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}$$

★ Left and right-handed squarks unphysical ➝ removed from the Lagrangian and replaced

★ Definitions ≡ the replacement rules (inversion of the above relation)

➝ Rotations performed automatically

➝ Lagrangian written in the gauge basis (easier with sqL / sqR)

Parameters

The model requires to implement three parameters

- Masses and widths handled automatically

$$\begin{aligned}\mathcal{L} = & \boxed{-\frac{1}{4} g_{\mu\nu} g^{\mu\nu} + \frac{i}{2} \bar{\tilde{g}} \not{D} \tilde{g} + D_\mu \tilde{q}_L^\dagger D^\mu \tilde{q}_L + D_\mu \tilde{q}_R^\dagger D^\mu \tilde{q}_R + i \bar{q} \not{D} q} \\ & - m_{\tilde{q}_L}^2 \tilde{q}_L^\dagger \tilde{q}_L - m_{\tilde{q}_L}^2 \tilde{q}_R^\dagger \tilde{q}_R - m_q \bar{q} q - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}} \tilde{g} \\ & - \frac{\boxed{g_s^2}}{2} \Big[- \tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R \Big] \Big[- \tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R \Big] \\ & + \sqrt{2} \boxed{g_s} \Big[- \tilde{q}_L^\dagger T^a \left(\bar{\tilde{g}}^a P_L q \right) + \left(\bar{q} P_L \tilde{g}^a \right) T^a \tilde{q}_R + \text{h.c.} \Big]\end{aligned}$$

- The strong coupling explicit and implicit
  - ➙ g_s needed
  - ➙ α_s as required by MC tools
- Squark mixing angle θ to add

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}$$

Parameters: strong coupling

Parameters are declared as elements of the list *M$Parameters*

- One declaration ≡ one set of MATHEMATICA replacement rules
- Strong coupling: α_s and g_s not independent

```
M$Parameters = {
  aS    == {
    ParameterType    -> External,
    Value            -> 0.1184,
    InteractionOrder -> {QCD, 2}
  },
  gs == {
    ParameterType    -> Internal,
    Value            -> Sqrt[4 Pi aS],
    InteractionOrder -> {QCD, 1},
    ParameterName    -> G
  },
```

★ Internal and External parameters
  → External ≡ free param. (numerical)
  → Internal ≡ dependent param (formula)

★ InteractionOrder: specific to some MC
  → more efficient diagram generation

★ ParameterName: QCD special

See the manual for more details on parameter declarations

Parameters: squark mixing

The squark mixing angle

- Nothing special (standard external parameter)

```
theta == {
  ParameterType     -> External,
  Value             -> Pi/4
}
```

More options (assuming matrix-form mixing)

- Tensor parameters can be implemented too (Indices, Unitary)
- Complex parameters can be implemented too (ComplexParameter)

See the manual for more details on parameter declarations

Parameter organisation

Les Houches blocks to organise external parameters

- Les Houches parameter card by MC programmes
  ➞ BlockName + OrderBlock
- Unspecified: the 'FRBlock' block

Complete α_s implementation as an example

- Many options (TeX, Description, BlockName, OrderBlock)

```
aS    == {
  ParameterType    -> External,
  BlockName        -> SMINPUTS,
  OrderBlock       -> 3,
  Value            -> 0.1184,
  InteractionOrder -> {QCD,2},
  TeX              -> Subscript[\[Alpha],s],
  Description      -> "Strong coupling constant at the Z pole"
},
```

Outline

1. FEYNRULES in a nutshell
2. Implementing supersymmetric QCD in FEYNRULES
3. **Using FEYNRULES with supersymmetric QCD model**
4. Summary
5. Appendix: advanced model implementation techniques

Getting started

Starting a FEYNRULES MATHEMATICA session

Step 1: loading FEYNRULES into MATHEMATICA

- The FEYNRULES path
- Loading FEYNRULES

```
In[1]:= $FeynRulesPath = SetDirectory["~/Work/tools/FeynRules/branch/feynrules-current"];
        << FeynRules`

         - FeynRules -
        Version: 2.3.32 ( 12 March 2018).
        Authors: A. Alloul, N. Christensen, C. Degrande, C. Duhr, B. Fuks

        Please cite:
            - Comput.Phys.Commun.185:2250-2300,2014 (arXiv:1310.1921);
            - Comput.Phys.Commun.180:1614-1641,2009 (arXiv:0806.4194).

        http://feynrules.phys.ucl.ac.be

        The FeynRules palette can be opened using the command FRPalette[].
```

The output:

- Information on FEYNRULES, authors, version numbers, references, *etc.*

Loading the model implementation

Step 2: loading the SUSY-QCD implementation into FEYNRULES

- Moving to the right directory
- Loading the model itself

```
SetDirectory[NotebookDirectory[]];
LoadModel["susyqcd.fr"];


This model implementation was created by
Benjamin Fuks
Model Version: 1.0
For more information, type ModelInformation[].

   - Loading particle classes.
   - Loading gauge group classes.
   - Loading parameter classes.

Model SUSYQCD loaded.
```

The output:

- Preamble of the model are printed to the screen
- More information from ModelInformation[ ]

The vector Lagrangian

The SUSY-QCD vector Lagrangian

The gauge sector of the theory : one gauge supermultiplet

- One massive Majorana fermion: a gluino
- One massless gauge boson: the gluon

Dynamics embedded in the vector Lagrangian

$$\mathcal{L}_{\text{vector}} = -\frac{1}{4} g_{\mu\nu} g^{\mu\nu} + \frac{i}{2} \bar{\tilde{g}} \not{D} \tilde{g} - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}} \tilde{g}$$

- Kinetic terms (both fields)
- Mass terms for the gluino
- Gauge interactions (both fields, through gauge-covariant objects)

Implementing the vector Lagrangian

Textbook expression:

$$\mathcal{L}_{\rm vector} = -\frac{1}{4} g_{\mu\nu} g^{\mu\nu} + \frac{i}{2} \bar{\tilde{g}} \not{D} \tilde{g} - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}} \tilde{g}$$

Strengths of a proper gauge-group implementation

- Shortcut functions (DC, FS, *etc.*)
- Very compact

```
LVector1 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
             I/2 gobar.Ga[mu].DC[go,mu] -
             1/2 Mgo gobar.go;
```

★ Indices understood (automatic handling)

Indices could be noted explicitly

```
LVector2 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
             I/2 Ga[mu,s1,s2] gobar[s1,a].DC[go[s2,a],mu] -
             1/2 Mgo gobar[s1,a].go[s1,a];
```

Checks of the implementation

Textbook expression

$$\mathcal{L}_{\text{vector}} = -\frac{1}{4} g_{\mu\nu} g^{\mu\nu} + \frac{i}{2} \bar{\tilde{g}} \not{D} \tilde{g} - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}} \tilde{g}$$

FeynRules implementation

```
LVector1 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
          I/2 gobar.Ga[mu].DC[go,mu] -
          1/2 Mgo gobar.go;
```

Printing the vector Lagrangian

In[6]:= **LVector1**

Out[6]= $-\frac{1}{2}\,\text{Mgo}\,\bar{\text{go}}.\text{go} + \frac{1}{2}\,i\,\bar{\text{go}}.\gamma^{\text{mu}}.\left(\partial_{\text{mu}}[\text{go}] - i\,\text{gs}\,\text{FSU3C}^{\text{a\$658}}.\text{go}\,G_{\text{mu},\text{a\$658}}\right) - \frac{1}{4}\left(-\partial_{\text{nu}}[G_{\text{mu},\text{a}}] + \partial_{\text{mu}}[G_{\text{nu},\text{a}}] + \text{gs}\,f_{\text{a},\text{bb\$656},\text{cc\$656}}\,G_{\text{mu},\text{bb\$656}}\,G_{\text{nu},\text{cc\$656}}\right) \left(-\partial_{\text{nu}}[G_{\text{mu},\text{a}}] + \partial_{\text{mu}}[G_{\text{nu},\text{a}}] + \text{gs}\,f_{\text{a},\text{bb\$657},\text{cc\$657}}\,G_{\text{mu},\text{bb\$657}}\,G_{\text{nu},\text{cc\$657}}\right)$

The output:

- Covariant derivatives and field strength tensors automatically evaluated
- FSU3C ≡ structure constants ➙ to be replaced in a second step by f_{abc}

Expansion of the Lagrangian

All indices restored automatically

In[7]:= `ExpandIndices[LVector1]`

Out[7]=

$$-\frac{1}{4}\partial_{nu}[G_{mu,a}]^2+\frac{1}{2}\partial_{nu}[G_{mu,a}]\,\partial_{mu}[G_{nu,a}]-\frac{1}{4}\partial_{mu}[G_{nu,a}]^2-\frac{1}{2}\,\mathrm{Mgo}\,go_{i1\$667,i2\$667}\cdot go_{i1\$667,i2\$667}+$$
$$\frac{1}{4}gs\,\partial_{nu}[G_{mu,a}]\,f_{a,bb\$662,cc\$662}\,G_{mu,bb\$662}\,G_{nu,cc\$662}-\frac{1}{4}gs\,\partial_{mu}[G_{nu,a}]\,f_{a,bb\$662,cc\$662}\,G_{mu,bb\$662}\,G_{nu,cc\$662}+$$
$$\frac{1}{4}gs\,\partial_{nu}[G_{mu,a}]\,f_{a,bb\$663,cc\$663}\,G_{mu,bb\$663}\,G_{nu,cc\$663}-\frac{1}{4}gs\,\partial_{mu}[G_{nu,a}]\,f_{a,bb\$663,cc\$663}\,G_{mu,bb\$663}\,G_{nu,cc\$663}-$$
$$\frac{1}{4}gs^2\,f_{a,bb\$662,cc\$662}\,f_{a,bb\$663,cc\$663}\,G_{mu,bb\$662}\,G_{mu,bb\$663}\,G_{nu,cc\$662}\,G_{nu,cc\$663}+$$
$$\frac{1}{4}i\,go_{i\$667,i1\$669}\cdot\partial_{mu}[go_{j\$667,i1\$669}]\,\gamma_{i\$667,j\$667}{}^{mu}-$$
$$\frac{1}{4}i\,gs\,go_{i\$670,i\$669}\cdot go_{j\$670,j\$669}\,f_{a\$664,i\$669,j\$669}\,G_{mu,a\$664}\,\gamma_{i\$670,j\$670}{}^{mu}-$$
$$\frac{1}{4}i\,\partial_{mu}[go_{j\$667,i1\$669}]\cdot go_{i\$667,i1\$669}\,\gamma_{j\$667,i\$667}{}^{mu}+\frac{1}{4}i\,gs\,go_{j\$670,j\$669}\cdot go_{i\$670,i\$669}\,f_{a\$664,i\$669,j\$669}\,G_{mu,a\$664}\,\gamma_{j\$670,i\$670}{}^{mu}$$

- Kinetic terms (gluon and gluino)
- Mass terms (gluino)
- QCD interactions (with the proper structure constants)

Checks: Hermiticity

Textbook expression

$$\mathcal{L}_{\text{vector}} = -\frac{1}{4} g_{\mu\nu} g^{\mu\nu} + \frac{i}{2} \bar{\tilde{g}} \not{D} \tilde{g} - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}} \tilde{g}$$

FEYNRULES implementation

```
LVector1 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
        I/2 gobar.Ga[mu].DC[go,mu] -
        1/2 Mgo gobar.go;
```

The Lagrangian must be Hermitian

- Feynman rules of the Lagrangian minus its Hermitian conjugate → $\mathcal{L} - \mathcal{L}^\dagger$

```
In[8]:= CheckHermiticity[LVector1];
Checking for hermiticity by calculating the Feynman rules contained in L-HC[L].
If the lagrangian is hermitian, then the number of vertices should be zero.
Starting Feynman rule calculation.
Expanding the Lagrangian...
No vertices found.
0 vertices obtained.
The lagrangian is hermitian.
```

Checks: normalisation

Textbook expression

$$\mathcal{L}_{\text{vector}} = -\frac{1}{4} g_{\mu\nu} g^{\mu\nu} + \frac{i}{2} \bar{\tilde{g}} \not{D} \tilde{g} - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}} \tilde{g}$$

FEYNRULES implementation

```
LVector1 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
            I/2 gobar.Ga[mu].DC[go,mu] -
            1/2 Mgo gobar.go;
```

Kinetic and mass terms canonically normalised

- Normalisation of the quadratic terms (kinetic and mass terms)
- Absence of non-diagonal quadratic terms

```
In[9]:= CheckKineticTermNormalisation[LVector1];
        Neglecting all terms with more than 2 particles.
        All kinetic terms are diagonal.
        All kinetic terms are correctly normalized.

In[11]:= CheckDiagonalMassTerms[LVector1];
        All mass terms are diagonal.
```

- Other checks: *CheckDiagonalQuadraticTerms*, *CheckDiagonalKineticTerms*

Checks: the mass spectrum

FEYNRULES gluino implementation

```
F[1] == {
  ClassName        -> go,
  WeylComponents   -> gow,
  SelfConjugate    -> True,
  Indices          -> {Index[Gluon]},
  Mass             -> {Mgo,500},
  Width            -> {Wgo,10},
  PDG              -> 1000021
},
```

FEYNRULES Lagrangian implementation

```
LVector1 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
            I/2 gobar.Ga[mu].DC[go,mu] -
            1/2 Mgo gobar.go;
```

Consistent mass information (numerically)

- Masses extracted from the Lagrangian
- Masses fixed in particle declarations

```
In[12]:= CheckMassSpectrum[LVector1]

         Neglecting all terms with more than 2 particles.
         All mass terms are diagonal.
         Getting mass spectrum.
         Checking for less then 0.1% agreement with model file values.
Out[12]//TableForm=
         Particle    Analytic value    Numerical value    Model-file value
         go          Mgo               500.               500.
```

Feynman rules

Extract all *N*-point interactions from the Lagrangian (with *N*>2)

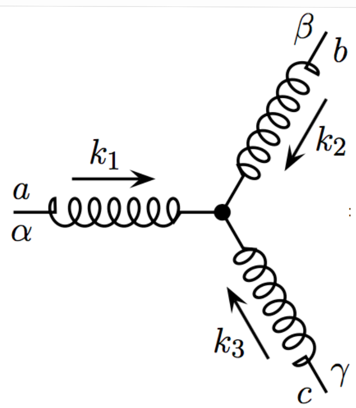
```
In[14]:= FeynmanRules[LVector1, ScreenOutput → True];

Starting Feynman rule calculation.
Expanding the Lagrangian...
Collecting the different structures that enter the vertex.
3 possible non-zero vertices have been found -> starting the computation: 3 / 3.
3 vertices obtained.
(* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *)
```

- Three vertices found in the Lagrangian
  - ★ One triple and quartic gluon interactions
  - ★ One gluino-gluon interaction

The triple gluon vertex

Literature



$$g_s f_{abc}\Big[(k_1-k_2)^\gamma\eta^{\alpha\beta}+(k_3-k_1)^\beta\eta^{\gamma\alpha}+(k_2-k_3)^\alpha\eta^{\beta\gamma}\Big]$$

[Incoming momenta]

FEYNRULES

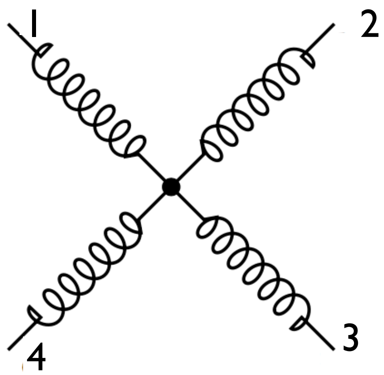
```
Vertex 1
Particle 1 : Vector , G
Particle 2 : Vector , G
Particle 3 : Vector , G
Vertex:
```

$$\text{gs}\, f_{a_1,a_2,a_3}\, p_1^{\mu_3}\,\eta_{\mu_1,\mu_2} - \text{gs}\, f_{a_1,a_2,a_3}\, p_2^{\mu_3}\,\eta_{\mu_1,\mu_2} - \text{gs}\, f_{a_1,a_2,a_3}\, p_1^{\mu_2}\,\eta_{\mu_1,\mu_3} + \text{gs}\, f_{a_1,a_2,a_3}\, p_3^{\mu_2}\,\eta_{\mu_1,\mu_3} + \text{gs}\, f_{a_1,a_2,a_3}\, p_2^{\mu_1}\,\eta_{\mu_2,\mu_3} - \text{gs}\, f_{a_1,a_2,a_3}\, p_3^{\mu_1}\,\eta_{\mu_2,\mu_3}$$

- Colour: index a_i related to the i^{th} particle(a = style for adjoint $SU(3)_C$ indices)
- Spin: index μ_i = Lorentz index of the i^{th} (vector) particle

The quartic gluon vertex

Literature



$$
\begin{aligned}
& i g_s^2\, f^{a_1 a_2 b} f^{b a_3 a_4}\ \left(\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4}\right)\\
&\quad + i g_s^2\, f^{a_1 a_3 b} f^{b a_2 a_4}\ \left(\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}\right)\\
&\quad + i g_s^2\, f^{a_1 a_4 b} f^{b a_2 a_3}\ \left(\eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}\right)
\end{aligned}
$$

FeynRules

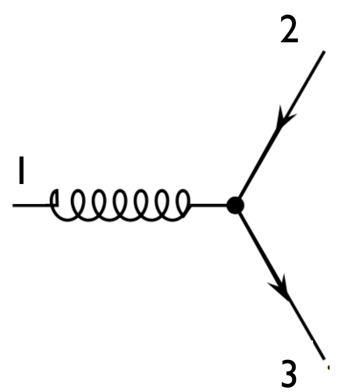
```
Vertex 2
Particle 1 : Vector , G
Particle 2 : Vector , G
Particle 3 : Vector , G
Particle 4 : Vector , G
Vertex:
```

$$
\begin{aligned}
& \mathrm{i}\,\mathrm{gs}^2 \big(\mathrm{f}_{a_1,a_3,\mathrm{Gluon\$1}}\,\mathrm{f}_{a_2,a_4,\mathrm{Gluon\$1}}\,\eta_{\mu_1,\mu_4}\,\eta_{\mu_2,\mu_3} + \mathrm{f}_{a_1,a_2,\mathrm{Gluon\$1}}\,\mathrm{f}_{a_3,a_4,\mathrm{Gluon\$1}}\,\eta_{\mu_1,\mu_4}\,\eta_{\mu_2,\mu_3} +\\
&\quad \mathrm{f}_{a_1,a_4,\mathrm{Gluon\$1}}\,\mathrm{f}_{a_2,a_3,\mathrm{Gluon\$1}}\,\eta_{\mu_1,\mu_3}\,\eta_{\mu_2,\mu_4} - \mathrm{f}_{a_1,a_2,\mathrm{Gluon\$1}}\,\mathrm{f}_{a_3,a_4,\mathrm{Gluon\$1}}\,\eta_{\mu_1,\mu_3}\,\eta_{\mu_2,\mu_4} -\\
&\quad \mathrm{f}_{a_1,a_4,\mathrm{Gluon\$1}}\,\mathrm{f}_{a_2,a_3,\mathrm{Gluon\$1}}\,\eta_{\mu_1,\mu_2}\,\eta_{\mu_3,\mu_4} - \mathrm{f}_{a_1,a_3,\mathrm{Gluon\$1}}\,\mathrm{f}_{a_2,a_4,\mathrm{Gluon\$1}}\,\eta_{\mu_1,\mu_2}\,\eta_{\mu_3,\mu_4}\big)
\end{aligned}
$$

- Colour: a_i related to the i^{th} particle (*Gluon$1* summed over)
- Spin: μ_i related to the i^{th} (vector) particle

Gluon and gluino interactions

Literature



$$g_s f_{a_1 a_2 a_3} \gamma^{\mu_1}$$

FEYNRULES

```
Vertex 3
Particle 1 : Majorana , go
Particle 2 : Majorana , go
Particle 3 : Vector , G
Vertex:
```

$$\text{gs } f_{a_1,a_2,a_3} \gamma_{s_1,s_2}{}^{\mu_3}$$

- Colour: a_i related to the i^{th} particle
- Spin: μ_i related to the i^{th} vector; s_i related to the i^{th} fermion

The FeynmanRules function

The function FeynmanRules has many options

- Restriction on the interactions to display: MaxParticles, MaxCanonicalDimension, *etc*.
- Selection of specific particles: Free, Contains, *etc*.
- ScreenOutput: displaying the vertices to the screen or not
- FlavorExpand: perform a flavour expansion (otherwise classes used)

See the manual for more details on the function *FeynmanRules*

The matter Lagrangian

The SUSY-QCD matter sector

Matter fields

- Two matter supermultiplets in the fundamental representation of $SU(3)_c$
  - ★ One massive Dirac fermion: a quark
  - ★ Two mixing massive scalar fields: two squark
  - ★ Gauge coupling to the $SU(3)_c$ gauge supermultiplet

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}$$

Lagrangian:

$$\begin{aligned}\mathcal{L}_{\rm matter} = &\ D_\mu \tilde{q}_L^\dagger D^\mu \tilde{q}_L + D_\mu \tilde{q}_R^\dagger D^\mu \tilde{q}_R + i\bar{q}\not{D} q - m^2_{\tilde{q}_i} \tilde{q}_i^\dagger \tilde{q}_i - m_q \bar{q} q \\ &- \frac{g_s^2}{2}\Big[-\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R\Big]\Big[-\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R\Big] \\ &+ \sqrt{2} g_s \Big[-\tilde{q}_L^\dagger T^a \big(\bar{\tilde{g}}^a P_L q\big) + \big(\bar{q} P_L \tilde{g}^a\big) T^a \tilde{q}_R\Big] + \text{h.c.}\end{aligned}$$

- Mixed use of mass / gauge bases (makes it easier to implement)
- Kinetic terms (first three terms) and D-terms (second line) in the gauge basis
- Mass terms (end of first line) in the mass basis
- SUSY gauge quark-squark-gluino interactions (fourth line) in the gauge basis

Kinetic and gauge interactions

Kinetic and gauge interactions, as well as mass terms

$$\mathcal{L}_{\rm matter} = D_\mu \tilde{q}_L^\dagger D^\mu \tilde{q}_L + D_\mu \tilde{q}_R^\dagger D^\mu \tilde{q}_R + i\bar{q} \not{D} q - m_{\tilde{q}_i}^2 \tilde{q}_i^\dagger \tilde{q}_i - m_q \bar{q} q$$

The implementation in FEYNRULES follows the textbook expression

- Shortcut functions (DC)
- Very compact implementation
- Repeated indices summed

```
Lkin := DC[sqLbar[cc],mu] DC[sqL[cc],mu] +
        DC[sqRbar[cc],mu] DC[sqR[cc],mu] +
        I qbar.Ga[mu].DC[q,mu] -
        Mq qbar.q -
        Msq1^2 sq1bar[cc] sq1[cc] -
        Msq2^2 sq2bar[cc] sq2[cc];
```

Gauge eigenstates are used

Mass eigenstates are used

Kinetic term Feynman rules

Feynman rules

```
In[18]:= Simplify[FeynmanRules[Lkin]] // MatrixForm
Starting Feynman rule calculation.
Expanding the Lagrangian...
Collecting the different structures that enter the vertex.
9 possible non-zero vertices have been found -> starting the computation: 9 / 9.
5 vertices obtained.
Out[18]//MatrixForm=
```

$$\begin{pmatrix} \{\{G,1\},\{sq1,2\},\{sq1^\dagger,3\}\} & i\,gs\,\left(p_2^{\mu 1}-p_3^{\mu 1}\right) T^{a1}_{m_3,m_2} \\ \{\{G,1\},\{sq2,2\},\{sq2^\dagger,3\}\} & i\,gs\,\left(p_2^{\mu 1}-p_3^{\mu 1}\right) T^{a1}_{m_3,m_2} \\ \{\{G,1\},\{G,2\},\{sq1,3\},\{sq1^\dagger,4\}\} & i\,gs^2\,\eta_{\mu 1,\mu 2}\left(T^{a1}_{m_4,Colour\$1}T^{a2}_{Colour\$1,m_3}+T^{a1}_{Colour\$1,m_3}T^{a2}_{m_4,Colour\$1}\right) \\ \{\{G,1\},\{G,2\},\{sq2,3\},\{sq2^\dagger,4\}\} & i\,gs^2\,\eta_{\mu 1,\mu 2}\left(T^{a1}_{m_4,Colour\$1}T^{a2}_{Colour\$1,m_3}+T^{a1}_{Colour\$1,m_3}T^{a2}_{m_4,Colour\$1}\right) \\ \{\{\bar{q},1\},\{q,2\},\{G,3\}\} & i\,gs\,\gamma_{s1,s2}{}^{\mu 3}\,T^{a3}_{m_1,m_2} \end{pmatrix}$$

- Field rotations have been performed automatically
  - ★ No more gauge eigenstates (sqL, sqR)
- All squark and quark QCD interactions derived
  - ★ The 3-point qqg interaction
  - ★ Two 3-point squark-gluon interactions
  - ★ Two 4-point squark-gluon interactions

D-terms

D-terms (in the gauge eigenbasis)

$$\mathcal{L}_{\rm matter} = -\frac{g_s^2}{2}\Big[-\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R\Big]\Big[-\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R\Big]$$

The FEYNRULES implementation follows the textbook expression

```
LD := -1/2 gs^2 *
       (sqRbar[cc1] T[a,cc1,cc2] sqR[cc2] - sqLbar[cc1] T[a,cc1,cc2] sqL[cc2]) *
       (sqRbar[cc3] T[a,cc3,cc4] sqR[cc4] - sqLbar[cc3] T[a,cc3,cc4] sqL[cc4]);
```

- Repeated indices (*cc1, cc2, cc3, cc4*) summed
- A single index can only be used twice

D-term Feynman rules

D-terms (in the gauge eigenbasis)

$$\mathcal{L}_{\rm matter} = -\frac{g_s^2}{2}\Big[-\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R\Big]\Big[-\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R\Big]$$

Feynman rules computation

```
Simplify[FeynmanRules[LD]] // MatrixForm
```

$$\begin{pmatrix}
\{\{\text{sq1},1\},\{\text{sq1},2\},\{\text{sq1}^\dagger,3\},\{\text{sq1}^\dagger,4\}\} & -i\,\text{gs}^2\,\text{Cos}[2\,\text{theta}]^2\left(T^{\text{Gluon\$1}}_{m_3,m_2}T^{\text{Gluon\$1}}_{m_4,m_1}+T^{\text{Gluon\$1}}_{m_3,m_1}T^{\text{Gluon\$1}}_{m_4,m_2}\right)\\
\{\{\text{sq1},1\},\{\text{sq1}^\dagger,2\},\{\text{sq1}^\dagger,3\},\{\text{sq2},4\}\} & \frac{1}{2}\,i\,\text{gs}^2\,\text{Sin}[4\,\text{theta}]\left(T^{\text{Gluon\$1}}_{m_2,m_4}T^{\text{Gluon\$1}}_{m_3,m_1}+T^{\text{Gluon\$1}}_{m_2,m_1}T^{\text{Gluon\$1}}_{m_3,m_4}\right)\\
\{\{\text{sq1}^\dagger,1\},\{\text{sq1}^\dagger,2\},\{\text{sq2},3\},\{\text{sq2},4\}\} & -i\,\text{gs}^2\,\text{Sin}[2\,\text{theta}]^2\left(T^{\text{Gluon\$1}}_{m_1,m_4}T^{\text{Gluon\$1}}_{m_2,m_3}+T^{\text{Gluon\$1}}_{m_1,m_3}T^{\text{Gluon\$1}}_{m_2,m_4}\right)\\
\{\{\text{sq1},1\},\{\text{sq1},2\},\{\text{sq1}^\dagger,3\},\{\text{sq2}^\dagger,4\}\} & \frac{1}{2}\,i\,\text{gs}^2\,\text{Sin}[4\,\text{theta}]\left(T^{\text{Gluon\$1}}_{m_3,m_2}T^{\text{Gluon\$1}}_{m_4,m_1}+T^{\text{Gluon\$1}}_{m_3,m_1}T^{\text{Gluon\$1}}_{m_4,m_2}\right)\\
\{\{\text{sq1},1\},\{\text{sq1}^\dagger,2\},\{\text{sq2},3\},\{\text{sq2}^\dagger,4\}\} & -i\,\text{gs}^2\left(\text{Sin}[2\,\text{theta}]^2\,T^{\text{Gluon\$1}}_{m_2,m_3}T^{\text{Gluon\$1}}_{m_4,m_1}-\text{Cos}[2\,\text{theta}]^2\,T^{\text{Gluon\$1}}_{m_2,m_1}T^{\text{Gluon\$1}}_{m_4,m_3}\right)\\
\{\{\text{sq1}^\dagger,1\},\{\text{sq2},2\},\{\text{sq2},3\},\{\text{sq2}^\dagger,4\}\} & -\frac{1}{2}\,i\,\text{gs}^2\,\text{Sin}[4\,\text{theta}]\left(T^{\text{Gluon\$1}}_{m_1,m_3}T^{\text{Gluon\$1}}_{m_4,m_2}+T^{\text{Gluon\$1}}_{m_1,m_2}T^{\text{Gluon\$1}}_{m_4,m_3}\right)\\
\{\{\text{sq1},1\},\{\text{sq1},2\},\{\text{sq2}^\dagger,3\},\{\text{sq2}^\dagger,4\}\} & -i\,\text{gs}^2\,\text{Sin}[2\,\text{theta}]^2\left(T^{\text{Gluon\$1}}_{m_3,m_2}T^{\text{Gluon\$1}}_{m_4,m_1}+T^{\text{Gluon\$1}}_{m_3,m_1}T^{\text{Gluon\$1}}_{m_4,m_2}\right)\\
\{\{\text{sq1},1\},\{\text{sq2},2\},\{\text{sq2}^\dagger,3\},\{\text{sq2}^\dagger,4\}\} & -\frac{1}{2}\,i\,\text{gs}^2\,\text{Sin}[4\,\text{theta}]\left(T^{\text{Gluon\$1}}_{m_3,m_2}T^{\text{Gluon\$1}}_{m_4,m_1}+T^{\text{Gluon\$1}}_{m_3,m_1}T^{\text{Gluon\$1}}_{m_4,m_2}\right)\\
\{\{\text{sq2},1\},\{\text{sq2},2\},\{\text{sq2}^\dagger,3\},\{\text{sq2}^\dagger,4\}\} & -i\,\text{gs}^2\,\text{Cos}[2\,\text{theta}]^2\left(T^{\text{Gluon\$1}}_{m_3,m_2}T^{\text{Gluon\$1}}_{m_4,m_1}+T^{\text{Gluon\$1}}_{m_3,m_1}T^{\text{Gluon\$1}}_{m_4,m_2}\right)
\end{pmatrix}$$

- All nine vertices automatically derived from the (very compact) Lagrangian

SUSY-gauge squark-quark-gluino couplings

The SUSY gauge gluino-quark-squark interactions (in the gauge basis)

$$\mathcal{L}_{\rm matter} = \sqrt{2} g_s \Big[- \tilde{q}_L^\dagger T^a \big(\bar{\tilde{g}}^a P_L q\big) + \big(\bar{q} P_L \tilde{g}^a\big) T^a \tilde{q}_R \Big] + {\rm h.c.}$$

The FEYNRULES again following the textbook expression

```
Lgosqq :=  Sqrt[2] gs ProjM[s1,s2] * (
    - sqLbar[cc1] T[a,cc1,cc2] gobar[s1,a].q[s2,cc2] +
      qbar[s1,cc1].go[s2,a] T[a,cc1,cc2] sqR[cc2]);
```

- All indices explicit (scalars not included in fermion chains)
- *ProjM* ≡ left-handed chirality projector (*ProjP* right-handed)
- The dot connects the elements of a fermion chain
  - ★ Remark: ProjM(s1,s2) is a scalar object

Feynman rules from the D-terms

The SUSY gauge gluino-quark-squark interactions (in the gauge basis)

$$\mathcal{L}_{\rm matter} = \sqrt{2} g_s \Big[- \tilde{q}_L^\dagger T^a \big(\bar{\tilde{g}}^a P_L q\big) + \big(\bar{q} P_L \tilde{g}^a\big) T^a \tilde{q}_R \Big] + \text{h.c.}$$

Feynman rules computation

- Including the Hermitian conjugate pieces with HC

```
In[19]:= Simplify[FeynmanRules[Lgosqq + HC[Lgosqq]]] // MatrixForm

Starting Feynman rule calculation.
Expanding the Lagrangian...
Collecting the different structures that enter the vertex.
4 possible non-zero vertices have been found -> starting the computation: 4 / 4.
4 vertices obtained.

Out[19]//MatrixForm=
```

$$\begin{pmatrix}
\{\{\bar{q},1\},\{go,2\},\{sq1,3\}\} & -i\sqrt{2}\,gs\,\left(\mathrm{Cos}[theta]\,P_{+s_1,s_2} - P_{-s_1,s_2}\,\mathrm{Sin}[theta]\right) T^{a2}_{m_1,m_3} \\
\{\{go,1\},\{q,2\},\{sq1^\dagger,3\}\} & -i\sqrt{2}\,gs\,\left(\mathrm{Cos}[theta]\,P_{-s_1,s_2} - P_{+s_1,s_2}\,\mathrm{Sin}[theta]\right) T^{a1}_{m_3,m_2} \\
\{\{\bar{q},1\},\{go,2\},\{sq2,3\}\} & i\sqrt{2}\,gs\,\left(\mathrm{Cos}[theta]\,P_{-s_1,s_2} + P_{+s_1,s_2}\,\mathrm{Sin}[theta]\right) T^{a2}_{m_1,m_3} \\
\{\{go,1\},\{q,2\},\{sq2^\dagger,3\}\} & i\sqrt{2}\,gs\,\left(\mathrm{Cos}[theta]\,P_{+s_1,s_2} + P_{-s_1,s_2}\,\mathrm{Sin}[theta]\right) T^{a1}_{m_3,m_2}
\end{pmatrix}$$

To phenomenology

From FEYNRULES to phenomenology

The SUSY-QCD model has been implemented into FEYNRULES

```
LMatter := Lkin + LD + Lgosqq + HC[Lgosqq];

LSUSYQCD := LVector1 + LMatter;
```

The Feynman rules have been extracted (and checked)

Ready to export the model to MC tools

- CALCHEP / COMPHEP
- FEYNARTS / FORMCALC
- UFO: MADGRAPH5_AMC@NLO / SHERPA / HERWIG++ / WHIZARD

```
WriteCHOutput[{LVector1, LMatter}];

WriteFeynArtsOutput[{LVector1, LMatter}];

WriteUFO[{LVector1, LMatter}];
```

Limitations and fineprints

Particle / parameter names

- The strong interaction has a special role
  - ★ Name for the gluon field, the coupling constant, *etc*.
  - ★ Which parameter is internal/external
  - ★ The numerical value of α_s at the Z-pole
- For some generators, the electroweak interaction has also a special role
  - ★ Name for the Fermi coupling, the Z-boson mass
  - ★ Which parameter is external/internal
  - ★ At which scale must the numerical values be given

Colour structures: up to octets in FEYNRULES, not in MC generators

- Interfaces discard non-supported vertices

Lorentz structures/spins: up to spin-2 in FEYNRULES, not in MC generators

- Interfaces discard non-supported vertices

Outline

Advanced techniques

- More about UFOs
- Extension / restriction of existing models
- The superspace module of FeynRules
- Mass diagonalisation
- Two-body decays
- Next-to-leading order module

Summary

The quest for new physics at the LHC

- Relies on event generators for background and signal modelling
- FEYNRULES facilitates the implementation of new physics models

FEYNRULES: http://feynrules.irmp.ucl.ac.be

- Straightforward implementation of new physics model in Monte Carlo tools
  - ★ Interfaces to many programs
- FEYNRULES shipped with its own computational modules
  - ★ A superspace module
  - ★ A decay package
  - ★ A mass diagonalisation module (ASPERGE)
  - ★ An NLO module

Try it on with your favorite model!

Outline

1. FEYNRULES in a nutshell
2. Implementing supersymmetric QCD in FEYNRULES
3. Using FEYNRULES with supersymmetric QCD model
4. Summary
5. **Appendix: advanced model implementation techniques**

Advanced techniques for FEYNRULES implementation

- ✦ More about the UFO
- ✦ Extension / restriction of existing models
- ✦ The superspace module of FEYNRULES
- ✦ Mass diagonalization
- ✦ Two-body decays
- ✦ Next-to-leading order module

The UFO

A step further: the Universal FEYNRULES Output

✦ The UFO in a nutshell

[Degrande, Duhr, BF, Grellscheid, Mattelaer, Reiter (CPC '12)]
[Degrande, Duhr, BF, Hirschi, Mattelaer, Shao *et al.* (*in prep.*)]

- ✤ UFO ≡ Universal FEYNRULES output
  - ★ Universal as not tied to any specific Monte Carlo program
- ✤ Consists of a set of PYTHON files to be linked to any code
- ✤ It contains all the model information
  - ★ Allows the models to contain generic color and Lorentz structures
- ✤ Can be employed for next-to-leading order calculations

✦ The UFO is now a standard and used by many other programs

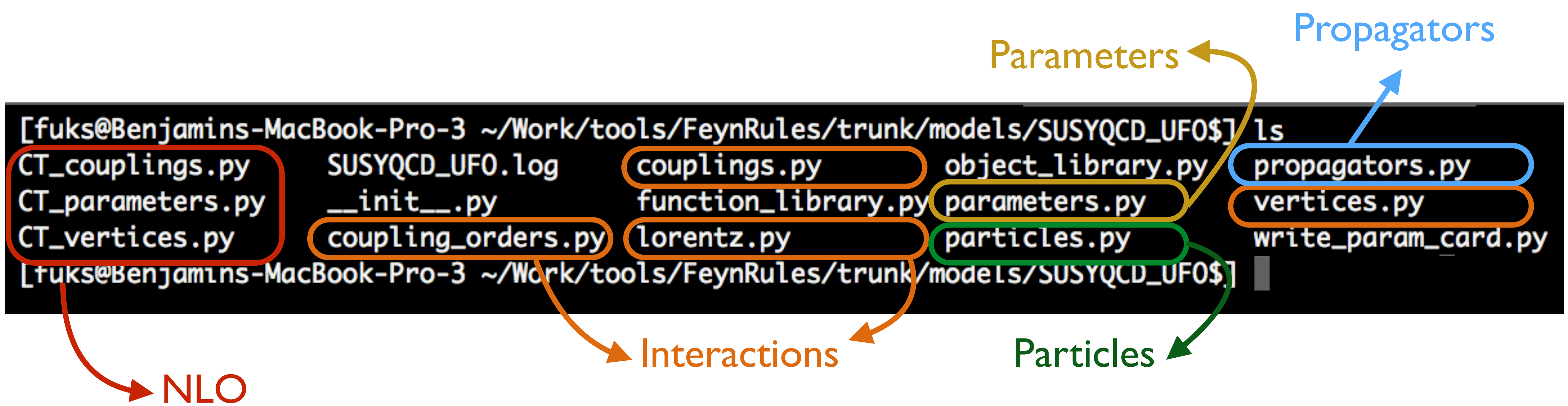
ALOHA | GOSAM | HERWIG ++ | MADANALYSIS 5 | SHERPA

MADGRAPH5_aMC@NLO | WHIZARD | LANHEP | SARAH

The UFO in practice

- **The UFO is a set of PYTHON files**
  - Factorization of the information: particles, interactions, propagation, parameters, NLO, *etc.*

- **Example**



Particles

- Particles are stored in the particles.py file
  - Instances of the particle class
  - Attributes: particle spin, color representation, mass, width, PDG code, *etc.*
  - Antiparticles automatically derived

```
G = Particle(pdg_code = 21,
             name = 'G',
             antiname = 'G',
             spin = 3,
             color = 8,
             mass = Param.ZERO,
             width = Param.ZERO,
             texname = 'G',
             antitexname = 'G',
             charge = 0)

go = Particle(pdg_code = 1000021,
              name = 'go',
              antiname = 'go',
              spin = 2,
              color = 8,
              mass = Param.Mgo,
              width = Param.Wgo,
              texname = 'go',
              antitexname = 'go',
              charge = 0)
```

```
sq1 = Particle(pdg_code = 1000006,
               name = 'sq1',
               antiname = 'sq1~',
               spin = 1,
               color = 3,
               mass = Param.Msq1,
               width = Param.Wsq1,
               texname = 'sq1',
               antitexname = 'sq1~',
               charge = 0)

sq1__tilde__ = sq1.anti()

sq2 = Particle(pdg_code = 2000006,
               name = 'sq2',
               antiname = 'sq2~',
               spin = 1,
               color = 3,
               mass = Param.Msq2,
               width = Param.Wsq2,
               texname = 'sq2',
               antitexname = 'sq2~',
               charge = 0)

sq2__tilde__ = sq2.anti()
```

```
q = Particle(pdg_code = 6,
             name = 'q',
             antiname = 'q~',
             spin = 2,
             color = 3,
             mass = Param.Mq,
             width = Param.Wq,
             texname = 'q',
             antitexname = 'q~',
             charge = 0)

q__tilde__ = q.anti()
```

Parameters

- Parameters are stored in the parameters.py file
  - Instances of the parameter class
  - External parameters are organized following a Les Houches-like structure (blocks and counters)
  - Python-compliant formula for the internal parameters

```
aS = Parameter(name = 'aS',
               nature = 'external',
               type = 'real',
               value = 0.1184,
               texname = '\\alpha _s',
               lhablock = 'SMINPUTS',
               lhacode = [ 3 ])

G = Parameter(name = 'G',
              nature = 'internal',
              type = 'real',
              value = '2*cmath.sqrt(aS)*cmath.sqrt(cmath.pi)',
              texname = 'G')
```

```
Mgo = Parameter(name = 'Mgo',
                nature = 'external',
                type = 'real',
                value = 500,
                texname = '\\text{Mgo}',
                lhablock = 'MASS',
                lhacode = [ 1000021 ])

Wq = Parameter(name = 'Wq',
               nature = 'external',
               type = 'real',
               value = 1.50833649,
               texname = '\\text{Wq}',
               lhablock = 'DECAY',
               lhacode = [ 6 ])
```

Interactions: generalities

✦ Vertices decomposed in a spin x color basis (coupling strengths ≡ coordinates)

✤ Example: the quartic gluon vertex can be written as

$$\begin{aligned} & i g_s^2\, f^{a_1 a_2 b} f^{b a_3 a_4}\, (\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4}) \\ & \quad + i g_s^2\, f^{a_1 a_3 b} f^{b a_2 a_4}\, (\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}) \\ & \quad + i g_s^2\, f^{a_1 a_4 b} f^{b a_2 a_3}\, (\eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}) \end{aligned} \quad \Rightarrow \quad \begin{aligned} & \left(f^{a_1 a_2 b} f^{b a_3 a_4},\ f^{a_1 a_3 b} f^{b a_2 a_4}, f^{a_1 a_4 b} f^{b a_2 a_3}\right) \\ & \times \begin{pmatrix} i g_s^2 & 0 & 0 \\ 0 & i g_s^2 & 0 \\ 0 & 0 & i g_s^2 \end{pmatrix} \begin{pmatrix} \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} \\ \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \\ \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \end{pmatrix} \end{aligned}$$

- ★ 3 elements for the color basis
- ★ 3 elements for the spin (Lorentz structure) basis
- ★ 9 coordinates (6 are zero)

✤ Several files are used for the storage of the information

Example: the quartic gluon vertex

✦ General information in vertex.py

```
V_2 = Vertex(name = 'V_2',
             particles = [ P.G, P.G, P.G, P.G ],
             color = [ 'f(-1,1,2)*f(3,4,-1)', 'f(-1,1,3)*f(2,4,-1)', 'f(-1,1,4)*f(2,3,-1)' ],
             lorentz = [ L.VVVV1, L.VVVV2, L.VVVV3 ],
             couplings = {(1,1):C.GC_4,(0,0):C.GC_4,(2,2):C.GC_4})
```

- ★ lorentz ≡ spin basis (in lorentz.py; common to all vertices)
- ★ color ≡ color basis
- ★ couplings ≡ coordinates (in couplings.py; common to all vertices)

$$\left(f^{a_1a_2b}f^{ba_3a_4},\ f^{a_1a_3b}f^{ba_2a_4},\ f^{a_1a_4b}f^{ba_2a_3}\right) \times \begin{pmatrix} ig_s^2 & 0 & 0 \\ 0 & ig_s^2 & 0 \\ 0 & 0 & ig_s^2 \end{pmatrix} \begin{pmatrix} \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} \\ \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \\ \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \end{pmatrix}$$

✦ Lorentz structures: straightforward implementations in lorentz.py

```
VVVV1 = Lorentz(name = 'VVVV1',
                spins = [ 3, 3, 3, 3 ],
                structure = 'Metric(1,4)*Metric(2,3) - Metric(1,3)*Metric(2,4)')
```

✦ Couplings: straightforward implementations in couplings.py

```
GC_4 = Coupling(name = 'GC_4',
                value = 'complex(0,1)*G**2',
                order = {'QCD':2})
```

Coupling orders: for selecting diagrams

Extending models

Merging and extending models (I)

- ✦ Many BSM models of interest are simple extensions of another model
  - ✤ FEYNRULES allows one to start from a given model
    - ★ Add new particles, parameters, Lagrangian terms
    - ★ Modify existing particles, parameters, Lagrangian terms
    - ★ Remove some particles, parameters, Lagrangian terms

- ✦ Simplified models
  - ✤ Special cases very relevant for LHC physics: Simplified Model Spectra
    - ★ The Standard Model + one or two new particles
    - ★ Often inspired by the MSSM or dark matter models
    - ★ *Ex.*: the SM + lightest stop and neutralino + relevant subset of MSSM interactions

Merging and extending models (2)

- Merged FEYNRULES model contains two .fr files
  - The parent model implementation
  - One extra file with the modifications
  - They must be loaded together (the parent model first)

```
LoadModel["SM.fr", "stops.fr"];
```

  ★ No need to re-implement what is common (gauge groups, *etc.*)

- One can start from the models available on the FEYNRULES database
  http://feynrules.irmp.ucl.ac.be

The FEYNRULES model database

- O(100) models are available online
  - Simple extensions of the Standard Model
    - ★ Simplified model spectra
    - ★ Four generation models
    - ★ Vector-like quarks
    - ★ Two-Higgs-Doublet Models, Hidden Abelian Higgs
    - ★ *etc.*
  - Supersymmetric models
    - ★ MSSM with and without R-parity
    - ★ The NMSSM
    - ★ R-symmetric supersymmetric models
    - ★ Left-right supersymmetric models
  - Extra-dimensional models
    - ★ Universal extra-dimensions
    - ★ Large extra-dimensions
    - ★ Heidi, Minimal Higgsless models
    - ★ Randall-Sundrum
  - Strongly coupled and effective field theories
    - ★ Technicolor
    - ★ Models with dimension-six and dimension-eight operators
    - ★ *etc.*

Restricting model implementations

- ✦ Many BSM models of interest are subset of other models
  - ✤ Equivalent to the parent model, with some parameters set to 0 or 1
    - ★ Example 1: the massless version of a model (massless light quarks in the SM)
    - ★ Example 2: a mixing matrix set to the identity (no-CKM matrix in the SM)
  - ✤ FEYNRULES allows one to start from a given model
    - ★ Write the restrictions under the form of a list of MATHEMATICA replacement rules
      - ➢ M$Restrictions
    - ★ Read them into FEYNRULES
    - ★ Apply them before the computation of any Feynman rule

- ✦ The output Feynman rules (and thus Monte Carlo model files)
  - ✤ Are free from the restricted parameters
  - ✤ Smaller files, more efficiency at the MC level
    - ★ *Ex.*: the MSSM has more than 10000 vertices; its flavor-conserving version ~1000

Example of restrictions

✦ One practical example: the Standard Model without CKM-mixing

```
M$Restrictions = {
            CKM[i_,i_] -> 1,
            CKM[i_?NumericQ, j_?NumericQ] :> 0 /; (i =!= j),
            cabi -> 0
}
```

```
LoadRestriction["DiagonalCKM.rst"]
```

Superspace module

See the manual for more details on the superspace module

Superfield declaration

- A module dedicated to calculations in superspace
  - Superfield declaration and links to the component fields

A left-handed squark superfield

```
CSF[1] == {
  ClassName      -> QL,
  Chirality      -> Left,
  Weyl           -> qLw,
  Scalar         -> sqL,
  Indices        -> {Index[Colour]}
}
```

★ Component fields to be declared too

★ Vector superfields can be linked to gauge groups

Supersymmetric model implementation

✦ Supersymmetric model implementation

- ✤ Declaration of the model gauge group
- ✤ Declaration of all fields and superfields
- ✤ Declaration of all model parameters
- ✤ Writing the Lagrangian (in a simplified way)

✦ Supersymmetric Lagrangian in superspace are very compact

$$\mathcal{L} = \Phi^\dagger e^{-2gV}\Phi_{|_{\theta^2\bar\theta^2}} + \frac{1}{16g^2\tau_{\mathcal{R}}}\mathrm{Tr}(W^\alpha W_\alpha)_{|_{\theta^2}} + \frac{1}{16g^2\tau_{\mathcal{R}}}\mathrm{Tr}(\bar W_{\dot\alpha}\bar W^{\dot\alpha})_{|_{\bar\theta^2}}$$
$$+ W(\Phi)_{|_{\theta^2}} + W^\star(\Phi^\dagger)_{|_{\bar\theta^2}} + \mathcal{L}_{\rm soft}$$

- ✤ First line: kinetic and gauge interaction terms for all fields
  - ➢ Model independent ⇒ can be automated
- ✤ Second line: superpotential and supersymmetry breaking Lagrangian
  - ➢ Model dependent ⇒ to be provided
- ✤ Series expansion in terms of component fields
- ✤ Automatic derivation of supersymmetric Lagrangians
- ✤ Solving the equations of motion of the unphysical fields

Implementing supersymmetric Lagrangians

✦ Supersymmetric Lagrangian in superspace are very compact

$$\mathcal{L} = \Phi^\dagger e^{-2gV}\Phi_{|_{\theta^2\bar\theta^2}} + \frac{1}{16g^2\tau_{\mathcal{R}}}\mathrm{Tr}(W^\alpha W_\alpha)_{|_{\theta^2}} + \frac{1}{16g^2\tau_{\mathcal{R}}}\mathrm{Tr}(\bar W_{\dot\alpha}\bar W^{\dot\alpha})_{|_{\bar\theta^2}} + W(\Phi)_{|_{\theta^2}} + W^\star(\Phi^\dagger)_{|_{\bar\theta^2}} + \mathcal{L}_{\rm soft}$$

- First line: kinetic and gauge interaction terms for all fields
  - ★ Model independent ⇒ can be automated (CSFKineticTerms / VSFKineticTerms)
  - ★ Dedicated methods to access the components of a superfield (Theta2Component, *etc.*)
- Second line: superpotential and supersymmetry-breaking terms
  - ★ Model dependent ⇒ to be provided by the user (*SuperW* and *LSoft*)

```
Lag = Theta2Thetabar2Component[ CSFKineticTerms[] ] +
   Theta2Component[ VSFKineticTerms[] ] +
   Thetabar2Component[ VSFKineticTerms[] ];
```

```
Lag2 = LSoft + Theta2Component[ SuperW ] +  Thetabar2Component[ SuperW ];
```

Implementing supersymmetric Lagrangians

✦ The implemented Lagrangian above must be post-processed

```
Lag = Theta2Thetabar2Component[ CSFKineticTerms[] ] +
   Theta2Component[ VSFKineticTerms[] ] +
   Thetabar2Component[ VSFKineticTerms[] ];
```

```
Lag2 = LSoft + Theta2Component[ SuperW ] +  Thetabar2Component[ SuperW ];
```

- Solving the equation of motion for the auxiliary fields
- Inserting the solution back into the Lagrangian
  - ★ Automated (SolveEqMotionF and SolveEqMotionD)
- Replacement of Weyl spinors in terms of Majorana and Dirac spinors
  - ★ Automated (WeylToDirac)
- Rotation to the mass basis
  - ★ Standard FEYNRULES function (ExpandIndices)

Mass matrix diagonalization

See the manual for more details on the ASPERGE module

Mass matrices and their diagonalization

✦ The problematics of the mass matrices

- ✤ Lagrangians are usually easily written in the gauge basis
- ✤ The included mass matrices are thus in general non-diagonal
  - ➢ diagonalization required
- ✤ The gauge basis must be rotated to the mass basis where the mass matrices are diagonal
- ✤ This diagonalization cannot in general be achieved analytically

✦ The ASperGe package of FeynRules

- ✤ A module allowing one to extract the mass matrices from the Lagrangian
- ✤ A generator of C++ code ➢ numerical diagonalization of all mass matrices (the generated code can be used in a standalone way)
- ✤ See: Alloul, D'Hondt, De Causmaecker, BF, Rausch de Traubenberg [EPJC 73 (2013) 2325]

Example of the Z-boson/photon mixing

✦ Example: the Z-boson and photon in the Standard Model

- Each mixing is declared as a set of replacement rules (in *M$MixingsDescription)*
- Each rule represent a property of the mixing relation

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = U_w \begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix} \Rightarrow$$

```
Mix["AZ"] == {
  MassBasis     -> {A, Z},
  GaugeBasis    -> {B, Wi[3]},
  MixingMatrix -> UW,
  BlockName     -> WEAKMIX
}
```

- ASPERGE can compute the mass matrices:
- ASPERGE can generate its standalone C++ version

```
ComputeMassMatrix[Lag];
WriteASperGe[Lag];
```

Decays

See the manual for more details on the decay module

Two-body decays

✦ The problematics of the decay widths and branching ratios

- ✤ Some MC tools need decay tables (widths and branching ratios) to decay particles
- ✤ Widths and branching ratios are not independent quantities
  - ➢ need to be calculated
- ✤ Some Monte Carlo tools compute these quantities on the fly
  - ➢ the procedure is repeated each time it is needed
- ✤ FEYNRULES offers a way to include analytical information on the two-body decay

Two-body decays

✦ Two-body decays in general

✤ Two-body decays can be directly read from three-point vertices ($\mathcal{V}$)

$$\Gamma_{1\to 2} = \frac{1}{2|M|S}\int \mathrm{d}\Phi_N\, |\mathcal{M}_{1\to 2}|^2 = \frac{\sqrt{\lambda(M^2, m_1^2, m_2^2)}}{16\,\pi\, S\,|M|^3}\mathcal{V}^{a_1 a_2 a_3}_{\ell_1\ell_2\ell_3}\,\mathcal{P}_1^{\ell_1\ell_1'}\,\mathcal{P}_2^{\ell_2\ell_2'}\,\mathcal{P}_3^{\ell_3\ell_3'}\,(\mathcal{V}^*)^{a_1a_2a_3}_{\ell_1'\ell_2'\ell_3'}$$

★ Partial width for the decay of a particle of mass M to two particles of masses m_1 and m_2

★ Includes a symmetry factor S and $\mathcal{P}$ denotes the polarization tensor of each particle

✦ The decay module of FEYNRULES

✤ FEYNRULES makes use of MATHEMATICA to compute all partial widths of the model

★ Ignores open and closed channels ➢ benchmark independent

★ The information is exported to the UFO (used, e.g, by MADWIDTH)

Running the decay module of FEYNRULES

✦ Automatic decay width computations

- ✤ All two-body decay widths can be easily computed from the Lagrangian

```
verts       = FeynmanRules[Lag];
vertsexp    = FlavorExpansion[verts];
results     = ComputeWidths[vertsexp];
```

- ✤ Many functions available for analytical calculations
  - ➢ PartialWidth, TotWidth, BranchingRatio
- ✤ The numerical value provided for the particle widths can be updated accordingly
  - ➢ UpdateWidths
- ✤ See: Alwall, Duhr, BF, Mattelaer, Öztürk, Shen [CPC (2015)]

Snippet of the UFO output

- The information is (by default) employed by the UFO interface
  - Can be turned of: (AddDecays → False)
  - The UFO contains an extra file *decays.py*
  - This file can be used by MC codes
  - Example of the Standard Model UFO: the top quark

```
Decay_t = Decay(name = 'Decay_t',
                particle = P.t,
                partial_widths = {(P.W__plus__,P.d):'((MT**2 - MW**2)*((3*CKM3x1*ee**2*MT**2*complexconjugate(CKM3x1))/(2.*sw**2) +
(3*CKM3x1*ee**2*MT**4*complexconjugate(CKM3x1))/(2.*MW**2*sw**2) - (3*CKM3x1*ee**2*MW**2*complexconjugate(CKM3x1))/sw**2))/
(96.*cmath.pi*abs(MT)**3)',
                                  (P.W__plus__,P.s):'((MT**2 - MW**2)*((3*CKM3x2*ee**2*MT**2*complexconjugate(CKM3x2))/(2.*sw**2) +
(3*CKM3x2*ee**2*MT**4*complexconjugate(CKM3x2))/(2.*MW**2*sw**2) - (3*CKM3x2*ee**2*MW**2*complexconjugate(CKM3x2))/sw**2))/
(96.*cmath.pi*abs(MT)**3)',
                                  (P.W__plus__,P.b):'(((3*CKM3x3*ee**2*MB**2*complexconjugate(CKM3x3))/(2.*sw**2) +
(3*CKM3x3*ee**2*MT**2*complexconjugate(CKM3x3))/(2.*sw**2) + (3*CKM3x3*ee**2*MB**4*complexconjugate(CKM3x3))/(2.*MW**2*sw**2) -
(3*CKM3x3*ee**2*MB**2*MT**2*complexconjugate(CKM3x3))/(MW**2*sw**2) + (3*CKM3x3*ee**2*MT**4*complexconjugate(CKM3x3))/(2.*MW**2*sw**2) -
(3*CKM3x3*ee**2*MW**2*complexconjugate(CKM3x3))/sw**2)*cmath.sqrt(MB**4 - 2*MB**2*MT**2 + MT**4 - 2*MB**2*MW**2 - 2*MT**2*MW**2 +
MW**4))/(96.*cmath.pi*abs(MT)**3)'})
```

NLO

Higher-order corrections (in QCD)

- ✦ NLO calculations matched to parton shower (for BSM) are automated
  - ✤ Model-dependent parts of calculations (on top of the tree-level information)
    - ★ Counterterms
    - ★ Finite pieces of the loop-integrals
  - ✤ Model independent contributions
    - ★ Subtraction of the divergences
    - ★ Matching to the parton showers

UFO @ NLO

[Degrande, Duhr, BF, Hirschi, Mattelaer, Shao *et al.* (*in prep.*)]

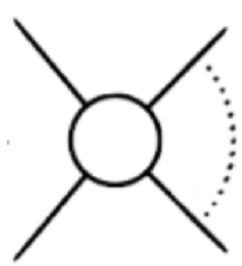
Recap' on NLO calculations

✦ Contributions to an NLO result in QCD

✤ Three ingredients: the Born, virtual loop and real emission contributions

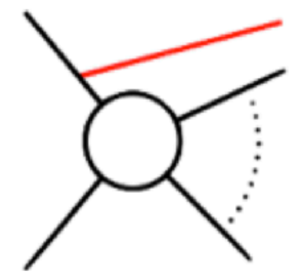
$$\sigma_{NLO} = \int \mathrm{d}^4\Phi_n \mathcal{B} \quad + \quad \boxed{\int \mathrm{d}^4\Phi_n \int_{\rm loop} \mathrm{d}^d\ell \; \mathcal{V}} \quad + \quad \int \mathrm{d}^4\Phi_{n+1} \; \mathcal{R}$$

Born



Virtuals: one extra power of α_s and divergent

Reals: one extra power of α_s and divergent



Extra information is needed

Virtual contributions

✦ Loop diagram calculations

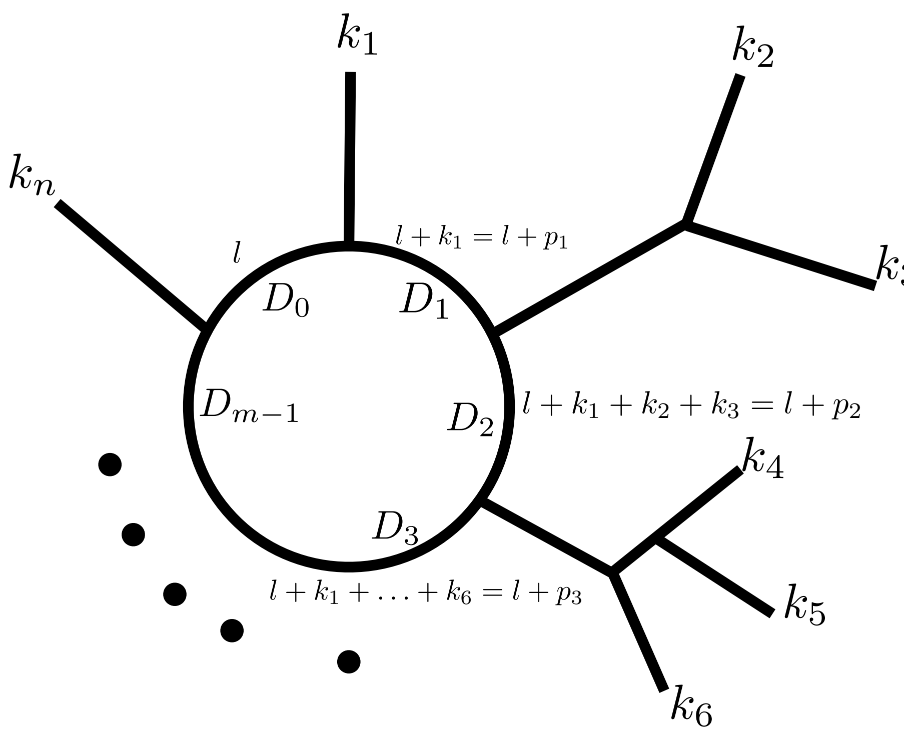
- ✤ Calculations to be done in *d=4-2ε* dimensions
  - ★ Divergences made explicit ($1/\varepsilon^2$, $1/\varepsilon$)

- ✤ Rewriting loop integrals with scalar integrals

$$\int \mathrm{d}^d\ell \frac{N(\ell)}{D_0 D_1 \cdots D_{m-1}} = \sum a_i \int \mathrm{d}^d\ell \frac{1}{D_{i_0} D_{i_1} \cdots}$$

  - ★ Involves integrals with up to four denominators
    - ➢ The decomposition basis is finite
    - ➢ Can be computed once and for all
  - ★ The reduction is the process-dependent part

m-point diagram with *n* external momenta



The rational terms (R_1 and R_2)

✦ The loop momentum lives in a *d*-dimensional space

✤ Reduction to be done in *d* dimensions

$$\int \mathrm{d}^d\ell \frac{N(\ell,\tilde{\ell})}{\bar{D}_0\bar{D}_1\cdots\bar{D}_{m-1}} \quad \text{with } \underbrace{\bar{\ell}}_{\text{D-dim}} = \underbrace{\ell}_{\text{4-dim}} + \underbrace{\tilde{\ell}}_{(-2\varepsilon)\text{-dim}}$$

✤ Numerical methods works in 4 dimensions: need to be compensated!

✦ The R_1 terms originates from the denominators

✤ Connected to the internal propagators

✦ The R_2 terms originates from the numerator

✤ Can be seen as extra diagrams with special Feynman rules

R_1 terms

✦ The R_1 terms originates from the denominators

$$\frac{1}{\bar{D}} = \frac{1}{D}\left(1 - \frac{\tilde{\ell}^2}{\bar{D}}\right)$$

✤ These extra pieces can be calculated generically (3 integrals in total)

$$\int \mathrm{d}^d\bar{\ell}\,\frac{\tilde{\ell}^2}{\bar{D}_i\bar{D}_j} = -\frac{i\pi^2}{2}\left[m_i^2 + m_j^2 - \frac{p_i - p_j)^2}{2}\right] + \mathcal{O}(\varepsilon)$$

$$\int \mathrm{d}^d\bar{\ell}\,\frac{\tilde{\ell}^2}{\bar{D}_i\bar{D}_j\bar{D}_k} = -\frac{i\pi^2}{2} + \mathcal{O}(\varepsilon)$$

$$\int \mathrm{d}^d\bar{\ell}\,\frac{\tilde{\ell}^2}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_l} = -\frac{i\pi^2}{6} + \mathcal{O}(\varepsilon)$$

✤ The denominator structure is already known at the reduction time

✤ The R_1 coefficients are extracted during the reduction

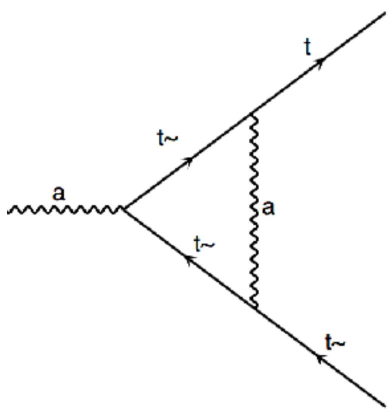
R_2 terms

✦ The R_2 terms originates from the numerator

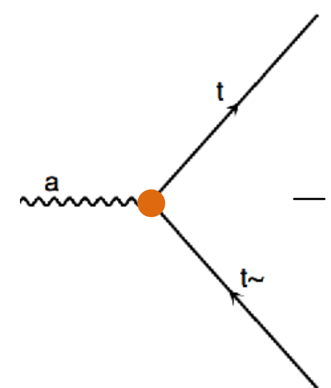
$$\underset{\text{D-dim}}{\bar{N}(\bar{\ell})} = \underset{\text{4-dim}}{N(\ell)} + \underset{(-2\varepsilon)\text{-dim}}{\tilde{N}(\tilde{\ell}, \ell, \varepsilon)} \quad \Rightarrow \quad R_2 \equiv \lim_{\varepsilon \to 0} \frac{1}{(2\pi)^4} \int \mathrm{d}^d \bar{\ell} \frac{\tilde{N}(\tilde{\ell}, \ell, \varepsilon)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

- ✤ Practically, we isolate the epsilon part
- ✤ There is only a finite set of loops for which it does not vanish

✦ They can be re-expressed in terms of R_2 Feynman rules



$$\propto \int \mathrm{d}^d \bar{\ell} \frac{\tilde{\ell}^2}{\bar{D}_i \bar{D}_j \bar{D}_k} = -\frac{i\pi^2}{2} + \mathcal{O}(\varepsilon) \quad \Rightarrow$$



$$-i\frac{\alpha}{2\pi} e \gamma^\mu$$

R_2 Feynman rules

- The R_2 are process dependent and model-dependent (like Feynman rules)
  - In a renormalizable theory, there is a finite number of them
  - They can be derived from the sole knowledge of the bare Lagrangian

[Ossala, Papadopoulos, Pittau (JHEP'08)]

- The R_2 calculation can be automated and performed once and for all
  - Development of the NLOCT package (extension of FEYNRULES)
  - Computation, for any model, of all R_2 and UV counterterms
    - ★ In the on-shell and MSbar schemes
  - Inclusion of the output in the UFO

[Degrande (CPC'15)]

Automated NLO simulations with MG5_aMC

✦ A comprehensive approach to Monte Carlo simulations at the NLO in QCD